A Systematic Approach to Microwave Amplifier Broadband Matching

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Abstract - An automatic synthesis process is described for wide-band matching of microwave amplifiers. Impedance-matching networks, yielding the required source and load impedances, are synthesised for maximum power transfer, for optimum noise performance or to achieve higher output power. The matching problem is solved in closed form also for conditionally stable devices.

I. INTRODUCTION

As it is well known, two purely resistive terminations are easily matched by means of simple LC networks over a bandwidth potentially larger than two octaves [1]. For GaAs FETs, the input and output impedances to be matched to the external impedance level are not purely resistive, although in some cases their reactive part can be absorbed into the network to be designed.

If the gain-bandwidth constraints are taken into account in the synthesis process, a suitable topology is selected [2] and the number of LC sections providing intermediate resistive transformations is determined. In most cases however, the absorption of parasites into the matching networks is simply not possible.

Moreover, at low frequencies, GaAs FETs and HEMTs usually exhibit a Rollet parameter k lower than unity, and the devices are therefore potentially unstable. Such devices can be stabilised by means of series or parallel feedback and the resulting network can be matched to the external impedance level as previously indicated. The amplifier performances however are reduced by the feedback network.

An alternative approach consists in designing input and output matching networks without stabilising the device and accepting a non optimum return loss at the device input or output. This is the usual approach also in the case of low-noise or power amplifier design, where the minimum noise figure or maximum output power have to be achieved at the expenses of input and output matching respectively. A design technique allowing an automatic synthesis also in the abovementioned case is therefore highly desirable, and can help to achieve better noise figure and power performances. The design of conditionally stable amplifiers is not a straightforward process. A typical design problem, making use of a conditionally stable device and with mismatched input and conjugately matched output [3], is shown in figure 1. The designer selects an appropriate stable source impedance $Z_s$ (i.e. selected to minimise or at least reduce the noise figure) and synthesises the resulting input matching network transforming $R_o$ into $Z_s$. Subsequently, the output network achieving conjugate matching at the device output is designed, matching the device output $Z_{out}$ to the desired impedance level $R_o$.

If $Z_s$ is selected outside the maximum stable gain circle, as a consequence $Z_m = Z_m^*$ will be located on the stable side of the load stability circle.

In the following, a systematic method for the direct synthesis of the matching networks involved in the design of conditionally stable devices will be presented, taking into account also the design bandwidth.

2. MATCHING NETWORKS SYNTHESIS PROCESS

The general topology of the proposed matching networks is composed by three sections, with a maximum of five lumped elements; from previous works [1], negligible bandwidth improvements can be obtained using a larger number of elements. The method will be presented for the lumped-element case only for the sake of
brevity, even if a similar approach can be attempted making use of distributed sections. The typical input matching problem for a single-stage amplifier consists in the synthesis of a two-port network, transforming the external impedance level $R_0$ into a previously selected stable source impedance $Z_s$, over a given design bandwidth $B = (\omega_h - \omega_l)/2\pi$, where $\omega_h$ and $\omega_l$ are the higher and lower design bandwidth edges respectively. A similar problem arises for the output matching network, transforming the external impedance level $R_0$ into a selected stable impedance $Z_L$. The series inductance $L_1$ of the matching network in figure 2 must be selected in order that $Y_1(\omega) = G_1 + jB_1$ fulfills the following condition:

$$G_1(\omega) = G(\omega) = G, \quad B_1(\omega) = B(\omega) < 0 \quad (1)$$

where

$$Y_1(\omega) = \frac{1}{Z_s + j\omega L_1} \quad \text{or} \quad Y_1(\omega) = \frac{1}{Z_L + j\omega L_1} \quad (2)$$

Condition (1) imposes that the admittance $Y_1$ assume identical values for the real part at the band extrema, while its imaginary part have opposite reactive behaviour. On a Smith Chart for admittances normalised to $R_0$ ($SCR_0$), condition (1) corresponds to $Y_1$ to lay on the same constant conductance circle on opposite sides at the bandwidth edges $\omega_h$ and $\omega_l$. A closed-form expression for $L_1$ can be easily found:

$$L_1 = \frac{1}{H} \left[ -K + \sqrt{K^2 - H \cdot J} \right] \quad (3)$$

where:

$$H = R_x(\omega_l) \cdot \omega_h^2 - R_x(\omega_l) - \omega_h^2$$

$$K = -R_x(\omega_l) \cdot X_x(\omega_l) \cdot \omega_l + R_x(\omega_h) \cdot X_x(\omega_h) \cdot \omega_h$$

$$J = R_x(\omega_l) \cdot R_x(\omega_l)^2 - R_x(\omega_l)^2 \cdot R_x(\omega_h) +$$

$$R_x(\omega_l) \cdot X_x(\omega_l)^2 - X_x(\omega_l)^2 \cdot R_x(\omega_h)$$

and $Z_s(\omega) = R_s(\omega) + jX_s(\omega)$. Analogous expressions can be obtained using $Z_s(\omega)$.

Depending on the value of the resulting real part of $Y_1(=G)$, one of the two following inequalities applies:

$$A \quad G > \frac{1}{R_s}$$

$$B \quad G < \frac{1}{R_s} \quad (5)$$

On a SCR$_0$ conditions A and B correspond to admittance values at the band extrema contained or external to the unit constant-conductance circle respectively. If condition A applies, a single L-cell matching approach suggests, starting from the final impedance level $R_0$, a parallel-series connection of the form shown on fig.2. On the contrary, if condition B applies, a series-parallel L-cell approach is mandatory, as shown on fig.3.

The purpose of the L$_1$-C$_s$ section is to transform the external impedance level $R_0$ into the admittance $Y_2(\omega) = G_2 + jB_2$ fulfilling the condition:

$$G_2(\omega) = G(\omega) = G \quad (6)$$

i.e. with the same real part as $Y_1$. Again, closed-form expressions for L$_2$ and C$_2$ can be derived; for the case of figure 2 (i.e. condition A), it is possible to find:

$$L_2 = \frac{R_0}{\omega_0^2} \cdot \sqrt{2 \omega_0 \omega_s (\gamma - (\omega_h^2 + \omega_l^2))} \quad (7)$$

$$C_2 = \frac{\gamma}{\omega_0 \omega_L} \quad (7)$$

where

$$\gamma = \frac{G \cdot R_0}{\sqrt{G \cdot R_0} - 1}$$

Similar expressions can be found if condition B applies.

Finally, the middle parallel resonant L$_2$-C$_2$ section allowing the desired change in susceptances from $B_1(\omega_h)$ and $B_1(\omega_l)$ to $B_2(\omega_h)$ and $B_2(\omega_l)$ respectively, can be computed according to (4)
\[ L_2 = \frac{(\omega_s^2 - \omega_b^2)/(\omega_s \cdot \omega_b)}{(B_1(\omega_s) - B_2(\omega_b)) \cdot \omega_b - (B_1(\omega_b) - B_2(\omega_s)) \cdot \omega_s} \]  

(8)

\[ C_2 = \frac{1}{\omega_b^2} \cdot \frac{B_2(\omega_b) - B_1(\omega_b)}{\omega_b} \]  

(9)

The effects of the various sections on the transformed admittances are presented on the Smith Chart in fig. 4, where \( Y_3 \) is the admittance defined in fig. 2 and 3.

As it is possible to note, the network in fig. 3 contains actually a parallel connection of two inductors that can be treated and realised as a single element, obtaining a four-element matching network shown in fig. 5. In this case an alternative strategy can be adopted, briefly summarised in the following. In the abovementioned case, a suitable condition to be imposed in order to find both \( C_3 \) and \( L_4 \) is:

\[ G_1(\omega_a) = G_2(\omega_b) \quad \text{and} \quad G_1(\omega_b) = G_2(\omega_a) \]  

(10)

It is to note that condition (10) is different from the already discussed approach corresponding to conditions (1) and (6), since different values for the real part of \( Y_1 \) and \( Y_2 \) are allowed at the band extrema.

Using (10) closed-form expressions for \( L_1 \) and \( C_3 \) can be derived, obtaining:

\[ L_1 = \frac{1}{H} \left[ K + \sqrt{K^2 - H \cdot J} \right] \]  

(11)

where:

\[ H = \left( \omega_s^2 R_0 - \omega_b^2 R_0 \cdot \frac{R_1(\omega_b)}{R_1(\omega_b)} \right) \]

\[ K = R_o \left( \omega_s^2 X_s(\omega_s) - \omega_b^2 X_b(\omega_b) \cdot \frac{R_1(\omega_b)}{R(\omega_s)} \right) \]

\[ J = \omega_s^2 R_0 R_1(\omega_b) \left( R_0 - R_1(\omega_b) \cdot X_b(\omega_b) \cdot \frac{R_1(\omega_b)}{R(\omega_s)} \right) - \omega_b^2 R_0 R_1(\omega_s) \left( R_0 - R_1(\omega_s) \cdot X_s(\omega_s) \cdot \frac{R_1(\omega_s)}{R(\omega_s)} \right) \]

\[ C_3 = \sqrt{\frac{R_1(\omega_a)/(\omega_s^2)}{R_1(\omega_a)/(\omega_b^2) - R_0 R_1(\omega_b) + X_s(\omega_s) + \omega_b L_2}} \]  

(13)

Finally, the middle parallel resonant \( L_2-C_2 \) can be computed by the same expressions in (8) and (9).

The computed parameters of the matching circuit represent a very good starting point for a circuit optimiser, resulting in enhanced circuit performances; the use of an optimiser can be necessary if component losses have to be taken into account. The picture in figure 4 refers to the various transformation steps applied to the series connection of a 20 \( \Omega \) resistor and 1 pF capacitor (modelling an active device input) that has been matched to a 50 \( \Omega \) source impedance over a bandwidth defined by \([\omega_a, \omega_b] = [3.7, 6.3] \text{ GHz}\). The resulting input return loss is plotted in fig. 6. The selection of two fixed topologies allows an automated synthesis approach, also for the layout generation process, where the lumped elements are realised by means of spiral inductors, MIM or interdigitated capacitors. In this case however, a final optimisation is mandatory to account for component parasitics and losses. It is to note that a parallel (or a series) resonant circuit is indeed beneficial also to guarantee out-of-band stability for the amplifier. Moreover, the circuit topologies has been selected to contain some fundamental elements of the amplifier; as a matter of fact the capacitor \( C_3 \) can be employed as DC blocking component and the inductance \( L_2 \) can be part of the biasing network. As a further remark, the choice of an intermediate parallel resonant LC section is however not mandatory, and a dual approach can be employed, making use of series resonant LC sections.

### III. DESIGN EXAMPLE

This section illustrates the use of the matching technique described above for the design of the matching network for a conditionally stable amplifier, operating at a center frequency of 5 GHz with a 40% bandwidth. The

![Fig. 5. Alternative matching network section B](image-url)
device used is a low-noise 4x50 GaAs FET manufactured by Alenia. Conjugately matched output is imposed. The design is based on measured S-parameters at $V_{DS} = 4$ V, $I_{DS} = 8$ mA. The matching network design begins selecting an appropriate available gain $G_a$ value (that must be lower than the device maximum stable gain, MSG). The input impedances to be matched $Z_S(\omega)$, $Z_S(\omega_0)$ are selected to ensure the same available gain at the band extrema. These values can be determined for the two frequencies by selecting $Z_S$ on the line connecting the center of the Smith Chart and the constant available gain circles' center so that $Z_S(\omega)$ and $Z_S(\omega_0)$ are at the maximum distance from their respective MSG circle. Such a choice can be easily automated using the well-known expressions giving the constant-gain circles.

On the other hand, to ensure greater stability, for a given available gain, it is possible to select the impedance values $Z_S(\omega)$ and $Z_S(\omega_0)$ in different ways but always on the circles corresponding to the same gain.

After the selection of the two source impedances as shown on fig.7, the input matching network element values are easily found by means of the expression given in the previous section. The active device output must be conjugately matched: consequently, the two impedances $Z_L(\omega)$ and $Z_L(\omega_0)$ are easily obtained by connecting the input matching network at the input of the FET and computing the output impedances $Z_{out}(\omega)$, $Z_{out}(\omega_0)$, obtaining, for the load impedances:

$$\Gamma_L(\omega) = \Gamma_{out}^*(\omega) \quad \Gamma_L(\omega_0) = \Gamma_{out}^*(\omega_0)$$ (14)

The expressions previously derived, using the impedances $Z_L(\omega)$ and $Z_L(\omega_0)$, directly provide the output matching network. The element values of the input and output matching networks are summarized in Table I.

The amplifier performances, in terms of output return loss and gain, are plotted in fig. 8.

IV. CONCLUSIONS

A systematic approach for input and output matching network design in conditionally stable microwave amplifiers has been described. Closed-form expressions and fixed topologies for the matching sections have been derived. An application to typical GaAs FET data demonstrates the usefulness of the proposed approach.

<table>
<thead>
<tr>
<th>Input Network</th>
<th>$L_1=1.6$ nH</th>
<th>$L_2=3.6$ nH</th>
<th>$C_2=0.2$ pF</th>
<th>$C_3=1.1$ pF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Network</td>
<td>$L_1=5.8$ nH</td>
<td>$L_2=2.3$ nH</td>
<td>$C_2=0.2$ pF</td>
<td>$C_3=0.5$ pF</td>
</tr>
</tbody>
</table>

TABLE I Matching network element values

V. REFERENCES