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STRUCTURAL PROPERTIES OF THE NEW QUARTERLY SERIES ON CONSUMPTION

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1. Introduction

As part of a more general revision of the quarterly national accounts, new estimates of consumer's expenditures have recently been made available by ISTAT, the Italian National Statistical Office, covering the period 1970:1-1990:4. The main feature displayed by the data is the presence of substantial cyclical growth, particularly accentuated during the the first decade.

In this paper we focus attention on the series at constant prices relative to eight expenditure classes, with the fundamental scope of characterising them from a structural standpoint; in other words, we will attemp to summarise the stylised facts concerning these series in terms of stochastic components which bear direct interpretation, such as trend, seasonals, cycle and irregular.

For this purpose we compare three classes of structural models which are based on a particular set of assumptions about the way those components interact. Thus the emphasis of the analysis is shifted to the representation which better interprets the data. The main finding is that the series in question do not admit the orthogonal decomposition trend+cycle+seasonals; rather a representation in which trend and cycle are related is preferred.

The plan of the paper is the following: the series are described in the next section; in section 3 we discuss three different classes of structural models that may account for the kind of behaviour displayed by the data, whereas section 4 is devoted to a brief review of estimation and model comparison and selection. The estimation results are presented and discussed in section 5 and a tentative explanation of the reason why one class of models proves to be almost systematically inferior with respect to its competitors is proposed. We then draw some conclusions.

2. Description of the data

Estimates of consumption making up the System of National Accounts are available on a yearly basis for 50 expenditure items at both current and constant (1985) prices. Several data sources are employed in this estimation process; among these, the Household Expenditure Survey carried out quarterly by ISTAT plays a major role.

The yearly estimates have been distributed across the quarters according to a variant of the Chow and Lin (1973) procedure described in Barbone *et al.* (1981); most indicator series employed are desumed from the Household Expenditure Survey.

The series considered in this paper are at constant prices. It it has been deemed adequate for a preliminary and exploratory analysis such as the present one to restrict the analysis to the data aggregated into the following expenditure categories:

1. *Food, Beverages and Tobacco*
2. *Clothing and Footwear*
3. *Rent, Fuel, Power*
4. *Furniture, Household Equipment and Services*
5. *Health*
6. *Transport and Communications*
7. *Recreation, Education, etc.*
8. *Other Goods and Services*

As can be seen from figure 1 the individual series trend upwards and are characterised by a regular seasonal behaviour; most of them show a break in regime occurring at the beginning of 1981. Figure 2 displays the seasonal differences of the logarithms of the series, which approximate the yearly growth rates; these have a very smooth appearance and their movements have a periodic nature and are strongly coherent with the Italian business cycle. For most consumption classes[1] growth rates underwent a marked reduction during the 1974-75 and 1981-82 depressions; two minor contractions followed the small crises in 1978 and 1985.

Another feature shared by all plots is the change in regime in 1981, when growth started to be less volatile; for series 2-5 the tendency to reduction in the growth rates which characterises the 70's was reverted.

Figure 3 shows the power spectrum of the series filtered by the operator Δ_4, estimated by adopting a Parzen window with truncation point at 15; the presence of a cyclical component whose period oscillates between 10 and 15

[1] Series (3) is somewhat of an anomaly since it lags the cycle.

quarters turns up rather clearly. In some cases the spectrum suggests that the cycle is the resultant of the overlapping of a *fundamental* and *harmonic* component.

3. A Model for the Data

The basic ingredients of a structural model are components which bear straightforward interpretation, such as trend, cycle, seasonals and irregular; there is also a variety of ways in which autoregressive effects can be brought into the model. By appropriately combining these building blocks we can envisage competing ways of interpreting the data previously described.

A trend component, denoted by μ_t, can be modelled according to the following local linear process:

$$\mu_t = \mu_{t-1} + \beta_{t-1} + \eta_t \qquad (1)$$

$$\beta_t = \beta_{t-1} + \zeta_t \qquad (2)$$

where η_t and ζ_t are mutually uncorrelated white noise disturbances with mean zero and variances σ_η^2 and σ_ζ^2 respectively; the underlying level is thus conceived as evolving over time according to a random walk with drift β_t which in turn evolves as a pure random walk.

The trend has an $IMA(2,1)$ reduced form, as can be seen by solving (1) and (2) with respect to μ_t: $\mu_t = \zeta_{t-1}/(1-L)^2 + \eta_t/(1-L)$; furthermore, its specification allows a fair degree of generality:

- if $\sigma_\zeta^2 = 0$ the trend is reduced to a random walk with constant drift ($\Delta\mu_t = \beta + \eta_t$);
- if also $\sigma_\eta^2 = 0$ then it is deterministic linear ($\mu_t = \mu_0 + \beta t$);
- if $\sigma_\eta^2 = 0$, but $\sigma_\zeta^2 > 0$, it becomes $\Delta^2\mu_t = \zeta_{t-1}$, resulting in a relatively smooth trend (Harvey and Jaeger, 1991).

As a process capable of generating a stochastic cycle, ψ_t, we consider:

$$\begin{bmatrix} \psi_t \\ \psi_t^* \end{bmatrix} = \rho \begin{bmatrix} \cos\lambda_c & \sin\lambda_c \\ -\sin\lambda_c & \cos\lambda_c \end{bmatrix} \begin{bmatrix} \psi_{t-1} \\ \psi_{t-1}^* \end{bmatrix} + \begin{bmatrix} \kappa_t \\ \kappa_t^* \end{bmatrix}, \qquad (3)$$

where ρ is a dumping factor, $0 \le \rho \le 1$, ψ_t^* appears by construction in order to form ψ_t, λ_c is the frequency in radians and κ_t and κ_t^* are two mutually uncorrelated white-noise disturbances with mean zero and variances σ_κ^2 and $\sigma_{\kappa^*}^2$. The single equation expression for ψ_t deduced from (3) is:

$$\psi_t = \frac{(1 - \rho\cos\lambda_c L)\kappa_t + \rho\sin\lambda_c L\kappa_t^*}{1 - 2\rho\cos\lambda_c L + \rho^2 L^2}. \qquad (4)$$

Hence $\psi_t \sim ARMA(2,1)$ with the AR parameters constrained to lie within the region corresponding to complex roots. The spectral generating function can be shown to be maximum at λ_c.

The cycle can otherwise be incorporated within the trend by defining a *cyclical trend*:

$$\mu_t^\dagger = \mu_{t-1}^\dagger + \beta_{t-1} + \psi_{t-1} + \eta_t \qquad (5)$$

where β_t and ψ_t follow (2) and (3) respectively.

The seasonal pattern is denoted by γ_t and is modelled by a set of nonstationary stochastic cycles defined at the seasonal frequencies $\lambda_i = 2\pi i/s, i = 1, \ldots, [s/2]$, s being the number of seasons in the data and $[s/2] = s/2$ for s even and $[s/2] = (s-1)/2$ for s odd; thus:

$$\gamma_t = \sum_{i=1}^{[s/2]} \gamma_{it}$$

where:

$$\begin{bmatrix} \gamma_{it} \\ \gamma_{it}^* \end{bmatrix} = \begin{bmatrix} \cos\lambda_i & \sin\lambda_i \\ -\sin\lambda_i & \cos\lambda_i \end{bmatrix} \begin{bmatrix} \gamma_{i,t-1} \\ \gamma_{i,t-1}^* \end{bmatrix} + \begin{bmatrix} \omega_{it} \\ \omega_{it}^* \end{bmatrix},$$

with ω_{it} and ω_{it}^* being two uncorrelated zero mean white-noises with variance $\sigma_{\omega_i}^2$ and $\sigma_{\omega_i^*}^2$ respectively.

When $s = 4$ (quarterly data), as in our case study, the seasonal component is the result of two cycles: the first is defined at the *fundamental frequency* $\pi/2$, corresponding to a period of 4 quarters, and has the single equation representation:

$$\gamma_{1t} = \frac{\omega_{1t} + \omega_{1,t-1}^*}{1 + L^2},$$

which is a nonstationary $ARMA(2,1)$ process with two complex conjugate roots i and $-i$, with i denoting the imaginary unit ($i^2 = -1$), of modulus one. The second is defined at the *harmonic* π (two cycles per year) and, as for $i = 2$, $\cos\lambda_i = -1$ and $\sin\lambda_i = 0$, it collapses to:

$$\gamma_{2t} = \frac{\omega_{2t}}{1+L},$$

and therefore $\gamma_{2t} \sim AR(1)$ process integrated at frequency π.

Finally, we can think of modelling the irregular component ε_t as a white noise variable with zero mean and variance σ_ε; to complete the specification it is assumed that the disturbances in all four components are mutually uncorrelated.[2]

A first way of putting the pieces together is to assume that the components combine orthogonally to yield the model:

$$y_t = \mu_t + \psi_t + \gamma_t + \varepsilon_t, \qquad (6)$$

which will be labelled as BSCM, since it is an extension of the Basic Structural Model (BSM) allowing for the presence of a cyclical component.

The second is the Cyclical Trend plus Seasonal Model (CTSM):

$$y_t = \mu_t^\dagger + \gamma_t + \varepsilon_t, \qquad (7)$$

The conditions $0 < \rho < 1$ and $0 \leq \lambda_c \leq \pi$ are sufficient to achieve the identifiability of both models: in fact, if $\rho = 0$ then $\psi_t = \kappa_t \sim WN$ and it cannot be distinguished from the irregular term; on the other hand, when $\rho = 1$ and $\lambda_c = 2\pi i/s$, $i = 1, \ldots, [s/2]$ the cycle cannot be separated from the seasonal component at frequency λ_i.

Since all starred quantities appear by construction and are devoid of meaningful interpretation, it is customary, for the sake of parsimony, to impose the overidentifying restrictions $\sigma^2_{\kappa^*} = \sigma^2_\kappa$ and $\sigma^2_{\omega_i^*} = \sigma^2_{\omega_i} = \sigma^2_\omega$, $i = 1, \ldots, [s/2]$.

An alternative way of modelling the short run dynamics consists in allowing y_t to follow a finite order autoregression:

$$\varphi(L) y_t = \mu_t + \gamma_t + \varepsilon_t, \qquad (8)$$

[2] We may otherwise assume that they are perfectly correlated, i.e. that there is a single disturbance driving all components. The basic restriction posed by identifiability is that we cannot estimate the degree of correlation among these disturbances.

where $\varphi(L)$ is a p-th order polynomial in the lag operator whose roots lie outside the unit circle. Model (8), which will be referred to as the Autoregressive Basic Structural Model (ABSM), may account for pseudo-cyclical behaviour providing the roots of the autoregressive polynomial are complex.

Both the trend and the seasonal are subject to the same autoregressive effects and there is no way of recovering the orthogonal decomposition into trend, cycle, seasonal and irregular.

4. Maximum Likelihood Estimation and Model Selection

Estimation of the structural parameters can be performed by casting the above models in the state space form which consists of a *measurement equation* for the "residual" $\xi_t = y_t - \boldsymbol{x}_t'\boldsymbol{\delta}$:

$$\xi_t = \boldsymbol{z}_t'\boldsymbol{\alpha}_t + \varepsilon_t, \qquad t = 1, \ldots, T, \qquad (9)$$

and a *transition equation*:

$$\boldsymbol{\alpha}_t = \boldsymbol{T}_t\boldsymbol{\alpha}_{t-1} + \boldsymbol{\eta}_t, \qquad t = 1, \ldots, T, \qquad (10)$$

where $\boldsymbol{z}_t$ is an $m \times 1$ vector, $\boldsymbol{\alpha}_t$ is an $m \times 1$ state vector, $\boldsymbol{x}_t$ is a $k \times 1$ vector of explanatory variables, $\boldsymbol{\delta}$ is a $k \times 1$ vector of unknown parameters, $\boldsymbol{T}_t$ is an $m \times m$ transition matrix and ε_t, $\boldsymbol{\eta}_t$ are independent $NID(0, \sigma^2_\varepsilon)$ and $NID(0, \boldsymbol{Q}_t)$.

Define:

$$\boldsymbol{T}_\mu = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix},$$

$$\boldsymbol{T}_\psi = \begin{bmatrix} \rho\cos\lambda_c & \rho\sin\lambda_c \\ -\rho\sin\lambda_c & \rho\cos\lambda_c \end{bmatrix},$$

$$\boldsymbol{T}_\gamma = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{T}_{\mu^\dagger} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

Then:

- for the BSCM: $\boldsymbol{\alpha}_t = [\mu_t\ \beta_t\ \psi_t\ \psi_t^*\ \gamma_{1t}\ \gamma_{1t}^*\ \gamma_{2t}]'$, $\boldsymbol{z}_t' = [1\ 0\ 1\ 0\ 1\ 0\ 1]$, $\boldsymbol{\delta} = \mathbf{o}$ and $\boldsymbol{T} = diag\{\boldsymbol{T}_\mu, \boldsymbol{T}_\psi, \boldsymbol{T}_\gamma\}$ $\boldsymbol{\eta}_t = [\eta_t\ \zeta_t\ \kappa_t\ \kappa_t^*\ \omega_{1t}\ \omega_{1t}^*\ \omega_{2t}]'$, $\boldsymbol{Q}_t = diag\{\sigma_\eta^2, \sigma_\zeta^2, \sigma_\kappa^2, \sigma_\kappa^2, \sigma_\omega^2, \sigma_\omega^2, \sigma_\omega^2\}$.
- for the CTSM: $\boldsymbol{z}_t' = [1\ 0\ 0\ 0\ 1\ 0\ 1]$, $\boldsymbol{T} = diag\{\boldsymbol{T}_{\mu^\dagger}, \boldsymbol{T}_\psi, \boldsymbol{T}_\gamma\}$, with the remaining quantities defined as before;
- for the ABSM $\boldsymbol{\alpha}_t = [\mu_t\ \beta_t\ \gamma_{1t}\ \gamma_{1t}^*\ \gamma_{2t}]'$, $\boldsymbol{z}_t' = [1\ 0\ 1\ 0\ 1]$, $\boldsymbol{x}_t = [y_{t-1}\ y_{t-2}]'$, $\delta = [\varphi_1\ \varphi_2]'$, $\boldsymbol{T} = diag\{\boldsymbol{T}_\mu, \boldsymbol{T}_\gamma\}$ $\boldsymbol{\eta}_t = [\eta_t\ \zeta_t\ \omega_{1t}\ \omega_{1t}^*\ \omega_{2t}]'$, $\boldsymbol{Q}_t = diag\{\sigma_\eta^2, \sigma_\zeta^2, \sigma_\omega^2, \sigma_\omega^2, \sigma_\omega^2\}$.

Under the further assumptions that $\boldsymbol{\alpha}_0 \sim N(\boldsymbol{a}_0, \boldsymbol{P}_0)$ independently of ε_t and $\boldsymbol{\eta}_t$, the likelihood function is obtained from the Kalman filter via the prediction error decomposition and can be maximised numerically with respect to the structural parameters. In the autoregressive model the lagged values of the dependent variable are treated as exogenous and the Generalised Least Square transformation method described in Harvey (1989), section 3.4.2, is used.

Let $\boldsymbol{a}_{t|s}$ denote the estimate of $\boldsymbol{\alpha}_t$ based on the information available up to time s and also let its covariance matrix be $\boldsymbol{P}_{t|s} = E[(\boldsymbol{\alpha}_t - \boldsymbol{a}_{t|s})(\boldsymbol{\alpha}_t - \boldsymbol{a}_{t|s})']$; then the Kalman filter consists of the two well know sets of equations, the *prediction equations*,

$$\begin{aligned} \boldsymbol{a}_{t|t-1} &= \boldsymbol{T}_t \boldsymbol{a}_{t-1|t-1}, \\ \boldsymbol{P}_{t|t-1} &= \boldsymbol{T}_t \boldsymbol{P}_{t-1|t-1} \boldsymbol{T}_t' + \boldsymbol{Q}_t, \end{aligned}$$

and the *updating equations*,

$$\begin{aligned} \boldsymbol{a}_{t|t} &= \boldsymbol{a}_{t|t-1} + \boldsymbol{P}_{t|t-1} \boldsymbol{z}_t f_t^{-1} \nu_t, \\ \boldsymbol{P}_{t|t} &= \boldsymbol{P}_{t|t-1} - \frac{\boldsymbol{P}_{t|t-1} \boldsymbol{z}_t \boldsymbol{z}_t' \boldsymbol{P}_{t|t-1}}{f_t} \end{aligned}$$

where $\nu_t = \xi_t - \boldsymbol{z}_t' \boldsymbol{a}_{t|t-1}$ is the one-step-ahead prediction error (*innovation*) with $f_t = E(\nu_t^2) = \boldsymbol{z}_t' \boldsymbol{P}_{t|t-1} \boldsymbol{z}_t + \sigma_\varepsilon^2$.

The Kalman filter thus recursively computes the optimal (in the mean square error sense) estimator of the unobserved state vector $\boldsymbol{a}_{t|t-1}$ based on the information at time $t-1$, together with its covariance matrix $\boldsymbol{P}_{t|t-1}$ and updates it on the basis of the information available at time t.

The Gaussian log likelihood is then (apart from a constant):

$$L = -\frac{1}{2} \sum_{t=d+1}^{T} \frac{\nu_t^2}{f_t} - \frac{1}{2} \sum_{t=d+1}^{T} \ln f_t.$$

The summation runs from $d+1$, d denoting the number of nonstationary elements in the state vector (which corresponds to the degree of the difference operator needed to achieve a stationary and invertible form), when the algorithm is initialised with a diffuse prior for the nonstationary elements in $\boldsymbol{\alpha}_t$; see also Harvey and Shephard (1991).

Once the model has been fitted, the main diagnostics are based on the one step ahead innovations $\nu_t = y_t - \boldsymbol{x}_t' \tilde{\boldsymbol{\delta}} - \boldsymbol{z}_t' \boldsymbol{a}_{t|t-1}$, which are standardised by division by the standars deviations, $f_t^{\frac{1}{2}}$, so as to yield the *standardised innovations*:

$$\tilde{\nu}_t = \frac{\nu_t}{f_t^{\frac{1}{2}}},. \qquad t = d+1, \ldots, T.$$

The time-domain diagnostic quantities that we use in the discussion below are:

- the Ljung-Box Q statistic based on the first P autocorrelations of the standardised innovations:

  $$Q(P) = T^*(T^*+2) \sum_{\tau=1}^{P} (T^* - \tau)^{-1} r^2(\tau),$$

  where $T^* = T - d$ and $r(\tau)$ is the residual autocorrelation at lag τ; Q is asymptotically χ^2 with $P - n$ degrees of freedom, n being the number of estimated hyperparameters.

- A test for heteroskedasticity constructed from the residuals:

  $$H(h) = \left[\sum_{t=T-h+1}^{T} \tilde{\nu}_t^2 \right] \left[\sum_{t=d+1}^{d+1+h} \tilde{\nu}_t^2 \right]^{-1},$$

  where h is the nearest integer to $T^*/3$; this statistic can be tested against an $F(h,h)$ distribution.

- A test for normality is based on the Bowman and Shenton statistic:

$$N = \frac{T^*}{6}b_1 + \frac{T^*}{24}(b_2-3)^2 = N_1 + N_2$$

where $\sqrt{b_1}$ is the third moment of the standardised innovations about the mean and b_2 is the fourth moment; its null distribution is χ^2_2. N_1 and N_2 thus provide separate tests of skewness and kurtosis.

The one-step-ahead prediction error variance ($p.e.v. = \tilde{\sigma}^2$), estimated by f_T, is an important measure of goodness of fit that can be used for model comparison and selection; for instance, the likelihood ratio test of g restrictions on the $n \times 1$ parameter vector θ can be based on the statistic:

$$LR \simeq T^* \ln(\tilde{\sigma}_0^2/\tilde{\sigma}^2)$$

where the subscript "0" denotes the value of the $p.e.v.$ in the restricted model, which is asymptotically distributed as a χ^2_g. Non-nested models can be compared on the basis of the Akaike information criterion:

$$AIC = \tilde{\sigma}^2 exp[2(n+d)/T].$$

For seasonal data a relative measure of goodness of fit is provided by the coefficient of determination:

$$R^2_s = 1 - T^*\tilde{\sigma}^2/SSDSM$$

where $SSDSM$ is the sum of squares of first differences around the seasonal means, i.e.

$$SSDSM = \sum_{t=2}^{T}(\Delta y_t - \sum_{j=1}^{s}\hat{\beta}_j z_{jt})^2$$

z_{jt}'s are seasonal dummies taking value one in season j and zero otherwise and the $\hat{\beta}_j$'s are least squares estimates.

5. Estimation Results

The estimation results for the three classes of models, along with diagnostics and goodness of fit, are reported in tables 1-6 (appendix).

The main findings are summarised below:

- The irregular term is not needed in any model (the estimated variance of ε_t is zero); this is the result of the excess smoothness which characterises the data.

- Normality is sometimes violated due to the presence of structural breaks and outlying observations.

- The seasonal component shows little variation (relative to the other components) for series (1) to (5), whereas for series (8) it stands for the most relevant source of variation in the data.

- The values of the Q statistic underline the fact that significant autocorrelation is still left in the data. This is attributable to the nature of the periodic component which results from the overlapping of two cycles, one defined at the fundamental frequency and the other at the harmonic.

- The estimated BSCM fall within two broad classes: models which have $\sigma^2_\eta = 0$ (series (1), (2), (5), (6)) and models for which the variance of the slope σ^2_ζ is either zero or close to zero. The frequency at which the cycle is estimated is in tune with what we expected from spectral analysis; however, the estimated dumping factors are fairly close to one implying a high concentration of power about this frequency.

- In the CTSM the ML estimates of σ^2_η and of σ^2_ζ are either zero or very close to zero. Note that the power spectrum is no longer maximum at λ_c but to the left of it.

- The estimated ABSM are capable of originating the kind of pseudo-cyclical behaviour displayed by the data, since the roots of the autoregressive polinomial are complex and less than unity in modulus; moreover, all models have $\sigma^2_\zeta = 0$.

- Coming to model selection, it is apparent from the diagnostic tables that the BSCM doesn't provide the best explanation of the data. The performance of the CTSM and the ABSM is fairly similar though sometimes the former is less satisfactory in interpreting the autocorrelation structure of the data; selection according to the AIC would tend to favour the CTSM in the case of series (4) (6) and (7), whereas the latter is preferred in the remaining cases. In the case of series (8) the BSCM model does not perform too badly with respect to its competitors, the main reason being that the seasonal variations capture most of the dynamic of the data, so as to blur any discrimination according to the nature of the remaining component. Moreover, the CTSM and the ABSM are to be preferred on the grounds of parsimony.

The possible reason why the CTSM and the ABSM gave a similar fit is better understood by comparing their stationary form. When $\sigma_\eta^2, \sigma_\zeta^2$ and σ_ε^2 are zero the stationary form of model (7) is

$$\Delta_4 y_t = 4\beta + S(L)\psi_{t-1} + \Delta[(1+L)(\omega_{1t} + \omega^*_{1,t-1}) + (1+L^2)\omega_{2t}], \qquad (11)$$

where $S(L) = 1 + L + L^2 + L^3$. For the ABSM (8) with $\sigma_\zeta = \sigma_\varepsilon = 0$ we have:

$$\varphi(L)\Delta_4 y_t = 4\beta + S(L)\eta_{t-1} + \Delta[(1+L)(\omega_{1t} + \omega^*_{1,t-1}) + (1+L^2)\omega_{2t}]. \qquad (12)$$

Then, replacing ψ_{t-1} in (11) by expression (4) lagged one period and dividing both sides of (12) by $\varphi(L)$, the link between the two models is apparent, provided $\varphi(L)$ is capable of generating pseudo-cyclical behaviour. The main difference is that in the ABSM the cycle does not affect the trend alone, but it is embedded in the seasonal movements too.

If we now consider the stationary form of the BSCM with $\sigma_\varepsilon^2 = 0$ we may give a tentative explanation of its poor performance:

$$\Delta\Delta_4 y_t = \Delta_4\eta_t + S(L)\zeta_t + \Delta\Delta_4\psi_t + \Delta^2[(1+L)(\omega_{1t} + \omega^*_{1,t-1}) + (1+L^2)\omega_{2t}],$$

with the first term on the right hand side vanishing when $\sigma_\eta^2 = 0$; when $\sigma_\zeta^2 = 0$ it becomes:

$$\Delta_4 y_t = 4\beta + S(L)\eta_t + \Delta_4\psi_t + \Delta[(1+L)(\omega_{1t} + \omega^*_{1,t-1}) + (1+L^2)\omega_{2t}].$$

In all cases for which $\sigma_\zeta^2 \neq 0$, then y_t is rendered stationary by $\Delta\Delta_4$ and its time series properties differ from those implied by (11) and (12). It might be the case that overdifferencing has taken place, which would raise identifiability problems.

6. Conclusions

In this paper we have attempted to describe the quarterly series on consumer's expenditures recurring to three classes of structural models: the BSCM, implying an orthogonal decomposition trend+cycle+seasonals; the CTSM, which interprets the series as a sum of a cyclical trend and a seasonal component; the ABSM, for which both the trend and the seasonal component are subject to the same autoregressive effects.

Model selection favours the last two classes of models which are shown to give similar responses. This is supposedly due to the fact that they are more apt to interpret phenomena whose growth is cyclical, whereas the BSCM postulates the presence of a cycle in the *level*.

The fundamental implication of this finding is that the traditional distinction between two independent forces driving the short and long run movements is not warranted by these data; trend variations and business cycle movements appear to be substantially related instead. We also note in passing that one can view the BSCM as the stochastic counterpart of the trend stationary process dealt with by Nelson and Plosser (1982) in a different setup. In fact, in their paper two conflicting representations were contrasted, the deterministic trend plus stationary cycle representation and one in which there is only one source of shocks driving both the trend and the cyclical component and for which the Beveridge-Nelson decomposition is appropriate [see Beveridge and Nelson (1981)]. These processes could be discriminated by performing unit roots tests. The analogy with their results lies in the finding that even though we allow the trend to be stochastic the data do not support the idea that trend and cycle are the expression of separate forces. Obvoiusly nothing prevents us from carrying out some kind of Beveridge-Nelson decomposition on the CTSM and the ABSM in order to extract the transitory component.

Finally, if we were to accept the representation furnished by the CTSM then there would be only one source of disturbances driving the cyclical trend, and the interpretation arising would be that cyclical fluctuations produce changes in long run growth. We may, however, interpret the trend-cycle (and seasonals-cycle) interactions by reverting the causal chain: innovations in growth are the source of the business cycle fluctuations. Unfortunately, the above univariate analysis cannot provide a solution to this dilemma and in order to gain more insight on the interactions among trend, cycle and seasonals we would have to bring in additional information on the sources of their variations.

Appendix

Table 1: *Estimates of BSCM*

Series	σ^2_η	σ^2_ζ	σ^2_κ	ρ	λ_c	$2\pi/\lambda_c$	σ^2_ω	σ^2_ε
(1)	0	30	43	0.93	0.58	10.81	1	0
(2)	0	235	826	0.92	0.55	11.35	0	0
(3)	1124	5	0	-	-	-	84	0
(4)	881	23	2370	0.83	0.55	11.46	51	0
(5)	0	23	679	0.94	0.43	14.60	5	0
(6)	0	243	707	0.92	0.44	14.30	89	0
(7)	2360	0	179	0.95	0.41	15.14	102	0
(8)	8	28	53	0.95	0.60	10.42	234	0

Notes: estimation was carried out using the package STAMP. All variance estimates have been multiplied by 10^7; $2\pi/\lambda_c$ is the period (in quarters).

Table 2: *BSCM - Diagnostics and goodness of fit*

Series	N_1	N_2	N	$H(h)$	$Q(12)$	$p.e.v.$	R^2_s	AIC
(1)	0.03	0.02	0.05	0.37	31.04	212×10^{-7}	0.26	282
(2)	2.34	9.52	11.86	0.20	16.15	3786×10^{-7}	0.33	5038
(3)	0.32	75.15	75.47	0.05	62.99	3165×10^{-7}	0.39	3921
(4)	1.85	9.06	10.91	0.25	22.42	7908×10^{-7}	0.04	10523
(5)	0.04	0.29	0.33	0.66	46.22	1773×10^{-7}	0.25	2359
(6)	2.48	22.71	25.19	0.78	49.12	5206×10^{-7}	0.68	6928
(7)	1.02	32.79	33.81	0.05	33.64	5768×10^{-7}	0.44	7676
(8)	1.44	0.17	1.61	2.01	22.22	3940×10^{-7}	0.90	5243

Table 3: *Estimates of CTSM*

Series	σ^2_η	σ^2_ζ	σ^2_κ	ρ	λ_c	σ^2_ω	σ^2_ε
(1)	0	1	69	0.82	0.61	1	0
(2)	0	10	1308	0.74	0.63	20	0
(3)	0	8	410	0.78	1.05	51	0
(4)	0	0	3111	0.68	1.11	35	0
(5)	0	0	703	0.78	0.56	5	0
(6)	0	0	882	0.77	0.48	84	0
(7)	0	0	1429	0.64	0.81	84	0
(8)	0	2	118	0.80	0.57	236	0

Notes: all variance estimates have been multiplied by 10^7.

Table 4: *CTSM - Diagnostics and goodness of fit*

Series	N_1	N_2	N	$H(h)$	$Q(P)$	*p.e.v.*	R^2_s	AIC
(1)	0.59	0.23	0.82	0.38	27.17	156×10^{-7}	0.46	198
(2)	2.19	6.38	8.57	0.18	8.84	3202×10^{-7}	0.44	4062
(3)	2.84	35.53	38.37	0.04	87.62	2414×10^{-7}	0.53	2991
(4)	0.04	0.50	0.54	0.26	16.74	6556×10^{-7}	0.21	7932
(5)	0.14	0.11	0.15	0.74	30.48	1329×10^{-7}	0.44	1608
(6)	3.24	16.37	19.61	0.84	46.84	2533×10^{-7}	0.85	3064
(7)	0.01	49.45	49.46	0.04	39.58	3303×10^{-7}	0.68	3996
(8)	0.29	0.00	0.26	2.10	22.98	3927×10^{-7}	0.90	4983

Table 5: *Estimates of ABSM*

Series	φ_1	φ_2	σ^2_η	σ^2_ζ	σ^2_ω	σ^2_ε
(1)	1.12	-0.56	95	0	2	0
(2)	1.03	-0.45	1168	0	89	0
(3)	0.77	-0.64	578	0	62	0
(4)	0.53	-0.40	3751	0	86	0
(5)	1.09	-0.53	636	0	13	0
(6)	1.27	-0.57	432	0	230	0
(7)	1.01	-0.51	725	0	203	0
(8)	0.84	-0.17	76	0	464	0

Notes: all variance estimates have been multiplied by 10^7; all estimates are significant at the 1% level.

Table 6: *ABSM - Diagnostics and goodness of fit*

Series	N_1	N_2	N	$H(h)$	$Q(12)$	*p.e.v.*	R^2_s	AIC
(1)	1.21	0.40	1.61	0.40	16.59	160×10^{-7}	0.55	194
(2)	1.19	6.34	7.53	0.14	5.38	3114×10^{-7}	0.48	3767
(3)	0.74	1.19	1.93	0.08	41.49	1794×10^{-7}	0.68	2170
(4)	0.56	0.00	0.56	0.29	10.81	6650×10^{-7}	0.18	8045
(5)	0.14	0.11	0.15	0.59	10.75	1106×10^{-7}	0.52	1338
(6)	3.17	2.84	6.01	0.66	23.81	3286×10^{-7}	0.81	3975
(7)	3.62	45.51	49.13	0.05	15.14	3708×10^{-7}	0.62	4486
(8)	2.67	5.89	8.56	1.75	14.27	3612×10^{-7}	0.91	4370

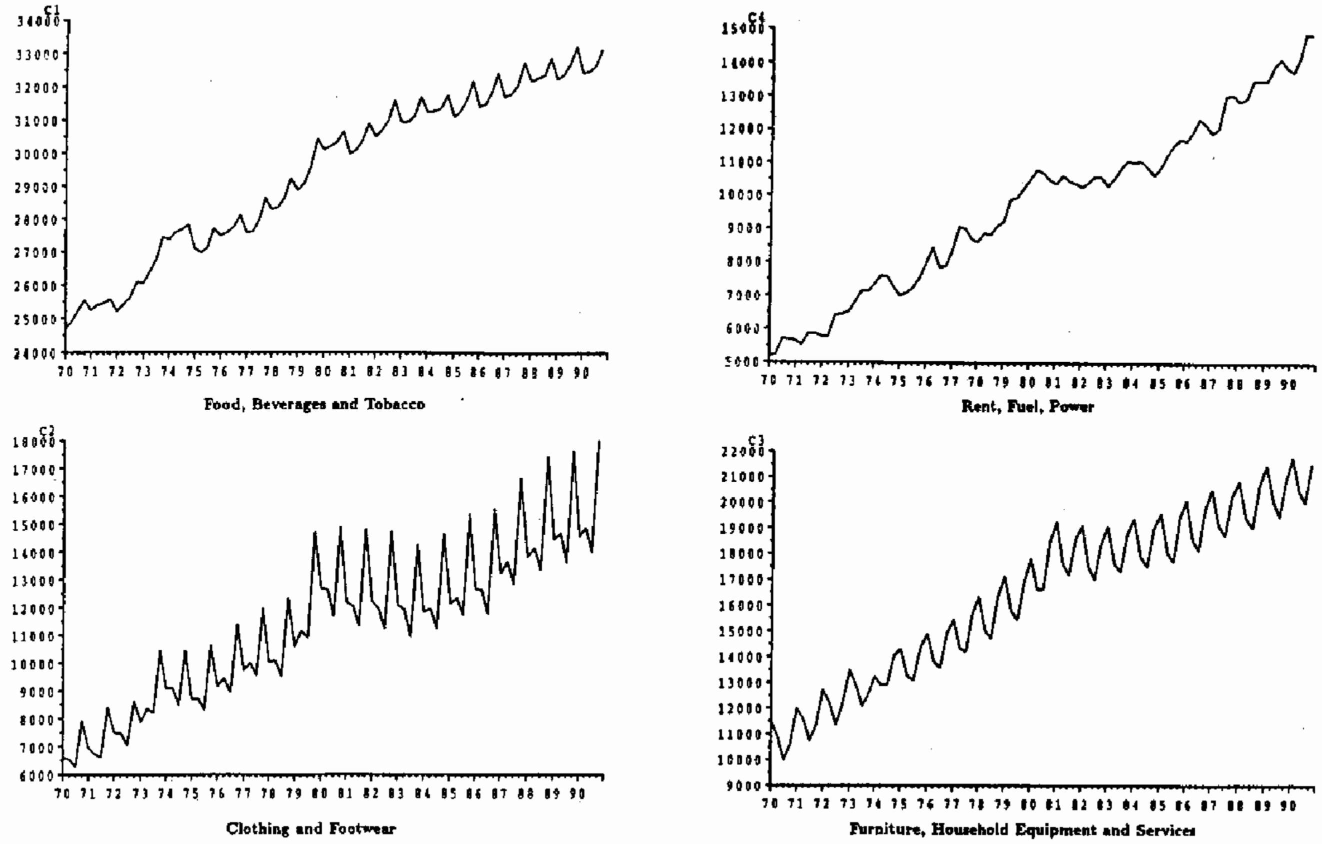

Figure 1: Original Series

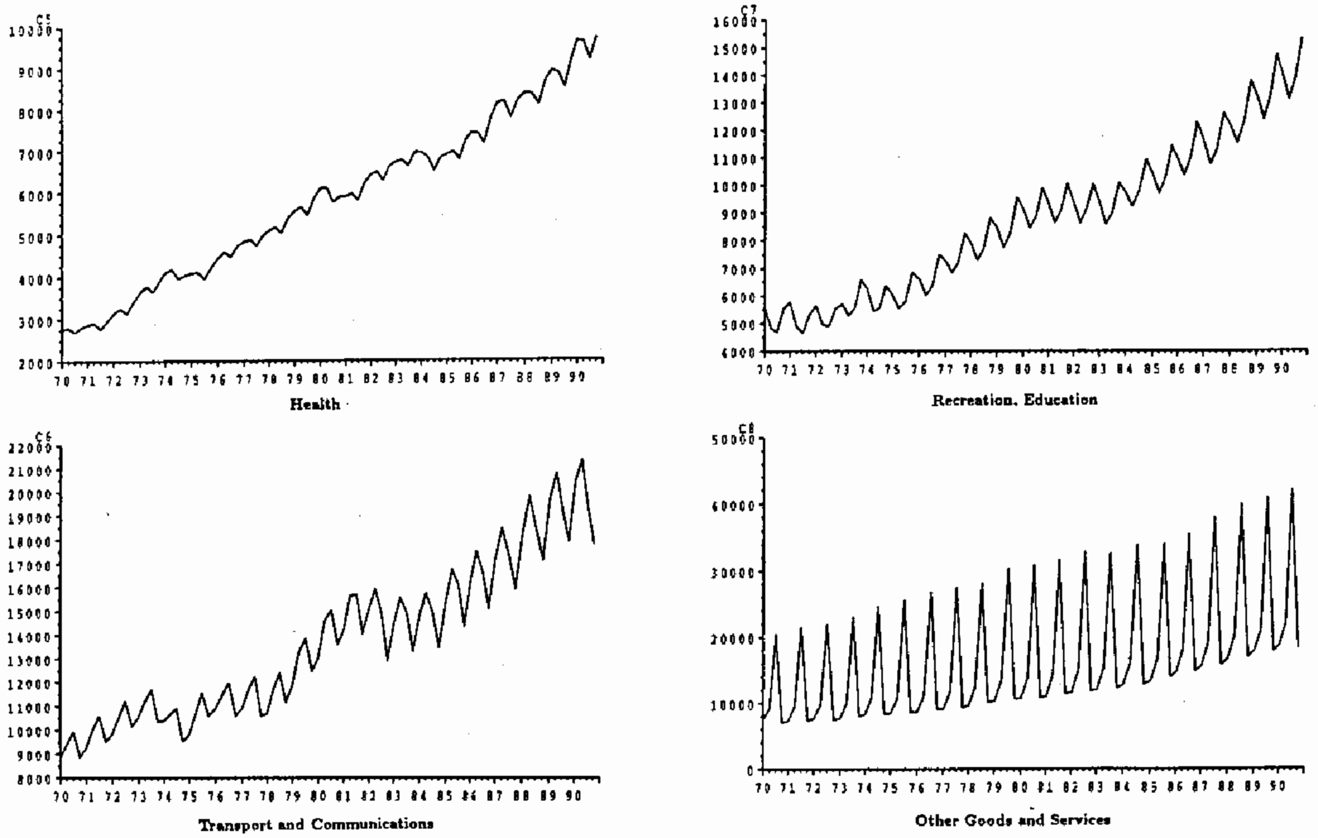

Figure 1 (continued): Original series

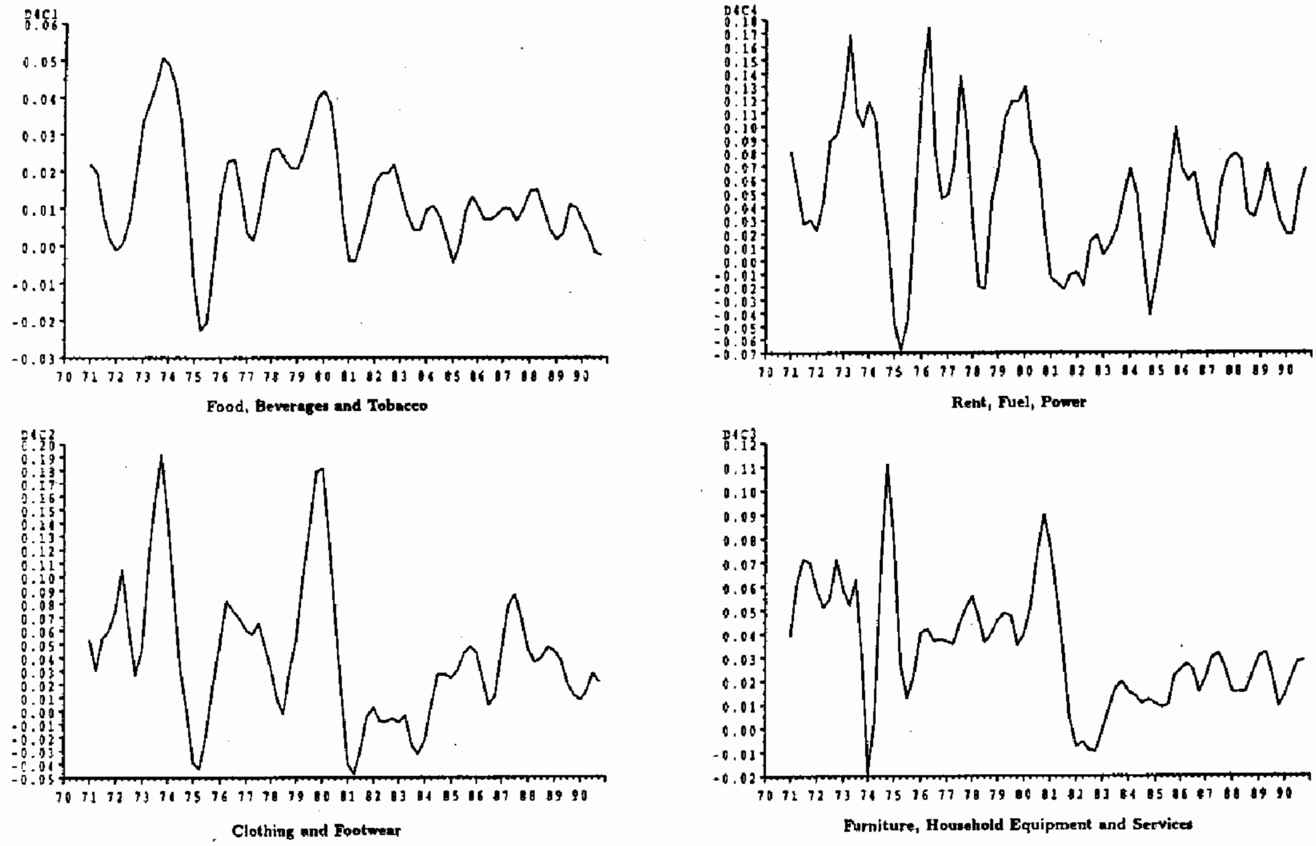

Figure 2: Seasonal differences

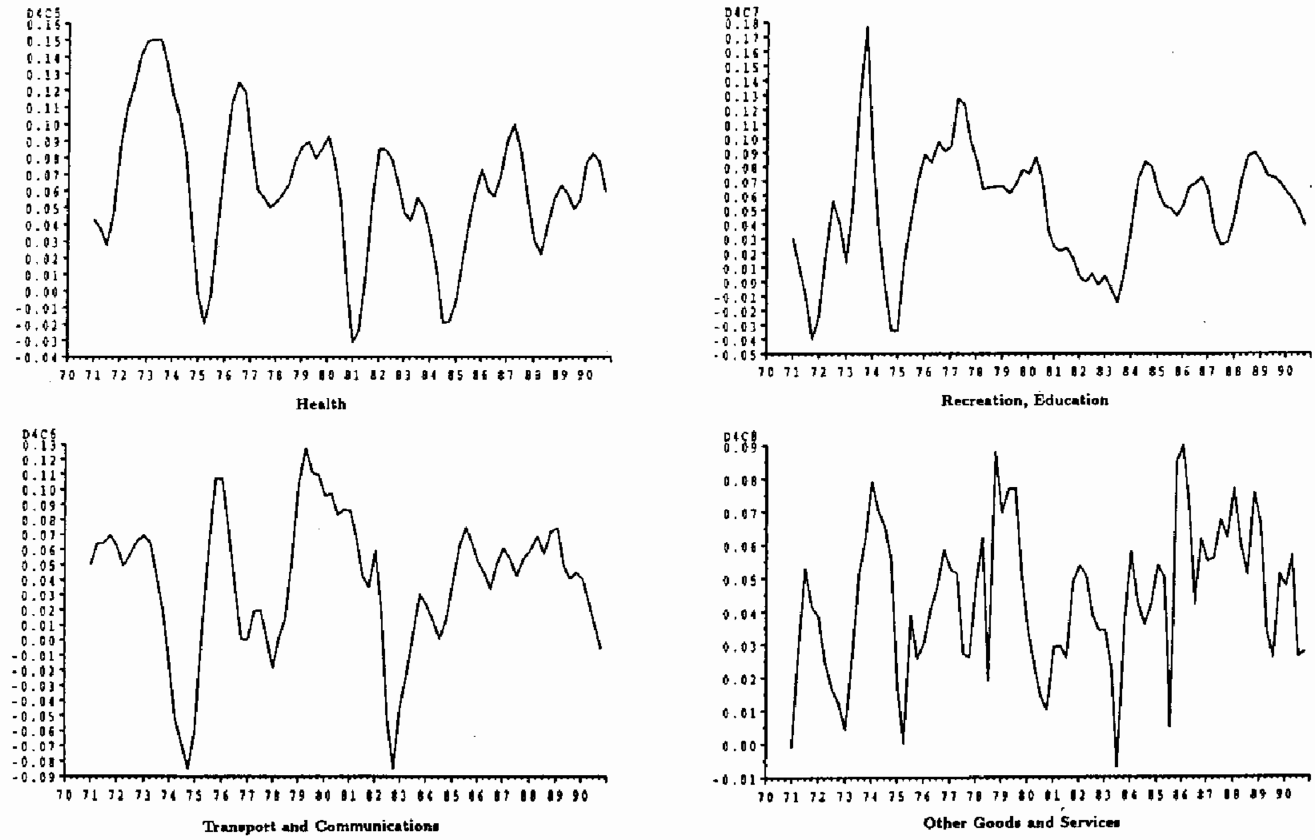

Figure 2 (continued): Seasonal differences

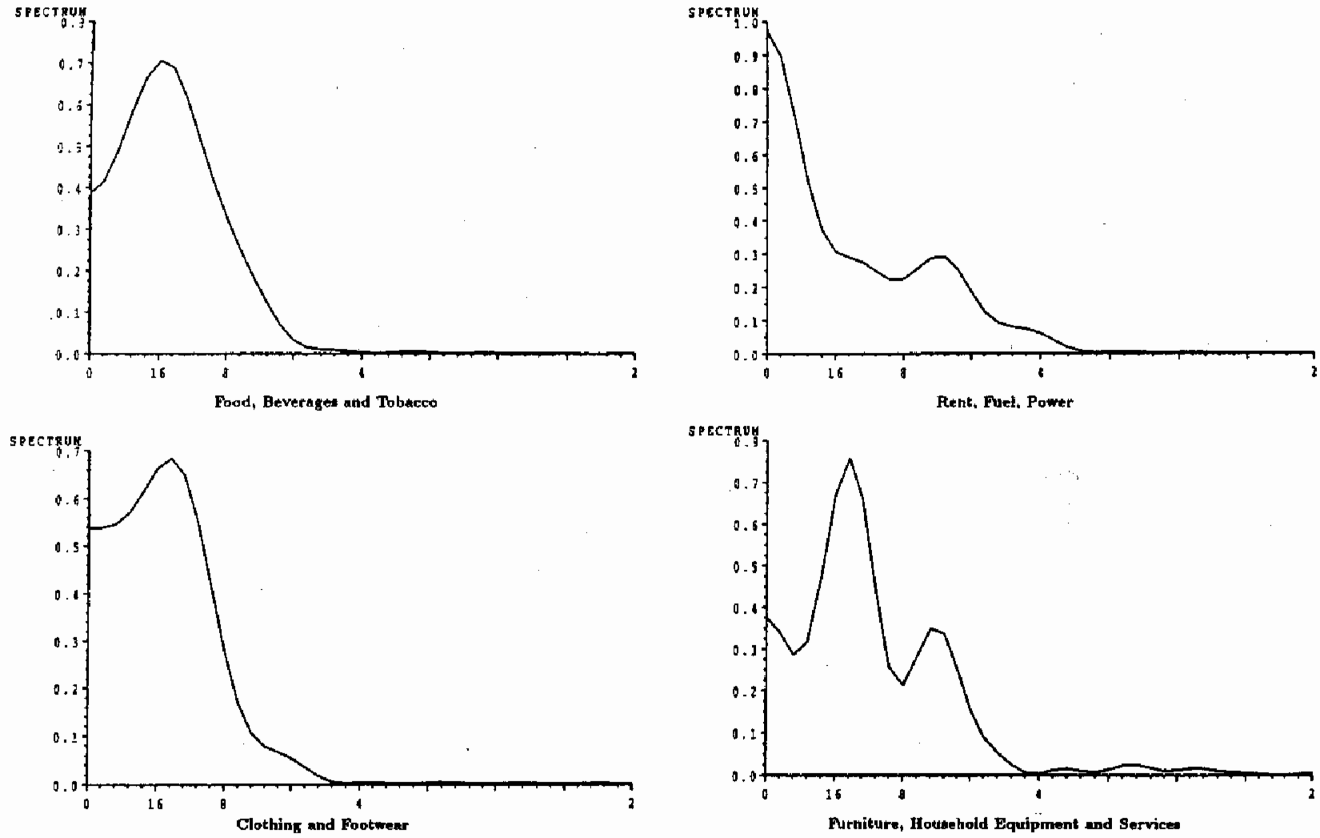

Figure 3: Power spectra (spectral ordinates are plotted versus period in quarters)

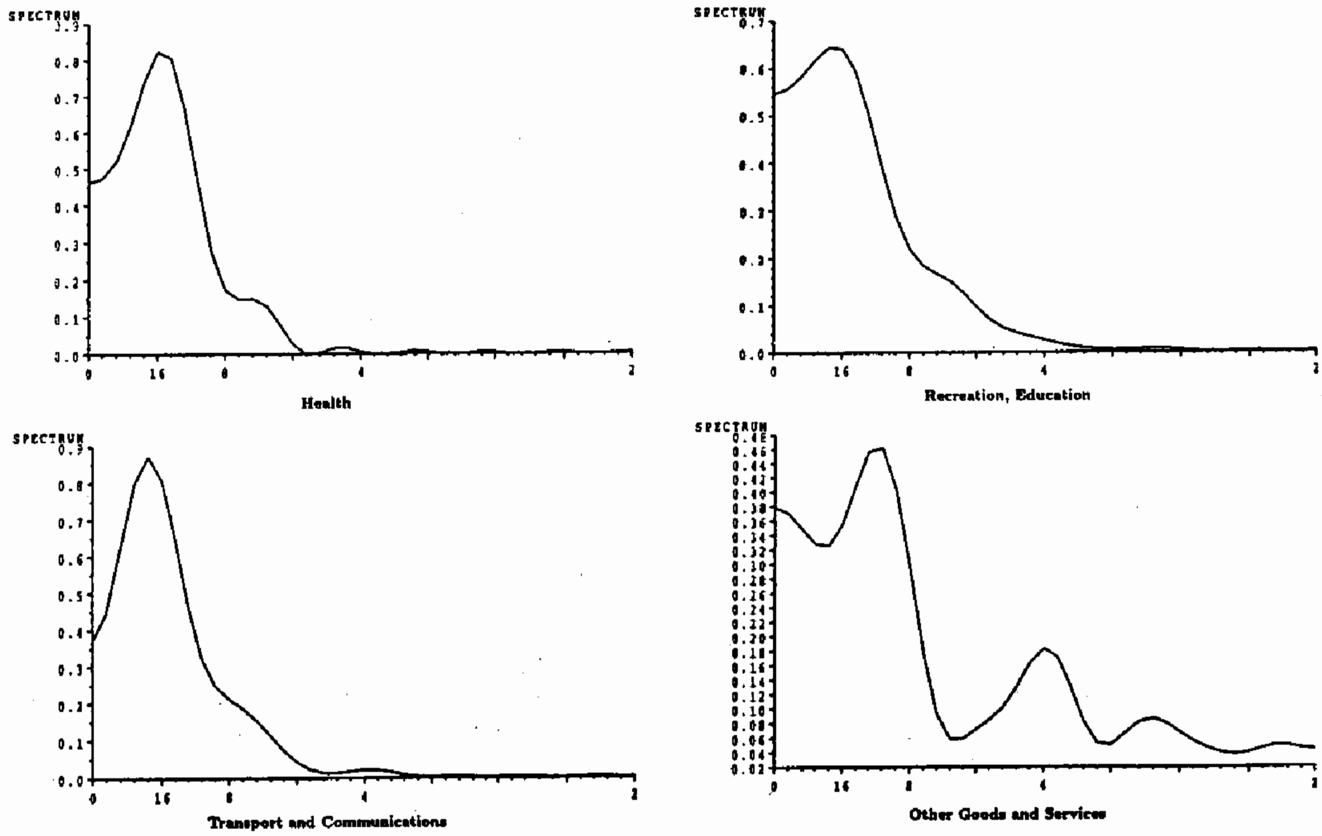

Figure 3 (continued): Power spectra

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