

# A Simplified Non-Linear Physical Model for High Frequency FET's

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*Direct numerical solution of device transport equations for a transistor, and device modelling approaches based on an equivalent circuit representation, are often seen as essentially competing approaches within non-linear high-frequency CAD. Each method has clear advantages and limitations. This contribution is an attempt to demonstrate the benefits of combining some of the best features of both, using a simplified physical FET model which is highly computable and yet retains key consistencies with the internal semiconductor dynamics in terms of both particle and displacement current. Results are presented which show the high frequency limitations of conventional equivalent circuit model architectures with respect to non-quasistatic behaviour.*

## INTRODUCTION

It is widely recognised that the most powerful and general approach to the modelling and simulation of high frequency transistors such as MESFET's and HEMT's is to perform a detailed numerical solution of the basic semiconductor transport equations for charge carriers in several spatial dimensions and time, consistent with the Poisson equation [1]. Nano-scale geometries and quantum mechanical aspects of operation mean that complex hydrodynamic-type formulations of carrier transport are often considered necessary, increasing the computational effort involved. While important advances have been made in the efficiency of such codes, they remain generally far too complex for many common circuit design tasks.

Most circuit design using non-linear CAD tools continues to rely on equivalent circuit models of the active devices. These models are broadly motivated by the basic physical principles of device operation but have a high empirical content. While fast and relatively simple to use, they have many limitations and potential inconsistencies (e.g. charge conservation problems [2], inaccurate modelling of non-quasistatic effects [3] etc). Indeed, it is not difficult to show that the standard  $\Pi$ -topology of intrinsic FET equivalent circuit models (see Fig 1) is fundamentally inadequate to represent the underlying dynamics of current flow in the semiconductor channel. In recognition of this, efforts have been made, for example, to use multiple channel sections in the circuit model to improve the accuracy of representation.

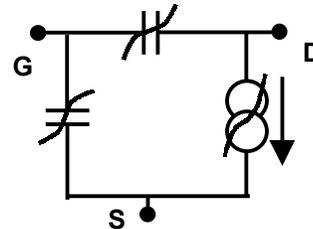


Fig. 1 Basic Intrinsic FET Model

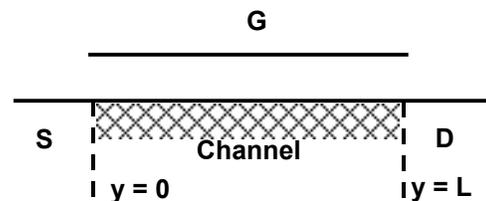


Fig. 2 Physical Structure of 'Generic' FET

In this work, we abandon any attempt to construct a *circuit-based* representation of the intrinsic device, even with an arbitrary number of non-linear charges, controlled sources etc. However, the replacement representation we seek to create is not greatly more complex in terms of computational effort. For example, the circuit of Fig. 1 leads to two non-linear ordinary differential equations: our alternative representation might require of the order of 10 such equations. Indeed, this number is scalable in our method, allowing a smooth trade-off between complexity and accuracy in the representation.

## MODEL FORMULATION

For simplicity we concentrate on a basic ‘generic’ FET device depicted in Fig. 2. If  $Q_n(y,t)$  represents the instantaneous channel charge density, then the controlling law connecting this parameter with the voltage on the Gate, will be different for a MESFET compared to a MOSFET device, for example.

As an illustration, we take the case of the MESFET. Then the following set of equations provides an elementary consistent representation of device physical behaviour, where  $i_{DS}(y,t)$  represents channel current and  $v(y,t)$  channel voltage:

$$i_{DS}(y,t) = \bar{\mu} \cdot Q_n(y,t) \cdot \frac{\partial v(y,t)}{\partial y} \quad (1)$$

where:

$$\bar{\mu} = \frac{\mu}{1 + \frac{\mu}{v_{sat}} \left| \frac{\partial v(y,t)}{\partial y} \right|}$$

and:

$$\frac{\partial Q_n(y,t)}{\partial t} = - \frac{\partial i_{DS}(y,t)}{\partial y} - i_{Gate}(y,t) \quad (2)$$

$$Q_n(y,t) = WqN_D \left[ a - \sqrt{\frac{2\epsilon(\phi_b - v_G(t) + v(y,t))}{qN_D}} \right] \quad (3)$$

The first equation represents drift current flow, using a simple model for velocity saturation. The second is a continuity equation with  $i_{Gate}$  allowing for forward Gate conduction, while the third equation expresses the controlling action of the Schottky Gate (‘a’ is the thickness of the epi-layer under the Gate). Note that these equations are coupled and non-linear, and that particle current and displacement current are closely interdependent throughout the channel. The common quasi-static assumption in equivalent circuit modelling corresponds to setting the time derivative to zero in Eq. (2). Now combining Eqs. (1)–(3) gives Eq. (4) below.

Our approach to solving this equation is to set up a grid of  $N$  internal points within the channel (which need not be equally spaced), and to re-formulate Eq. (4) in the channel as a system of  $N$  coupled non-linear ordinary differential equations. The state variables are taken as the channel voltages at the grid points. Furthermore, before a transient analysis can be performed, the steady-state or DC solution must be found, corresponding to finding the solution for a system of  $N$  non-linear coupled algebraic equations. The numerical details are omitted here, except to comment that a set of effective techniques has been developed which provide a very fast, stable and accurate solution to the physical behaviour described by Eqs. (1)–(3). A series of verification and convergence tests have been run to check numerical convergence etc.

and, for example, to assess the number of internal points required.

$$\begin{aligned} \frac{\partial v(y,t)}{\partial t} = & \frac{\partial v_G}{\partial t} + \frac{1}{W} \sqrt{\frac{2}{qN_D\epsilon}} \sqrt{(\phi_b - v_G + v)} \\ & \left\{ -\bar{\mu}^2 v_{sat} WqN_D \left[ a - \sqrt{\frac{2\epsilon(\phi_b - v_G + v)}{qN_D}} \right] \right. \\ & - \bar{\mu} W \sqrt{\frac{qN_D\epsilon}{2(\phi_b - v_G + v)}} \left( \frac{\partial v}{\partial y} \right)^2 \\ & \left. + \bar{\mu} WqN_D \left[ a - \sqrt{\frac{2\epsilon(\phi_b - v_G + v)}{qN_D}} \right] \frac{\partial^2 v}{\partial y^2} \right\} \\ & + I_s \left( e^{\left( \frac{q(v_G - v)}{nkT} \right)} - 1 \right) \quad (4) \end{aligned}$$

Very good results are achieved under a wide range of operating conditions with fewer than 10 equations (i.e.  $N < 10$ ). It is straightforward to add lumped parasitic element descriptions to this core description of intrinsic behaviour, to complete the large signal device model.

## RESULTS

A simple common-source single-ended GaAs MESFET amplifier (Fig. 3) has been used to test the model described above in a range of large-signal operating conditions. Some results are given in the following.

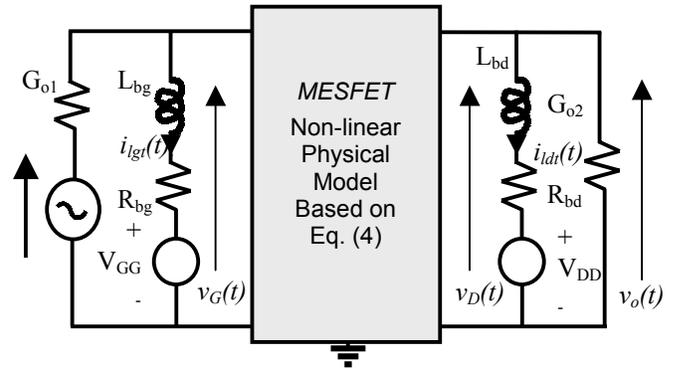


Fig. 3 Simple Amplifier Application of Non-linear Physical FET Model

Figure 4 shows a simulated power sweep for the amplifier of Fig. 3 under a Class A bias condition. The main parameters used for the device model were:  $N_D = 1.5 \times 10^{17} / \text{cm}^3$ ;  $W = 20 \mu\text{m}$ ,  $L = 1 \mu\text{m}$ ;  $V_p = -1.21\text{V}$  These results show that the model is capable of being used into

significant gain compression and that it can predict harmonic levels over a wide dynamic range.

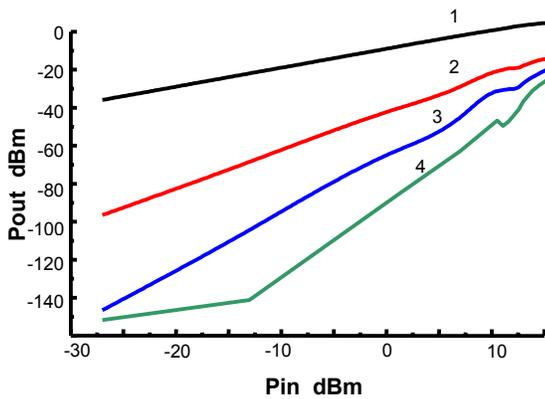


Fig. 4 50Ω Power Sweep at 2GHz, Fundamental through to Fourth Harmonic

A further detailed study has been carried out using the semi-physical model to provide insight into non-quasistatic behaviour, by calculating scattering parameters at a given bias point as a function of frequency, and then converting to a Y-parameter model of the kind shown in Fig. 5. It is then possible to see directly any evidence for dispersive behaviour and consequently assess the limitations of a quasistatic modelling approach.

As an example, Figs. 6-9 show the computed frequency response of the various intrinsic circuit model parameters in Fig. 5 at a typical Class A operating point.

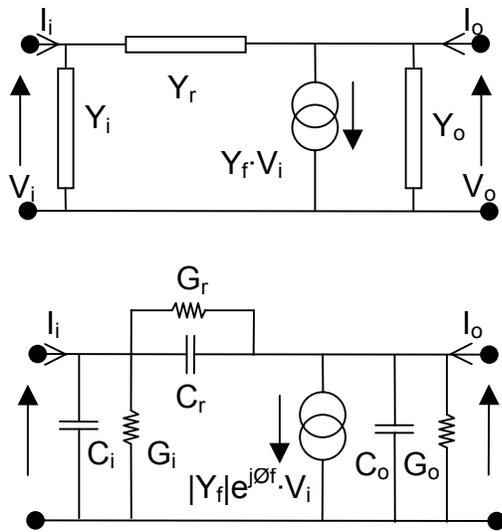


Fig. 5 Y-parameter model and corresponding circuit model for intrinsic device

The results show evidence of quite noticeable dispersive behaviour above about 1GHz for this relatively long-channel device.

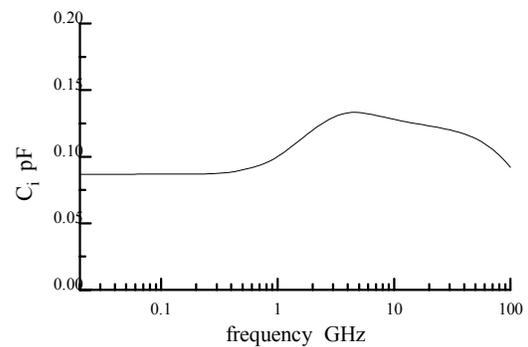
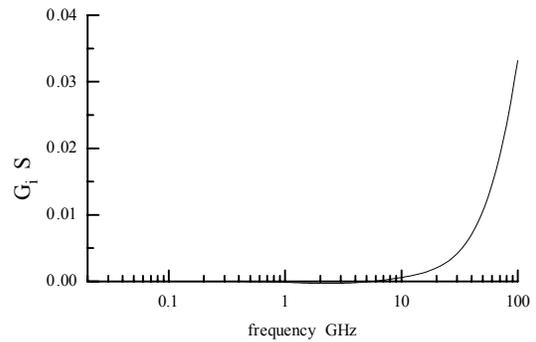


Fig. 6 Variation of  $G_i$  and  $C_i$  with frequency

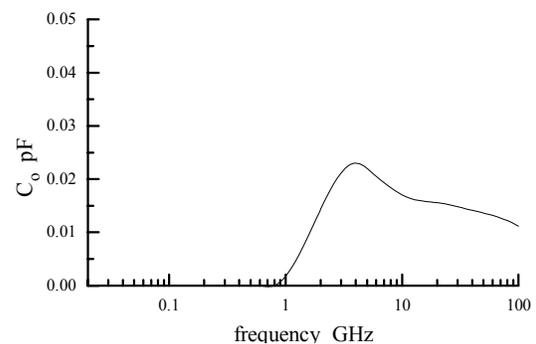
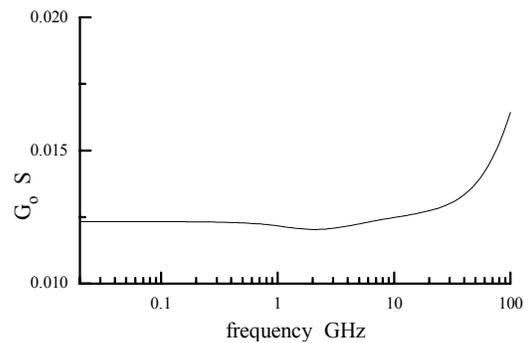


Fig. 7 Variation of  $G_o$  and  $C_o$  with frequency

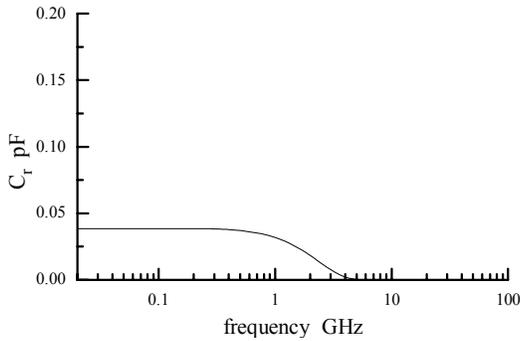
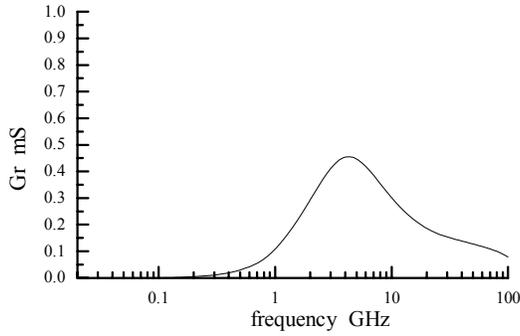


Fig. 8 Variation of  $G_r$  and  $C_r$  with frequency

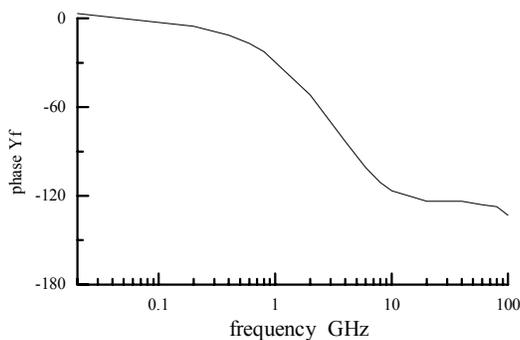
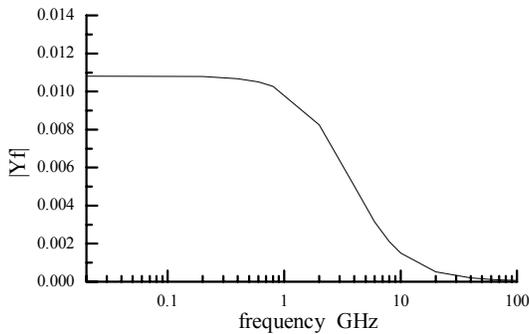


Fig. 9 Amplitude and phase of  $Y_f$

## CONCLUSIONS

We have presented a successful implementation of a physically-based non-linear FET modelling strategy, which is only moderately more complex than that used in conventional equivalent circuit models, and is therefore potentially of use in circuit design. The advantages of the proposed approach include: intrinsic consistency with physical processes, automatic accounting for charge conservation and non-quasistatic behaviour, potential scalability etc. A number of extensions are also conceivable: e.g. inclusion of channel trap dynamics and direct introduction of channel noise generation.

While the MESFET case has been presented here, a similar approach should be possible for the MOSFET. Because the model remains relatively crude (although greatly improved compared to standard equivalent circuit models), no great significance should be attached to the values of the physical parameters used. These would be expected to be reasonably close to the ‘true’ values, but are probably best regarded as fitting parameters, to be adjusted to obtain optimum fit to measured multi-bias S-parameters. The results presented here show that the model can operate reliably under strongly non-linear conditions, and show that the model can give useful insights into the degree of dispersion produced by non-quasistatic effects in FET’s.

## REFERENCES

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