# Injection locked laser modelling in presence of noise

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## Abstract

Injection locking is a way of sinchronizing one free-running oscillator to a stabilized master oscillator. When considering applications utilizing this technique in semiconductor lasers, the noise properties of the locked lasers are of vital importance. In this paper we present and analyze some clarifying simulation results about determinate aspects related to the effect of noise on injection-locking. In this aim, a new semiconductor injection-locked laser modelling technique that includes this important effect of noise, both from the injected laser and from the external source, has been implemented. This modelling technique is based on a description of the coherent rate equations in terms of a funcional blocks diagram that can be analized by means of a suitable computer simulation program

## Introduction

The phenomenon of injection-locking takes place when a weak narrow-bandwidth optical beam emitted from a master laser is coupled into a second slave laser. If the optical frequency of the master is close to the eigenfrequency of the unperturbed slave laser, the latter will adjust its frequency and coherence properties to those of the injected light [1].

In recent years, considerable attention has been paid to injection-locking in semiconductor lasers, because of the great variety of its applications in long-range high-speed optical communication systems and for coherent transmission systems, such as: linewidth reduction of the free-running slave laser, suppression of mode partition noise and mode-hopping, direct optical-frequency conversion by four wave mixing (FWM), generation and amplification of optical FM and PM signals or optical bistability. In all these applications, the noise properties of the locked lasers are of great importance. Particularly, in those applications that make use of the locking condition, as FSK and PSK modulations, their operation bandwidths are limited by the time it takes to lock, what significantly depends on noise. Even in "steady-state" applications, the relative phase between both lasers fluctuates due to the spontaneous emission noise and so, there is a certain probability for the locked lasers to momentarily unlock [2].

In this paper, a new semiconductor injection-locked laser modelling technique that includes this important effect of noise, both from the slave and from the master lasers, is presented.

This modelling technique is based on a description of the coherent rate equations in terms of a funcional blocks diagram that can be analized by means of a suitable computer simulation program.

## Rate equations for the injected laser in presence of noise

In a semiclassical analysis of the laser, the noise due to spontaneous emission and carrier generation and recombination is included in the rate equations by adding appropiate Langeving driving terms. If the external ligth injection is also considered, the resulting differential equations for the electric field and for the carrier density are:

$$\frac{dE_{SL}(t)}{dt} = [\, j\omega_N(n) + \frac{1}{2}\,(\, g(n) - \frac{1}{\tau_p}\,)\,]\, E_{SL}(t) + k_c\, E_{inj}(t) + E_N(t) \tag{1}$$

$$\frac{dn}{dt} = \frac{I}{e} - \frac{n}{\tau_s} - g(n)\, S + F_c(t) \tag{2}$$

$E_{SL}(t)$ is the complex electric field within the slave laser, considering that the laser is single-longitudinal-mode, and normalized as $|E_{SL}(t)|^2 = S$, with S denoting the number of photons within the laser cavity. $|E_{inj}(t)|^2$ corresponds to the externally injected photons, with $k_c$ the coupling coefficient. $\omega_N(n)$ and $g(n)$ are the angular resonance frequency and the gain, respectively, both depending on the carrier number n. $E_N(t)$ is a Langevin noise term, assumed to be Markoffian, i.e.: $\langle E_N(t)\, E_N(t+\Delta t)\rangle = 2D\delta(\Delta(t))$, where D is the diffusion coefficient. On an equal basis, $F_c(t)$ stands for the carrier noise.

The slowly varying amplitudes approximation is applied to these equations and, for entering them into a computer simulation program, the sources of noise are discretized, giving the following set of modified equations [3]:

$$\frac{dS(t)}{dt} = (\, g(n) - \frac{1}{\tau_p}\,)\, S(t) + R + 2k_c\sqrt{S(t)}\, E_{inj}(t)\, \cos(\phi_{ML}(t) - \phi_0(t)) + \sqrt{\frac{2S(t_i)\, R}{\Delta t}}\, x_e \tag{3}$$

$$\frac{d\phi_0(t)}{dt} = \frac{1}{2}\alpha\frac{dg}{dn}\,(n - n_{th}) - (\omega_{ML} - \omega_N\,(n_{th}\,)) +$$
$$+k_c\frac{E_{inj}(t)}{\sqrt{S(t)}}\;sen\;(\;\phi_{ML}(t) - \phi_0(t)\;) + \frac{1}{S(t)}\sqrt{\frac{S(t_i)}{2\Delta t}}Rx_{\Phi} \tag{4}$$

$$\frac{dn}{dt} = \frac{I}{e} - \frac{n}{\tau_s} - g(n)\;S(t) - \sqrt{\frac{2S(t_i)\;R}{\Delta t}}\,x_e +$$
$$+ \sqrt{\frac{2n(t_i)}{\tau_s\Delta t}}\,x_n \tag{5}$$

where $\Phi_0(t)$ and $\Phi_{ML}(t)$ are the phases of the slave and master laser fields, and $\alpha$ is the linewidth enhancement factor. R is the spontaneous emission rate, $x_e$, $x_\Phi$ and $x_n$ represent Gaussian random variables with zero mean and unity variance and $\Delta t$ is the time slot used in the numerical integration method.

## Model description

The laser model implementation here presented is based on a description of the set of equations above in terms of signal flow-graph. So, the equivalent block diagram can be directly analized with the computer program SIMULINK, that is an extension of MATLAB for WINDOWS that provides with a set of specific tools for dynamic system simulation. This program offers a wide variety of functional blocks (integrators, function and subsystem definition, generation of random number sequences...) that can be joined in a proper manner to define the dynamic behaviour of the laser.

With regard to the simulation, there are different integration routines and the option to modify its parameters. The results can be shown "in real time" or be saved into a file to be processed with MATLAB afterwards. This facilities as well as the immediacy in obtaining the results make this model of very friendly use.

The model allows to simulate the effect of noise, both from the slave and from the master laser. To obtain the time domain behaviour of the latest, the injected field term must be set to zero in the general model. The results can then be used as an input in the slave laser simulation.

## Analisys of results

By taking into account the effect of noise in the model, there has been possible to study the effect of external light injection on the noise-related characteristics of the slave laser.

The semiconductor laser chosen for the simulation was the Hitachi HLP1400. As this laser is often referenced, we disposed not only of the values of its parameters but also of detailed measurements of its characteristics (threshold current, optical power, spectrum with and without light injection ...). This has made possible to make interesting comparations that prove the validity of the here presented results.

### *A. Effect of injection on the optical spectrum of the slave laser*

Fig. 1.a) shows the optical spectrum of the unperturbed laser, operating at a current of 65mA. (1.2 times the threshold current, approximately). The spectrum consists of a Lorentzian central line, corresponding to the eigenfrequency of the laser (zero in the frequency axes), and two sidebands at a frequency distance from the former equal to the relaxation oscillation frequency. Also, one can observe the peculiar asymmetry beetween the sidebands, due to the coupling beetween the phase and the modulus of the field (linewidth enhancement factor $\alpha$).

The remaining figures show the optical spectrum of the laser upon injection. The operating current is taken the same that in the previous figure and the level of injection is -50dB.

In Fig.1.b), 1.c) and 1.d) the detuning is taken beyond the locking range ($|\Delta f| >> |\Delta f_L|$, $|\Delta f_L| \approx 350$ MHz ) and near the relaxation oscillation frequency ($f_{OR}$=2.9GHz). As the detuning is beyond the locking range, the laser emits most of its light around the eigenfrequency, as is reflected in the spectrum. Furthermore, two new components appear in the spectrum: one at the injected frequency and one at the "mirror frequency", apparently produced by four wave mixing. When $\Delta f$ matches the $f_{OR}$ (Fig.3.a)) the two additional components are maximum - the relaxation oscillation apparently acts as a resonance-. On the other hand, the components are greater if

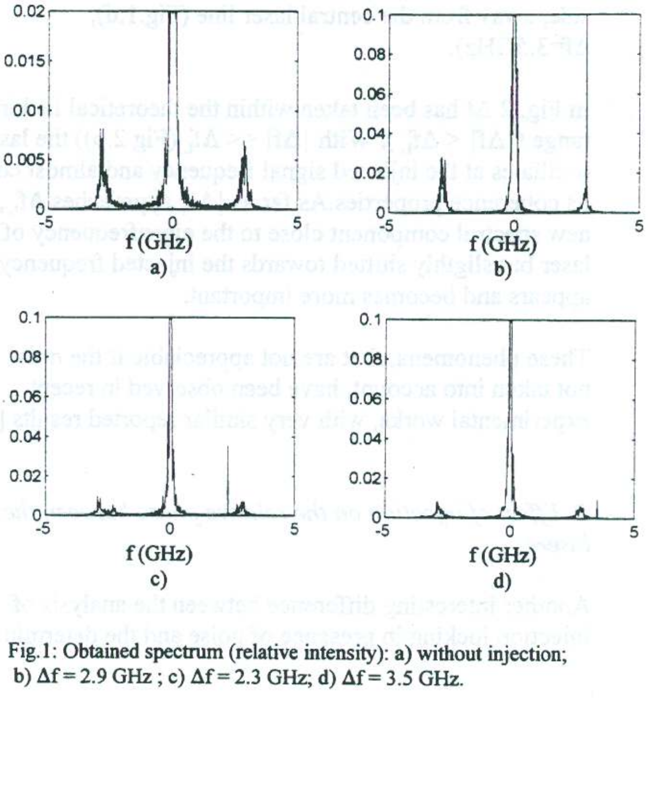


Fig.1: Obtained spectrum (relative intensity): a) without injection; b) $\Delta f$ = 2.9 GHz ; c) $\Delta f$ = 2.3 GHz; d) $\Delta f$ = 3.5 GHz.

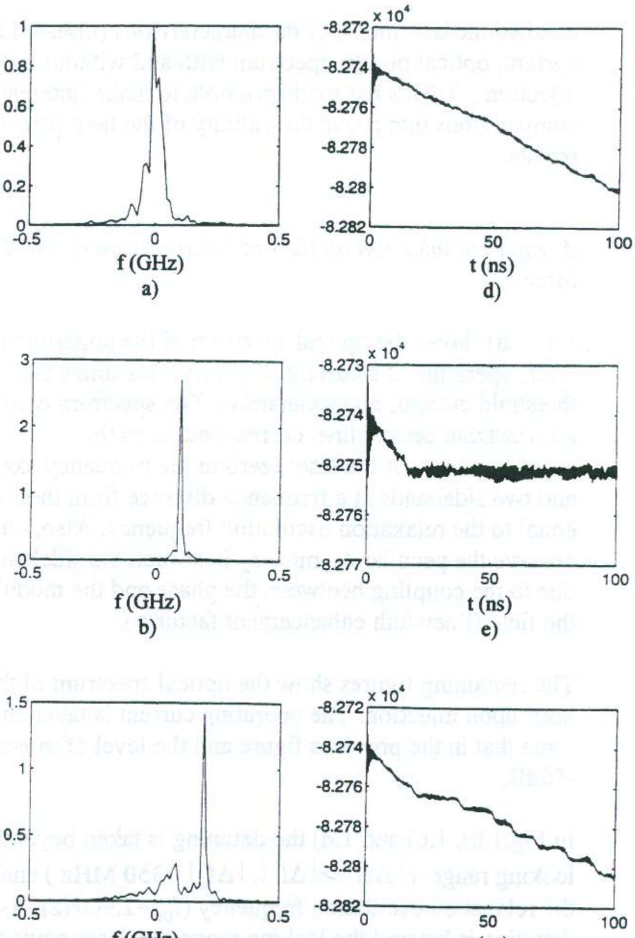


Fig.2: Obtained spectrum (relative intensity) and relative phase: a), d) $\Delta f = 100$ MHz, without injection; b), e) $\Delta f = 100$ MHz, upon injection; c), f) $\Delta f = 200$ MHz, upon injection.

the injected frequency is at the inner side, towards the central laser line (Fig.1.c), $\Delta f$=2.3GHz) , than at the outer side, away from the central laser line (Fig.1.d), $\Delta f$=3.5GHz).

In Fig. 2 $\Delta f$ has been taken within the theoretical locking range ($|\Delta f| < \Delta f_L$ ). With $|\Delta f| << \Delta f_L$ (Fig 2.b)) the laser oscillates at the injected signal frequency and almost copies its coherence properties.As far as $|\Delta f|$ approaches $\Delta f_L$ ,a new spectral component close to the eigenfrequency of the laser but sligthly shifted towards the injected frequency appears and becomes more important.

These phenomena, that are not appreciable if the noise is not taken into account, have been observed in recent experimental works, with very similar reported results [4].

*B. Effect of injection on the relative phase between the lasers*

Another interesting difference between the analysis of injection locking in presence of noise and the deterministic analysis is the relative to the definition of the locking range.

In the deterministic analysis is shown that the lasers lock together if $|\Delta f/\Delta f_L| \leq 1$. In this case, the lasers remain locked as long as the above condition is mantained and the difference between their phases is held constant.

Actually, however, the relative phase between the lasers fluctuates due to the spontaneous emission noise and so there is a certain probability for the locked lasers to momentarily unlock.

This fact has been made clear in several simulations. Fig.2 d) shows the time domain behaviour of the phase difference between the master laser and the slave for a detuning of 100MHz without injection. When the slave laser is injected (t=15ns), it adjustes its frequency to that of the master and their relative phase becomes almost constant (Fig.2e), $\Delta f$=100MHz). However, for a detuning closer to $\Delta f_L$ ($\Delta f$ =200MHz, Fig.2.f)) this condition is not longer true and the phase jumps $2\pi$. Moreover, the larger the value of $\Delta f$, the number of jumps increases.

This phenomenon, that has been observed recently by other researchers [2], may conduce to a "redefinition" of the locking range conditions as those for which the relative phase is held almost constant for a fixed mean time. Furthermore, there appears to be a correlation between the number of phase jumps and the amount of spectral component near the laser free-running frequency.

*C. Effect of injection on the relative intensity noise (RIN)*

In analog optical communication systems, the relative intensity noise (RIN) is the factor limiting the carrier-to-noise ratio at high levels of received power. Injection can affect this parameter either increasing or decreasing it. For example, in Fig.3 RIN increases upon injection conditions.

Moreover, an additional peak appears at a frequency twice the relaxation oscillation frequency.

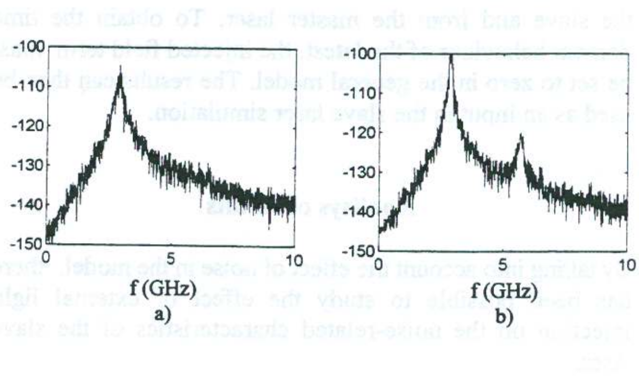


Fig.3: Relative intensity noise (RIN) (dB /Hz).
a) Unperturbed laser; b) laser upon injection, $\Delta f = 0$ MHz.

## Conclusions

By means of an specially developed model of an injected laser including the noise, several phenomena related to injection-locking not noticeable in a deterministic analysis have been made clear and studied in detail.

First, the effect of injection on the optical spectrum has been analyzed. An apparent resonance occurs when $\Delta f$ matches $f_{OR}$. Within the locking range, an additional component in the laser spectrum, close to the eigenfrequency, appears when $\Delta f$ approaches $\Delta f_L$.

Second, discrete phase jumps in the relative phase between both lasers have been observed, even within the locking range, apparently due to the momentarily unlock of the lasers because of the spontaneous emission. Moreover, there appears to be a clear correlation between this phenomenon and the additional component in the spectrum.

We have also investigated the effect of injection on the RIN.

Finally, the obtained results are in good agreement with recently published experimental works, what clearly supports the validity of the study here presented.

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