Vertical integration in a stochastic framework
and a nonsymmetric bargaining equilibrium

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Abstract

We go through the decision to vertically integrate or outsource in an uncertain framework. We consider two different market strategies, price setting and quantity setting and two different vertical relationships: a Stackelberg one and a bargaining one. In the first scenario, with certainty, price and quantity settings are alike, while with uncertainty the ranking changes. If the bargaining framework is adopted instead, quantity setting under uncertainty leads to an asymmetric distribution of realized gains along the vertical chain.

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1 Introduction

Vertical integration (VI) and its contrary, i.e. outsourcing (OS), have become quite a hot issue in both literature (see for instance recent contributions of Antràs and Helpman (2004), Grossman and Helpman (2002)) and policy discussions. OS has become a relevant phenomenon nowadays both within national areas and crossborder. The decision concerning the vertical arrangement to adopt, OS or VI, is a strategic choice that occurs in different market environments. As most choices of this kind, is affected by several externalities.

Two of them are quite common and worth mentioning.

The first concerns the act of going VI or OS by a firm. This action generates a negative or positive externality to other rivals according to whether it leads to a more or less competitive market altering the profitability of going OS or VI for subsequent firms (McLaren, 1999, 2000).

The second is an inner externality regarding the vertical relationship between a downstream (D) firm manufacturing a final good and an upstream (U) firm producing an intermediate good that enters the final product. As already pointed out in the literature (Spengler, 1950; Williamson, 1971; Tirole, 1988; Perry, 1989), OS is subject to the "double marginalization" shortcoming, i.e.: when the price of the final good increases the profit of the U firm decreases. This externality arises in all market structures but perfect competition. As a result VI turns out to be superior from both a private and a social point of view. There are however some circumstances, related to either different objectives of firms (Rossini, 2003), or differentiation (Lambertini and Rossini, 2003; Pepall and Norman, 2001) or market strategic substitutability (Buehler and Schmutzler, 2003) where this externality is neutralized or even
Empirically the question of the private and social superiority of VI vis à vis OS has been analyzed by Slade (1998a, b) who casts doubts on some past stances of antitrust agencies.

Further externalities in vertical relationships are related to R&D. Theoretical analyses have been provided in Rossini and Lambertini (2003), Brocas (2003), Banerjee and Lin (2001), among others, while empirical investigations date back to fundamental contributions of Teece (1976) and Armour and Teece (1981) all the way through more recent investigations, such as Nemoto and Goto (2004). R&D vertical spillovers may add a new source of external benefits to either vertical arrangement according to which one is thought to be more spillover prone.

Going back to the second externality it seems that it is canceled by adopting a particular vertical relationship. The one that gives rise to the externality is a Stackelberg-like link between a U (leader) and a D (follower). As an alternative it may be adopted a cooperative approach by assuming that U and D bargain among them. This approach opens the way to the use of a Nash Bargaining Solution (Nash, 1950), in one of its many versions and refinements.

The use of a cooperative approach rather than a Stackelberg solution concept is quite a matter of judgement. Stackelberg introduces a vertical asymmetry and is affected by an externality that is swept away if there is VI due to internalization.

Assuming that firms bargain along the vertical chain leads to a close replication of the VI result. In other words firms symmetrically share the surplus, as the NBS dictates, mimicking vertical collusion. Hence, the result

\footnote{See Rubinstein (1982) or, for a good survey, Petrosjan and Zenkevich (1996).}
is quite close to that of a vertical cartel, even though here the distribution of the joint surplus is not necessarily a symmetric one as with NBS.

Most of these conclusions are taken for granted in an environment in which there is market certainty. Here we extend the analysis to an uncertain environment. We shall see that a great deal of difference depends upon which strategic variable is adopted by the D firm in both the Stackelberg case and in the NBS case. In the first case we find a private superiority of quantity setting for the D firm, which extends to the U firm if nonlinear costs are common to both stages of the vertical production process.

With bargaining we find that in some circumstances the NBS equilibrium is no longer symmetric in realized gains and with convex costs price setting may be either superior or inferior to quantity setting according to the stochastic scenario and technology parameters.

The paper is organized as follows. In section 2 we go through the cases of a Stackelberg vertical relationships. In section 3 we provide comparisons between price and quantity settings. In section 4 we use the NBS. In Section 5 we draw some concluding remarks.

2 Outsourcing with a Stackelberg solution

We now consider a vertical production process of a good that is sold in a final market by a monopoly which needs an intermediate input to produce it.

The vertical technological relationship is one of perfect vertical complementarity: i.e. only one unit of the intermediate good enters the production of one unit of the final good.

In terms of the organizational character of the vertical relationship we
can have either Vertical Integration (VI), whereby only one firm own both the D and the U sections of production, or outsourcing (OS) with two distinct firms in the D and U sections respectively.

The vertical \textit{market} relationship can be modeled in at least two different modes: a non cooperative Stackelberg solution or a bargaining. We first consider Stackelberg while in the next section we go through the bargaining solution$^2$.

When we consider a vertical relationship adopting a noncooperative stance it is immaterial whether the monopolist firm in the DW section is a price or a quantity setter. However, the results change when we introduce uncertainty. We know from literature (Leland, 1972; Klemperer and Meyer, 1986; Lambertini, 2004; Rossini, 1993; Malliaris and Brock, 1982) that market uncertainty drives a wedge between the profits a monopolist gets according to whether the decision variable is price or quantity. Here we wish to see whether the choice of different controls has an effect also in the case of a vertical relationship.

We then consider the general framework.

With \textit{price setting}, the demand function for the final good is uncertain and linear in price ($p$), the size of the market ($a$) and a shock term ($e$):

\[ q = (a - p + e) / b \]

where $b$ stands for the slope of the demand function and $q$ is the quantity sold. We assume that the additive shock term has zero expected value and constant second moment, i.e.:

\[ Ee = 0; \quad Ee^2 = \sigma^2, \]

\[ ^2\text{To define a bargaining as a market solution is a bit imprecise since competition is quite far from bargaining which is, on the contrary, very close to collusion.} \]
where \( E \) is the expectation operator.

In case of \textit{quantity setting} the uncertain market demand is:

\[
p = a - bq + e. \tag{3}
\]

As far as the technology is concerned we adopt the same approach of Klemperer and Meyer (1986) for the D stage of production, while we keep linear technology for the U stage. Then total cost \((C)\) in the D stage is a quadratic function of quantity with \(c\) and \(d\) technological parameters:

\[
C = cq - dq^2. \tag{4}
\]

In the U stage we assume that production has to bear a constant marginal cost \(z\) and that the intermediate input is sold to the D firm at a price \(g\).

\subsection{2.1 Price setting}

The profit function of the D monopolist is:

\[
\pi_D = pq - cq - dq^2 - gq. \tag{5}
\]

We take the expected value and derive it with respect to the market price to find a maximum\(^3\) from which we get the set price, which is a nonstochastic magnitude:

\[
\frac{\partial E\pi_D}{\partial p} = 0 \implies p_S = \frac{a(b + 2d) + b(c + g)}{2(b + d)}. \tag{6}
\]

Once we substitute \(p_S\) in the demand function we obtain the stochastic quantity that we plug in the profit of the U firm:

\[
\pi_U = gq - zq. \tag{7}
\]

\(^3\)Second order conditions (SOCs) are satisfied for the chosen \(p\). In the subsequent parts of the paper we shall not mention SOCs unless they impose restrictions on the solutions.
Then we take the expected value of $\pi_U$ and maximize it with respect to $g$. Setting it equal to zero we get the FOC and the price set in the U stage, i.e.:

$$\frac{\partial E\pi_U}{\partial g} = 0 \implies g = \frac{1}{2} (a - c + z).$$

As a result the optimal endogenous quantity is:

$$q^* = \frac{ab + 4de - b(c - 4e + z)}{4(b + d)}.$$

We are then able to find the equilibrium expected profits of the two firms.

Defining $A = a - c - z$, for the D firm we have:

$$E\pi_{DP}^* = \frac{A^2}{16(b + d)} - \frac{d}{b^2}Ee^2 = \pi_{DP}^* - \frac{d}{b^2}Ee^2.$$

We see that the expected profit is lower than the corresponding certainty profit ($\pi_{DP}^*$).

For the U firm we have:

$$E\pi_{UP}^* = \frac{A^2}{8(b + d)} = \pi_{UP}^*$$

which is equal to the corresponding certainty profit ($\pi_{UP}^*$). The above arguments lead to the following

**Proposition 1** With price setting and Stackelberg mode of behavior along the vertical chain in an uncertain market framework the expected value of the profit of U is equal to the certainty profit, while that of D is lower for any finite level of the variance of the stochastic shock $e$. The higher is the degree of convexity of D costs and the lower the slope of the demand the larger is the premium paid to uncertainty by D profits.
2.2 Quantity setting

We now go through the quantity setting framework. In this case the demand for the final good is:

\[ p = a - bq + e. \]  

(12)

Then the profit of the D firm is:

\[ \pi_D = pq - cq - dq^2 - gq. \]  

(13)

If we take the FOC of expected profit with respect to the quantity we get:

\[ \frac{\partial E\pi_D}{\partial q} = 0 \implies q_S = \frac{a - c - g}{2(b + d)}, \]  

(14)

where the set quantity is a nonstochastic magnitude.

The profit of the U firm is:

\[ \pi_U = gq - zq. \]  

(15)

Following a similar procedure as in the previous subsection we get the price set by the U firm:

\[ \frac{\partial \pi_U}{\partial g} = 0 \implies g_S = \frac{a - c + z}{2}. \]  

(16)

from which we get the quantity sold:

\[ q^* = \frac{A}{4(b + d)}. \]  

(17)

The final stochastic price is:

\[ p = a + e + \frac{b(-A)}{4(b + d)}. \]  

(18)

The profit of U is nonstochastic since U takes from D the quantity set which is nonstochastic. Therefore:

\[ \pi_{UQ}^* = \frac{A^2}{8(b + d)} = E\pi_{UP}^*. \]  

(19)
while the realized profit of D is:

\[ \pi_{DQ}^{R^*} = \frac{A(A + 4e)}{16(b + d)} \]  

(20)

whose expected value is equal to

\[ E\pi_{DQ}^* = \frac{A^2}{16(b + d)} = \pi_{DQ}^* \]  

(21)

which is equal to the corresponding certainty level.

We are then able to write:

**Proposition 2** The comparison of price with quantity setting in the final stage of the vertical chain establishes that quantity setting is superior for the D firm since its expected profits are higher in an uncertain environment with quantity rather than with price setting. For the U firm there is indifference since Q and P setting lead to the same expected profit for U. The D firm gains from a positive shock, while the U firm does not.

### 2.3 Non linear costs in U

A further comparison can be undertaken if we adopt non linear costs also in the U stage, i.e.:

\[ C_U = zq + wq^2. \]  

(22)

In that case we get that, with *price setting* expected profits of D are:

\[ E\pi_{DP} = \left[ \frac{A^2 (b + d)}{4(2b + 2d + w)^2} - \frac{dEe^2}{b^2} \right]. \]  

(23)

While for U we have

\[ E\pi_{UP} = \left[ \frac{A^2}{4(2b + 2d + w)} - \frac{wEe^2}{b^2} \right]. \]  

(24)
If we go through the *quantity setting* expected profits we see that they are equal to the certainty profits, despite non linear costs.

Then we may write the following

**Corollary 1** If we adopt non linear costs in both stages also the expected profits of the U firm are affected by the variance of the shock. Both firms in U and D suffer in a way that depends on their respective cost parameters \( w \) and \( d \).

### 3 Vertical integration: comparison between quantity and price setting

Here we go through the case of VI for both price and quantity settings, using the same demand functions and the same U and D technologies. The profit of the VI monopoly is:

\[
\pi_{VI} = pq - cq - dq^2 - zq
\]

since the intermediate good is internally transferred at its opportunity cost equal to the marginal cost of production, i.e. \( z \).

In the price setting case we get:

\[
\frac{\partial E\pi_{VIP}}{\partial p} = 0 \implies p_S = \frac{b(a + c + z) + 2ad}{2(b + d)}. \tag{26}
\]

We take \( p \) and get the optimal quantity

\[
q^* = \frac{A}{2(b + d)} + \frac{e}{b}. \tag{27}
\]

Therefore the expected value of profit is

\[
E\pi_{VIP} = \frac{A^2}{4(b + d)} - Ee^2 \frac{d}{b^2}. \tag{28}
\]
Turning to quantity setting we get:

$$\frac{\partial E\pi_{V\!IQ}}{\partial q} = 0 \implies q_S = \frac{A}{2(b+d)}.$$  \hfill (29)

Substituting $q_S$ in the demand function we get

$$p = a + e - \frac{Ab}{2(b+d)}$$  \hfill (30)

and the optimal profit:

$$\pi_{V\!IQ} = \frac{A(A + 2e)}{4(b+d)}.$$  \hfill (31)

Whose expected value is

$$E\pi_{V\!IQ} = \frac{A^2}{4(b+d)}$$  \hfill (32)

and is equal to the certainty outcome.

Immediate comparison between the two settings leads to:

$$E\pi_{V\!IQ} - E\pi_{V\!IP} = Ee^2\frac{d}{b^2},$$

Then we can write the following

**Proposition 3** With vertical integration and market uncertainty expected profits are higher with quantity setting rather than with price setting (This result closely replicates Klemperer and Mayer, 1986).

Consider now non linear costs also in the U stage.

With **price setting** we have:

$$E\pi_{V\!IP} = \frac{A^2}{4(b+d+w)} - \frac{d + w}{b^2}Ee^2 = \pi_{V\!IP} - \frac{d + w}{b^2}Ee^2.$$  

With **quantity setting** we get:

$$E\pi_{V\!IQ} = \frac{A^2}{4(b+d+w)} = \pi_{V\!IQ}.$$  

In words: when costs are non linear in U and D, the loss due to market uncertainty in terms of profits is the same no matter whether we have vertical integration or outsourcing.
4 Vertical bargaining

A very common way to model vertical relationships is via a bargaining between the U and D firm. This arrangement is mostly set in the framework of the Nash Bargaining Solution and its refinements (Nash, 1950; Petrosjan and Zenkevich, 1996; Rubinstein, 1982). With certainty the bargaining solution gives rise to aggregate profits equal to those of VI. Their distribution is perfectly symmetric among the two firms as the bargaining solution dictates.

Here we confine to vertical bargaining solutions in two scenarios paralleling the above sections, i.e.: price setting and quantity setting.

Demand structure is the same as above. As far as technologies are concerned we consider the same as above, linear and nonlinear. Finally we assume that outside options are equal to zero.

QUANTITY SETTING

We replicate the demand in (3). Then we have to specify the features of the bargaining in the uncertain setting. We design the decision procedure as one whereby both the D firm and the U firm maximize their expected profit. As a result the bargaining requires the maximization of the geometric average of expected D and U profit. This assumption is extended to the price setting case below.

Since

\[ \pi_{DQ} = q(p - c - g) = q(a + e - bq) - q(c + g) \]  

we have that

\[ E\pi_{DQ} = q(a - bq) - q(c + g). \]

Moreover

\[ E\pi_{UQ} = q(g - z). \]  

(34)
The bargaining objective is:

\[ \nabla_Q = E \pi_{UQ} E \pi_{DQ} \quad (35) \]

that has to be maximized with respect to the controls of the two rivals.

We then go through the two simultaneous FOCs:

\[ \begin{align*}
\frac{\partial \nabla_Q}{\partial q} &= 0 \\
\frac{\partial \nabla_Q}{\partial g} &= 0
\end{align*} \quad (36) \]

From this we get

\[ q = \frac{A}{2b}, \quad (37) \]

where \( a - c - z = A \), and

\[ g = \frac{1}{4}(a - c + 3z). \quad (38) \]

Therefore we have:

\[ p = \frac{1}{2}(a + c + z + 2e). \quad (39) \]

Then, we get:

\[ \pi_{UQ} = \frac{1}{8b} A^2 = E \pi_{UQ} \quad (40) \]

and

\[ \pi_{DQ} = \frac{A}{8b} (A + 4e). \quad (41) \]

To sum up we see that:

\[ E \pi_{DQ} = \frac{A^2}{8b} = E \pi_{UQ}. \quad (42) \]

Yet if we compare the realizations of profits we get:

\[ \pi_{DQ} - \pi_{UQ} = \frac{A}{2b} e. \quad (43) \]

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This implies that, with quantity setting we get a symmetric distribution of expected profits along the vertical chain. However their realization is not. In particular, if the shock is positive profits will be greater in D while if it is negative U will be better off.

This is quite an important result. First it has an asymmetric content despite the symmetric bargaining. Second it may explain the different incentives to vertically integrate of the D section vis à vis the U section in the downturn and in the upturn of the business cycle. With vertical integration we may in fact figure out that each firm gets exactly one half of the whole profit which is \( \frac{A^2}{4b} \).

**PRICE SETTING**

We adopt the same sequence of decisions as above. Therefore the maximand of the bargaining is made again of \( E\pi_{DP} \) and \( E\pi_{UP} \). We shall have:

\[
\begin{align*}
\frac{\partial E\pi_{DP}}{\partial p} &= 0 \\
\frac{\partial E\pi_{DP}}{\partial q} &= 0 \\
\frac{\partial E\pi_{UP}}{\partial p} &= 0 \\
\frac{\partial E\pi_{UP}}{\partial q} &= 0
\end{align*}
\]

from which we get the FOCs as:

\[ p = \frac{A}{2} \] (45)

and

\[ g = \frac{A}{4} + z \] (46)

Then the endogenous quantity is

\[ q = \frac{A + 2e}{2b} \] (47)

Realized profits are equal:

\[ \pi_{UP} = \pi_{DP} = \frac{A(A + 2e)}{8b} \] (48)
and therefore also

\[ E\pi_{DP} = E\pi_{UP}. \]  

(49)

We are then able to write the following

**Proposition 4**  With bargaining along the vertical chain and market uncertainty, quantity setting provides a symmetric distribution of expected profits between U and D, while realizations are not symmetric, showing distinct incentives to outsource in the downturn and in the upturn of the business cycle for the D and the U sections. With price setting the symmetry occurs for both the expected and the realized profits. In this last case the gain in the realized profits is equally shared between the two firms\(^4\).

**NON LINEAR COSTS**

We assume that costs are non linear in both D and U as in the previous section.

With *quantity setting* that expected profits are equal between U and D, yet again their realizations are not:

\[ \pi_{DQ} = \frac{A(A + 4c)}{8(b + d + w)} \]

while

\[ \pi_{UQ} = \frac{A^2}{8(b + d + w)}. \]

Their respective expected values coincide.

\(^4\)In the case of linear costs the profit with VI and Q setting is equal to that with P setting in both the realized and expected values:

\[ \pi_{QVI} = \frac{A(A + 2c)}{4b} = \pi_{PVI}. \]
With price setting we have that realized profits and expected profits are symmetric. In particular:

\[ E\pi_{UP} = E\pi_{DP} = \frac{A^2}{8(b + d + w)} - Ee^3 \frac{d + w}{2b} + Ee^3 \frac{(d - w)(b + d + w)}{Ab^3}. \]

As it can be seen the expected profits suffer from market uncertainty due to the variance of the shock. However, if the convexity parameters of the costs in U and D differ \((d \neq w)\) the third moment will enter the picture and uncertainty could even make expected profits larger than in the certainty case, provided

\[ -Ee^3 \frac{d + w}{2b} + Ee^3 \frac{(d - w)(b + d + w)}{Ab^3} \geq 0. \]
5 Concluding remarks

We have investigated the issue of vertical integration and outsourcing in a stochastic framework by using two different equilibrium concepts: Stackelberg and Nash Bargaining Solution.

With the Stackelberg solution we find that price setting in an uncertain environment is always inferior for the D firm. If we allow for non linear costs also in the U stage of production the inferiority extends to the U firm that will then prefer the D firm to set the quantity in the market for the final good rather than the price.

An analogous result can be found when we consider a vertically integrated monopoly.

A second solution concept, the NBS, has been adopted to model the interaction among the vertically related firms. In this case price setting is still inferior to quantity setting. It provides a symmetric solution in both realized and expected profits. This solution is equal to an even split of profits of vertical integration. With quantity setting we have a non symmetric result for the realized profits. If there is a positive demand shock the D firm is better off than the U firm. The opposite happens in the downturn, providing different incentives to integrate or to outsource of D and U over the business cycle. If we introduce non linear costs in both stages of production we find that quantity setting replicates the closely the outcome with linear costs. With price setting firms are worse off than with quantity setting as far as their expected profits are concerned if costs have the same convexity parameters. If they differ \(d \neq w\) uncertainty may increase or decrease expected profits.
References


