Small-Signal and Noise Two-Dimensional Modeling of Submicrometer High Speed Bipolar Transistor

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Abstract—In the first part of the paper, the Y matrix representation is used to directly and accurately extract all the parameters of the $\pi$-hybrid model. In the second part the noise sources are added to the equivalent model and the analytical expression of noise figure is obtained. This expression is used to get a full set of noise parameters to characterize the behaviour of the bipolar microwave transistors from numerical simulation.

I. INTRODUCTION

SINCE trial manufacturing of a VLSI device is always more expensive and time consuming, a better way to investigate the performance and effects of process/structure changes is to use device modeling. Recently, the submicron bipolar transistors for high speed digital and analog circuit applications are becoming increasingly popular. For these devices it is very important to get full small-signal characterization and obtain the device parameters that affect the behaviour of the bipolar circuits. In this paper, the results of the AC/DC measurements were used to get a complete characterization of a Double Polysilicon Spacer Self Aligned Transistor (DPSSAT).

II. SMALL-SIGNAL EQUIVALENT CIRCUIT

In order to characterize the high-frequency behaviour of bipolar transistors the hybrid-$\pi$ model [1] (figure 1) is generally used. The technique for extracting the hybrid parameters used in this work is based on a direct method and all the parameters will be extracted at the same time, only from the Y-matrix. Applying Kirchhoff’s laws to the equivalent circuit, and using the emitter as the reference point, the analytical expression of the $Y$ parameters can be obtained in the definite form.

\[
\begin{bmatrix}
\frac{1}{r_{eb}} - \frac{1}{r_{eb}^2 r_{bc}} & Y_{ce} \\
\frac{1}{r_{be}} & -\frac{1}{r_{be}} + Y_{bc}
\end{bmatrix}
\begin{bmatrix}
\frac{V_{be}}{r_{be}} \\
\frac{1}{r_{be}} + V_{bc}
\end{bmatrix}
= Y_{ce} + Y_{bc}
\]

and:

\[
\begin{align*}
Y_{ce} &= g_{ce} + j\omega C_{ce} \\
Y_{bc} &= g_{bc} + j\omega C_{bc} \\
Y_{be} &= j\omega C_{be} \\
\Delta &= \frac{1}{r_{be}} + Y_{ce} + Y_{bc}
\end{align*}
\]

By equating the definite $Y$-matrix obtained from measurement, in numeric form, to that obtained in analytical form, we have a complex non linear system of 4 equations from which the 8 real parameters of the small-signal circuit are determined using a numerical technique such as the Gauss-Newton method or directly solving the nonlinear system with the aid of a symbolic solver.

III. SMALL-SIGNAL CHARACTERIZATION

The $\pi-\text{hybrid}$ model could be considered frequency independent and, once it is known at low frequencies, it can be used to analytically simulate...
the performance of the device up to frequencies less than \( \frac{f}{2} \). For higher frequencies the effects due to distributed phenomena in the base region introduce some errors. These errors can be seen as an extra phase delay.

IV. NOISE MODEL OF THE BIPOLAR TRANSISTORS

Once the \( \pi \)-model is known, the full small-signal equivalent circuit including noise sources can be derived (Figure 2). The current generator \( i^2 \) from the collector to emitter represents a full shot noise. Other random processes in the base are represented by a shot noise current generator; this generator is combined with the Flicker and Burst noise sources, which in bipolar transistors, were experimentally found to be represented by current generators across the internal base and the emitter. A thermal noise generator is due to the base resistance, which in the model represents a physical resistor. The collector series resistor shows thermal noise too, but this source could be neglected, and is not included in the model because it is in series with the high-impedance of the collector node. We have no thermal noise due to \( g_{be} \), \( g_{bc} \) and \( g_{ce} \) because they are fictitious conductances only used for modeling purposes. Then:

\[
\begin{align*}
\bar{i}^2_e &= 4kTq\bar{V}_{bb'}\Delta f \\
\bar{i}^2_c &= 2qI_c\Delta f \\
\bar{i}^2_b &= 2qI_b\Delta f + K_1 \frac{I_b}{f_c^2} \Delta f + K_2 \frac{I_b}{1 + \frac{f_c^2}{f^2}} \Delta f
\end{align*}
\]

where \( k \) is the Boltzmann’s constant, \( T \) the temperature, \( q \) the electronic charge, \( f \) the frequency and \( \Delta f \) is the bandwidth frequency, while the parameters \( K_1 \), \( a \), \( b \), \( K_2 \) and \( c \) due to Flicker and Burst Noise are constants for a particular device.

In order to evaluate the noise performance, it is necessary to take into account the thermal noise due to the internal conductance \( (G_s) \) of the signal source driving of the transistor.

\[
\bar{i}^2_s = 4kTG_s\Delta f
\]

All these noise generators are due to independent factors, so they are uncorrelated to one another.

V. EXPRESSION OF NOISE FIGURE

The definition of the noise figure of a two-port network is well known. It can be also expressed in the form of the ratio between the actual total mean squared noise current in the AC-short-circuited output \( i^2_0(T) \) and the portion which results from the thermal noise originating in source conductance \( i^2_0(S) \).

\[
\bar{i}^2_0(S) = 4kTG_s \frac{Y_{21}}{Y_{11} + Y_{22}}
\]

The \( \bar{i}^2_0(T) \) can be computed resolving the full small-signal equivalent circuit, including noise sources. This can be done using Kirchhoff’s laws and considering that all the source noise generators are uncorrelated.

\[
\bar{i}^2(T) = \left( \frac{a_1}{\Delta_2} \right)^2 \bar{i}^2_0 + \frac{\Delta_2}{\Delta_1} \bar{i}^2_0 \left( \frac{\Delta_3}{\bar{i}^2_0} \right) + \frac{\Delta_4}{\Delta_1} \bar{i}^2_0 \left( \frac{\Delta_3}{\bar{i}^2_0} \right) + \bar{i}^2_0
\]

where

\[
\begin{align*}
Y_b &= Y_{bb'} + Y_{bc} \\
\Delta &= \Delta_1 = \frac{1}{\Delta_2} + Y_b \\
Y_y &= g_m - Y_{bc} - Y_{bb'} \\
Y_c &= Y_{bc} - g_m \\
\Delta_2 &= Y_{bc} + Y_s + \frac{Y_s}{\Delta_1} \\
\Delta_4 &= Y_s + r_{bb'} \frac{Y_s}{\Delta_1} \\
\Delta_5 &= Y_c - r_{bb'} \frac{Y_s}{\Delta_1}
\end{align*}
\]

VI. NOISE PARAMETERS

An alternative expression of the noise figure of two-port amplifiers is given by the formula [2]:

\[
F = F_{min} + \left( \frac{R_m}{G_s} \right) |Y_s - Y_0|^2
\]
Where $F_{\text{min}}$, $R_n$ and $Y_0$ are known as **Noise Parameters**. If we know the full set of noise parameters we can completely determine the noise behaviour of two-port amplifiers.

### A. Determination of Noise Parameters

After the extraction of the $\pi$-model, the noise parameters can be estimated using the formulae given by Fukui [3]. These formulae are derived from the full small-signal equivalent circuit, neglecting the present capacitances and with further approximations. If we want to determine the noise parameters in a more accurate way, we can select a range $[Y_1, Y_2]$ of $Y_0$ around $Y_0 (G_0, B_0)$ given by Fukui’s formulae and using the equations which we have obtained in order to compute the noise figure in $n$ nodes. These values of noise figure will be fitted to the equation (1), in order to determine more accurate values of the noise parameters.

### VII. Results

The proposed method was used to evaluate the noise performance of a submicron self aligned bipolar device made in HSB technology (figure 3). To verify the model, the microwave behaviour [e.g. Cut-Off frequency ($f_T$) and S-Parameters] is compared against the experimental data (figure 4, 5). The noise model was verified using measurement at $Z_0$ of 50 $\Omega$ as the figure 6 shows. The figure 7 and 8 draw the noise performance in term of Minimum Noise Figure as function both of frequency and current.

### VIII. Conclusions

In order to complete the information provided by the device characterization, the small-signal model was extracted from the $\chi$ matrix. This allows us to make a full characterization of the microwave transistors by all kinds of network parameters and useful characteristic attributes. The model was completed by adding the sources of noise for obtaining a complete characterization in term of noise parameters. The proposed technique and expression was implemented and verified with a submicron self aligned bipolar device (DPSSAT) using the data obtained from measurements.

### REFERENCES

Fig. 5. S-Parameters Comparison at $V_{be}=0.87$ and $V_{cc}=0.8$ Volt between Simulation Data and Measurement, in Bode Plot

Fig. 6. Comparison of Noise figure versus frequency at $Z_o=50$ Ω between measurements and model prediction

Fig. 7. Minimum Noise figure versus frequency at $V_{be}=0.805$ Volt and $V_{cc}=0.8$ Volt

Fig. 8. Minimum Noise figure versus collector current $I_c$ for different frequencies