

# Modelling of thermal dispersive effects in electron devices based on an equivalent voltage approach

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*A model for thermal dispersive effects due to self-heating in electron devices at different “case” temperatures is here presented. The model is based on an equivalent voltage approach, which has already been used for taking into account dispersive effects due to charge trapping phenomena in FETs. According to this approach a virtual non-dispersive associated device controlled by equivalent port voltages is defined, in such a way to be compatible with modelling based on standard non-linear dynamic approaches. The equivalent-voltage description of dispersive effects can be identified on the basis of conventional measurements carried out under static and low-frequency small-signal operating conditions and takes into account both charge trapping effects in FETs and self-heating in a comprehensive way. A preliminary experimental validation of the proposed approach is presented.*

## INTRODUCTION

Important efforts have been made in the last years by different research groups to take into account low-frequency dispersion both in mathematical and equivalent circuit models (2-8). As well known, low-frequency dispersive phenomena due to charge trapping and device self-heating are responsible of often important deviations between static and dynamic (e.g., pulsed) measurements of the dc characteristics, or, if we think in terms of differential parameters, frequency dependent behaviour of the trans-admittance and output impedance even at low frequencies (e.g., lower than 1MHz). Since microwave large-signal performance prediction involves accurate modelling of both dc and ac components, the modelling of dispersive phenomena can not be neglected whenever very accurate predictions are required.

A recently proposed approach, namely the Equivalent Voltage Model (EVM), is here considered and a complete formulation including thermal dispersive effects due to self-heating, also taking into account “case” temperature dependency, is provided. The proposed empirical model provides good predictive capabilities of bias-dependent dynamic drain current deviations due to traps and thermal effects in FETs on the basis of standard identification data, i.e., static characteristics and low-frequency small-signal parameters evaluated at different biases. Moreover, the proposed approach can be easily embedded into any non-linear dynamic model for high-frequency predictions, without requiring modifications of the corresponding equations.

In the following, the complete EVM formulation is presented, by considering an FET device not only affected by dispersive phenomena due to charge trapping as in (1), but also by important self-heating effects. Case temperature dependency is also taken into account.

Preliminary experimental validation confirming the validity of the proposed approach is also provided.

## THE EQUIVALENT VOLTAGE MODEL INCLUDING THERMAL EFFECTS

Let us consider first an ideal intrinsic field effect transistor<sup>1</sup>, where no low-frequency dispersive phenomena take place, so that a purely algebraic non-linear relationship can be assumed between charges and voltages. Such a device can be properly described by adopting the following charge-controlled quasi-static vector model formulation:

$$i(t) = \Phi\{q(t)\} + \frac{dq(t)}{dt} \quad (1)$$

$$q(t) = \psi\{v(t)\} \quad (2)$$

where:

$i = [i_S \ i_D]^T$ ,  $q = [q_{GS} \ q_{GD}]^T$ ,  $v = [v_{GS} \ v_{GD}]^T$   
 represent the source and drain currents, the gate-source and gate-drain charges, which are dealt with as state-variables, and the intrinsic port voltages respectively. Moreover,

$$\Phi(.) = [\Phi_1(.) \ \Phi_2(.)]^T, \ \psi(.) = [\psi_1(.) \ \psi_2(.)]^T$$

are suitable purely-algebraic non-linear functions.

Alternatively, by substituting (2) in (1) the equivalent voltage-controlled model formulation is obtained:

$$i(t) = F\{v(t)\} + C\{v(t)\} \frac{dv(t)}{dt} \quad (3)$$

where :

<sup>1</sup> The FET device is here considered as a general case, since it is usually affected by dispersion due to both traps and self-heating. The same kind of approach could be obviously adopted, for example, for a BJT device where no macroscopic charge trapping effects are usually observable

$$F\{v\} = \phi\{\psi\{v\}\} \text{ and } C\{v\} = \frac{d\psi\{v\}}{dv}$$

are purely-algebraic functions.

Let us now consider a “real” intrinsic field effect device. This device is typically affected by low-frequency dispersive phenomena due to self-heating and traps, possibly present in inter-electrode surface regions and in channel-substrate interface deep layers. The following discussion shows how an intrinsic non-dispersive associated device can be defined when dispersive phenomena are separately taken into account by means of “extrinsic” series controlled voltage sources, as shown in Fig.1 (a).

For the sake of simplicity, thermal effects and charge trapping will be dealt with as separate phenomena in the following, by first considering the presence of dispersion due to self-heating only and, then, including charge trapping phenomena. Thus, let us now consider a device only affected by dispersive effects due to self-heating. In this case, both currents and charges have to be considered as temperature-dependent quantities, so that (1) and (2) must be replaced by:

$$i(t) = \Phi_{\theta}\{q(t), \Delta\theta(t)\} + \frac{dq(t)}{dt} \quad (4)$$

$$q(t) = \psi_{\theta}\{v(t), \Delta\theta(t)\} \quad (5)$$

where  $\Delta\theta(t) = \theta(t) - \theta_R$  represents the difference between a suitable instantaneous “equivalent mean channel temperature”  $\theta(t)$ , assumed constant along the channel, and an arbitrary reference case temperature  $\theta_R$ . Notice that the  $\Phi(\cdot)$  and  $\psi(\cdot)$  functions have been changed into  $\Phi_{\theta}(\cdot)$  and  $\psi_{\theta}(\cdot)$  denoting now a *thermally dispersive* device. By temporarily neglecting the  $dq/dt$  term in (4), for the simplicity sake and without loss of generality, it is possible to relate currents and charges of the dispersive device to the corresponding quantities of an associated non-dispersive device, i.e.,:

$$i(t) = \Phi_{\theta}\{q(t), \Delta\theta(t)\} = \Phi\{q(t)\} + \Delta i_{\theta}\{q(t), \Delta\theta(t)\} \quad (6)$$

$$q(t) = \psi_{\theta}\{v(t), \Delta\theta(t)\} = \psi\{v(t)\} + \Delta q_{\theta}\{v(t), \Delta\theta(t)\} \quad (7)$$

where:  $\Phi\{q\} = \Phi\{q\}_{\theta=\theta_R}$ ,  $\psi\{q\} = \psi\{q\}_{\theta=\theta_R}$ .

The  $\Delta i_{\theta}$ ,  $\Delta q_{\theta}$  terms in (6) and (7) are perturbation functions of the non-dispersive device characteristics  $\Phi(\cdot)$  and  $\psi(\cdot)$ , where  $\Delta\theta(t)$  also depends on past time values  $p(t-\tau)$  of the power dissipated in the device. Eqs. (6) and (7) can also be expressed in an alternative form, where the perturbations  $\Delta i_{\theta}$  and  $\Delta q_{\theta}$  are replaced by equivalent voltage perturbations  $\Delta v_{\theta_q}$ ,  $\Delta v_{\theta_i}$ , which satisfy the following equivalence conditions:

$$q(t) = \psi_{\theta}\{v(t), \Delta\theta(t)\} = \psi\{v(t) + \Delta v_{\theta_q}\} \quad (8)$$

$$\begin{aligned} i(t) &= \Phi_{\theta}\{\psi\{v(t) + \Delta v_{\theta_q}\}, \Delta\theta(t)\} = \\ &= F\{v(t) + \Delta v_{\theta_q}(t) + \Delta v_{\theta_i}(t)\} = F\{\tilde{v}(t)\} \end{aligned} \quad (9)$$

where:  $\tilde{v} = v + \Delta v_{\theta_q}(t) + \Delta v_{\theta_i}(t)$ . In fact, due to the monotonic behaviour of the device electrical characteristics, the two voltage perturbation functions  $\Delta v_{\theta_q}$  and  $\Delta v_{\theta_i}$  can be defined as:

$$\Delta v_{\theta_q}\{v, \Delta\theta\} = \psi^{-1}\{\psi_{\theta}\{v, \Delta\theta\}\} - v \quad (10)$$

$$\Delta v_{\theta_i}\{v, \Delta\theta\} = F^{-1}\{\Phi_{\theta}\{\psi\{v + \Delta v_{\theta_q}\}, \Delta\theta\}\} - v - \Delta v_{\theta_q} \quad (11)$$

Using the equivalent voltage perturbations  $\Delta v_{\theta_q}$  and  $\Delta v_{\theta_i}$  defined by (8) - (11), instead of the perturbations  $\Delta i_{\theta}$ ,  $\Delta q_{\theta}$  in (6) - (7), is convenient since the latter would be strongly dependent on the actual charges and voltages respectively. In fact, the  $\Delta i_{\theta}$ ,  $\Delta q_{\theta}$  terms have a relevant amplitude in the on-state device operation, while they become vanishing small in or near to the off-state. Instead, the equivalent voltage perturbations  $\Delta v_{\theta_q}$  and  $\Delta v_{\theta_i}$ , whose dependence on the operating voltages  $v(t)$  is weak and nearly negligible, can describe the same type of behaviour as it will be confirmed by experimental validation. More precisely, by considering a relatively small  $\Delta\theta$ , (10) and (11) can be expressed in the form:

$$\Delta v_{\theta} = \Delta v_{\theta_q} + \Delta v_{\theta_i} \cong \gamma\{v\}\Delta\theta \quad (12)$$

where:

$$\gamma\{v\} = \left. \frac{\partial \Delta v_{\theta_q}}{\partial \Delta\theta} \right|_{\theta=\theta_R} + \left. \frac{\partial \Delta v_{\theta_i}}{\partial \Delta\theta} \right|_{\theta=\theta_R}$$

is only weakly dependent on  $v$ . In particular, as a first approximate approach, a constant value for  $\gamma$  has been assumed in this work. Thus, by considering that RF operation involves frequencies well beyond the thermal cut-off, the thermal equation can be expressed as:

$$\Delta\theta(t) \cong \Delta\theta_0 \cong R_{\theta} P_0 + \Delta\theta_C \quad (13)$$

where  $\Delta\theta_0$ ,  $P_0$  represent respectively the dc components of  $\Delta\theta(t)$  and of the dissipated power  $p(t)$ , while  $\Delta\theta_C = \theta_C - \theta_R$  accounts for possible variations in the case temperature.

According to the results presented in (1), dispersive phenomena due to charge trapping effects also cause modifications of the charge-based state variables and lead to charge perturbations, which can be coherently replaced by voltage deviations  $\Delta v_t$  also dependent on past values  $v(t-\tau)$  of the voltages. By considering a linear approximation and RF operation above cut-off of charge trapping effects, according to the study in (1), we have:

$$\Delta v(t) \cong \Delta v_{t_0} \cong A_0 \cdot V_0 \quad (14)$$

where  $V_0$  is the vector of the dc components of the operating voltages  $v(t)$  and  $A_0$  represents a suitable matrix of coefficient to be determined.

This shows that any intrinsic field effect transistor, affected by dispersive effects due to both self-heating and charge trapping and excited by port voltages  $v(t)$ , can be described in terms of a virtual non-dispersive associated device excited by *equivalent port voltages*:

$$\tilde{v} = v + \Delta v_{\theta_q}(t) + \Delta v_{\theta_i}(t) + \Delta v_i$$

Circuit schematic in Fig.1 (a) is coherent with the model definition outlined above, where the  $\Delta v$  terms correspond to series controlled voltage sources, yet to be identified. Anyway, since all the dynamic drain current characteristics give  $i_D = 0$  for any  $v_{GS}$  when  $v_{DS} = 0$ , model equations (12)-(14) are more conveniently evaluated in a common-source device configuration (1), where all of the controlled voltage sources at the drain port can be neglected, as shown in Fig.1 (b).

Thus, when a suitable identification procedure exists for the four coefficients ( $s$  apex meaning *common-source* configuration):

$$\gamma^s, k = \gamma^s R_\theta, A_0^s = \begin{bmatrix} A_{11_0}^s & A_{12_0}^s \end{bmatrix},$$

the non-linear modelling problem of a dispersive device is transformed into the modelling of the associated non-dispersive device (e.g., any non-linear dynamic approach can be adopted such as, for example, widely available lumped-component equivalent circuits).

The model parameters can be identified by means of a straightforward procedure consisting in the solution of an over-determined linear system of equations deriving from conventional dc and low-frequency S-parameter measurements at different bias conditions and possibly (in order to identify the  $\gamma$  coefficient) at different case temperatures.

In particular, by differentiating the drain current around a generic voltage pair  $\hat{V}_{GS_0}$ ,  $\hat{V}_{DS_0}$  and a case temperature  $\hat{\theta}_C$ , the following linear system of equations is obtained, which involves the static ( $\hat{g}_m^{DC}$ ,  $\hat{g}_d^{DC}$ ) and low frequency ( $\hat{g}_m^{AC}$ ,  $\hat{g}_d^{AC}$ ) conductances and, moreover, the static drain current sensitivity with respect to case temperature deviations  $\hat{s}_{\theta_c}^{DC}$ :

$$\begin{cases} \hat{g}_m^{AC} A_{11_0}^s + \hat{g}_m^{AC} \hat{g}_m^{DC} V_{D_0} k = \hat{g}_m^{DC} - \hat{g}_m^{AC} \\ \hat{g}_m^{AC} A_{12_0}^s + \left( \hat{I}_d^{DC} + \hat{g}_d^{DC} V_{D_0} \right) \hat{g}_m^{AC} k = \hat{g}_d^{DC} - \hat{g}_d^{AC} \\ \hat{s}_{\theta_c}^{DC} \left( 1 - k \hat{g}_m^{AC} V_{D_0} \right) = \hat{g}_m^{AC} \gamma \end{cases}$$

The imposition of the above equations on a set of different bias points and case temperatures leads to an over-determined linear system, which can be solved for the four unknown parameters adopting a closed-form analytical least square algorithm.

## EXPERIMENTAL VALIDATION

The proposed equivalent-voltage approach has been preliminary applied for the prediction of low-frequency dispersive effects of a GaAs MESFET device. In Table I and II predicted and measured dynamic differential parameters are reported for a wide set of quiescent voltages. Moreover, in Figs. 2,3 the comparison is presented between predicted dynamic drain current characteristics at  $\theta_R = 20^\circ C$  and measurements obtained by applying short, simultaneous voltage pulses at the gate/drain electrodes starting from different quiescent conditions (10). Finally, after a complete model identification using different case-temperature measurement data, dynamic drain current characteristics have been predicted at  $\theta_C = 0^\circ C$  and  $\theta_C = 50^\circ C$ . The corresponding results are presented in Fig. 4.

## CONCLUSION

A new approach for the modelling of low-frequency dispersive phenomena in FETs based on the definition of a non-dispersive associated device controlled by equivalent port voltages, has been presented. The model can be identified on the basis of conventional dc and small-signal S-parameter measurements and provides accurate predictions of bias-dependent, low-frequency dynamic current characteristics. The associated non-dispersive device is suitable for modelling based on conventional non-linear dynamic approaches in order to take into account also high-frequency junction charge-storage phenomena.

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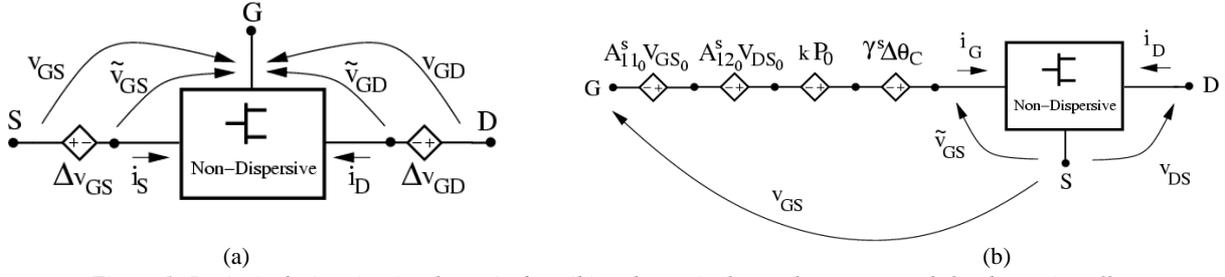


Figure 1: Intrinsic device circuit schematic describing the equivalent-voltage approach for dispersive effects and thermal phenomena modelling.

$g_m$ [mS]	$V_{GS}$	-3V	-2V	-1V	0V
		$V_{DS}$			
1V	meas.(dc)	65.089	78.254	70.209	54.607
	meas.(ac)	68.973	82.281	69.806	49.904
	sim.(ac)	67.385	81.460	72.840	56.288
3V	meas.(dc)	65.089	76.791	83.373	97.512
	meas.(ac)	74.146	84.369	93.823	105.623
	sim.(ac)	71.245	85.372	93.514	111.506
5V	meas.(dc)	64.385	65.821	70.209	78.985
	meas.(ac)	72.624	79.927	86.013	95.345
	sim.(ac)	74.597	76.553	82.503	94.780

$g_d$ [mS]	$V_{GS}$	-3V	-2V	-1V	0V
		$V_{DS}$			
1V	meas.(dc)	12.311	28.888	75.694	153.704
	meas.(ac)	23.425	40.195	90.234	176.923
	sim.(ac)	19.485	40.462	88.981	165.986
3V	meas.(dc)	7.679	3.657	-3.291	-11.336
	meas.(ac)	13.237	12.110	10.701	7.647
	sim.(ac)	16.234	15.438	12.030	8.701
5V	meas.(dc)	4.022	0.731	-4.388	-11.336
	meas.(ac)	9.940	8.520	6.491	3.702
	sim.(ac)	12.658	11.821	8.988	5.019

Table I: Comparison between dynamic low-frequency trans-conductances – left - and output-conductances – right - measured and predicted by the equivalent-voltage-approach at the reference case temperature of 20°C.

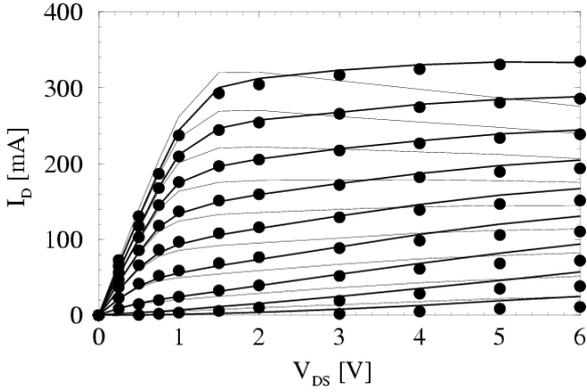


Figure 2: Pulsed drain current characteristics at the reference case temperature  $T_C=20^\circ\text{C}$  (quiescent condition:  $V_{G0}=-1\text{V}$ ;  $V_{D0}=3\text{V}$ ). Measurements ( $\bullet$ ) versus predictions ( $\text{—}$ ) including the proposed self-heating model. The static characteristics at the same temperature (thin  $\text{---}$ ) are also shown.

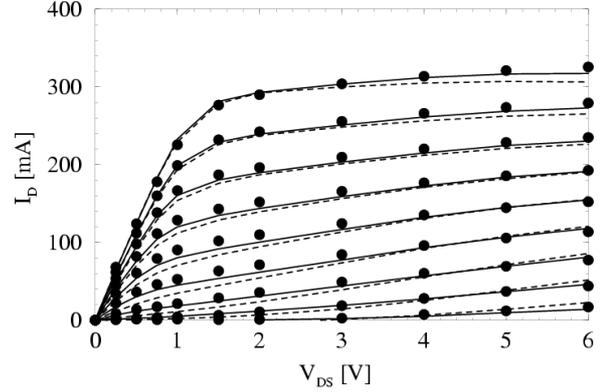


Figure 3: Pulsed drain current characteristics at the reference case temperature  $T_C=20^\circ\text{C}$  (quiescent condition:  $V_{G0}=-2\text{V}$ ;  $V_{D0}=5\text{V}$ ). Measurements ( $\bullet$ ) versus predictions obtained with ( $\text{—}$ ) and without ( $\text{- -}$ ) the proposed self-heating model.

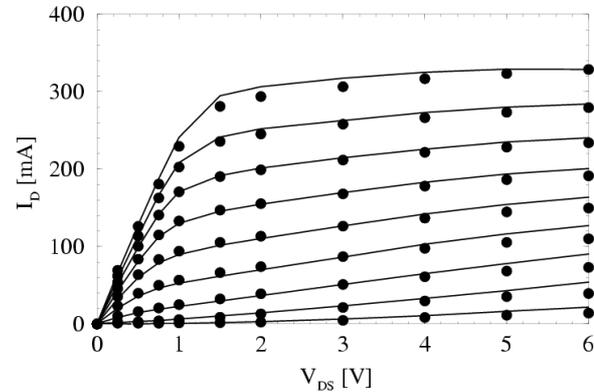
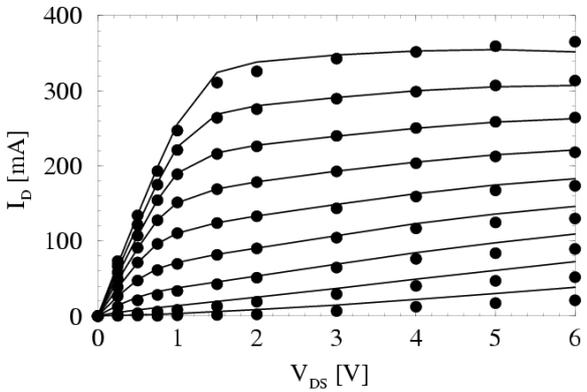


Figure 4: Pulsed drain current characteristics (quiescent condition:  $V_{G0}=-2\text{V}$ ;  $V_{D0}=3\text{V}$ ) at two different case temperatures:  $T_C=0^\circ\text{C}$  - left;  $T_C=50^\circ\text{C}$  - right. Measurements ( $\bullet$ ) versus predictions ( $\text{—}$ ) obtained with the equivalent-voltage approach including the proposed self-heating model.