A Fully CAD Consistent Model of MESFETs and HEMTs
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ABSTRACT
An empirical large-signal model of MESFETs and HEMTs has been developed which ensures the full charge conservation, numerical stability and small-signal consistency during computer-aided simulations. For a maximum of accuracy, the charges are directly extracted from the raw gradient experimental data and represented by optimized polynomial functions. The model is valid in reverse operation of the device too. It has been verified for a 0.5 μm gate length, 2 x 50 μm and 4 x 50 μm gate width PHEMTs and implemented into the Hewlett-Packard’s MDS software.

INTRODUCTION
The device model must not only represent the device behaviour as accurately as possible, but it must respect the constraints imposed by the circuit simulation. These constraints are more critical than the model accuracy, because their violation may result in catastrophic failure of the simulation. Concerning the HEMTs and the MESFETs, the following conditions must be fulfilled [1], [2], [3]:
1) charges (not voltages) must be exactly represented as state variables;
2) the functions \( Q(V) \) and \( I(V) \) must be continuous, and more particularly at \( V_d = 0 \) when the source and the drain of FETs may change their roles;
3) the matrix capacitances and conductances of the device must exactly satisfy the path independent integration conditions:
   \[
   \frac{\partial C_{ij}}{\partial V_k} = \frac{\partial C_{ik}}{\partial V_j} \quad \text{and} \quad \frac{\partial C_{ij}}{\partial V_k} = \frac{\partial C_{ij}}{\partial V_j} \quad ;
   \]
4) the large-signal model must be consistent with the small-signal model. Naturally, the model must be simple and easy to extract from data and to insert in the circuit simulator too.

None of existing models meets all the requirements of the circuit simulator enumerated above, which limits the field of their applications. The capacitance models described in [4] to [15] use voltages as state variables. In [4], [5], [6] and [14], only the dependence of the gate-source capacitance \( C_{gs} \) on the gate-source voltage \( V_g \) is taken in account. In [7], [8], [9], [10], [11], [12] and [15], the drain-source capacitance \( C_{ds} \) is assumed to be constant and even equal to zero [9]. These assumptions are good only in the saturated region. In the lookup-table models ([2], [16], [17], [18]), the terminal charges are extracted in a table form by numerical integration of the capacitance data, and then interpolated by spline functions ([2], [16], [17]) or fitted by analytical functions [18]. They are excellent, if the data and the integration are exact. Very often, the capacitance data are "noisy". And there are errors in the quadratures. So, the integration becomes path dependent [16]. The analytical charge models ([19], [20]) use transcendental empirical functions for the charges. The transcendental functions are somewhat simpler to write, but longer to compute than the polynomial ones, and special measures must be taken to avoid divisions by zero or imaginary results when using them [8]. The number of their coefficients is not sufficient to get a good fit to the data. In [20], only the gate charge is modeled, and the drain-source capacitance \( C_{ds} \) is assumed to be constant. Both models are not continuous at \( V_d = 0 \).

In this paper, we develop an empirical large-signal model of HEMTs and MESFETs which ensures the complete charge conservation, convergence and compatibility during the transient and small-signal simulations. The model is simple but rather accurate in all the saturation, linear and inverse device operating regions.

NODAL CHARGES EXTRACTION PROCEDURE
The large signal equivalent circuit of the device (Fig.1) is that used for example in [17], [18] and [19]. As analytical expressions for the charges and the capacitances, we suggest polynomial functions of the type \( \sum_{i,j} a_{ij} V_g^i V_d^j \) \( (i = 0, 1, 2, ...; j = 0, 1, 2, ...) \), because they are continued, as well as their derivatives; they can follow more closely the experimental data than transcendental functions, which in addition consume considerable computer time [8]; they can have enough coefficients to ensure a good fit, while respecting the constraints imposed by the simulation.

The danger of the fitting with polynomial functions is that ripples may occur when their order arises. To avoid this, and to meet all the requirements, we use the following procedure to find the optimal charge and capacitance expressions:

1. Choose analytical functions \( C_{12} (V_g, V_d) \) and \( C_{21} (V_g, V_d) \) which are able to fit closely the experimental data for \( C_{12} \) and \( C_{21} \). Keep their coefficients literal (do not calculate their numerical values).
2. Integrate the functions \( C_{12} (V_g, V_d) \) and \( C_{21} (V_g, V_d) \) to obtain literal expressions for the nodal charges:
   \[
   Q_g = \int C_{12} \, dV_d + a (V_g), \quad (1)
   
   Q_d = \int C_{21} \, dV_g + b (V_d), \quad (2)
   
   \text{where } a (V_g) \text{ and } b (V_d) \text{ are the integrating constants.}
3. Differentiate \( Q_g \) and \( Q_d \) to find the literal expressions \( C_{11} (V_g, V_d) \) and \( C_{22} (V_g, V_d) \). Try to fit the experimental data with them. If the fit is not good, change the functions \( a (V_g) \) and \( b (V_d) \) appropriately and restart the
procedure from Point 2. If a good fit is not found after many cycles, change suitably the functions $C_{ij}(V_g, V_d)$ and $C_{2j}(V_g, V_d)$ and restart the procedure from Point 1.

4. Calculate the coefficients of the functions $C_{ij}(V_g, V_d)$ in order to obtain the best fit to all the experimental data. During the optimization, ensure the continuity of the functions $Q_g(V_g, V_d)$ and $Q_d(V_g, V_d)$ at $V_d = 0$ in order to keep the model correct in the case of reverse operation of the transistor. To do this, the charge expressions $Q_g(V_g, V_d)$ and $Q_d(V_g, V_d)$ in reverse operation have to be found and to be made equal to $Q_g(V_g, V_d)$ and $Q_d(V_g, V_d)$ respectively at $V_d = 0$. Supposing that the device is symmetric, in reverse operation the gate charge $Q_g$ will be done by the same expression as $Q_g$, and the drain charge $Q_d$, by the expression of the source charge $Q_s = -Q_g - Q_d$, in which $V_g$ must be substituted by $V_g - V_d$, and $V_d$ by $-V_d$. As a result, $k$ equations $\varphi_k(a_{ij}, b_{ij}) = 0$ which link some coefficients $a_{ij}$ and $b_{ij}$ of the charge expressions will be found.

The error function to minimize is:

$$f = \Sigma(1 - \frac{C_{ij}^{\text{exp}}}{C_{ij}})^2 + \Sigma g_k \varphi_k(a_{ij}, b_{ij}) .$$

where $C_{ij}^{\text{exp}}$ are the literal values of $C_{ij}$ for a set of operating points $(V_g, V_d)$; $C_{ij}^{\text{exp}}$ are the experimental (extracted) values of $C_{ij}$ for the same set of operating points $(V_g, V_d)$; $a_{ij}$ and $b_{ij}$ are the polynomial coefficients of $Q_g$ and $Q_d$ respectively; $\varphi_k$ are Lagrange multipliers; to ensure the continuity of $Q_g$ and $Q_d$ at $V_d = 0$, the functions $\varphi_k(a_{ij}, b_{ij})$ should cancel at the end of the minimization.

The minimization is realized by solving numerically the system of equations:

$$\frac{\partial f}{\partial a_{ij}} = 0, \quad \frac{\partial f}{\partial b_{ij}} = 0, \quad \frac{\partial f}{\partial g_k} = 0 .$$

One or more solutions are possible. The global minimum corresponds to the solution for which $f$ is minimal.

MODEL REALIZATION AND EXPERIMENTAL RESULTS

In order to compare the simulated with the experimental results, the model was completed with the Curtice expression for the drain current [21]. However, the time constant was neglected, because it is implicitly enclosed in the matrix capacitances of our model:

$$I_g = (a_0 + a_1 V_I + a_2 V_I^2 + a_3 V_I^3)(1 + a V_d) \tanh(g V_D)$$

with $V_u = V_g (1 + b(V_d - V_d))$ and $V_d = 2 V$.

A special preliminary procedure was introduced to find the initial values of the coefficients in order to obtain the global optimization. The condition 3) was respected.

The experiments were made with $0.5 \mu m$ gate length, $2 \times 50 \mu m$ gate width pseudomorphic high electron mobility transistors (PHEMTs). The device $S$ -parameters were obtained by small-signal on-wafer measurements with LRM calibration in the frequency range from 0.1 to 40 GHz with a Hewlett-Packard equipment. The values of the extrinsic elements were obtained as in [22], except for $C_{pds}$, which was separated from $C_{ds}$ as suggested in [23] by extrapolation of the measurements made on transistors with different gate width ($2 \times 50 \mu m$ and $4 \times 50 \mu m$). The intrinsic voltages $V_g$ and $V_d$ were calculated for each operating point from the extrinsic ones $V_g'$ and $V_d'$ and the DC current $I_D$ as follows:

$$V_g = V_{g}' - I_D R_s, \quad V_d = V_{d}' - I_D (R_s + R_d) .$$

All the measurements were made at 101 frequencies for each of the 96 operating points determined by $V_g' = -1, -0.8, -0.6, -0.4, -0.2, 0, 0.2$ and 0.4 V and $V_d' = 0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.6, 2, 3, 4$ and 5 V combinations. The threshold gate-source voltage of the device was found to be equal to -0.82 V.

Then the incremental capacitances $C_{ij}$ and conductances $g_m = g_{21}$ and $g_d = g_{22}$ were calculated from the intrinsic $Y$-parameters for each frequency and operating point:

$$C_{ij} = \frac{\text{lmd}(Y_{ij})}{\omega}, \quad g_{ij} = \text{Re}(Y_{ij}) .$$

In order to minimize the "noise" of the measurements and their frequency dispersion, their average values have been calculated for each operating point in the frequency range from 0.5 to 40 GHz.

Following the procedure described here, the optimal expressions for the charges have been found to be:

$$Q_g = a_{10}V_g + a_{20}V_g^2 + a_{40}V_g^4 + a_{50}V_g^5 + a_{60}V_g^6 + a_{70}V_g^7 + a_{11}V_g V_d + a_{12}V_g V_d + a_{14}V_g^4 V_d = a_{51}V_g^5 V_d + a_{61}V_g^6 V_d + a_{71}V_g^7 V_d + a_{12}V_g V_d + a_{12}V_g V_d + a_{22}V_g^2 V_d^2 + a_{32}V_d^3 + a_{13}V_g V_d^3 + a_{14}V_g V_d + a_{14}V_g V_d^2 + a_{15}V_g V_d^5 + a_{16}V_g V_d^6 .$$

$$Q_d = a_{20}V_d + a_{40}V_d^4 + a_{50}V_d^5 + a_{60}V_d^6 + a_{70}V_d^7 + a_{11}V_g V_d + a_{11}V_g V_d + a_{14}V_g^4 V_d = a_{51}V_g^5 V_d + a_{61}V_g^6 V_d + a_{71}V_g^7 V_d + a_{12}V_g V_d + a_{12}V_g V_d + a_{22}V_g^2 V_d^2 + a_{32}V_d^3 + a_{13}V_g V_d^3 + a_{14}V_g V_d + a_{14}V_g V_d^2 + a_{15}V_g V_d^5 + a_{16}V_g V_d^6 .$$

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\[ Q_d = b_{10}V_g + b_{20}V_g^2 + b_{40}V_g^4 + b_{50}V_g^5 + b_{60}V_g^6 + b_{70}V_g^7 + b_{01}V_d + b_{11}V_g V_d + b_{21}V_g^2 V_d + \]
\[ b_{31}V_g^3 V_d + b_{41}V_g^4 V_d + b_{51}V_g^5 V_d + b_{61}V_g^6 V_d + b_{02}V_d^2 + b_{12}V_g V_d^2 + b_{22}V_g^2 V_d^2 + b_{03}V_d^3 + b_{13}V_g V_d^3 + \]
\[ b_{23}V_g^2 V_d^3 + b_{04}V_d^4 + b_{14}V_g V_d^4 + b_{24}V_g^2 V_d^4 + b_{05}V_d^5 + b_{15}V_g V_d^5 + b_{25}V_g^2 V_d^5 + b_{06}V_d^6 + b_{16}V_g V_d^6 + \]
\[ b_{26}V_g^2 V_d^6 + b_{36}V_g^3 V_d^6 + b_{46}V_g^4 V_d^6 + b_{07}V_d^7 + b_{17}V_g V_d^7 + b_{27}V_g^2 V_d^7 + b_{37}V_g^3 V_d^7 + b_{47}V_g^4 V_d^7 \] (8)

Fig. 2 to 5 compare the experimental (extracted) and the computed values of the matrix capacitances \( C_{ij} \).

Fig. 6 and 7 show the behaviour of \( Q_g \) and \( Q_d \) around \( V_d = 0 \). They are continuous and even monotonous functions.

In order to improve these results even more, a further optimization was made with, as variables, all the coefficients of \( Q_g \), \( Q_d \) and \( I_0 \) plus the extrinsic parameters of the device. Thus, the difference between the measured and the computed \( S \)-parameters was minimized for all operating points. Table 1 gives the values of the model parameters before (which corresponds to the figures 2 to 7) and after this optimization. Fig. 8 compares the measured and the computed scattering parameters for \( V_d = 0 \) to 5 V and \( V_d' = -1 \) to 0.4 V at 20 GHz after the optimization. Fig. 9 shows the measured and the computed scattering parameters for \( F = 0.5 \) to 40 GHz at \( V_d = 1 \) V and \( V_d' = -0.2 \) V.

Similar results have been obtained for a \( 4 \times 50 \mu \text{m} \) gate width PHEMT.

The model has been implemented into the Hewlett-Packard's Microwave Design System as a simple two-port device. The same implementation is valid for both forward and reverse operation of the transistor, only the functions \( Q_g(V_g, V_d), Q_d(V_g, V_d) \) and \( I_0(V_g, V_d) \) have to be switched at \( V_d = 0 \).

CONCLUSION

The model extraction procedure developed here allows to avoid any problem with charge nonconservation which is crucial for the CAD with MESFETS and HEMTs. The numerical stability during the large-signal simulations is ensured by the choice of polynomial functions for the charges and by their optimization under constraints in order to warrant their continuity even when \( V_d \) becomes negative. The applied optimization method gives all the minima of the error function and makes possible the choice of the solution which corresponds to the global one.

In spite of its simplicity and of the numerous constraints it has to satisfy, our model is rather accurate in all the saturated, linear and inverse transistor operation, because the charge expressions are directly deduced from the raw capacitance (nonintegrated) data.

The same (current generator) equivalent circuit may be used for AC small-signal simulations. This small-signal equivalent circuit does not include any branch capacitance and time constant, and, therefore, has no need of their extraction! So does the large-signal model, which is fully consistent with it.

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**Fig. 1.** Large-signal equivalent circuit of FET

**Fig. 2** (a) Measured \( C_{12} \times 10^{-13} \) F

**Fig. 2** (b) Modeled \( C_{12} \times 10^{-13} \) F

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Fig. 3. Measured $C_{21} \times 10^{-13}$ F

Fig. 4. Measured $C_{11} \times 10^{-13}$ F

Fig. 5. Measured $C_{22} \times 10^{-13}$ F (extended scale)

Fig. 6. Gate charge $Q_g \times 10^{-13}$ C vs $V_{d}$ with $V_g = -1$ V (the lower curve) to 0.2 V (the upper curve) by step of 0.2 V as a parameter

Fig. 7. Drain charge $Q_d \times 10^{-13}$ C vs $V_{d}$ with $V_g = -1$ V (the upper curve) to 0.2 V (the lower curve) by step of 0.2 V as a parameter
Fig. 8. Measured and computed S-parameters for $V_{d'}=0$ to 5 V and $V_{g'}=-1$ to 0.4 V at 20 GHz

Fig. 9. Measured and computed S-parameters for $f=0.5$ to 40 GHz at $V_{d'}=1$ V and $V_{g'}=-0.2$ V
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**REFERENCES**


