SOME ASPECTS OF NON-LINEAR MODELING
AND SIMULATION OF MICROWAVES CIRCUITS

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INTRODUCTION

The design of optimized non-linear microwave circuits remains a difficult task although commercial CAD tools come to maturity.

The aim of our paper is to provide some insights on different aspects encountered by designers confronted with the problem of modelling and simulation of non-linear circuits.

Non-Linear modelling of transistors at millimeters waves

Non-linear millimeter wave circuit design needs accurate modelling of semiconductor devices. For very high frequencies applications, the distributed effects along the width of the FET-fingers must be taken into account, the more the device works at high input levels and harmonic frequencies are generated.

Recently [1], [2], [3] linear distributed FET models have been proposed.

Figure 1 shows the non-linear distributed model of a FET which may be implemented in an harmonic balance simulator. A FET-finger is modeled by N sliced sections shown in the figure.

Each section includes a non-linear two-port, inserted between two linear four-ports.

- The non-linear two-port describes the active region (intrinsic FET) of the section,
- The two linear four-ports describe the coupling between electrodes and the distributed effects along the width of the finger,
- The losses of a section are modeled by lumped resistances along the gate and drain lines, and across the width of the source.

Element-values of the non-linear two-port are derived from the lumped model [4], by appropriate scaling rules. Element-values of the linear four-port are derived from an electromagnetic analysis of the transverse structure of the FET (figure 2), which includes coupling and distributed effects along the electrodes.

The procedure has been applied successfully to FET modelling at high frequencies.

Figure 3 shows the non-linear behaviour of a $0.3 \mu m \times 32 \mu m$ FET finger described by seven sections, where each elementary drain-current source load-line is drawn, for a generator frequency of 47 GHz, in function of the gate and drain voltages of the corresponding section: note that the FET is loaded by 50 $\Omega$.

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Figure 4 shows an example of resulting waveforms for an increased input voltage level. The non-linear and distributed effects are clearly shown.

Practical limits of the lumped model will be discussed.

**Design tools for non-linear microwave circuits**

- A simple verification tool

Today, harmonic-balance analysis is proved to be the most efficient technique to found the steady-state of a non-linear circuit driven by a periodic signal [5], [6], [7].

This technique is now well known and many papers have been published on this topic. However, the simulation results are not always easy to explain and practically, it may be difficult to verify that the result obtained is near optimized.

Fortunately, there is an easy to use verification tool for transistors: the drawing of the transistor load-line on the Ids(t) = f(vgs(t - τ)), vds(t) plane. In fact, it may be shown that a transistor which is in an optimum state has a load-line characterized by a closed curve without surface. So the load-line drawing may be an attractive tool for the verification of numerical optimizations; practical examples will be shown.

- Analysis of autonomous circuits

The analysis of an autonomous circuit is quite difficult, because the oscillation frequency is unknown, moreover the circuit may contains spurious frequencies, so parasitic oscillations may build-up.

The design of an oscillator requires the use of a set of analysis which must characterize completely the electrical behaviour of the circuit.

Those calculations are:

* Determination of all frequencies where oscillations may start. After modification of the topology, verification that oscillations may only start at a frequency close to the desired one.
* Determination of the accurate steady state and the oscillation frequency f0, (waveforms, power and harmonic).
* Study of the stability of the solution under bias and the variation value of an element.
* Evaluation of FM noise spectrum around ω0, and eventually around nω0 (case of multiplier-oscillator).

Such software has been developed in our laboratory.

The unicity of formalism used in each step ensures coherence of the set, and makes its integration in a general non-linear CAD software easier.

**Steady state search**

The now classical harmonic balance method requires solving the non-linear system of equations (1) in the frequency-domain:

\[ D(C) = C - A_1 \mathbf{NL} - A_2 \mathbf{E}_g = 0 \]  \hspace{1cm} (1)

where \( C \) and \( \mathbf{NL} \) are split vectors of real and imaginary parts of control variables and non-linear variables, respectively, \( \mathbf{E}_g \) is the external source vector, and \( A_1 \) and \( A_2 \) are
matrices representing linear parts of the circuit, at \( o = \omega_0 \), ..., \( N \omega_0 \). The iterative process leading to a solution (Newton Raphson method) involves a Jacobian evaluation.

\[ J = I - A_1 U \] (2) with \( U = \frac{\partial \text{NL}}{\partial C_j^L} \) (3)

\( i = 1, \) number of control variables
\( j = 1, \) number of nonlinear variables
\( k = 0, N \)

The harmonic balance equation must be modified to allow autonomous circuit analysis:

- The \( E_g \) vector contains only DC generators so it is necessary to introduce frequency as an unknown.
- In order to keep a particular steady state a phase origin must be chosen.

The new system of equations becomes:

\[ F(C, \omega) = 0 \]

with:

\[ C = C_j^R, C_j^C, C_j^L, C_j^R, C_j^N, C_j^N \]

fixing \( C_j^L = 0 \)

It must be noticed that the new Jacobian contains terms in the form of \( \frac{\partial \text{NL}}{\partial \omega} \). The evaluation of these terms implies matrix \( A_1 \), sensitivity evaluation with respect to \( \omega \). So, at each iteration, it is necessary to evaluate \( A_1(\omega) \) and \( \frac{\partial A_1(\omega)}{\partial \omega} \). By means of symbolic analysis this step can be achieved without time penalty.

**Starting frequency range**

For a given oscillator, finding the steady state (frequency, electrical state) by an iterative process requires an initial frequency estimation as close as possible to the accurate solution. Robustness and convergence speed of the process depend on this estimation.

To achieve this goal, the circuit stability is studied, around DC \( [8] \) by drawing the graph of the locus of the determinant for the following linear system:

\[ I - A_1(\omega) U \] (4), with \( U = \frac{\partial \text{NL}}{\partial C_j} \) (5)

and \( \omega : 0 \rightarrow \infty \)

If the Nyquist locus involves the origin, that is to say if the circuit may oscillate, two frequencies: \( f_{\text{inf}} \) and \( f_{\text{sup}} \), near the oscillation frequency may be defined, bounded by:

\[ f_{\text{inf}} = \begin{cases} \text{(Re(det) = 0)} & \text{and by } f_{\text{sup}} = \begin{cases} \text{(Re(det) < 0)} \\ \text{(Im(det) < 0)} \\ \text{(Im(det) = 0)} \end{cases} \end{cases} \]

This method necessitates the evaluation of matrix \( A_1 \), for a set of \( f \) values between DC and 100 GHz. Also, for speed improvement, this matrix has been generated by means of symbolic analysis, from a bloc circuit description.
Noise analysis

To perform the noise analysis, the circuit is represented by figure [5]. One may shown:

- The external generator access-port (for mixer and synchronised oscillator analysis).
- The load access where the resulting noise voltages will be calculated.

The noise sources are separated in linear and nonlinear generators and are represented by their correlation matrices.

- The linear noise sources depend on the topology and temperature of the circuit.
- The non-linear sources depend also on the semiconductor bias and on the oscillation-signal. This signal has been previously calculated by the harmonic-balance algorithm.

The purpose of the noise analysis is to find first the noise voltage converted around each harmonic-frequency of the fundamental oscillation frequency \( f_o \), at a distance \( \Omega \) from the carrier. The noise voltage in a branch may be represented by a vector \( \mathbf{v} \):

\[
\mathbf{v} = \begin{bmatrix}
V_{Kl}(k\omega_o - \Omega) \\
V_{ll}(k\omega_o - \Omega) \\
V_l(\Omega) \\
V_{llu}(k\omega_o + \Omega) \\
V_{kl}(k\omega_o + \Omega)
\end{bmatrix}
\]

- The phase noise spectrum at a distance \( \Omega \) from \( k\omega_o \) is defined as \([9]\):

\[
S_{k\omega_o} = \frac{V_o^2}{V_{kl}^2 + V_{kl}^2 - 2 \Re[V_{kl} V_{kl}^* e^{j2f_o}]}.
\]

- The amplitude noise spectrum is defined as:

\[
S_{A_o} = \frac{V_o^2}{V_{kl}^2 + V_{kl}^2 + 2 \Re[V_{kl}^* V_{kl} e^{j2f_o}]}.
\]

- The amplitude-phase noise correlation spectrum is:

\[
S_{A_o} = \frac{V_o^2}{V_{kl}^2 - |V_{kl}^2| + 2 \Im[V_{kl}^* V_{kl}^* e^{2j\phi_o}]}.
\]

where \( V_o e^{j\phi_o} \) is the carrier voltage at \( k\omega_o \), in the branch where the noise is calculated. A practical example of an oscillator analysis will be given during the presentation.

CONCLUSION

Today, the effort placed in the developement of non-linear modeling and simulation tools allows to simulate accurately many active circuits; however non-linear CAD applied to GaAs MMICS is still beginning and many problems must be solved to get automated design and optimization softwares. Efficient methods of optimization must be found, in frequency and time domain, since more and more microwave systems handle with pulsed signals. A projection into the future allows to think that the work of the microwave designers will be facilitated by the introduction of expert systems in the next years.
REFERENCES


An elementary section of the distributed non-linear model.
A FET-finger is described by N cascaded sections.