Pitfalls in private and social incentives of vertical crossborder outsourcing *

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Abstract

Vertical production processes take increasingly place in a crossborder fashion with two distinct patterns. Either a multinational firm (MNF) controls the whole vertical chain spreading production over many countries or vertically separated firms, belonging to different countries, operate independently in distinct stages. Which arrangement emerges is a matter of incentives. On the private side, the decrease of transport costs may expand crossborder outsourcing, due to

the incentives to disintegrate that emerge alternatively for the Upstream and the Downstream sections of production. Even though there remains a social superiority of vertical integration (VI) this becomes questionable since the benefits are spread over more than one country, and some country may rather like a vertically disintegrated (VD) arrangement, which is often more trade oriented. Finally, we consider an international duopoly with a vertical restraint, coming either from a competition or a trade policy. Additional private incentives to go VD, due to some fresh drawbacks of VI, arise and countries may show distinct patterns of VI according to their relative size.

*JEL Classification:* F12, L13, O31, R40.

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1 Introduction

The ongoing wave of international “outsourcing” raises macro - such as employment in rich countries - and micro worries. Among the latter ones, some concern the private incentives of outsourcing as technological, market and R&D conditions change.

Most production processes are technically and organizationally decomposable into more than one vertical independent stage. Each firm at its birth, or along its operative life, has to choose how many portions of the vertical production process to control, or alternatively, “outsourcing” to other enterprises at home or abroad. This decision determines the degree of vertical integration ($VI$) of a firm.

During the last decade outsourcing or vertical disintegration ($VD$) has become quite a common practice among enterprises of both industrialized and emerging countries\(^1\) even though outsourcing has a long tradition. For instance, the dawn of United Provinces (Nederland) industrial revolution in the XVII century was based on dyeing and finishing wool fabric imported from English small artisan factories (Wallerstein, 1980). The decision as to how much $VI$ to adopt is fairly complex and is affected by technical, institutional, governance and market issues.

The ongoing wave of outsourcing is adding fresh interrogatives as to the pros and cons of $VI$ vis à vis $VD$. Mainstream interpretation emphasizes the cost advantage of countries where outsourced production occurs, but this interpretation does not appear exhaustive.

Theoretically speaking, once we assume away perfect competition, vertical market relationships suffer from a first negative externality (Spengler, 1951). Each time the downstream ($D$) firm increases its price, the profit of the upstream ($U$) enterprise goes down. The externality, dubbed double marginalization, vanishes when firms opt for the “make” rather than the “buy” action, i.e. when they integrate vertically and transfer the intermediate product internally at marginal cost. In this circumstance, an imperfect market vertical relationship is taken over by a clone of perfect competition,

making VI privately and socially superior due to higher profits and lower prices (Williamson, 1971; Perry, 1989).

Recently, a second vertical externality has been investigated. This emerges whenever a firm producing a final good increases or decreases the number of vertical stages of production it controls. If an enterprise keeps only the final D stage, while abandoning the U stage, the market for the intermediate good becomes more competitive, shaving the incentive for other VI firms to maintain their own U section. Broadly speaking, a kind of bandwagon effect emerges in closed and open economies, making for corner equilibria, where either VD or VI prevails within an industry. (McLaren, 1999, 2000; Grossman and Helpman, 2002; Markusen, 2002; Chen, Ishikawa and Yu, 2003; Rey and Tirole, 2004; Antràs and Helpman, 2004)\(^2\). This conclusion raises a few perplexities at theoretical and empirical tiers (Buehler and Schmutzler, 2003, 2005; Slade, 1998, a; Slade, 1998, b) since in most sectors we observe firms with heterogeneous degrees of VI\(^3\).

Latest contributions (Antràs and Helpman, 2004) amend the effects of the above externality by introducing productivity differentials across and within countries along the vertical chain of production.

To sum up: if we stick to the first externality, it is awkward to explain observed outsourcing, except on the basis of large cost differentials, while the second should lead to vertically homogeneous industries.

Are there ways to weaken these conclusions to make theory and reality come closer?

The reply is a partial “yes”.

First of all, when a firm splits into two vertically independent companies the incentives to do it are not uniform along the vertical chain. There are cases in which the U section may loose more than the D section, or vice versa. In other circumstances one section reaps more than 1/2 of the profits of the previously VI firm. If we introduce R&D, private and public desirability of VI may change, according to whether there are spillovers in the production of knowledge and depending on how these spillovers are channelled through markets vis à vis within VI firms\(^4\). Also product differentiation, by shielding

\(^2\)Broad compendiums of theoretical and empirical issues in VD or, more simply, outsourcing can be found in the reading of Arndt and Kierzkowski (2001) and in the monograph of Jones (2000).


\(^4\)Literature on vertical R&D begins with contributions of Teece (1976), Armour and
firms from competition, may change the incentives to $VI$, provided differenti-
ation is extended to inputs (Pepall and Norman, 2001; Rey and Tirole, 2004).

Finally, the profitability of $VI$ may be reduced if there is a cutoff of market opportunities for the $U$ section not allowed to sell to other firms but its $D$ section, since, that way, the $U$ section does not receive any market stimulus (Lee Heavner, 2004).

Worldwide outsourcing has prompted a variety of vertical organization modes. Two arrangements loom largest: $a$) a vertically integrated multinational firm ($MNF$) scattering production crossborder; $b$) a domestic enterprise buying inputs from foreign producers.

Within these two organizational schemes we wish to explain why many firms, rather than staying $VI$, undertake crossborder $VD$, regardless of the first negative externality and international transport and communication ($TC$) costs. Some of our arguments will go through the distributional effects occurring when a firm splits into two independent enterprises producing in distinct sections of the vertical chain, as a different aggregate profit and a fresh vertical distribution of it follows. The same issue has already been investigated in the case of institutionally heterogeneous firms along the vertical chain (Rossini, 2003) since, in those cases, not just the vertical distribution, but also the $VI$ social superiority becomes questionable.

Here, two scenarios are investigated: $i$) a monopoly where part of the production process is outsourced abroad and $ii$) a Cournot duopoly where trade or competition policies introduce a vertical restraint..

We shall see that private incentives to go $VD$ (or $VI$) are not symmetric along the vertical chain and may raise questions as to the feasibility of $VD$ ($VI$) from the point of view of the support within the formerly integrated (disintegrated) firm - the internal “political economy” of a $VI$ ($VD$) firm. Trade opening makes these questions even more intriguing since the private incentives to go $VD$ (or $VI$) may be affected.

The paper is organized as follows: In next section we compare an international monopoly alternatively with $VI$ and with $VD$, first in a simple setting and, then, with process R&D. In the third part we investigate an international duopoly where firms are obliged to buy part of their inputs in the country of destination of their exports. Epilogue is in the last section.

2 The simple crossborder monopoly

2.1 The disintegrated case (crossborder outsourcing)

Consider an industry where production requires a $U$ stage, where an input is manufactured, and a $D$ stage, where a final homogeneous good is assembled using the intermediate good produced in $U$. Two monopolies, based in two countries, Home ($h$) and Foreign ($f$), produce respectively the input ($U$ stage) and the final good ($D$ stage). Their location is the result of an exogenous comparative advantage embedded in the international vertical chain of production. Trade is necessary, otherwise, production, split between $h$ and $f$, could not be delivered. The final good, assembled only in $h$, is simultaneously sold in $h$ and exported to $f$, while bearing a $TC$ cost of the traditional iceberg type (Samuelson, 1954; Lambertini and Rossini, 2005), whereby only a fraction $t \in (0, 1]$ of the good reaches the final buyer abroad. The intermediate good, produced only in $f$, is wholly exported to $h$, and the $TC$ cost is borne by its buyer.

Linear demand functions for the final good in the two countries are:

\[ p_h = 1 - x_{hh} \]  \hspace{1cm} (1)

and

\[ p_f = b - t \cdot x_h \]  \hspace{1cm} (2)

where $x_{hh}$ and $x_h$ are the quantities of the final good sold in $h$ and $f$ respectively, while $b \in (0, \infty)$ stands for relative market size of country $f$. Consumers in $f$ get only $t \cdot x_h$ of the final good, due to $TC$ costs.

Assembly of the final good in $h$ requires an input, whose $fob$ (free on board) price is $g$. The $D$ firm bears a $TC$ cost $1 - t$ to get the input shipped to its own facilities in $h$. Marginal cost of production is $c$ in $U$, while in $D$ is - for the sake of simplicity - the price of the input. Then, total cost of production in $D$ is:

\[ C_{hD} = \left( \frac{g}{t} \right)(x_h + x_{hh}) \]  \hspace{1cm} (3)

where $g/t$ is the $cif$ (cost insurance and freight) price paid by $D$ for the input.

Then, the profit functions of the two independent firms along the vertical chain of production are:

\[ \pi_{hD} = x_{hh}(p_h - \frac{g}{t}) + x_h(p_f t - \frac{g}{t}) \]  \hspace{1cm} (4)
and
\[ \pi_{fU} = (x_{hh} + x_h)(g - c). \] (5)

Market decisions of the two firms follow the sequential procedure that mimics a Stackelberg market relationship, whereby the \( D \) firm plays the role of a quantity follower, while \( U \) that of a price leader\(^5\). The alternative to this game is a bargaining between \( U \) and \( D \), that parallels quite closely the vertically integrated arrangement we shall see in the next subsection\(^6\). As it is customary in the literature (Tirole, 1988; Spencer and Jones, 1991), we assume perfect vertical complementarity (1 unit of input for each unit of output). Profit maximization leads to three optimal\(^7\) controls, the first two of the \( D \) firm, while the third one of the \( U \) firm:

\[ x_h^* = \frac{1}{4} \left( \frac{b - t + 2bt^2}{t + t^3} - \frac{c}{t^3} \right) \] (6)
\[ x_{hh}^* = \frac{2t - c + t^3 - t^2(b + c)}{4(t + t^3)} \] (7)
\[ g^* = \frac{c + (b + c)t^2 + t^3}{2(1 + t^2)}. \] (8)

Equilibrium prices and profits are:

\[ p_f^* = \frac{1}{4} (2b + \frac{c}{t^2} + \frac{b + t}{1 + t^2}) \] (9)
\[ p_h^* = \frac{c + t(2 + 3t^2) + (b + c)t^2}{4(t + t^3)} \] (10)
\[ \pi_{hD}^* = \frac{c^2(1 + t^2)^2 + t^4(4 - 6bt + t^2 + b^2(1 + 4t^2) - 2ct^2(b + t)(1 + t^2))}{16(t^4 + t^6)} \] (11)
\[ \pi_{fU}^* = \frac{(t^2(b + t) - c(1 + t^2))^2}{8(t^3 + t^5)}. \] (12)

By inspection and comparison of the above controls and equilibrium values we may write:

\(^6\)In other contributions (Antràs and Helpman, 2004, among others) a Nash Barganing Solutions is adopted. Yet this solution is equivalent to a cartel that is not really the market interaction we are after.
\(^7\)Second order conditions (SOCs) are all met without restrictions on the parameters.
**Lemma 1 Scenery:** a monopoly sells in two markets separated by $TC$ costs a final good requiring an input manufactured abroad by an independent firm. **Act 1:** There is a home bias ($x_{hh} \geq x_h$) for most parameters range; as $f$ gets larger the home bias effect vanishes; **Act 2:** $\pi_{fU} \geq \pi_{hD}$ for low levels of $TC$ costs, low production costs in $U$ and countries of similar size; however, when $f$ gets smaller the profit inequality reverses. **Act 3:** market prices in $f$ are mostly larger than in $h$.

**Proof.** See Appendix 1. ■

### 2.2 The integrated case

VI means no outsourcing. A MNF, owned and headquartered in $h$, produces both the intermediate input and the final product. Yet, the final product is manufactured in $h$, while input production is delocalized in $f$.

Therefore the profit of the $VI$ MNF is:

$$\pi_{hVI} = p_{hVI}x_{hhVI} + p_{fVI}tx_{hVI} - (c/t)(x_{hhVI} + x_{hVI}).$$  \hspace{1cm} (13)

The input is imported and pays a $TC$ cost. The $MNF$ simultaneously sets sales in the two markets, i.e.:

$$x_{hhVI}^* = \frac{t - c}{2t} \quad \text{and} \quad x_{hVI}^* = \frac{bt^2 - c}{2t^3}$$  \hspace{1cm} (14)

leading to equilibrium values:

$$p_{fVI}^* = \frac{bt^2 + c}{2t^2} \quad \text{and} \quad p_{hVI}^* = \frac{t + c}{2t}$$  \hspace{1cm} (15)

$$\pi_{hVI}^* = \frac{(1 + b^2)t^4 - 2ct^2(b + t) + c^2(1 + t^2)}{4t^4}.$$  \hspace{1cm} (16)

### 2.3 Comparison between $VD$ and $VI$

**Proposition 1 Scenery:** same as Lemma 1. **Act 1:** The $VD$ arrangement is more trade prone: there is an area of the parameters space, as the foreign country gets smaller, where the $VI$ sells only in $h$ while the $VD$ sells in both markets. **Act 2:** In a large section of this area the final price is lower with $VD$ than with $VI$. **Act 3:** $VI$ delivers larger
aggregate profits, i.e. $\pi_{hVI}^* \geq \pi_{hD}^* + \pi_{fU}^*$. However, a) if countries have the same size $\pi_{hD}^* \leq \pi_{fU}^*$, b) as the country gets smaller there exists a parameters range where $\pi_{hD}^* \geq \pi_{fU}^*$, and, in a subset of this area, $\pi_{Dh}^* \geq \frac{1}{2}\pi_{hVI}^*$, making for an incentive to go $VD$ by the $D$ section of the $MNF$.

**Proof.** See Appendix 2

**DISCUSSION**

A few implications are worth emphasizing.

First, the $VD$ arrangement exports to a small country even when the corresponding $VI$ does not. Outsourcing allows market penetration when an MNF refrains from that. Secondly, when countries have the same size, with $VD$, $U$ is always able to make larger profits than $D$. This inequality reverses as $f$ gets smaller. In the more extreme cases, the profit of $D$ is larger than half the profit of the $VI$ arrangement. This introduces an incentive for the $D$ section to go $VD$, despite the larger aggregate profits of the $VI$ arrangement.

### 2.4 The monopoly case with R&D

#### 2.4.1 A crossborder $VD$ monopoly

Assume that both vertical sections carry out R&D activities and face the same marginal production cost.

The $D$ firm in $h$ carries out process R&D, with convex costs (d’Aspremont - Jacquemin, 1988), to decrease the marginal cost, i.e.:

$$k_h = \frac{\gamma y_h^2}{2} \quad (17)$$

where $k_h$ represents the commitment, $y_h$ the cost reduction and $\gamma$ is a parameter of R&D. The marginal cost becomes:

$$c = \overline{c} - y_h \quad (18)$$

where $y_h \in [0, \overline{c}]$ and $\overline{c}$ is the cost in the absence of R&D. The profit of the $D$ firm in $h$ is:

$$\pi_{hD} = x_{hh} \left( p_h - \frac{q}{t} - c \right) - \frac{\gamma y_h^2}{2} + x_h \left( t p_f - \frac{q}{t} - c \right). \quad (19)$$

The controls are $x_{hh}$, $x_h$, $y_h$. 

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Also $U$ in $f$ carries out process R&D. Then:

$$\pi_{fU} = (g - z)(x_{hh} + x_h) - \frac{\gamma y_f^2}{2} \quad (20)$$

where

$$z = \bar{\sigma} - y_f \quad (21)$$

with $y_f \in [0, \bar{\sigma}]$ and controls $g$ and $y_f$.

Profit maximization follows a similar procedure as in the previous subsection. Now there is a further control, the amount of R&D. Therefore, $U$ and $D$ interact in two stages. First they set R&D and then solve for market controls. To secure subgame perfection, we proceed backwards, first the market and then R&D. Optimal controls and equilibrium variables are in Appendix 3.

### 2.4.2 A crossborder VI monopoly

Here is the $VI$ counterpart of the previous subsection. A $MNF$ stands as the sole producer of the input in $f$ and the final good in $h$. R&D occurs in both stages as before. The input is transferred within the $MNF$ at cost $z$. A $TC$ cost is born to ship the final good to $f$ and to transfer the input internally, yet crossborder, since the $MNF$ has its vertical branches in two countries.

The profit of the $VI$ $MNF$ is:

$$\pi_{VI} = x_{hhVI}(p_{hVI} - \frac{z}{t} - c) - \frac{\gamma y_{hVI}^2}{2} + x_{hVI}(t \cdot p_{fVI} - \frac{z}{t} - c) - \frac{\gamma y_{fVI}^2}{2} \quad (22)$$

where

$$c = \bar{\sigma} - y_{hVI} \quad (23)$$

and

$$z = \bar{\sigma} - y_{fVI} \quad (24)$$

with usual constraints on $y_{hVI}$ and $y_{fVI}$. Social welfare in $f$ is:

$$SW_{fVI} = \frac{1}{2}((h - p_{fVI}^*) (tx_{hVI})) \quad (25)$$

which does not include profits since the $MNF$ belongs to $h$. Welfare in $h$ is:

$$SW_{hVI} = \pi_{hVI}^* + \frac{1}{2}(1 - p_{hVI})(x_{hVI}) \quad (26)$$

Therefore, we can write:
Proposition 2 Scenery: Process R&D takes place along the vertical chain.

- **General background:** $VI$ retains private superiority.

- **Act 1:** $TC$ costs decrease. **Reactions:**
  1. Market prices tend to converge across countries, but in different manners: with $VD$, $p_h$ grows and $p_f$ goes down; with $VI$ both prices increase.
  2. With $VD$ profits increase in both countries yet relatively faster in $f$. $iii)$ With $VI$ quantities decrease, while with $VD$ there is an increase in the quantity exported to $f$ and a decrease in that sold in $h$. $iv)$ With $VD$ R&D grows, while it decreases with $VI$. $v)$ With $VD$ social welfare increases in both countries but relatively more in $f$. With $VI$ social welfare increases in $h$ and decreases in $f$.

- **Act 2:** The size of $f$ increases. **Reactions:**
  1. With both $VD$ and $VI$ the inequality $p_f \leq p_h$ reverses.
  2. With $VD$, the inequality $\pi_{hD} \geq \pi_{fU}$ reverses, while, with $VI$, profit increases.
  3. With $VD$, $x_{hh}$ goes down and $x_h$ goes up, while with $VI$ they both go up.
  4. R&D increases overall but remains higher with $VI$.
  5. Welfare increases, but with $VD$ welfare in $f$ becomes larger than in $h$.

**Proof.** See Appendix 3 ■

**DISCUSSION**

In the first Act of Proposition 2 we see the effects of trade liberalization, proxied by the reduction in $TC$ costs. With $VD$, lower $TC$ costs boost $p_h$ and reduce $p_f$. $p_h$ was down due to the large quantity dumped in $h$ since $f$ was less accessible. With $VI$ both prices increase as opening allows the firm to increase its monopoly power, previously restrained by $TC$ costs which were playing the role of a tax. As for profits, with $VD$ both countries benefit, but $f$ gains relatively more from lower $TC$ costs, while with $VI$ one country gains and the other loses. Even though this framework is far from general, it highlights a large chunk of cases where only one country, $f$ with the $U$ section of production, may be in favor of further liberalization.

In the second Act we see that the increase in the size of $f$ benefits relatively more $f$, if the $VD$ arrangement is on, as the quantity sold in $h$ goes down and that in $f$ increases. With $VI$, both quantities increase distributing the effect of the growth in $f$ more evenly over the two countries.

As a sort of partial conclusion, we may say that $VI$ is more efficient, but liberalization is more welcome with $VD$ than with $VI$. Moreover as sizes of
countries change the distribution of the benefits is more even with $VD$, since the $MNF$ is headquartered in only one country.

These Acts may be altered by vertical spillovers (Rossini, 2004), unless they are associated with $VI$, since, in this case, there are not many exciting novelties. If the vertical externality occurs with $VD$, the desirability of $VI$ may fade or even reverse for some country. The reason is simple. $VI$ superiority arises as it internalizes an externality that plagues $VD$. Once $VD$ enjoys a positive vertical externality, absent in $VI$, the country ranking of $VD$ versus $VI$ may reverse, provided the spillover is able to counterbalance the disadvantage of $VD$. Nonetheless, to maintain that vertical spillovers occur only with $VD$ is a very strong statement with an anti-Schumpeterian flavor.. In some circumstances this hypothesis could simply be aftermath of more efficient R&D induced by market incentives that turn out to be more robust than internal incentives. In Appendix 4 we provide some analytical treatment and a Remark concerning the spillover case.

3 The duopoly case: differentiation and vertical restraints

We leave the monopoly framework and we turn to an international differentiated duopoly that can be either $VI$ or $VD$. A $VI$ duopoly has been analyzed in a different framework by Spencer and Jones (1991) to evaluate the impact of trade policies. We walk along a different route introducing product differentiation and a vertical restraint. We still have the same two countries. Each one has a firm selling its own final good in $h$ and $f$. The two products assembled in distinct countries are horizontally differentiated. $s \in (0, 1]$ is the parameter measuring the marginal rate of substitution between the two goods.

3.1 $VI$

Consider first a symmetric international duopoly made up by two $VI$ firms. Demand functions in the two countries are:

$$p_h = a - x_{hh} - st x_f$$  \hspace{1cm} (27)

$$p_f = b - x_{ff} - st x_h,$$  \hspace{1cm} (28)
where \(a\) and \(b\) are the two market sizes. Each firm uses an input internally produced and transferred from \(U\) to \(D\) at the marginal cost.

There is a vertical restraint. Each firm must use the input produced by the foreign rival when it manufactures its exports, while sticking to the internally produced input to assemble the good sold domestically. This may be the outcome of either \(i\) a policy to let firms compete not just in the production of the final good but also in inputs, or \(ii\) a trade policy to foster the domestic production of the input or to preserve some national features of a good sold at home, but manufactured by a foreign firm\(^8\).

With these hypotheses the profit functions are:

\[
\pi_{hVI} = (p_h - c - z)x_{hh} + p_f tx_h - c x_h - (p_{mf}/t)x_h + (p_{mh} - z)x_f 
\]

(29)

and

\[
\pi_{fVI} = (p_f - c - z)x_{ff} + p_h tx_f - c x_f - (p_{mh}/t)x_f + (p_{mf} - z)x_h
\]

(30)

where \(p_{mf}\) is the price of the input bought by the firm in \(h\) from the foreign rival to produce its exports \(x_h\) to \(f\), while \(p_{mh}\) is the price of the input bought by the firm in \(f\) from the rival so as to produce \(x_f\). The input bought from the foreign rival has to be shipped home incurring the usual \(TC\) cost.

### 3.2 VD international duopoly

Here is the parallel arrangement with \(VD\) with four firms. Now each \(D\) firm has to buy the input needed for the production of its exports from the foreign \(U\). Demand functions replicate (27) and (28) while profit functions are now four since there are two \(D\) firms and two \(U\) firms.

Profit functions of the \(D\) firms are:

\[
\pi_{hD} = (p_h - c - g_h)x_{hh} + p_f tx_h - c x_h - g_f x_h/t
\]

(31)

\[
\pi_{fD} = (p_f - c - g_f)x_{ff} + p_h tx_f - c x_f - g_h x_f/t
\]

(32)

while those of the \(U\) firms are:

\[
\pi_{hU} = (g_h - z)(x_{hh} + x_f)
\]

(33)

\[
\pi_{fU} = (g_f - z)(x_{ff} + x_h).
\]

(34)

\(^8\)A similar policy can be found in Ishikawa (1999).
As before, inputs are sold *fob* to the *D* firms headquartered where they manufacture the final good. There is a Cournot game between the two *D* firms, who play on their turn Stackelberg as price followers with their *U* counterparts. Optimal controls and equilibrium values are reported in Appendix 5.

We now consider the effects of changes respectively in *TC* cost, degree of substitutability among the final goods produced by the duopolists, relative size of countries and relative production costs along the vertical chain.

3.2.1 Changing transport costs

We assess, by calibrated simulations, - Appendix 5 - the effect of changing *TC* costs on incentives either to go *VD* or *VI* and sum up the results in:

**Remark 1 Scenery:** Differentiated firms buy part of their inputs from the foreign rival and *TC* costs decrease. **Act 1:** *VD* duopolists export a larger share of production making for a deeper trade integration than a *VI* international duopoly. **Act 2:** With *VD* the mark up on marginal costs of inputs goes up, while the opposite happens for *VI*. **Act 3:** Aggregate profits become larger for *VD* than *VI* and *U* branches have an increasing incentive to go *VD*, while *D* branches would rather go *VI*.

The proof is in Appendix 5.

3.2.2 Changing the degree of differentiation

Here we see how incentives change as final goods become closer substitutes, i.e. as *s* increases:\footnote{The proofs of Remarks 2 and 3 are not reported in the text for the sake of brevity. We just confine to numerical samples coming from Tables A4, A5. More detailed diagramatic and analytical proofs can be provided upon request from the author.}

**Remark 2 Scenery:** *s* increases. **Act 1:** The quantity sold at home increases while exports decrease for *VI*. *VD* sells less in both markets. **Act 2:** Final products and input prices go up in all arrangements. **Act 3:** *SW* and profits decline everywhere. There is an incentive for *U* branches to go *VD*. However, aggregate profits of *VD* are no longer larger than with *VI*. 
3.2.3 Changing relative country size

Remark 3 Scenery: Country h gets larger. Act 1: Firms increase sales only in h (independence property as in Brander and Krugman (1983)). Only $p_h, q_h$ and $p_{mh}$ increase, while $p_f, q_f$ and $p_{mf}$ stay constant. Act 2: Aggregate profits get higher with $VD$ than with $VI$ in $f$, while they are lower in $h$ with $VD$ than with $VI$. In $f$ firms may go $VD$ sharing the higher aggregate profit. In $h$ only $U$ has an incentive to go $VD$. $D$ opposes it. Asymmetric countries make firms follow distinct vertical arrangements. This is an example of coexistence in the same industry across countries of firms with heterogeneous levels of $VI$.

4 Epilogue

We have gone through distinct vertical arrangements in the presence of trade and $TC$ costs. Our curiosity has been stimulated by the great deal of international outsourcing taking place, first among high cost and low cost countries and, secondarily, among countries with close standards of living (as for instance Japan, the US and the EU). Outsourcing has a long history and generates opposite reactions. England in XVII century shipped wool fabric to Holland for dyeing and finishing (Wallerstein, 1980). Protectionist reactions followed and trade relationships between England and Holland suffered.

Leaving aside traditional cost differentials we have gone through the effects and the desirability of outsourcing vis à vis its opposite, i.e. $VI$.

In the simple monopoly framework $VI$ enjoys a wide range of desirability. However, the distribution of production over more than one country raises some questions about the desirability of $VI$ once we consider each country separately. Also the canonical advantage of $U$ remains, but it may fade away if $TC$ costs are high and $f$ gets very small. In this last case the $D$ section has a private incentive to go $VD$ since it is able to reap more than half the profit of the $VI$ arrangement. Moreover $VD$ is more trade oriented and is able to serve both markets even when $VI$ confines only to the one where the $MNF$ belongs. In this case the price set by the $VD$ firm in $h$ is lower than that of the $VI$.

With process R&D along the vertical chain we have that $f$ gains more from a decrease in $TC$ costs and may, therefore, be in favor of trade liberalization when $VI$ is not. Country $f$ benefits anyway from lower $TC$ costs as
consumers are able to get a larger quantity of the final good. These conclusions challenge some traditional wisdom that $VI$ can always do better than $VD$ and may cast some further light on trade related expansion of outsourcing.

Then, we have gone through the duopoly case with a vertical restraint imposing each firm to buy the inputs for its exports in the foreign market. As $TC$ costs decrease there is a higher incentive to go $VD$, since aggregate profits are higher with $VD$ than with $VI$. This is quite a remarkable outcome since it changes the private incentives to $VI$. Secondly, as the degree of differentiation declines export go down and prices go up in all markets. Lower differentiation pushes $VD$ firms to look for residual rents in foreign markets. The incentive to go $VD$ disappears for the $D$ section, yet stays alive for the $U$ section. Thirdly, when size of countries differs and $h$ becomes quite larger than $f$ vertical arrangements follow distinct paths in equilibrium across countries. The larger country may prefer $VI$ - even though the $U$ section would rather go $VD$, while $D$ opposes it. The smaller country prefers $VD$. This makes for the coexistence of heterogeneous vertical organizations across countries of different size.

Some of these results adds to the already rich set of trade and competition policies that may be legitimated in imperfectly competitive markets. A general result that surfaces in the paper is that increased openness boosts $VD$. Countries producing in the $U$ sections stand to gain. Then, competition and trade policies should be carefully calibrated to avoid welfare losses. In some circumstances competition policies may - ironically - favor firms which pocket higher aggregated profits when they go $VD$. However, these firms tend to be more trade oriented than their $VI$ counterparts. Since trade may be thought beneficial for many other reasons, $VD$ may be the preferred arrangement. Last but not least, $VD$ provides a more equal geographical distribution of profits and increases welfare where outsourcing takes place.
References


5 Appendix Box

5.1 Appendix 1

Here we go through the proof of Lemma 1. First, we assume \( b = 1 \), i.e. perfect symmetry between the two countries.

5.1.1 Quantities

First consider nonnegative requirements on quantities produced.

\[ x_{hh} \geq 0 \quad \text{if} \quad 2t - c - (1 + c)t^2 + t^3 \geq 0, \quad \text{i.e.}: \]

\[ c \leq t - 1 + \frac{1 + t}{1 + t^2} = c_{hh1}. \tag{35} \]

\[ x_h \geq 0 \quad \text{if} \quad \frac{1 - t + 2t^2}{t + t^3} - \frac{c}{t^5} \geq 0, \quad \text{i.e.}: \]

\[ c \leq \frac{t^2(1 + t(2t - 1))}{1 + t^2} = c_{h1}. \tag{36} \]

Then, compare the two quantities and get:

\[ x_{hh} - x_h = \frac{c + (t - 1)^3t^2 - ct^4}{4(t^3 + t^5)}. \]

Then \( x_{hh} \geq x_h \) if

\[ c \geq \frac{(t - 1)^2t^2}{1 + t + t^2 + t^3} = c_{h2}. \tag{37} \]

We can draw \( c_{hh1}, c_{h1}, c_{h2} \) in the same space getting picture A1 below, where line I corresponds to (35), II to (36), III to (37).
Figure A1: Quantities of VD as c and t vary

As it can be seen the area where both quantities are non negative is below the curve II. We have a reversal of the home bias effect in the tiny area below curve III.

As b increases the area, over which $x_{hh} \geq x_h$, shrinks and the home bias effect reverses. As b goes down the area where $x_{hh} \leq x_h$ disappears.

5.1.2 Prices

With $b = 1$,

$$p_h - p_f = \frac{(t - 1)(c(1 + c)t^2 + t^3)}{4(t^2 + t^4)}$$

which is negative. As b changes (38) remains negative except for very low values of b.

5.1.3 Profits

For $b = 1$, we have that:

$$\pi^{*}_{fU} - \pi^{*}_{hD} =$$

$$[c^2(1 + t^2)^2(2t - 1) + t^4(2b(3 + b)t - 4 - b^2 -$$

$$(1 - 2b)^2t^2 + 2t^3) - 2ct^2(b + t)(2t - 1)(1 + t^2)]/16(t^4 + t^6).$$

(39)
The area where this is nonnegative in the space \( t, c \) is in picture A2\(^{10}\).

**Figure A2:** Profits of U minus profits of D

In the picture (39) is assessed for 3 different levels of \( b = 0.5, 1.0, 2.0 \). When \( b = 1, 2 \) the difference is positive in the area below the corresponding lines. When \( b = 0.5 \), this occurs above the corresponding line, i.e.: when \( f \) gets smaller, there is an area corresponding to low TC costs and low production costs where \( \pi_{hD}^* \geq \pi_{fU}^* \).

### 5.2 Appendix 2

Here we go through the proof of Proposition 1.

#### 5.2.1 Prices

As for prices we have that:

\[
p_{fVI}^* - p_f^* = \frac{-t^2(b + t) + c(1 + t^2)}{4(t^2 + t^4)} \tag{40}
\]

while

\[
p_{hVI}^* - p_h^* = \frac{-t^2(b + t) + c(1 + t^2)}{4(t + t^3)} \tag{41}
\]

\(^{10}\)Meaningful comparisons require \( t \geq 0.5 \) to get real numbers.
For values of $b \geq 1$ prices are lower with $VI$. For low values of $b$ prices show the same pattern. However, the $VI$ arrangement sells in both markets in a narrower range of parameters. In the range of parameters where only $VD$ sells in $h$ and $f$ while $VI$ sells only in $h$, the price of the final good in $h$ may be lower with $VD$ in a subset of the feasible set of $VD$.

Consider for instance $b = 0.1$ in the picture A3.

**Figure A3**: Prices: $VD$ vs $VI$

On line I, $c = t$. Below it $x_{hVI} \geq 0$. Line II corresponds to $c = \frac{t^2(0.1+t)}{1+t^2}$. Below it $p_{VhVI}^* - p_{Vf}^*$ and $p_{fVI}^* - p_{h}^*$ are negative. Line III corresponds to $c = 0.1t^2$. Below it $x_{hVI}^* \geq 0$. Then, between line I and II the $VD$ sells in both markets and prices are lower than with $VI$ which sells only in $h$. This area gets larger as $b$ goes down and shrinks as $b$ goes up.

### 5.2.2 Profits

The $VI$ arrangement always leads to higher profits. This can be seen easily from

$$\pi_{hVI}^* - \pi_{hD}^* - \pi_{fU}^* = \frac{(3 - 2t)(t^2(b + t) - c(1 + t^2))^2}{16(t^4 + t^6)}$$

which is always $\geq 0$. To prove the remaining part of the proposition we have to compute:

$$\pi_{fU}^* - \pi_{hD}^* = \frac{1}{16(t^4 + t^6)}(2ct^2(b + t)(2t - 1)(1 + t^2) + c^2(2t - 1))$$

(42)
\[ \pi_{fU} - \frac{1}{2} \pi_{hVI} = \frac{(t^2(b + t) - c(1 + t^2))^2}{8(t^3 + t^5)} - \frac{125((1 + b^2)t^4 - 2ct^2(b + t) + c^2(1 + t^2))}{t^4} \]  
(43)

\[ \pi_{hD} - \frac{1}{2} \pi_{hVI} = \frac{1}{(t^4 + t^6)}(125c^2(b + t)(1 + t^2) - .062c^2(1 + t^2)^2 + t^4(.125 - .375bt - .062^2 + b^2(.125t^2 - .062))) \]

Close inspection of the three above expressions easily confirm the second part of the proposition.

### 5.3 Appendix 3

Proof of Proposition 2. Optimal controls and equilibrium variables are:

\[ y^*_f = \frac{-16t(\varpi(1 + t)(1 + t^2) - (b + t)t^2)}{\Omega} \]  
(45)

\[ x^*_{hh} = -[(bt - 1)(1 + t^2) - 4(t + t^3)(\varpi - 4 + (4b - 4 + \varpi)t + (3b + \varpi)t^2 + (\varpi - 1)t^3)\gamma^2]/2(1 + t^2)\Omega \]  
(46)

\[ x^*_h = [-t(bt - 1)(1 + t^2)^2 - 4(t + t^3)(\varpi - 4 + (4b - 4 + \varpi)t + (3b + \varpi)t^2 + (\varpi - 1)t^3)\gamma + 32t^3(\varpi + (\varpi - 2)t + (b + \varpi)t^2 + (\varpi - 1)t^3)\gamma^2]/2(1 + t^2)\Omega \]  
(47)

\[ \pi^*_h = \left[\left(2\varpi^2(1 + t + t^2)^2 + 4a\varpi t^2(1 + t)^2(1 + t^2)\right)\left(t^2(2\gamma - 1) - 1\right) + a^2(1 + t(2 + t(-1 + t(-8\gamma + t[-1 - 2\gamma + t(2\gamma - 1)[6 + t\{10\gamma - 5 + 2t(2 - 6\gamma + t(5\gamma - 2))\}])])])\right]/\left\{4(1 + t^2)\left[1 + t(2 + t(2 - 4\gamma))\right]^2\right\} \right] \]  
(48)

\[ y^*_h = \frac{-(-t^2(b + t) + \varpi(1 + t)(1 + t^2))(t^2(8\gamma - 1) - 1)}{t\Omega} \]  
(49)
where

\[ \Omega = (1 + t^2)^2 - 16t(1 + t)(1 + t^2)\gamma + 64t^4\gamma^2. \]

Profits are:

\[ \pi^*_f = (8t(t^2(b + t) - \bar{c}(1 + t)(1 + t^2))\gamma^2/(1 + t^2)\Omega \]

\[ \pi^*_h = (t^2(b t - 1)^2(1 + t^2)^4 - 2(1 + t^2)^3(-2\bar{m}t^2(1 + t)(b + t)(1 + t^2) + +b^2(1 + 16t(1 + t))))(1 + t^2)\gamma^2 - 128t^4(1 + t^2)(-6\bar{m}t^2(1 + t)(b + t)(1 + t^2) + +3\bar{m}^2(1 + t)^2(1 + t^2)^2 + t^2(16 + t)(32 + 24t + 3t^3 + b^2t(19 + 8t(4 + 3t)) - +2b(16 + t(32 + 21t))))(1 + t^2) + +3\bar{m}^2(1 + t)^2(1 + t^2)^2 + t^3(16 + t(16 + 3t^2 + b^2(1 + 4t)(3 + 4t)) - +2b(16 + 13t)))\gamma^3 + 1024t(-2\bar{m}t^2(1 + t)(b + t)(1 + t^2) + +3\bar{m}^2(1 + t)^2 + t^4(4 - 6bt + t^2 + b^2(1 + 4t^2))\gamma^4)/(4t^2(1 + t^2)\Omega \]

Prices of the final good in \( h \) and \( f \) are, respectively:

\[ p^*_f = ((1 + t^2)(t + b(2 + t^2)) - 4(1 + t^2)(8bt + \bar{c}(1 + t)(1 + t^2) + t^2(4 + +3t + b(7 + 4t(1 + t)))))(1 + t^2) + t^2(t + b(3 + 2t^2)))\gamma^2)/ 2(1 + t^2)\Omega \]

(50)

\[ p^*_h = ((1 + t^2)(1 + t(b + 2t)) - 4(t + t^3)(4 + \bar{c} + (4 + 4b + \bar{c})t + (8 + 3b + \bar{c})t^2 + +(7 + \bar{c})t^3)\gamma + 32t^2(\bar{c}(1 + t)(1 + t^2) + t^2(t + b(3 + 2t^2)))\gamma^2)/2(1 + t^2)\Omega \]

(51)

Second best social welfare (SW) in the two countries\(^{11}\), are

\[ SW_h = \pi^*_h + \frac{1}{2}(1 - p^*_h)(x^*_h), \]

\[ SW_f = \pi^*_f + \frac{1}{2}(b - p^*_f)(t x^*_f). \]

When we have VI, optimization leads to equilibrium\(^{12}\) values:

\[ x^*_{hhVI} = \frac{-(bt - 1)(1 + t^2) - 2t^3(\bar{c} + (\bar{c} - 1)t)\gamma}{2\Psi} \]

\(^{11}\)Detailed formulas are available upon request.

\(^{12}\)SOCs are all met in the feasible set of parameters.
\[ x_{hVI}^* = \frac{1 + t(t - 2\tau(1 + t)\gamma + b(t^2(2\gamma - 1) - 1))}{2\Psi} \]  

(53)

\[ y_{hVI}^* = -\frac{t(-t^2(b + t) + \tau(1 + t)(1 + t^2))}{\Psi} \]  

(54)

\[ y_{fVI}^* = \frac{t^2(b + t) - \tau(1 + t)(1 + t^2)}{\Psi}. \]  

(55)

Equilibrium profits, prices and SW are:

\[ \pi_{VI}^* = \frac{[-(bt - 1)^2(1 + t)^2 + 2((1 + b^2)t^4 - 2\tau t^2(1 + t)(b + t) + +\tau^2(1 + t)^2(1 + t^2))\gamma]/4\Psi}{2\Psi} \]  

(56)

\[ p_{fVI}^* = b - \frac{t(1 + t(t - 2\tau(1 + t)\gamma + b(-1 + t^2(2\gamma - 1))))}{2\Psi} \]  

(57)

\[ p_{hVI}^* = \frac{-(1 + t^2)(1 + t(b + 2t)) + 2t^3(\tau + t + \tau)\gamma}{2\Psi} \]  

(58)

where

\[ \Psi = -2(1 + t^2)^2 + 4t^4\gamma \]

To prove Proposition 2, we have to resort to numerical calibration of exogenous parameters. In Table A1 below, we provide just an excerpt of numerical simulations taking place within the parameters sets consistent with second order conditions and nonnegativity constraints. Market size of \( h \) is normalized to \( 1, \tau = .2, \gamma = 9 \). TC costs include all costs and duties to sell a good abroad. We assume that they vary between zero and 30%, i.e. \( t \in [0.7, 1] \). This is consistent with the twin observation that i) the average tariff rate for OECD countries is some 4%, while in other areas varies around an average 5-6% (Laird and Yeates, 1990; WTO, 2003); ii) pure transport costs are some 5% of the final price (Obstfeld and Rogoff, 2000). In integrated areas, such as the EU, crossborder transaction costs are some 5%. Outside they range between a maximum 30% and a lower bound of 10%\(^\dagger\).  

\(^\dagger\) A more radical view can be found in Anderson and van Wincoop (2004) where TC costs loom quite larger than what we assume.
### Table A1

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### 5.4 Appendix 4

R&D carried out by $D$ and $U$ exerts a reciprocal beneficial effect, since we assume that the market provides better incentives than $VI$ for the diffusion of R&D knowledge. This is a non-Schumpeterian hypothesis that can be contrasted on many grounds. We analyze one way and two ways spillovers. The analytical presentation is confined to two way spillovers\(^{14}\). Cost equations become:

$$c_S = \tau - y_h - \beta y_f$$

(59)

and

$$z_S = \tau - y_f - \beta y_h.$$  

(60)

with $y_h - \beta y_f \in [0, \tau]$ and $y_f - \beta y_h \in [0, \tau]$.

In Table A2 below we show results of numerical simulations. Calibration replicates Table A1, but with $\beta = 0.7$ and $\gamma = 16$.

\(^{14}\)One way spillover can easily be obtained from the two ways. We consider only one case of one way spillover, i.e. from $D$ to $U$. 

28
We compare three cases: 1) a $VD$ crossborder arrangement with one way spillover from $D$ to $U$, 2) a similar $VD$ case with two ways spillover 3) a $VI$ without spillover.

We may the write:

Remark A1 As $TC$ costs decrease there is an incentive for the $U$ section of the $VI$ $MNF$ to go $VD$. This occurs with two ways spillover, since profits of $U$ are larger than those imputed to it in the $VI$ arrangement (i.e.: $\pi_f \geq 1/2\pi_{VI}$). This private incentive is reinforced by the public incentive owing to the decrease of welfare of $f$ that takes place with $VI$ as $TC$ costs subside and welfare of $h$ goes up.

| Table A2$^{15}$ | $VD - 2ws$ | $VD - 2ws$ | $VD - 1ws$ | $VD - 1ws$ | $VI$ | $VI$
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$^{15}$ $VD - 2ws$ stands for $VD$ with two ways vertical spillover, while $VD - 1ws$ stands for $VD$ with one way spillover.
5.5 Appendix 5

Equilibrium\textsuperscript{16} controls for the VI case are:

\[
p_h^* = \frac{2z + t(c(2 - 2s + 5t) - 2s z + 5t(a + z))}{2t(6t - s)} \tag{61}
\]

\[
p_f^* = \frac{2z + t(c(2 - 2s + 5t) - 2s z + 5t(b + z))}{2t(6t - s)} \tag{62}
\]

\[
x_h^* = \frac{2z - t(b(t - s) + c(s + t - 2) + z(s + t))}{s(s - 6t)t^2} \tag{63}
\]

\[
x_{hh}^* = \frac{t(7t z - 5a t + c(7t - 2)) - 2z}{2(s - 6t)t} \tag{64}
\]

\[
x_f^* = \frac{2z - t(a(t - s) + c(s + t - 2) + z(s + t))}{s(s - 6t)t^2} \tag{65}
\]

\[
x_{ff}^* = \frac{t(7t z - 5b t + c(7t - 2)) - 2z}{2(s - 6t)t} \tag{66}
\]

\(U\) firms set prices for inputs enjoying a quasi-monopoly position since the foreign \(D\) must buy from them an amount determined by their exports of the final good.

\[
p_{mh}^* = \frac{t(c(s(2 - 4t) + 3(t - 2)t) + 6z + t(2a s + 3a t - 4s z + 3t z))}{2(6t - s)} \tag{67}
\]

\[
p_{mf}^* = \frac{t(c(s(2 - 4t) + 3(t - 2)t) + 6z + t(2b s + 3b t - 4s z + 3t z))}{2(6t - s)} \tag{68}
\]

Reduced form equilibrium profits and welfare can be easily found by simple substitution\textsuperscript{17}.

\textsuperscript{16}SOCs are met provided that:

\[ t \geq \frac{1}{6} s. \]

\textsuperscript{17}Formulas are not reported because they are too long.
For the $VD$ case, assuming Cournot competition in $D$, we get optimal controls\textsuperscript{18}:

$$x_{hh}^* = \frac{[at^2(5 - st - 4t^2 + 2st^3) + ct(2 - (7 + s)t + +2(1 + s)t^2 + 2(1 + s)t^3 - -4st^4) - (2t^2 - 1)(2 - st - t^2 + 2st^3)z] / [6t^2(2 - st - t^2 + 2st^3)]}{\ldots}$$

$$x_h^* = \frac{bt^2(2 - 4st - t^2 + 5st^3) + ct(-4 + 2(1 + s)t + 2(1 + s)t^2 - (1 + 7s)t^3 + 2st^4) + (t^2 - 2)(2 - st - t^2 + 2st^3)z] / [6st^3(2 - st - t^2 + 2st^3)]}{\ldots}$$

$$g_h^* = \frac{2z - 2ct + t(t(a + c + cs + (a - 2c)st) + (s(2t^2 - 1) - t)z)}{2[2 - t(s + t - 2st^2)]}$$

$$p_h^* = \frac{[at^2(5 - t^2 + st(5t^2 - 1)) + ct(2 + t((1 - t)(5 + t) + +s(-1 + t(-4 + t(5 + 2t)))) + +(1 + t^2)(2 - t^3 + st(2t^2 - 1))z] / [6t^2(2 - st - t^2 + 2st^3)]}{\ldots}$$

We just provide a brief proof of \textbf{Act 3} of Remark 1 resorting to the diagram below, where $\pi_{VD}$ is the sum of profits of the $U$ and $D$ firms operating in one country, while $\pi_{VI}$ is the profit of the $VI$ firm. As it can be seen, the gap between the two increases as $TC$ costs go down.

\textsuperscript{18}Socs and stability conditions are always met. Variables for country $F$ are not reported for sake of brevity.
Figure A4: Profits of VD and VI as $t$ varies

We omit proofs for the other remarks and confine to numerical simulations in the ensuing Tables.
### Table A3: VI and VD Duopoly as $t$ increases

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Table A4: VI and VD Duopoly as $s$ changes

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Table A5: VI and VD Duopoly as $a/b$ changes

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$t = 0.9 \quad c = z = 2 \quad b = 10 \quad s = 0.5$