Optimal Monetary Policy in a Simple Distorted Economy

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Abstract. In this paper I search for an optimal configurations of parameters for variants of the Taylor rule by using an Accurate Second-Order Welfare based method within a fully microfounded Dynamic Stochastic model, with price rigidities, without capital accumulation. Money is inserted via a transaction cost function, price rigidities are modelled via quadratic cost of price adjustment. A version of the model with distortionary taxation is also explicitly tested. The model is solved up to Second Order solution. Optimal rules are obtained by maximizing a conditional welfare measure, differently from what has been done in the current literature. Optimal monetary policy functions turn out to be characterized by inflation targeting parameter lower than in empirical studies. In general, the optimal values for moentary policy parameters depend from the degree of nominal rigidities and from the role of fiscal policy. When nominal rigidities are higher, optimal monetary policy becomes more aggressive towards inflation. With a tigther fiscal policy, optimal monetary policy turns out to be less inflation-aggressive. Moreover, the results show that relying conditional welfare measure avoids the problems related with first-order or unconditional welfare measures. Impulse Response functions based on second order model solution show a non-affine pattern when the economy is hit by shocks of different magnitude.

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1. Introduction

The present paper derives the optimal monetary policy function, in a simple model with nominal rigidities. The kind of monetary policy reaction function here analyzed belongs to the Taylot-type class, where short term nominal interest rate reacts with respect to inflation rate, output (or output gap) together with a coefficient of a lagged interest rate itself.

The search of the optimal monetary policy function identified by a collection of parameter values maximizing a specific welfare criterion, has recently become one of the most debated topics in monetary economics.

The crucial novelty of the approach followed in this paper is given by the type of welfare measure derived from a solution algorithm based on a second order approximation of the whole model, including constraints and first order conditions, as described by Kim, Kim, Schaumburg and Sims (2003). Differently from other existing studies, the metric for welfare is here offered by a second order expansion of the utility function conditional to the non-stochastic steady state[1].

The optimal monetary policy combination found in this model lies within the usual range of parameters, accepted by the empirical literature, but the size of the inflation targeting coefficient is smaller than what has been found in the current literature. Moreover, the results show that the usage of first order based welfare measures or unconditional welfare might deliver highly misleading results. The advantage of conditional second order based welfare measure is related with the correct evaluation of welfare during the transitional period from a steady state to another. Impulse Response functions based on second order approximation show a non-affine pattern when the economy is hit by shocks of different magnitude.

The approach undertaken in this paper closely follows the recent developments of Dynamic Stochastic General Equilibrium monetary models, known as "New Neoclassical Synthesis" according to Goodfriend and King (1997). This literature mixes up the fully rational expectation Real Business Cycle modelling framework with the microfoundations of nominal rigidities, belonging to the standard apparatus of Keynesian models. A collection of remarkable papers aiming to find optimal monetary policy reaction function is contained in a famous book edited by Taylor (1999)[2].

The only nominal rigidity included in the present paper is represented by price stickiness. Such choice is motivated by the need to make the model as much as comparable as possible with the existing literature, especially becasue optimal results are obtained under a different solution method and a different welfare method. In what follows, money is inserted via the transaction cost approach: this makes the construction of the model much more general and avoids misunderstandings in the interpretation of money demand shocks.

A further element contrasting with respect to the existing literature is given by the explicit consideration of fiscal policy: in the existing literature, fiscal policy is only *assumed* to be 'Ricardian', i.e. including a solvency constraint on the government, as for example in Rotemberg and Woodford (1999). On the other hand, the present paper takes explicitly into consideration the

[1] Sutherland (2002), Schmitt Grohè and Uribe (2002) and Kollmann (2002) all use welfare measures based on unconditional welfare measures.

[2] Other important contributions in this area are those by King and Watson (1996), Ohanian et al. (1995), Chari et al. (2000).

role of the government budget constraint into the model solution for the comparative evaluation of the various monetary policy rules.

According to Leeper (1991), Sims (1994) fiscal policy requirements on the future path of primary surpluses are a necessary complement to monetary policy rules in order to achieve full price stability. In the present model, I assume a fiscal policy rule making taxes reacting to the outstanding stock of real public debt. Such policy rule is defined 'Passive' in Leeper's terminology, or 'Ricardian' in Woodford's terminology. These assumptions are considered on both cases with and without distortionary taxation.

The rest of the paper is organized as follows. The next section discusses the main assumptions underlying the model. A separate section studies the assumptions underlying Government's behavior. Moreover, the reaction functions of monetary authority are presented in a separate section. Three additional sections discussing, respectively, the characteristics of the equilibrium, calibration and the assumptions underlying the variant of the basic model with distortionary taxation. The solution method including a discussion of welfare measures and impulse-responses is discussed in a subsequent section. Another section discusses the empirical findings. Concluding remarks close the paper.

2. The model

The model is populated by j-th measured on the real line between [0,1]. The utility function of each agent j is:

$$U_t^j = E_0 \sum_{t=0}^{\infty} \beta^t u\left(C_t^j, L_t^j\right) \tag{1}$$

where the instantaneous utility function $u\left(C_t^j, L_t^j\right)$ is:

$$u\left(C_t^j, L_t^j\right) = \frac{\left[C_{jt}^{(1-\gamma)}\left(1-L_{jt}\right)^{\gamma}\right]^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \tag{2}$$

with $\sigma > 0$. In (2) I considered a weakly separable utility function in both consumption C_t^j and labor effort L_t^j. This allows a better propagation mechanism for the shocks included in the model. The representative agent's budget constraint is given by:

$$P_t C_t^j\left[1+\xi_t^j f\left(V_t^j\right)\right] + B_t^j + M_t^j = \left(1+i_{t-1}\right) B_{t-1}^j + M_{t-1}^j + w_t^j L_t^j + P_t \Omega_t^j - P_t T_t^j \tag{3}$$

From (3) each household j receive income from investing in nominal bonds B_t^j (the superscript indicates that bonds are owned by agent j) paying nominal interest i_t, and from labor income $w_t^j L_t^j$. Additionally, each agent pays real lump sum taxes T_t^j. Money holdings M_t^j enter the budget constraint in two ways: one direct, and one indirect via the definition of the money velocity $V_t^j = \frac{P_t C_t^j}{M_t^j}$. Moreover, Ω_t^j indicates the profit gained by agent j in participating to firm j.

According to the transaction cost approach, to buy an amount of goods $P_t C_t^j$ each household must spend an additional amount given by $P_t C_t^j \xi_t^j f\left(V_t^j\right)$. In (3), $f\left(V_t^j\right)$ is the transaction cost function and ξ_t^j is a transaction cost shock represented by the following AR(1) process:

$$\log\left(\xi_t^j\right) = +\left(1-\rho_\xi\right)\log\left(\xi^j\right) + \rho_\xi \log\left(\xi_{t-1}^j\right) + \varepsilon_t^\xi \tag{4}$$

with $E\left(\varepsilon_t^\xi\right) = 0$, $Var\left(\varepsilon_t^\xi\right) = \sigma_\xi^2$. The transaction cost approach has the advantage of making money demand shocks as transaction cost shocks, differently from preference shocks, as it occurs when money enters directly into the utility function. The transaction cost function $f\left(V_t^j\right)$ is increasing in velocity V_t^j, i.e. $f'\left(V_t^j\right) > 0$. Here I assume convex transaction cost, by setting $f''\left(V_t^j\right) > 0$. The assumption of convex transaction costs rules out indeterminacy under *active* monetary policy, which, in our setting is equivalent to require that monetary policy follows an interest rate pegging rule with an inflation targeting coefficient bigger than one. However, this is equivalent to assume that there is no barter equilibrium for this economy. A concave transaction cost function, with $f''\left(V_t^j\right) < 0$, instead makes possible the existence of a barter equilibrium with a zero money in steady state and a positive nominal interest rate.

In what follows, I adopt a concave transaction cost function like:

$$f\left(V^j\right) = \frac{V^j}{1+V^j} \tag{5}$$

Function (5) has been suggested by Sims (1994).

Intertemporal First Order Conditions. Given the above assumptions, the representative agent's problem consists in maximizing the utility function (1)-(2) subjected to the budget constraint (3), with respect to consumption C_t^j, labor income L_t^j, money M_t^j and bond holdings B_t^j. The first order conditions with respect to C_t^j, L_t^j, M_t^j, B_t^j are respectively given by:

$$\lambda_t^j\left[1+\xi_t f\left(V_t^j\right)+\xi_t f'\left(V_t^j\right)V_t^j\right] = (1-\gamma)\, C_t^{(1-\gamma)\left(1-\frac{1}{\sigma}\right)-1}\left(1-L_t\right)^{\gamma\left(1-\frac{1}{\sigma}\right)} \tag{6}$$

$$\gamma C_t^{(1-\gamma)\left(1-\frac{1}{\sigma}\right)}\left(1-L_t\right)^{\gamma\left(1-\frac{1}{\sigma}\right)-1} = \lambda_t^j W_t \tag{7}$$

$$\beta E_t \frac{\lambda_{t+1}^j}{P_{t+1}} = \frac{\lambda_t^j}{P_t}\left[1-\xi_t f'\left(V_t^j\right)\left(V_t^j\right)^2\right] \tag{8}$$

$$\beta\left(1+i_t\right)E_t\frac{\lambda_{t+1}^j}{P_{t+1}} = \frac{\lambda_t^j}{P_t} \tag{9}$$

In (6)-(9) λ_t^j indicates the Lagrange multiplier associated to the representative agent budget constraint (3). The household's problem is completed by the inclusion of a Transversality condition.

Condition (6) is the usual first order condition on consumption, while condition (7) equates the marginal disutility from working to labor remuneration discounted with the marginal utility of consumption. Equation (8) nests a money demand function, and condition (9) derives from the optimal allocation of bonds. By mixing together equation (8)-(9) we find the following money demand function:

$$\frac{1}{\left(1+i_t\right)} = \left[1-\xi_t f'\left(V_t^j\right)\left(V_t^j\right)^2\right] \tag{10}$$

In order to explore the characteristics of (10), I assume that the economy is in a non-stochastic steady state[3], with $\pi_{t+1} = \pi$, $V_t^j = V$, $\xi_t = \xi$, for all t and j. Furthermore, we can simplify money demand function (10), as follows:

$$\frac{\beta}{\pi} = 1 - \xi f^{'}(V) V^2 \tag{11}$$

with V is the steady state money velocity. In particular, the steady state real money balances $m = M/P$ are given by:

$$m = \frac{(1-\Delta)}{\Delta} C$$

where:

$$\Delta = \left[\frac{1-\beta\pi^{-1}}{\xi}\right]^{1/2}$$

In order to find a solution of the model for a wide range of policy parameter values, I impose that $\xi < 1$, as in Sims (1994).

A variant of the model includes a distortionary tax on labor income: $\tau_t^L W_t L_t$, with τ_t^L as tax rate. In this case, the first order condition (7) is replaced by:

$$\gamma C_t^{(1-\gamma)\left(1-\frac{1}{\sigma}\right)} (1 - L_t)^{\gamma\left(1-\frac{1}{\sigma}\right)-1} = \lambda_t^j \left(1 - \tau_t^L\right) W_t$$

The presence of distortionary taxation makes the model comparable to other papers considered in the literature, like, for example Schmitt-Grohé and Uribe (2003). Distortionary taxation adds a real rigidity: this sheds additional light on the interactions between fiscal and monetary policy, in the definition of the optimal monetary policy rule.

The intra-temporal choice problem. Given the presence of a large number of final goods, each agent chooses the composition of a basket of differentiated goods. On this ground, the variable C_t^j represents an index of all the differentiated goods produced in this economy. Following Dixit and Stiglitz (1977) the basket C_t^j is defined by a CES aggregator over all the $i \in [0,1]$ final goods as:

$$C_t^j = \left[\int_0^1 \left(C_t^j(i)\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \tag{12}$$

where $\theta > 1$, indicates the elasticity of substitution between different final goods varieties. Let X_t^j be the total expenditure of agent j. The optimal composition of varieties within basket C_t^j in (12) can be derived by maximizing index (12) over $C_t^j(i)$ for all $i \in [0,1]$ subjected to an expenditure constraint. The solution to the *intra-temporal* defines the aggregate price index P_t:

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta} di\right]^{\frac{1}{1-\theta}} \tag{13}$$

Moreover, the demand of variety i expressed by each agent j is:

$$\frac{C_t^j(i)}{C_t^j} = \left[\frac{p_t(i)}{P_t}\right]^{-\theta} \tag{14}$$

[3] A variable at time t, X_t, in steady state is indexed as X, without time subscript, for all i and $j \in [0,1]$.

where θ is the elasticity of good i with respect to its price.

For what concerns public expenditure, I assume that government reveals a demand for goods G_t which has to be allocated in terms of j-varieties of differentiated goods. The total demand for goods G_t is given by:

$$G_t = \left[\int_0^1 G_t^{\frac{\theta-1}{\theta}} (j)\, dj \right]^{\frac{\theta}{\theta-1}}$$

Thus, by using an optimization process similar to (12)-(**??**), we find that the demand for variety j expressed by the government is:

$$\frac{P_t (j)}{P_t} = \left[\frac{G_t (j)}{G_t} \right]^{-1/\theta} \tag{15}$$

The aggregate demand for variety j can be expressed as:

$$C_t (j) + G_t (j) = Y_t (j) \tag{16}$$

So that:

$$\frac{P_t (j)}{P_t} = \left[\frac{Y_t (j)}{Y_t} \right]^{-1/\theta} \tag{17}$$

with $Y_t (j)$ defined as in (16). After aggregating (16) over all j-varieties we get: $Y_t = C_t + G_t$.

2.1. Firms. I assume the existence of a large number of imperfectly competitive firms indexed on the real line, each producing a single variety $j \in [0, 1]$. Thus, each firm has control on the price of each variety j, $P_t(j)$, but not on the aggregate price level P_t. Nominal rigidities are introduced via cost of price adjustment, described by the following function:

$$PAC_t^p (j) = \frac{\phi_p}{2} \left(\frac{P_t(j)}{P_{t-1}(j)} - \pi \right)^2 \tag{18}$$

where π is the steady state inflation rate. According to (18) every time a firm decides to change the price of variety j has to pay a cost in terms of output. In particular, the cost of changing price is positive when the rate of price change of variety j differs from steady state inflation rate π. A similar approach has been adopted by Kim (2000) and Hairault and Portier (1993). In the *menu cost* literature *à la* Rotemberg (1982) π is set to $\pi = 1$, implying that each firm pays a cost of changing price at all, not necessarily according to the steady state level of the inflation rate.

The degree of price stickiness in the economy is entirely governed by the magnitude of parameter $\phi_p \geq 0$. Under (18) cost of price adjustment are zero in steady state.

The production function is assumed to be of Cobb-Douglas type:

$$Y_t (j) = A_t \left(g_y L_t (j) \right)^{1-\alpha} \tag{19}$$

where g_y is the exogenous growth rate of labor-augmenting technical progress, and A_t indicates a time dependent technological shock, for which I assume the following AR(1) process:

$$\log A_t = (1 - \rho_A) \log A + \rho_A \log A_{t-1} + \varepsilon_t^A \tag{20}$$

with $\varepsilon_t^A \sim N\left(0, \sigma_A^2\right)$.

The profit maximization problem for each firm j consists in finding the optimal amount of labor input $L_t\left(j\right)$ by maximizing the following profit function:

$$\Omega_t\left(j\right) = \sum_{t=0}^{\infty} \rho_t \left[\frac{P_t\left(j\right)}{P_t} Y_t\left(j\right) - W_t L_t\left(j\right) - PAC_t^p\left(j\right)\right] \tag{21}$$

subjected to the production function (19), the total demand for variety j given in (17) and the price adjustment cost function (18). In (21) ρ_t indicates the intertemporal rate of discount employed by each firm to evaluate future profit streams.

As it is customary in the recent literature on microfounded monetary models, I posit an homogeneity condition on the behavior of all firms, by setting $X_t\left(j\right) = X_t$ for all $j \in [0,1]$. This assumption allows to simplify algebra and avoids the need to keep track of the entire price distribution across firms.

The first order conditions for firms' profit maximization, after the imposition of the homogeneity assumption, delivers the following expression for the equilibrium wage rate (in real terms):

$$W_t = \left(1-\alpha\right)\left(1 - \frac{1}{\theta_t^y}\right)\frac{Y_t}{L_t} \tag{22}$$

where:

$$\frac{1}{\theta_t^y} = \frac{1}{\theta}\left[1 - \phi_p\left(\pi_t - \pi\right) + E_t \frac{\rho_{t+1}}{\rho_t}\phi_p\left(\pi_{t+1} - \pi\right)\pi_{t+1}^2 \frac{Y_{t+1}}{Y_t}\right] \tag{23}$$

The variable θ_t^y defined in (23) is the output demand elasticity augmented by the cost of price adjustment weighted by the utility. In steady state, with zero cost of price adjustment the output demand elasticity equates the elasticity of substitution across differentiated goods, i.e.: $\theta_t^y = \theta^y$.

From (23) the *mark up* over marginal costs is $\mu_t = \left(1 - \frac{1}{\theta_t^y}\right)^{-1}$. Thus, with perfectly flexible prices when $\phi_p = 0$, the markup is constant. In the perfectly competitive case, when $\theta_t^y \to \infty$, the markup is equal to unity.

An interesting feature of the formulation of nominal rigidities adopted by (18) is that with sticky prices the markup becomes endogenous and works as a transmission channel for real and nominal shocks hitting both inflation and output via equation (23): a shock decreasing θ_t^y lowers the mark up, because of the lower degree of monopolistic market power.

3. Fiscal Policy and Government behavior

The government budget constraint in nominal terms is:

$$B_t - \left(1 + i_{t-1}\right)B_{t-1} + M_t - M_{t-1} = P_t G_t - P_t T_t \tag{24}$$

From (24) the primary deficit (surplus) $G_t - T_t$ plus the interest rate proceedings paid by Government to the owner of government debt $i_{t-1}B_{t-1}$ is financed either by printing new money $M_t - M_{t-1}$ or by issuing new debt $B_t - B_{t-1}$.

Of course, the equilibrium conditions on both debt and money market are such that demand meets supply at any instant t, so that the total amount of debt and money floating in the market are entirely owned by j-*th* households. In fact:

$$B_t = \int_0^1 B_t^j dj, \quad M_t = \int_0^1 M_t^j dj, \text{ and } \quad T_t = \int_0^1 T_t^j dj \tag{25}$$

As pointed out by Sims (1994), the inflation can be viewed as fiscal phenomenon: if the government is perceived to adopt a loose fiscal policy, the inflation rate will explode right from today, discounting the future increase of money supply needed to wash out the level of debt. This occurs *in expectation* even if seigniorage revenues are very small and the commitment to avoid the usage of money printing is very strong.

To prevent an explosive solution for price level, I include the following fiscal policy reaction function:

$$T_t = \psi_0 + \psi_1 \frac{B_{t-1}}{P_t} \tag{26}$$

As in Leeper (1991) and Sims (1994), given (26), a fiscal policy is defined to be 'passive' when ψ_1 lies in the following range:

$$\left(\beta^{-1} - 1\right) < \psi_1 < \left(\beta^{-1} + 1\right) \tag{27}$$

For an equilibrium price level to be determinate, condition (27) has to be respected and coupled with an 'active' monetary policy rule, identified as an interest rate pegging rule whose inflation targeting coefficient is set to be bigger than one.

In case of distortionary taxation, the tax rule considered in (26) becomes:

$$\tau_t^L W_t L_t = \psi_0 + \psi_1 \frac{B_{t-1}}{P_t} \tag{28}$$

Finally, I assume that the aggregate level of public expenditure G_t follows an AR(1) process, given by

$$\log(G_t) = (1 - \rho_G) \log G + \rho_G \log(G_{t-1}) + \varepsilon_{t+1}^G \tag{29}$$

with ε_{t+1}^G i.i.d. variable normally distributed with zero mean and constant variance, σ_G^2.

4. Monetary Policy Rules

I consider alternative monetary policy rules whose behavior has been carefully studied in both theoretical and applied works. All the rules discussed here are reported in log-linear framework.

4.1. Taylor Rules. All the rules considered in the present paper have been obtained as variants of the following general interest-rate pegging rule:

$$i_t = i + \sum_{n=1}^{T_i} \phi_{in} (i_{t-1} - i) + \sum_{n=1}^{T_\pi} \phi_{\pi n} (\pi_{t-1} - \pi) + \sum_{n=1}^{T_x} \phi_{xn} (Y_{t-1} - Y) + \eta_t^{mp} \tag{30}$$

where i_t is a measure of the nominal interest rate (the Federal Funds Rate in the empirical literature on US economy). The advantage of making use of rules embedded in (30) is given by the restrict number of parameters to be controlled by monetary authority. Under rules like (30), money supply becomes endogenous, since monetary authority sets interest rate by letting the quantity of money to be determined by market clearing conditions.

In equation (30) I also included a shock η_t^{mp} modelled as an AR(1):

$$\log\left(\eta_t^{mp}\right) = \left(1-\rho_{mp}\right)\log\left(\eta^{mp}\right) + \rho_{mp}\log\left(\eta_{t-1}^{mp}\right) + \varepsilon_t^{mp} \tag{31}$$

where ε_t^{mp} is an i.i.d. process distributed as $\varepsilon_t^{mp} \sim N\left(0, \sigma_{mp}^2\right)$. This assumption allows for a better internal propagation mechanism, as suggested by Furher and Moore (1995).

In this work I am going to study the impact on welfare induced by the following Taylor Rules:

$$\widetilde{i}_t = \phi_\pi \widetilde{\pi}_t + \phi_y \widetilde{Y}_t + \phi_i \widetilde{i}_{t-1} + \eta_t^{mp} \tag{32}$$

$$\widetilde{i}_t = \phi_\pi E_t \widetilde{\pi}_{t+1} + \phi_y \widetilde{Y}_t + \phi_i \widetilde{i}_{t-1} + \eta_t^{mp} \tag{33}$$

where η_t^{mp} has been defined in (31). Equation (32)-(33) indicates the set of basic Taylor Rules to be considered in the optimality analysis.

The standard Taylor rule is given by equation (32), with $\phi_i = 0.0$. This is the classical rule assumed by Taylor (1993,1999), where $\phi_\pi = 1.5$, $\phi_y = 0.5$, $\phi_i = 0.0$, obtained after estimating the rule after 1979 for US economy.

Rotemberg and Woodford (1999) in a simple model show that a rule like in (32) with $\phi_\pi = 1.22$, $\phi_y = 0.06$, $\phi_i = 0.0$ is the optimal Taylor Rule for the US economy.

The forward-looking variant of Taylor Rule is represented by equation (33), where monetary authority sets current nominal rate as reacting to future inflation and output has been suggested - among others - by Clarida, Galì and Gertler (2000), on the basis of empirical results.

4.2. Rules based on monetary aggregate targeting. The evaluation of Taylor-type monetary policy rules should also be considered against the alternative represented by monetary aggregate targeting rules. According to Kim (2000) and Ireland (1997), I adopt the following specification for a monetary aggregate targeting rule:

$$\begin{aligned}\log(\chi_t) = {} & \rho_M \log(\chi_{t-1}) + (1-\rho_M)\log(g_m) + (1+\nu)\log(g_{mt}) + \\ & +\nu \log\left(\frac{R_t}{R}\right) - \upsilon\rho_i \log\left(\frac{R_{t-1}}{R}\right)\end{aligned} \tag{34}$$

where:

$$\chi_t = \frac{M_t}{M_{t-1}} \tag{35}$$

and:

$$\log(g_{mt}) = \rho_m \log(g_{mt-1}) + \varepsilon_t^m \tag{36}$$

with $\varepsilon_t^m \sim N\left(0, \sigma_m^2\right)$. From (34) monetary authority targets a mix between a monetary aggregate and nominal interest rate, and nominal interest rate is residually determined within the system, if one accepts - as in the present paper - the vision which sees rule (34) as determining nominal money. In (34) g_{mt} indicates a shock to money growth rate, represented via the AR(1) process given in (36).

The sensitivity of money supply to interest rate movements is identified by parameter ν: higher ν, smoother will be the response of interest rate to money growth shocks. A pure monetary aggregate targeting rule is obtained with $\nu = 0$: in this case monetary authority cares only about money supply. If $\nu = 1$, there is a one-to-one correspondence between nominal rate changes and money growth rule. When $\nu = \infty$, monetary authority cares only about interest rate movements. The role of interest rate smoothing is subsumed by parameter ρ_i: with $\rho_i = 0$, monetary authority targets only current nominal rate. If $\rho_i > 0$ monetary authority smooths interest rate by targeting both actual and last period's interest rate.

5. Equilibrium and Calibration

5.1. Equilibrium. I posit an homogeneity assumption by setting $X_t(j) = X_t$ for all $j \in [0,1]$, for each variable of the system. This assumption avoids to keep track of the entire price distribution across different final goods varieties.

From Euler equation the steady state level of gross inflation rate is:

$$g_x \pi = \beta_x (1+i) \tag{37}$$

where β_x indicates the discount rate of the transformed economy, whose relationship with the discount rate for the non-transformed one β is given by: $\beta_x = \beta^{(1-\gamma)\left(1-\frac{1}{\sigma}\right)} g_x$.

Furthermore, I assume that each agent has access to a set of complete markets for contingent claims. This that the discount factor for consumers must equal that of firms:

$$\frac{\rho_{t+1}}{\rho_t} = \beta_x \frac{\lambda_{t+1}}{\lambda_t} \tag{38}$$

We can rationalize condition (38) by supposing the existence of a representative agent who can freely exchange shares of whatsoever firm operating in this economy, without paying any sort of transaction cost at all. The inclusion of an additional first order condition for the optimal allocation of shares would produce a similar result.

To save notation, I indicate the variables of the transformed economy associated to the non-stochastic steady state with the same variable without the time subscript t.

5.2. Calibration. The model is calibrated on the basis of data of the US economy relatively to the sample period 1959:1-2001:4, quarterly observations.

The 'core' parameters of the model are reported in Table 1, while in Table 2 I reported the policy parameters, i.e. the parameters of the reaction functions of both monetary and fiscal authorities. The discount rate of the transformed economy has been set equal to 0.9978, implying a discount factor for the non-transformed economy equal to 1.0071.

The growth rate of real GDP of the US economy over the specified sample period is 1.76% on an yearly basis, which corresponds to a quarterly growth rate equal to 1.0044 (in gross terms). To measure the steady state nominal interest rate, I considered the mean of the Federal Funds Rate which is equal to 6.47% per year, equivalent to a 1.61% in quarterly terms.

Parameter	β_x	g_y	ξ	γ	θ_y	σ	α
Value	.9978	1.0044	.07723	.6	10	0.1	0.33

Table 1: Basic Parameters

Parameter	ψ_1	ϕ_π	ϕ_y	ϕ_i	ρ_i	ν	ρ_M
Value	0.05	free	free	free	0.999	0.576	-0.158

Table 2: Policy Parameters

From First Order Condition on Bonds computed in steady state (37), we find a quarterly inflation rate (in gross terms) equal to 1.009499, implying an annual inflation rate equal to 3.8% (the value of the annual inflation rate computed over the specified sample period by using the GDP deflator is equal to 3.78%).

The real interest rate is $\frac{g_y}{\beta_x} = 1.0066$, equivalent to 2.64% in annual terms.

As in Rotemberg and Woodford (1999), I assume a share of Aggregate Consumption over GDP S_c equal to 0.57. Moreover, I assume that the total amount of transaction costs is 2% of GDP in year terms, equivalent to 0.5% on a quarterly basis: this determines a value of ξ equal to .07723.

From data on M_1, consumption over non-durable goods and CPI we get a value for velocity[4] equal to 0.46. With the above parameters, the value of money velocity implied by the model is given by $V = 0.38$. The steady state level of debt is calibrated in order to have a debt/GDP ratio equal to 0.44, as in Schmitt-Grohè and Uribe (2001).

The amount of labor effort L supplied by each agent is calibrated by considering the ratio of market to non-market activities obtained from data. As in the current literature, this ratio is $\frac{L}{1-L} = 0.289017$, implying a value for $L = 0.2243$. With these number from equation (**??**) we derive the value of parameter γ matching the ratio $\frac{L}{1-L}$. This value is 0.6, as reported in Table 1.

Parameter A is calibrated by matching the steady state value of output implied by the model with the value of Y obtained from US data[5], given by 20.12.

The elasticity of intertemporal substitution σ has been set equal to 0.1[6].

[4] Christiano, Eichenbaum and Evans (2001) show a velocity value equal to 0.44. Their definition of velocity is computed with respect to output, not consumption. Taking into account the quantitative relationship existing between consumption and output here described, it is possible to obtain a value for money velocity very close to the number showed by Christiano et al. (2001).

[5] The value for GDP from US data equal to 20.12 is the mean over the sample 1959:1-2001:4 for the real GDP (seasonally adjusted) transformed according to the following methodology: $i_t = 1 + (FFR_t/400)$; $M_t = 1000(M2_t/N)$; $P_t = GNPDE_t$, $Y_t = 1000(GNP_t/N)$, where FFR_t is the Federal Funds Rate, $M2_t$ is M2, N is the population defined as the total civilian noninstitutional, $GNPDE_t$ is the Implicit Price deflator of Gross National Product, and GNP_t, is the Gross National Product.

[6] The value proposed by Rotemberg and Woodford (1999) is $\sigma = 0.17$ and that proposed by Kim (2000) for an estimated model $\sigma = 0.08$.

The elasticity of substitution across differentiated goods has been set equal to $\theta_y = 10$, implying a steady state level of the markup equal to 1.1, as it is common in the literature on monopolistic competition.

The parameters of the monetary policy reaction function are considered as free: the sensitivity analysis will shed light on the role of the policy parameters in welfare evaluation and second order model's fit. The parameter values for the monetary aggregate rule have been obtained from Kim (2000).

The parameters of the AR(1) process driving the stochastic side of the model are reported in Table 3.

Parameter	ρ_A	ρ_ξ	ρ_ζ	ρ_G	ρ_m
Value	0.98	0.99	0.74	0.95	0.487

Table 3: AR coefficients

The variance-covariance matrix is:

$$\Sigma_\varepsilon = \begin{bmatrix} \sigma_A^2 & 0 & 0 & 0 \\ 0 & \sigma_\xi^2 & 0 & 0 \\ 0 & 0 & \sigma_i^2 & 0 \\ 0 & 0 & 0 & \sigma_G^2 \end{bmatrix} = \begin{bmatrix} .0003 & 0 & 0 & 0 \\ 0 & .00196 & 0 & 0 \\ 0 & 0 & 0.001109 & 0 \\ 0 & 0 & 0 & .000126 \end{bmatrix} \tag{39}$$

The value for ρ_A, has been obtained from Kim (2000), as well as the value for the variances of the stochastic process for technology and fixed cost shock, reported in the variance-covariance matrix (39). The values for ρ_ξ has been estimated, after having considered the definition for ξ derived from the calibration exercise, as well as the value for the variance[7] σ_ξ^2.

To get the volatility parameters of the monetary policy reaction function, I estimated the following equation:

$$\widetilde{R}_t = \phi_R \widetilde{R}_{t-1} + \sum_{l=0}^{4} \phi_{\pi l} \widetilde{\pi}_{t-l} + \sum_{l=0}^{2} \phi_{yl} \widetilde{Y}_{t-l} + \eta_t^{mp} \tag{40}$$

over the sample period 1979:3-2001:4, quarterly observations.

I estimated equation (40) by using the Generalized Method of Moments proposed by Hansen (1982), over the sample 1979:3-2001:4, quarterly observations. From this estimate I have obtained the volatility of the residuals η_t^{mp} and I have tested for the degree of autocorrelation of η_t^{mp}.

A selective report of the estimated coefficients is collected in Table 4 (t-statistics in parenthesis):

[7] The regression estimated is: $\log(\xi_t) = \rho_\xi \log(\xi_{t-1}) + \varepsilon_t^\xi$. From IV estimation (instruments: four lagged values of ξ_t) we get (standard errors in brackets): $\rho_\xi = .99$ (.00160), D.W.: 1.059. The variance of residuals is $\sigma_\xi^2 = 0.00196$.

$\phi_{\pi 0}$	$\phi_{\pi 1}$	$\phi_{\pi 2}$	ϕ_{y0}	ϕ_{y1}	ϕ_R
1.25	1.2	0.638	0.378	1.04	0.65
(2.86)	(2.84)	(2.93)	(2.13)	(2.27)	(10.47)

Table 4: Estimated Coefficients

The $\overline{R}^2$ of the regression is 0.836; the overidentifying restrictions are passed with a 1% of significance. The results reported in Table 4 are perfectly in line with what has been reported by Clarida, Galì and Gertler (2000). I tested for the level of autocorrelation of order 1 for η_t^{mp}. In this case, I adopted an LM test which produces a value for the test statistics equal to 29.481671, which, after comparison with $\chi^2(1)$, forces to strongly reject the null of absence of autocorrelation of order 1.

I also estimated another regression in order to pin down both the AR(1) coefficient ρ_{mp} of equation (31) and the variance σ_i^2. From this simple regression we get $\rho_{mp} = 0.74$ (t-statistics: 8.8), and a value of the volatility of the overall regression given by $\sigma_i^2 = 0.001109$.

The values of the parameters of the money supply rule (34)-(36) ϕ_i, ρ_m, ν, ρ_M reported in Tables 2 and 3 have been obtained by Kim (2000). The same criteria applies to the volatility of the growth rate of money supply from (36) for which I set $\sigma_m^2 = .00002$.

6. Solution Method

The model is solved via an Accurate Second Order Solution, as outlined by Sims (2001a) and Kim et al. (2003) adapted to a large scale dynamic general equilibrium model with a multiplicity of state variables. The reason for that is given by the need to have a framework useful to make accurate welfare comparisons across policies that do not have first order effects on the deterministic steady state. The recent literature on monetary policy finds its main contribution in the cross comparisons among different types of monetary policy functions. The metric for this comparison is offered by the utility reached by representative agent.

In the current literature there are examples in a very specific context where it is possible to explicitly obtain an exact expression for the utility -based welfare criterion. These contributions are: Obstfeld and Rogoff (2002), Erceg, Henderson and Levin (2000). An explicit second-order approximation has been derived by Rotemberg and Woodford (1997,1999). The general strategy pursued by these authors is to obtain a solution of the full model up to first order and to insert this solution into an expanded version of the utility function expanded up to second order. This strategy delivers accurate welfare results only if the model possesses a steady state exactly coincident with the first best. Since the economy under study contains several elements of rigidity which do not disappear in steady state, we need a more accurate welfare measure, like that obtained from a second order approximation of the whole model. Following the details contained in a companion paper (see Marzo 2004b) the solution can be expressed as follows:

$$\begin{aligned} dy_t^{(2)} &= \mathrm{F}^{st}\{1\}\{1\}\, dy_{t-1}^{(2)} + \mathrm{F}^{st}\{1\}\{2\}\,\sigma\varepsilon_t + \mathrm{F}^{st}\{2\}\{1\}\left(dw_{t-1}^{(1)} \otimes dw_{t-1}^{(1)}\right) \\ &\quad + \mathrm{F}^{st}\{2\}\{2\}\,\sigma\left(\varepsilon_t \otimes dw_{t-1}^{(1)}\right) + \mathrm{F}^{st}\{2\}\{3\}\,\sigma^2\left(\varepsilon_t \otimes \varepsilon_t\right) + \mathrm{F}^{us}_{con}\sigma^2 \end{aligned} \tag{41}$$

where $dy_t^{(2)}$ is the second order solution of the model, here intended as the deviation from the steady state, while $dw_{t-1}^{(1)}$ is the vector of disturbances having an AR(1) representation (see Appendix .. for further details). Moreover, $\mathrm{F}^{st}\{1\}\{1\}$, $\mathrm{F}^{st}\{1\}\{2\}$, $\mathrm{F}^{st}\{2\}\{1\}$, $\mathrm{F}^{st}\{2\}\{2\}$, $\mathrm{F}^{st}\{2\}\{3\}$ are matrices defined in Marzo (2004b); σ is the standard deviation of i.i.d. errors ε_t.

6.1. An utility-based welfare measure. Given the solution to the model, with representation in (41), in general, a second order expansion of the utility $u(y)$ around the non-stochastic steady state vector of variables y is:

$$\begin{aligned} U_0(y_0) &= E_0\left[\sum_{t=0}^{\infty}\beta^t u(y_t)\right] \approx \\ &\approx E_0\left[\sum_{t=0}^{\infty}\nabla u(y)\, dy_t^{(2)} + \frac{1}{2}vec\left(\nabla^2 u(y)\right)'\left(dw_t^{(1)} \otimes dw_t^{(1)}\right)\right] \end{aligned} \tag{42}$$

The above expression can be rewritten as:

$$U_2(y_0) = \begin{bmatrix} \nabla u(y) & \frac{1}{2}vec\left(\nabla^2 u(y)\right)' \end{bmatrix} \left\{ \begin{array}{c} \left[I - \beta_x G_1\right]^{-1}\begin{pmatrix} dy_0 \\ dy_0 \otimes dy_0 \end{pmatrix} + \\ \beta_x\left(1-\beta_x\right)^{-1}\left[I - \beta_x G_1\right]^{-1} G_2\sigma^2 \end{array} \right\} \tag{43}$$

Equation (43) is the accurate welfare expression based on the full second order solution of the model, given by equation (41). It is a conditional welfare measure, because it is conditional to the initial steady state perturbation dy_0 . The unconditional welfare measure is given by:

$$U^{NC} = \left(1-\beta_x\right)^{-1}\begin{bmatrix} \nabla u(\overline{y}) & \frac{1}{2}vec\left(\nabla^2 u(\overline{y})\right)' \end{bmatrix}\left[I - G_1\right]^{-1} G_2\sigma^2 \tag{44}$$

From (43) the first order welfare measure can be obtained by setting equal to zero all second order moments.

The large majority of contributions in the litereature employ conditional welfare to evaluate policy alternatives. From (41), it is clear that unconditional welfare evaluates the level of welfare at the steady state. However, since moving from one steady state to another, becasue of policy effects takes time, the unconditional welfare measures do not take into account the welfare effects during the transition periods.

6.2. Second Order Impulse-Responses. Given the full second order solution (41), it is possible to recover the expression for the accurate forecasting matrices, which will serve as a basis for the impulse response analysis. After using matrix Z, the second order accurate evolution of

dy_t is:

$$\begin{bmatrix} dy_t^{(2)} \\ \left(dy_t^{(1)} \otimes dy_t^{(1)}\right) \end{bmatrix} = G_1 \begin{bmatrix} dy_{t-1}^{(2)} \\ \left(dy_{t-1}^{(1)} \otimes dy_{t-1}^{(1)}\right) \end{bmatrix} + G_2\sigma^2 + \tag{45}$$
$$+G_3 \begin{bmatrix} \sigma I_m & 0 & 0 \\ 0 & \sigma I_{m(n-q)} & 0 \\ 0 & 0 & \sigma^2 I_{m^2} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \widetilde{\nu}_t \end{bmatrix}$$

where $\widetilde{\nu}_t = \left((\varepsilon_t \otimes dy_{t-1})', (\varepsilon_t \otimes \varepsilon_t - \overline{\varepsilon})'\right)'$. The size of matrices in (45) is defined as $G_1\left(n + n^2 \times n + n^2\right)$, $G_2\left(n + n^2 \times 1\right)$, $G_3\left(n + n^2 \times m + mn + m^2\right)$. Finally, the T-step ahead forecast is given by:

$$E_0\left[dy_T^{(2)}\right] = \begin{bmatrix} Z & 0 \end{bmatrix} E_0 \begin{bmatrix} dw_T^{(2)} \\ \left(dw_{1T}^{(1)} \otimes dw_{1T}^{(1)}\right) \end{bmatrix} \tag{46}$$

with $dw_0 = Z'dy_0$. Equation (46) represents our expression for impulse response analysis. An interesting feature of this method lies in the fact that the T-period ahead expectations do not diverge as the horizon increases. This is very important, and it is different from what has been proposed in the current literature (see, for example Schmitt-Grohé and Uribe, 2001b).

7. RESULTS

The analysis proceeds under two steps: the discussion about the optimal monetary policy function and about the pattern of impulse-response functions. The goal of this section is to show what is the combination of parameters for the monetary policy function *à la* Taylor which maximize the conditional welfare measure above described. For completeness, I am going to consider the results for three welfare measures: a second order-based conditional welfare measure (given by U_2), based on (41), the unconditional welfare measure (U_{UN}) based on (44), and a welfare measure based on the first order solution method U_1.

In order to have a simple and intuitive measure usable to rank policy alternatives, the results derived from welfare analysis are reported in terms of units of forgone consumption, or certainty equivalent consumption. Therefore, C_2 C_1 C_{UN} indicate, respectively, the units of certainty equivalent consumption obtained from second order Conditional welfare solution (C_2), the units of forgone consumption from first order solution (C_1), and the certainty equivalent consumption measure from the Unconditional welfare measure (C_{UN}). Basically, the level of consumption associated to each welfare measure is derived from the utility function as follows:

$$C_i^A = \left[\frac{\left(1 - \frac{1}{\sigma}\right) U_i^A}{(1-L)^{\gamma\left(1-\frac{1}{\sigma}\right)}}\right]^{\frac{1}{(1-\gamma)\left(1-\frac{1}{\sigma}\right)}} \tag{47}$$

with $i = 1, 2, UN$. From (47) we recover the level of consumption associated to a given configuration policy, derived from the welfare measures based on first order approximation (U_1^A), conditional second order (U_2^A) and unconditional (U_{UN}^A).

[In other words, the unit of certainty equivalent consumption makes the representative agent indifferent between a situation where she can consume that specific amount of C (C_1, C_2 or C_{UN}) without risk and living in a risky environment, characterized by a specific monetary policy configuration]. All the numbers reported in tables 5-8 can be compared with the level of consumption associated to the non-stochastic steady state, which in this model is given by $C = 11.47124$.

		C_2	C_{UN}	C_1	σ_π	σ_y	σ_i
Taylor 1987-1997	LS	6.123	6.21	6.26	.0382	.2352	.26
$\phi_\pi = 1.53$; $\phi_y = 0.77; \phi_i = 0.0$	DT	6.52	6.45	6.49	.0168	.1126	.041
Clarida et al. 1979-1996	LS	5.9	6.07	6.32	.0216	.195	.165
$\phi_\pi = 2.15$; $\phi_y = 0.93; \phi_i = 0.79$	DT	6.5	6.74	6.65	.026	.109	.256
Optimal							
$\phi_\pi = 1.22$; $\phi_y = 0.1; \phi_i = 0.3$	LS	6.6	6.25	6.21	.039	.231	.18
$\phi_\pi = 1.15$; $\phi_y = 0.92; \phi_i = 0.21$	DT	6.71	6.51	6.53	.012	.12	.151
Money Targeting Rule	LS	6.03	6.6	6.38	.034	.214	.067
	DT	5.12	5.42	5.33	.14	.25	.12
US Economy					.0106	.1369	.0321

Table 5: results for the Contemporaneous Targeting Rule, Lump Sum Taxes (LS) and Distortionary Taxation Taxes (DT)

In Table 5 I reported the welfare values reached under several parameter configurations. The Optimal parameter configuration has been obtained after a grid search over the interval between (0 and 4) for all the parameter values ϕ_π, ϕ_y and ϕ_i. This range of variations for parameters has been found to be the most empirical plausible range for the parameter of monetary policy reaction function, as reported by the empirical literature. In Table 5 I have also reported the welfare results obtained under some benchmark parameter configurations, obtained from the estimated results by Taylor (1999) and by Clarida, Galì and Gertler (2000). Associated to each rule, I also report the standard deviation of inflation, σ_π, output, σ_y, and interest rate, σ_i, as well as the values for those statistics computed for the US economy.

Each rule result in Table 5 has been computed for a simple Taylor rule like (32) in an economy with Lump Sum taxation (LS) and with Distortionary Taxation (DT) on the basis of labor taxes, as described in (26), (28), respectively.

If we compare the value for the parameters characterizing an optimal monetary policy reported in Table 5 with those proposed by empirical estimates, we find that in general the inflation targeting coefficient is higher in the optimal case than in the empirical estimation cases. Moreover, the value of the income targeting is bigger than what has been showed to be empirically plausible. In the same guise, the coefficient on interest rate smoothing is lower than what has been found in the empirical literature. This suggests that monetary authority in the past year has been successful in fighting against the inflation rate, but their policy actions were too focused on inflation.

The model with distortionary taxation delivers a combination of policy parameters where the inflation targeting coefficient is bigger than in the lump sum taxation case ($\phi_\pi = 1.15$, vs. $\phi_\pi = 1.22$) and the output targeting coefficient is $\phi_y = 0.92$, bigger than $\phi_y = 0.8$, the number delivered by the optimal rule in the lump sum tax case. Thus, more distorted is the economy lower is to be the welfare maximizing inflation targeting coefficient, and bigger is the output targeting coefficient. Intuitively, a larger level of distortion requires a stronger policy reaction towards income, relaxing the tension with respect to inflation targeting. Since in the model is nested an AS curve of the type described by Roberts (1995) and Rotemberg and Woodford (1997, 1998), a too tight inflation targeting policy in an economy with an high level of distortions can determine a recession, or an excess volatility of output. A less severe inflation targeting regime determines a more flexible management of fluctuations.

A further qualifications of the above results concerns role of fiscal policy. In what follows I assumed a 'passive' fiscal policy, given by the range (27) for ψ_1. Since all the rule highlighted in Table 5 report inflation targeting coefficients bigger than one (i.e. active monetary policy'), this condition coupled with (27), ensures full determinacy of the Rational Expectations Equilibrium (REE, henceforth).

In the model with lump sum taxes, we note that according to the unconditional welfare measure, the money targeting rule would have been considered as the optimal rule (with a level of consumption C_{UN} equal to 6.6, against the value 6.25 associated to the rule optimal under measure C_2). Moreover, a similar argument applies also for the rule based on first order solution here represented by the values assumed by C_1. Similar considerations can be formulated for the model with distortionary taxation for which the rule with the parameterization provided by CGG provides an higher level of consumption than the rule obtained with C_2. Such This is an example of the contradictory results provided by different welfare measures, classified as 'spurious welfare reversal' as in Kim and Kim (2002).

Let us consider now the results for the expected inflation targeting rule, given by (33). In Table 6 I reported the optimal configuration of parameters obtained after a grid search for $\phi_\pi, \phi_y, \phi_i \in [0, 4]$, together with welfare values associated to the same parametrization considered in Table 5.

		C_2	C_{UN}	C_1	σ_π	σ_y	σ_i
Taylor 1987-1997	LS	6.12	6.21	6.25	.018	.1179	.0229
$\phi_\pi = 1.53$; $\phi_y = 0.77; \phi_i = 0.0$	DT	6.45	6.38	6.42	.0166	.1124	.0412
Clarida et al. 1979-1996	LS	6.03	6.22	6.26	.038	.1489	.0732
$\phi_\pi = 2.15$; $\phi_y = 0.93; \phi_i = 0.79$	DT	6.37	6.24	6.27	.026	.109	.2547
Optimal							
$\phi_\pi = 1.6$; $\phi_y = 0.83; \phi_i = 0.17$	LS	6.8	6.24	6.31	.015	.091	.095
$\phi_\pi = 1.32$; $\phi_y = 0.89; \phi_i = 0.24$	DT	7.15	6.44	6.47	.0125	.156	.115

Table 6: results for the Expected Inflation Targeting Rule, Lump Sum Taxes (LS) and Distortionary Taxation Taxes (DT)

From the numbers reported in Table 6, we find that with a rule with expected inflation, the pattern of the results are not too dissimilar from what we have already seen with a contemporaneous targeting rule. The optimal parameters combination for rule (33) identifies an inflation targeting coefficient lower than the estimated empirical values (both Taylor and CGG estimates) and the output targeting coefficient bigger than the estimated values. At the same time, the interest-rate smoothing parameter turns out to be lower than the estimated values. Even in this case, we can report the same sort of considerations raised before: a too high inflation targeting coefficient can create roo for recession because of the excess deflationary effect. For what concerns the interest rate smoothing parameter, the optimal values (reported for both the Lump Sum case and the Distortionary Tax case), imply a lower value than what has been found in the empirical literature. Overall, even with the rule with expected inflation (both in Lump Sum case and the Distortionary Tax case) we can say that the optimal value for the inflation targeting coefficient is lower than empirical estimates. Such type of results are close to what has been found by Schmitt-Grohé and Uribe (2004) under a Ramsey command optimum, where inflation targeting coefficient is lower than one, and the output targeting coefficient is negative. Although such results strictly depend from the specific fiscal policy rule considered (i.e. whether fiscal policy is passive or active), they emphasize the role of an inflation targeting coefficient lower than the estimated value.

For what concerns fiscal policy, the results reported in Table 6 have been obtained under the assumption of passive fiscal policy, as it was for Table 5. Since the values for ϕ_π in Table 6 are all bigger than one, the REE is once againa determined.

7.1. Sensitivity Analysis.

Nominal Rigidities. InTable 7I report the welfare maximizing parameter combination for the monetary policy rule, when the degree of price stickiness has been raised to $\phi_p = 20$ (the benchmark value was $\phi_p = 3$). The welfare optimization procedure is based on a grid search over the parameter space within the interval: $\phi_\pi, \phi_y, \phi_i \in [0, 4]$. The welfare metric is still given by the second order conditional welfare, as in (41). The optimal parameter values are reported in Table 7.

	Lump Sum				Dist. Taxation			
	C_2	ϕ_π	ϕ_y	ϕ_i	C_2	ϕ_π	ϕ_y	ϕ_i
Current Inflation	5.95	1.85	.25	.15	5.73	2.05	.67	.12
Expected Inflation	5.42	1.73	.91	.05	5.56	1.9	.74	.08

Table 7: Optimal Monetary Policy when $\phi_p = 20$.

In Table 7 I reported the welfare maximizing parameters for the two monetary policy rules (with current and expected inflation targeting) studied in this paper together with the value of C_2 derived from the conditional welfare measure, for both lump sum taxes and distortionary taxation case. From Table 7 we observe that inflation targeting and output targeting coefficients are higher than their values reported in Tables 5-6 for the benchmark case. Therefore, higher is the degree of price stickiness, bigger are the inflation and output targeting parameters.

The intuition behind this result can be explained as follows: in an economy with price flexibility and perfect competition, the Friedman rule is optimal, i.e. the interest rate is zero and constant. On the other hand, the economy under study is far from being located on an optimal path: in this case the Friedman rule is no longer optimal. If taxes on profits were available, full Friedman allocation would be restored after a 100 per cent tax rate on profits. In absence of that, The Ramsey planner uses inflation as indirect tax on profits in order to restore the optimal allocation. However, with costly price adjustment the benevolent government wants to keep the price level constant, but he faces a trade-off: from one side, the planner would like to use unexpected price changes as lump sum tax, or wealth transfers in order to restore the first best allocation. From the other side, the planner wants to stabilize the price level to minimize the cost associated to price changes. With the former alternative, the planner avoids to use distortionary taxation and interest rate changes as a mean of restoring optimal allocation.

All such considerations explain the reason why optimal monetary policy under a strong degree of price stickiness is characterized by a more aggressive behavior with respect to inflation rate. In fact, the Planner tries to restore optimal allocation by keeping the price level constant and by minimizing the cost of price adjustment. The optimal rule indicated in Table 7 implies that to keep under control inflation we need small changes of nominal interest rates. As discussed previously, such results are in line with the results and the theoretical considerations formulated by Schmitt-Grohé an Uribe (2004) and Correia, Nicolini and Teles (2001).

Fiscal Policy Parameter. In what follows I am going to consider a similar exercise for the sensitivity with respect to fiscal policy parameter ψ_1. I computed welfare effects for the model when the fiscal policy reaction function has been raised to 0.8 starting from the benchmark value given by 0.05, as reported in Table 4. Given β, it is not difficult to check that this value still respect the range established by the inequality (27), which defines a Passive fiscal policy.

The optimized values for the monetary policy parameters are given in Table 8.

	Lump Sum				Dist. Taxation			
	C_2	ϕ_π	ϕ_y	ϕ_i	C_2	ϕ_π	ϕ_y	ϕ_i
Current Inflation	6.18	1.05	.22	.12	5.32	1.02	.56	.15
Expected Inflation	6.20	1.28	.89	.21	5.43	1.14	.68	.18

Table 8: Optimal Monetary Policy when $\psi_1 = 0.8$. Results based on Second Order Conditional Welfare Measure.

I reported in Table 8 only the welfare computed on the basis of the Conditional Second Order measure. From the numbers reported in Table 8, we observe that the welfare maximizing parameter value for ϕ_π and ϕ_y are lower than in the case examined in Tables 5 and 6. The reason can be explained with the logic of the interactions between Fiscal and Monetary policies. According to Fiscal Theory of the Price Level (FTPL), a tighter fiscal policy (an increase in ψ_1), absorbs the large part of the burden for price level determination: therefore, monetary policy can relax the tension in fighting inflation. In fact, with a tighter fiscal policy, it turns out to be optimal a lower level of inflation targeting parameter, since now taxes adjust more intensively in order to respect the solvency criteria of the Government Budget Constraint. It is worth to remind that monetary policy parameters are still bigger than one and the REE is still determinate.

7.2. Impulse response functions. In figures 1-3 I reported the graph of impulse-response functions for some, selected variables of the model. The impulse response functions are highly non-linear functions derived from the solution method, as reported in equation (46). In figure 1 I reported the response of the model with current inflation and the parameter combinations which turned out to be optimal after the grid search (see Table 5) where $\phi_\pi = 1.22$, $\phi_y = 0.1$, $\phi_i = 0.3$. In Figure 1 two lines are represented for each graph: the dark line is obtained after simulating the impulse responses for one standard deviation of the technological shock, while the dotted line is obtained for a shock which is 1/2 smaller.

From Figure 1 we observe that after a positive technological shock , both output and labor effort go up. Consumption raises, but only gradually: the graph shows a good internal persistence. After the shock, nominal rate raises gradually, because of the smoothing parameter involved in the monetary reaction function. Since inflation targeting coefficient is bigger than one and $\phi_i > 0$, the inflation rate decreases[8]. At the same time, the joint increase of nominal rate and the reduction of inflation makes public debt to steadily increase and to return back to steady state in the long run. It is worth to note that the impulse responses showed lack of the liquidity effect: the present model does not contain a sufficiently high degree of nominal rigidity (combined with real rigidity) to generate the liquidity effect. As proved by Kim (2000) and Marzo (2004a) the liquidity effect can be obtained only by introducing a substantial degree of inertia.

[8] The long run impact coefficient on the inflation rate equal to 1.74.

In Figure 2 I report the evolution of impulse-responses for the model including expected inflation and lump sum taxes. The dark line indicates the response pattern for each variable with the configuration of parameters associated to Clarida, Galì and Gertler (CGG) parametrization (i.e., $\phi_\pi = 2.15$, $\phi_y = 0.93$, $\phi_i = 0.79$), while the dashed line is obtained under the optimal configuration setting, with $\phi_\pi = 1.6$, $\phi_y = 0.83$, $\phi_i = 0.17$. From Figure 2 we observe that Impulse-Responses for the optimal monetary policy allows for a better response with respect to an expansionary technological shock: the response of the model are much smaller, allowing lower deviations from steady state than in the CGG case.

Finally, in Figure 3, I reported the same kind of exercise for the current inflation targeting rule with distortionary taxation. As before, the dark line indicates the response pattern for the CGG configuration, while the dashed line indicates the response pattern for the optimal monetary function, here characterized by $\phi_\pi = 1.15$, $\phi_y = 0.92$, $\phi_i = 0.21$. As in previous pictures, we observe that the implementation of the optimal monetary policy rule allows a smaller response patterns of variables, closer to their steady state values.

To sum up, these pictures show that Impulse Responses based on Second Order solution of the model provide the same qualitative information than first order response. However, with second order Impulse-Response functions the magnitude of the shock hitting the economy is crucial to determine the long run forecasts of each variable.

A final remark is about the very long run pattern: the long run impulse responses do not explode, but they tend to come back to the original steady state position, sign of intrinsic stability of the model.

8. Concluding Remarks

In this paper it has been shown how the introduction of a new solution method for dynamic stochastic general equilibrium model up to second order changes the analysis of welfare associated to different monetary policy configurations. The model is kept simple in order to make the results as much comparable as possible with the existing literature. The model is presented in two versions: one with lump taxation and the other with distortionary taxation on labor income. The results confirm that the optimal parameter configurations for a monetary policy reaction function lies within the range specified by the most celebrated empirical studies. Sensitivity analysis shows that by raising the degree of nominal rigidities, the optimal inflation targeting coefficient is slightly bigger than what has been suggested by empirical work.

When fiscal policy becomes tighter, the magnitude of the inflation targeting parameter becomes smaller, as prescribed by the Fiscal Theory of the Price Level. The analysis reveals that first order-based welfare measures are often misleading in indicating the welfare maximizing parameters, if compared to conditional second order ones.

Impulse response function are showed to be non-explosive in the long run and in general show a better degree of persistence of the model. The results here showed need to be generalized for models with a better design of nominal and real rigidities, in order to properly detect the inertial behavior of real and nominal variables.

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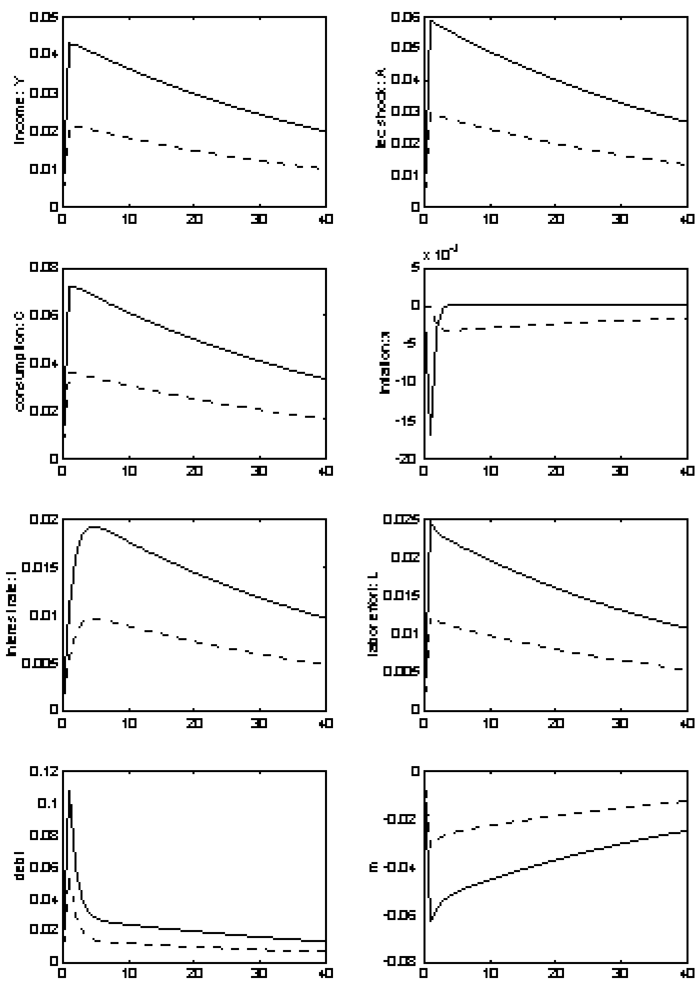

Figure 1: Second Order Impulse-Responses. Current inflation lump sum taxes

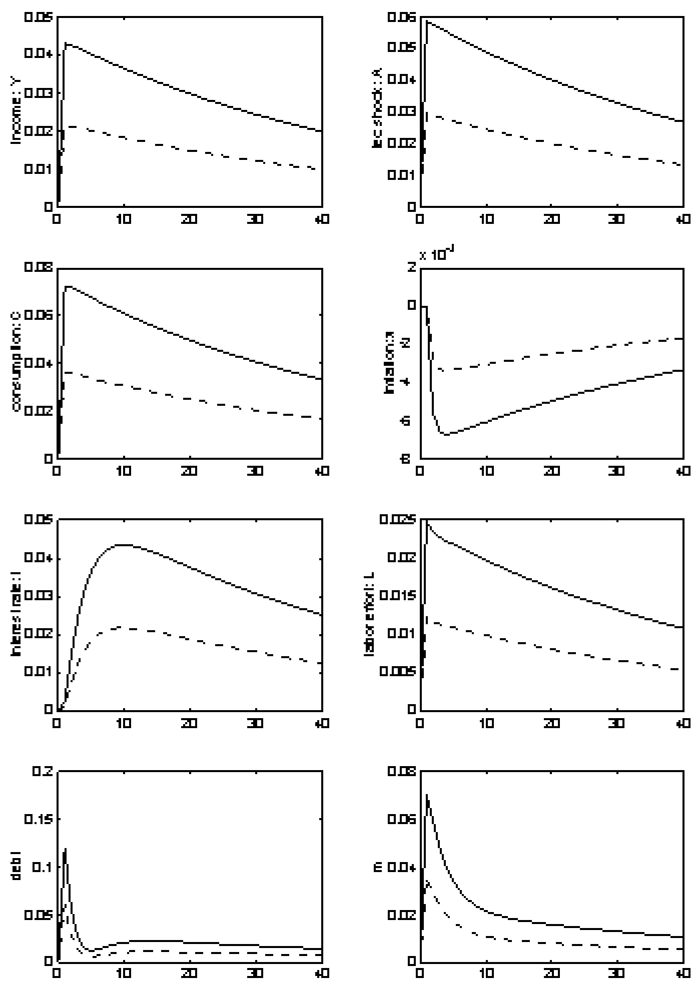

Figure 2: Second Order IRF, ecpected infl. lump sum taxes.

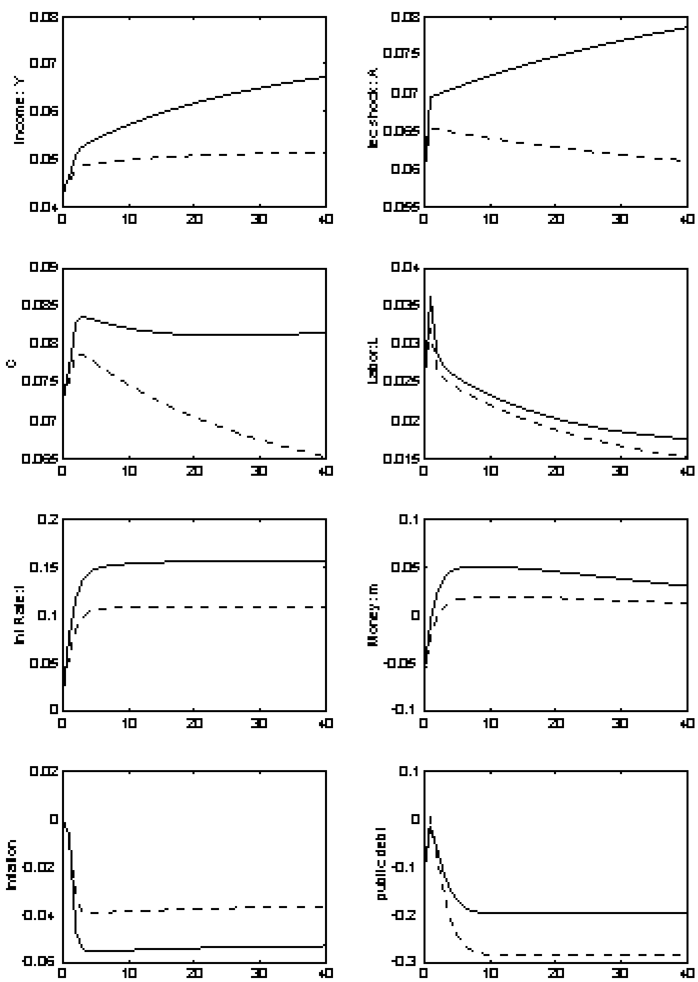

Figure 3: Second Order Impulse-Responses: expected inflation targeting