Process R&D in Monopoly under Demand Uncertainty[1]

Luca Lambertini
Dipartimento di Scienze Economiche
Università degli Studi di Bologna
Strada Maggiore 45, 40125 Bologna, Italy
phone 39-051-2092600, fax 39-051-2092664
lamberti@spbo.unibo.it
ENCORE, Faculty of Economics
University of Amsterdam
Roetersstraat 11, 1018 WB Amsterdam

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Abstract

I investigate R&D efforts for process innovation in a monopoly with uncertain demand. Two different models are proposed, where either (i) the reservation price is affected by an additive shock and the marginal production cost is increasing, or (ii) a multiplicative shock on the slope of demand combines with a flat marginal production cost. In either case, price-setting behaviour generates a larger R&D investment than quantity-setting behaviour. An Arrowian interpretation of the first result and a Schumpeterian interpretation of the second are proposed.

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1 Introduction

The incentives to invest in R&D (either for process or for product innovation) have been extensively investigated in the existing literature. The available contributions focus upon either (i) the role of uncertainty in the R&D activity, given the prize to be awarded to the winner of the race, or (ii) the role of the type of market competition, either Cournot or Bertrand, and market structure in shaping firms' incentives (for exhaustive accounts of both strands see, e.g., Reinganum, 1989; and Martin, 2001, ch. 14).

To the best of my knowledge, the interplay between demand uncertainty and firms' R&D efforts has not been investigated thus far, although there exist several influential contributions dealing with demand uncertainty either in monopoly (see Leland, 1972; Klemperer and Meyer, 1986) or in oligopoly (Weitzman, 1974; Klemperer and Meyer, 1986) or in perfect competition (Sandmo, 1971). All of these contributions focus upon the optimal price or quantity choice and the relative profitability of such strategies. In particular, Klemperer and Meyer (1986) show that if techology is characterised by an increasing marginal production cost, then the monopolist is better off using the output level rather than the price.

Here, I rely on Klemperer and Meyer's analysis to model the relationship between the monopolist's incentive to invest in process innovation and demand uncertainty. In particular, I propose two alternative models. In the first, an additive shock appears in the demand function and the cost function is convex. Under these conditions, expected profits (gross of R&D costs) are larger under quantity-setting behaviour. In the second, a multiplicative shock affects the slope of the demand function, while production costs are linear in the output level. Under these conditions, instead, expected profits (gross of R&D costs) are larger under price-setting behaviour. In both cases, the monopolist invests in R&D in order to reduce marginal cost. I show that, irrespective of the assumptions adopted regarding the type of uncertainty and the shape of the cost function, the optimal R&D investment is larger when the monopolist sets the price than when it sets the output level. In the first model, this is due to the fact that increasing the R&D effort amounts in fact to a decrease in the uncertainty affecting the profits generated by fixing the price; it is indeed an optimal response to the expected profit loss associated with the variance of the shock, that the monopolist foresees when setting the price. In the second model, R&D cannot contribute to reduce the effects of uncertainty on the expected profits associated with quantity-setting behaviour; therefore, the larger funds available under price-setting behaviour drive the result.

The two alternative models are laid out and investigated in section 2. Concluding comments are in section 3.

2 The setup

Consider a single-product monopolist operating in a market over $t \in [0, \infty)$. At $t = 0$ the firm invests in R&D for process innovation; then, over $t \in [1, \infty)$, she supplies the good to the market by setting either the price or the output level so as to maximise profits. Define as π the instantaneous profits, gross of R&D costs, which are $\Gamma(x)$, x being the R&D effort prouced by the monopolist. The R&D cost function is characterised by the following properties: $\Gamma'(x) > 0$ and $\Gamma''(x) \leq 0$.

Assuming a constant discount factor $\delta \in [0,1]$, the expected net profit flow is:

$$E\Pi = \sum_{t=1}^{\infty} \delta^t E\pi - \Gamma(x) = \frac{\delta E\pi}{1-\delta} - \Gamma(x). \tag{1}$$

As to the issue of modelling production costs and the demand function, I will consider two alternative cases based upon Klemperer and Meyer (1986):

- **Model I:** At any $t \in [1,\infty)$, the market demand function is $p = a - Q + \varepsilon$. The additive shock ε has $E(\varepsilon) = 0$ and $E(\varepsilon^2) = \sigma^2$. The cost function is $C(Q) = cQ^2/2$, with $c = c(x)$ and $c'(x) < 0$; $c''(x) \geq 0$.[1]

- **Model II:** At any $t \in [1,\infty)$, the market demand function is $p = a - Q/\theta$. The shock on the slope of demand, θ, has $E(\theta) = 1$ and $E(\theta^2) = s^2 > 1$. Accordingly, I may define $z \equiv E(1/\theta)$, which is larger than one (by Jensen's inequality). The cost function is $C(Q) = \gamma Q$, with $\gamma = \gamma(x)$ and $\gamma'(x) < 0$; $\gamma''(x) \geq 0$.

In both models, the monopolist may use either price or quantity as the market variable to maximise per-period profits.

2.1 Equilibrium analysis: Model I

Under the additive shock on the vertical intercept of the demand function (i.e., the reservation price), the per-period monopoly profits are:

$$E\pi_Q = \frac{a^2}{2[2+c(x)]} \; ; \; E\pi_P = E\pi_Q - \frac{\sigma^2}{2} c(x) \tag{2}$$

[1] As is known from Klemperer and Meyer (1986, Lemma 1), if the marginal production cost were constant, then the additive shock on demand would exert no effects on the equilibrium behaviour of the firm.

under quantity- and price-setting behaviour, respectively (cf. Klemperer and Meyer, 1986, pp. 636-37). Therefore, the larger the variance, the larger the difference $E\pi_Q - E\pi_P$, all else equal. Profits (2) can be plugged into (1) in order to derive the first order conditions (FOCs) pertaining to the R&D activity at $t = 0$, in the two cases:

$$\frac{\partial E\Pi_Q}{\partial x} = \frac{\delta}{1-\delta} \cdot \frac{\partial E\pi_Q}{\partial x} - \Gamma'(x) = 0 \Leftrightarrow$$

$$-\frac{\delta a^2 c'(x)}{2(1-\delta)[2+c(x)]^2} - \Gamma'(x) = 0; \tag{3}$$

$$\frac{\partial E\Pi_P}{\partial x} = \frac{\delta}{1-\delta} \cdot \frac{\partial E\pi_P}{\partial x} - \Gamma'(x) = 0 \Leftrightarrow$$

$$-\frac{\delta a^2 c'(x)}{2(1-\delta)[2+c(x)]^2} - \frac{\sigma^2}{2} \cdot c'(x) - \Gamma'(x) = 0. \tag{4}$$

From (3-4), one obtains:

$$\frac{\partial E\Pi_P}{\partial x} - \frac{\partial E\Pi_Q}{\partial x} = -\frac{\sigma^2}{2} \cdot c'(x) > 0 \tag{5}$$

given that $c'(x) < 0$.

2.2 Equilibrium analysis: Model II

Now examine the setup where the shock affects the slope of the demand function. In this case, per-period profits are (cf. Klemperer and Meyer, 1986, pp. 636-37):

$$E\pi_Q = \frac{[a-\gamma(x)]^2}{4z}\ ;\ E\pi_P = z \cdot E\pi_Q = \frac{[a-\gamma(x)]^2}{4}. \tag{6}$$

Here, $E\pi_Q < E\pi_P$ since $z > 1$. Proceeding as in the previous subsection, one has to calculate the FOCs pertaining to the R&D phase at $t = 0$:

$$\frac{\partial E\Pi_Q}{\partial x} = -\frac{\delta [a-\gamma(x)]\gamma'(x)}{2(1-\delta)z} - \Gamma'(x) = 0; \tag{7}$$

$$\frac{\partial E\Pi_P}{\partial x} = -\frac{\delta\left[a-\gamma\left(x\right)\right]\gamma'\left(x\right)}{2\left(1-\delta\right)} - \Gamma'\left(x\right) = 0, \tag{8}$$

implying:

$$\frac{\partial E\Pi_P}{\partial x} - \frac{\partial E\Pi_Q}{\partial x} = -\frac{\delta\left(z-1\right)\left[a-\gamma\left(x\right)\right]\gamma'\left(x\right)}{2\left(1-\delta\right)z} > 0 \tag{9}$$

given that $\gamma'\left(x\right) < 0$.

The discussion of the two models can be summarised in the following Proposition:

Proposition 1 *Irrespective of whether (i) the marginal cost is increasing and demand is affected by an additive shock, or (ii) the marginal cost is flat and uncertainty affects the slope of demand, the R&D investment is larger when the monopolist sets the price than when it sets the output level.*

However, the source of the result is different in the two cases. With increasing marginal cost and an additive shock on the vertical intercept of demand, gross profits are larger under quantity-setting behaviour, so that the firm invests more under price-setting in order to reduce the negative bearings of the reservation price variance on profits. In the limit, as the marginal cost tends to zero, the effect of the shock disappears altogether. That is, by increasing the intensity of the R&D effort, the monopolist gets two eggs in one basket: a more efficient technology as well as a reduction in the negative effects of uncertainty on profits. This is an insurance policy against uncertainty which has an Arrowian flavour, as lower profits call for more intense R&D efforts. On the contrary, in the presence of a constant marginal cost coupled with a multiplicative shock on the slope of the demand function, the interpretation of the result is Schumpeterian, as setting the price allows for higher expected gross profits than setting the output level. Given that in this case a larger investment does not bring about a reduction in the degree of uncertainty while it entails an increase in gross profits for any given z, the incentive to invest in R&D is driven by a 'deep purse' argument.

3 Concluding remarks

I have modelled R&D efforts for process innovation in a monopoly with uncertain demand. Two different models have been considered: one where an additive shock on the reservation price couples with an increasing marginal production cost, and the other with a multiplicative shock on the slope of demand and a constant marginal production cost. In either case, price-setting behaviour generates a larger R&D investment than quantity-setting behaviour. The reason for this result is that process R&D provides the firm with un insurance policy against uncertainty in the first model, while it cannot do so in the second model.

Extending the above analysis to oligopoly models may represent a productive perspective, which is left to future research.

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