ITERATIVE FULL-VECTORIAL "FINITE DIFFERENCES - VARIATIONAL PRINCIPLE" APPROACH FOR THE DETERMINATION OF THE MODAL SPECTRUM OF PHOTONIC WAVEGUIDES

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1.Introduction

There are several methods to determine numerically modal spectrum of photonic waveguides. The problem itself can be formulated using appropriate discretisation scheme as a well-known algebraic problem of eigenvalues and eigenvectors. The difficulty consists in the large matrix dimensions which requires special approach to the determination of the eigenvalues and eigenvectors of the sparse matrices and complicates effective calculation due to immense demands on the computer memory. Recently proposed iterative method [1] modified for the use in full vectorial formulation can be efficiently employed. The feasibility of the method together with some improvements will be presented.

2.Algorithm

For the full-vectorial solution of the modal spectrum only two components of the ***E*** and ***H*** vectors are necessary. It is suitable to take two transversal components $\varphi = H_x$ and $\psi = H_y$ of the ***H*** vector because of theirs continuity/discontinuity properties. The wave equations for these two components read

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial y^2} + \left[k_0^2 n^2 - \beta^2\right]\varphi + \frac{1}{n^2}\frac{\partial n^2}{\partial y}\left[\frac{\partial\psi}{\partial x} - \frac{\partial\varphi}{\partial y}\right] = 0 \qquad (1a)$$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \left[k_0^2 n^2 - \beta^2\right]\psi + \frac{1}{n^2}\frac{\partial n^2}{\partial x}\left[\frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x}\right] = 0 \qquad (1b)$$

where $k_0 = 2\pi/\lambda$; $n^2 = n^2(x,y)$ and λ is the vacuum wavelength, n the refractive index of the medium and β the propagation constant of the modus to be determined. Refractive index of the medium is taken to be constant within the rectangular segments defined by a computational grid with discontinuities on the boundaries.

On the boundaries between segments with different permittivity the continuity/discontinuity conditions have to be met. Here the indices *w*(-est), *e*(-ast), *n*(-orth), *s*(-outh) mean values and derivatives on the boundary from the left, right, upper or lower side. For example on the horizontal (i.e. *x*-) boundary with ε_n from the upper (northern) side and with ε_s from the lower (southern) side it holds

$$\left[\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y}\right]_{y=const}^{s} = \left[\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y}\right]_{y=const}^{n} = \mathrm{j}\beta H_z \qquad (2a)$$

$$\frac{1}{\varepsilon_n}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]_{y=const}^{n} = \frac{1}{\varepsilon_s}\left[\frac{\partial H_y}{\partial x} - \frac{\partial Hx}{\partial y}\right]_{y=const}^{s} = \mathrm{j}\omega E_z \qquad (2b)$$

The finite differences approximation of the second derivative of $\varphi = H_x$ in the *i*-th point on the boundary from the southern side is then given

$$\left.\frac{\partial^2\varphi}{\partial y^2}\right|_i^s = 2\frac{\varepsilon_s\varphi_{i+1} - 2(\varepsilon_n + \varepsilon_s)\varphi_i + \varepsilon_n\varphi_{i-1}}{\Delta^2(\varepsilon_n + \varepsilon_s)} + \frac{\varepsilon_s}{\varepsilon_n + \varepsilon_s}k_0^2(\varepsilon_n - \varepsilon_s)\varphi_i + 2\frac{\varepsilon_n - \varepsilon_s}{\varepsilon_n + \varepsilon_s}\left.\frac{\partial\psi}{\partial y}\right|_i \tag{3}$$

This leads to the following formulas for the discrete representation of (1a) for $\varphi = H_x$

$$2\frac{\varphi_{i+1,j} - 2\varphi_{ij} + \varphi_{i-1,j}}{\Delta_x^2} + \left[\frac{\varepsilon_{se}}{\varepsilon_{ne} + \varepsilon_{se}} + \frac{\varepsilon_{sw}}{\varepsilon_{nw} + \varepsilon_{sw}}\right]\frac{2(\varphi_{i,j+1} - \varphi_{ij})}{\Delta_y^2} + \left[\frac{\varepsilon_{ne}}{\varepsilon_{ne} + \varepsilon_{se}} + \frac{\varepsilon_{nw}}{\varepsilon_{nw} + \varepsilon_{sw}}\right]\frac{2(\varphi_{i,j-1} - \varphi_{ij})}{\Delta_y^2} +$$

$$+\frac{2}{\Delta_y}\frac{\varepsilon_{ne} - \varepsilon_{se}}{\varepsilon_{ne} + \varepsilon_{se}}\left.\frac{\partial\psi}{\partial x}\right|_i^e + \frac{2}{\Delta_y}\frac{\varepsilon_{nw} - \varepsilon_{sw}}{\varepsilon_{nw} + \varepsilon_{sw}}\left.\frac{\partial\psi}{\partial x}\right|_i^w + \left[2\left(\frac{\varepsilon_{ne}\varepsilon_{se}}{\varepsilon_{ne} + \varepsilon_{se}} + \frac{\varepsilon_{nw}\varepsilon_{sw}}{\varepsilon_{nw} + \varepsilon_{sw}}\right)k_0^2 - \beta^2\right]\varphi_{ij} = 0 \tag{4a}$$

and discrete representation of (1b) for $\psi = H_y$

$$2\frac{\psi_{i+1,j} - 2\psi_{ij} + \psi_{i-1,j}}{\Delta_y^2} + \left[\frac{\varepsilon_{nw}}{\varepsilon_{ne} + \varepsilon_{nw}} + \frac{\varepsilon_{sw}}{\varepsilon_{se} + \varepsilon_{sw}}\right]\frac{2(\psi_{i+1,j} - \psi_{ij})}{\Delta_x^2} + \left[\frac{\varepsilon_{ne}}{\varepsilon_{ne} + \varepsilon_{nw}} + \frac{\varepsilon_{se}}{\varepsilon_{se} + \varepsilon_{sw}}\right]\frac{2(\psi_{i+1,j} - \psi_{ij})}{\Delta_x^2} +$$

$$+\frac{2}{\Delta_y}\frac{\varepsilon_{ne} - \varepsilon_{nw}}{\varepsilon_{ne} + \varepsilon_{nw}}\left.\frac{\partial\varphi}{\partial y}\right|_i^n + \frac{2}{\Delta_y}\frac{\varepsilon_{se} - \varepsilon_{sw}}{\varepsilon_{se} + \varepsilon_{sw}}\left.\frac{\partial\varphi}{\partial y}\right|_i^s + \left[2\left(\frac{\varepsilon_{ne}\varepsilon_{nw}}{\varepsilon_{ne} + \varepsilon_{nw}} + \frac{\varepsilon_{se}\varepsilon_{sw}}{\varepsilon_{se} + \varepsilon_{sw}}\right)k_0^2 - \beta^2\right]\psi_{ij} = 0 \tag{4b}$$

where the permittivities in four quadrants relevant to the central point with indices i,j are showed in Fig.1. The other components can be inferred using standard formulas

$$H_z = \frac{1}{j\beta}\left[\frac{\partial H_x}{\partial x} + \frac{\partial H_y}{\partial y}\right];\quad E_x = \frac{1}{\omega\varepsilon}\left[\beta H_y - j\frac{\partial H_z}{\partial x}\right];\qquad E_y = \frac{1}{\omega\varepsilon}\left[j\frac{\partial H_z}{\partial x} - \beta H_x\right];$$

$$E_z = \frac{1}{j\omega\varepsilon}\left[\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right]$$

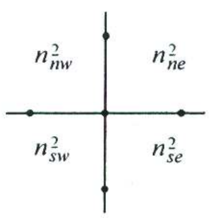

Fig.1

Starting from the first guess of the modus profile $\varphi_{ij}^{(n)}$, $n = 1$, where φ denotes one of either H_x or H_y and $\psi_{ij}^{(n)}$ denotes the second quantity, together with the first guess of the propagation constant $\beta^{(n)}$ the next guess of the modus profile $\varphi_{ij}^{(n+1)}$, $\psi_{ij}^{(n+1)}$ is calculated by the subsequent iterations using (4) from the neighbouring values of φ and ψ

$$\left\{\varphi_{i-1,j}^n, \varphi_{i+1,j}^n, \varphi_{i,j-1}^n, \varphi_{i,j+1}^n, \psi_{i-1,j}^n, \psi_{i+1,j}^n, \psi_{i,j}^n\right\} \Rightarrow \varphi_{i,j}^{(n+1)} \tag{5a}$$

$$\left\{\varphi_{i,j}^{(n+1)}, \varphi_{i,j-1}^{(n+1)}, \varphi_{i,j+1}^{(n+1)}, \psi_{i-1,j}^n, \psi_{i+1,j}^n, \psi_{i,j-1}^n, \psi_{i,j+1}^n\right\} \Rightarrow \psi_{i,j}^{(n+1)} \tag{5b}$$

until the equilibrium at the chosen decimal place (e.g. 12th) is being reached. Then the new guess of the propagation constant is calculated using the Galerkin method

$$
\left(\beta^{(n+1)}\right)^2 = \frac{\iint \left\{ -\left(\frac{\partial \varphi^{(n+1)}}{\partial x} \right)^2 - \left(\frac{\partial \varphi^{(n+1)}}{\partial y} \right)^2 + k_0^2 n^2 \left(\varphi^{(n+1)}\right)^2 \right\} dxdy}{\iint \left(\varphi^{(n+1)}\right)^2 dxdy} \qquad (6)
$$

This is repeated until the propagation constant reaches is equilibrium state up to the chosen decimal place. The method splits the solution of the vectorial problem into two subsequent iterations of smaller problems improving thus computational efficiency.

3.Results and Conclusions

As a testing structure a simple rectangular waveguide and a coupled pair of two rectangular waveguides have been used with refractive indices of the core n_{core}=3.24, and of the cladding n_{clad}=3.16. The results for H_x and H_y profiles are illustrated in Figures 2 through 7. The calculated effective index n_{eff} is smaller than n_{clad} due to rather small computational window with zero boundary conditions. Calculated values confirm a good applicability of the method used. The results of test calculations achieved by this method show good convergence properties and accuracy as compared to of other numerical methods and thus confirm feasibility of the method.

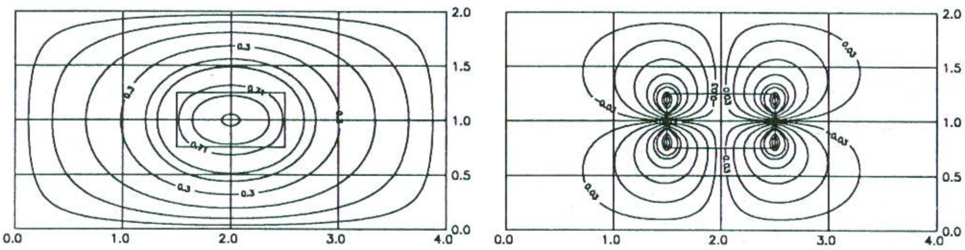

Fig.2 Rectangular waveguide, H_x, H_y components,fundamental modus, n_{eff}=3.1591

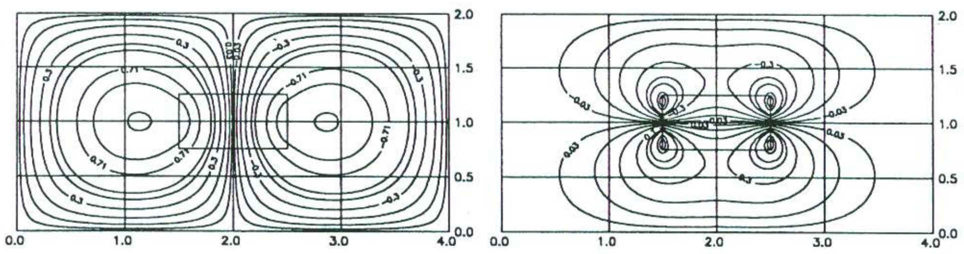

Fig.3 Rectangular waveguide, H_x, H_y components, second mode in x direction, n_{eff}=3.1202

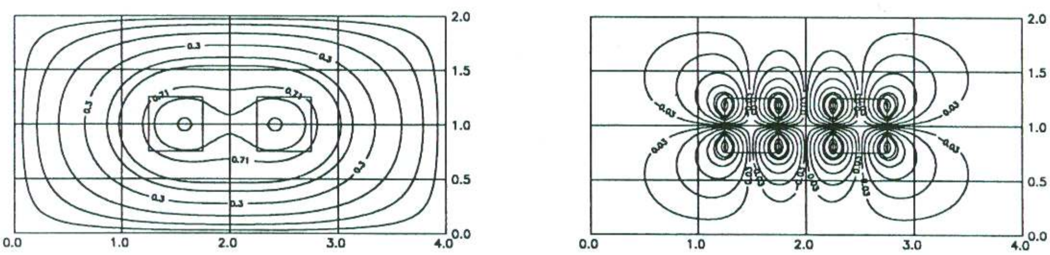

Fig.4 Coupled rectangular waveguides, H_x, H_y components, fundamental mode, n_{eff}=3.1523

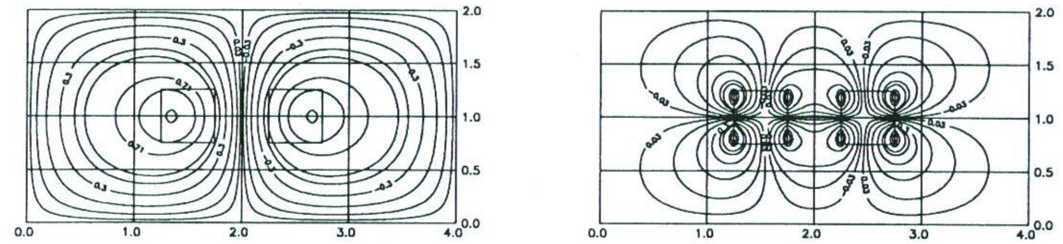

Fig.5 Coupled rectangular waveguides, H_x, H_y components, second mode in x direction, n_{eff}=3.1294

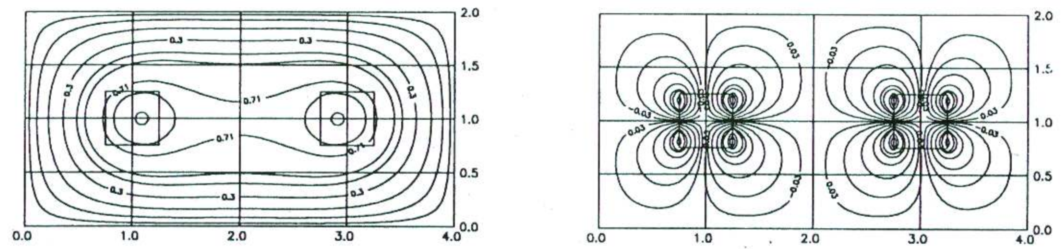

Fig.6 Coupled rectangular waveguides, H_x, H_y components, fundamental mode, n_{eff}=3.1460

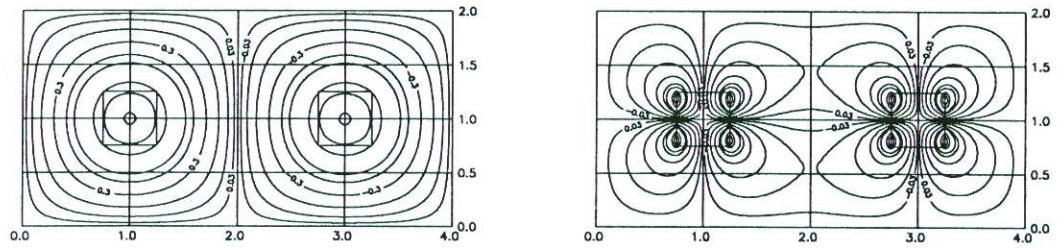

Fig.7 Coupled rectangular waveguides, H_x, H_y components, second mode in x direction, n_{eff}=3.1374

REFERENCES

[1] Benson T.M., Bozeat R.J., Kendall P.C. (1994): Complex finite difference method applied to the analysis of semiconductor lasers, *IEE Proc.-Optoelectron.*, No 2, pp-97-101.