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A SEASONAL INTEGRATION ANALYSIS OF THE ITALIAN CONSUMPTION QUARTERLY TIME SERIES

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1. Introduction

The empirical analysis of consumer behaviour has been extensively researched in the literature for a considerable time, giving rise to a number of issues which, to a great extent, can be grouped into two main research approaches. [1]

The first concerns the attempt to represent the preference of the consumers, through the estimation of increasing general and flexile functional forms, considering alternative levels of disaggregate expenditure data and commodities. These contributions have emphasised the nature and limits of the different generalizations of the functional forms, assuming almost the same base assumptions. According to these hypotheses the consumer tends, in a *static context* and with predefined budget constraints, to optimise an objective function.

The second research approach deals with the informative and "technological" changes in the economic consumer environment, focusing on how budget constraints, dynamic structures and heterogeneity of preferences may provide a direct influence on consumption.

From a more specific statistical point of view, demand analysis has focused on the investigation of the empirical relationship between consumption and the following: the rate of saving, the intersectorial and production activity linkage, the dynamics with other macroeconomic aggregates, the economic and social regional systems, the household characteristics, with the aim of pointing out the most evident structural and evolving patterns.

The quarterly time series of consumption data recently made available for the period 1970:1-1990:4 (benchmark 1985), are divided into 50 groups of commodities and services, and 8 categories [2], collected from diary records of the Family Expenditure Survey. These, along with and the corresponding quarterly time series estimates of the national economic accounts, allow for further research on consumer behaviour.

Coincidental to the interest in developing research in demand analysis, since the end the seventies there has been increasing concern among statisticians and econometricians in the long-run equilibrium behaviour of time series that are *difference stationary* processes (see, among others, Granger 1981, Hendry 1986).

In the light of the recent data on Italian consumption, and the methodological context of integration and cointegration, which defines a uniform approach within which it is possible to analyse time series that present common stochastic trends and seasonals, the purpose of this paper is to examine the quarterly time series of

[1] For a recent survey on demand analysis referring to Italian contributions, see Bollino and Rossi (1987), and the papers collected in the special issue of *Studi e Informazioni*, 1987 n.4. As regards to the new topics on the theory of consumption in relation to the aspect of statistical survey adequacy, see Filippucci (1992).

[2] Our analysis focuses on the following expenditures categories at 1985 constant prices: 1. *Food, Beverage and Tobacco*; 2. *Clothing and Footwear*; 3. *Rent, Fuel, Power*; 4. *Furniture, Household Equipment and Services*; 5. *Health*; 6. *Transport and Communications*; 7. *Recreation, Education, etc.*; 8. *Other Goods and Services* .

consumption for the eight categories of expenditures, made available by Istat, for both the time series of family budget surveys and national accounts. The main aim is to estimate the order of integration at zero and seasonal frequencies of the series to determine whether the different conceptual definitions existing between the two sources of data indicate a common pattern of actual coherence. It should be pointed out that both the statistical methods of estimation, which have slowly changed through time, as well as the limits imposed by the availability of time series for only a relatively short period of time and the contextual presence of seasonal and trend components in the data, call for some caution in evaluating our results, which should be considered as a first step in the ongoing statistical analysis of the time series on consumption in Italy.

The paper is organised as follows. Section 2 briefly examines the available data on private consumption, pointing out their main characteristics and statistical aspects. Section 3 introduces the notion of seasonal integration. Section 4 reviews the tests proposed for testing the presence of unit roots at both seasonal and zero frequencies. Section 5 explains how these can be put together so as to form a testing sequence. Section 6 presents the empirical results and section 7 contains a few concluding remarks.

2. Some general remarks on the Italian consumption quarterly time series

The quarterly time series drawn from the Household Expenditure Survey differ in many respects from those reported in the National Accounts. This circumstance limits the comparability of the two sources, even if both aim at the measurement of private consumption as economic aggregate. As it is known, while the Family Budget estimates are the results of questionnaires which are designed to gather exaustive detail on household expenditures, the National Accounts data are obtained through an indirect estimation procedure and fully represent the aggregate expenditure level which a community bears to directly satisfy individual needs.

The consumption estimates determined by the Household Expenditure Survey should constitute the most informative data set for the National Accounts estimates. This linkage is, however, weaker than it may seem because the former, besides referring to different populations, tend to underestimate the expenses for durable and semidurable commodities. This aspect rather limits the possibilities of the survey as an exhaustive information base, with the consequence that only the National Accounts category of *Food, Beverage and Tobacco* depends, to a great extent, on the observed household expenditures (Innocenzi 1989 a, 1989 b; Istat 1990 a, 1990 b).

Another aspect which prevents comparability is the different procedures used to estimate the quarterly series. While, in fact, the quarterly estimates for the Household Expenditure Survey are obtained by aggregating the monthly reported survey data to the relevant quarter, the National Accounts consumption aggregates are the result of a procedure which distributes the annual estimates across the quarters by using specific indicators.

Generally, the indicator series drawn from the Household Expenditure Survey are extensively employed to estimate the distribution of the National Account series over time, with the exception of the *Rent, Fuel, Power* and *Transport and Communications* categories, for which they are used to distribute series whose weight is less than one third.

Moreover, the latter differ even for the backward-in-time estimates, given benchmarck revisions and a different aggregation level for the expenditure categories as well. This implies that, in spite of the fact that the two sources of data have a common conceptual definition, they underlie estimation methods which imply differences not always and not only connected with the consumer related populations.

In particular, while the diversification in the two sources of data here considered (benchmarck 1985) is rather reasonable for the *Food, Beverage and Tobacco*, and *Rent Fuel and Power* categories for which the Household Expenditure Survey data are on average 10% lower, the same percentage rises up to 30% for durable goods consumptions (Tassinari e Mantegazza, 1992). These concluding remarks are largely in agreement with the results of Tassinari and Viviani (1990). Their empirical investigations have suggested that while there exists a substantial agreement between the two sources as regards expenditures for commodities with an income elasticity less than unit, the same does not hold for the expenditure groups which include the consumer durables and the so-called household inventories and services which present an elastic income response (*Furniture, Household Equiment* and *Services, Recreation, Education, etc.*). Moreover other contributions suggest that the time structure of some consumption categories is affected by step outliers connected with survey methodology changes which imply a different weight to the total variance of the series of the trend and seasonal components (Daddi and Viviani, 1992).

3. Seasonal Integration

The following definition, by Engle *et al.* (1989), provides a useful extension of the notion of integration, initially introduced for non-seasonal stochastic processes:

Def. 1 Let y_t denote an indeterministic process; then y_t is said to be *integrated* of order d at frequency λ, denoted $y_t \sim I_\lambda(d)$, if it has a spectrum, $f(\omega)$, taking the form:

$$f(\omega) \propto (\omega - \lambda)^{-2d},$$

in a neighbourhood of λ.

According to this definition the random walk is a process integrated at frequency $\lambda = 0$; as another example consider the cyclical process $(1 - \phi_1 L - \phi_2 L^2)y_t = \epsilon_t$ with $\phi_1 = 2\cos\lambda$, $\phi_2 = -1$ and $\epsilon_t \sim WN(0, \sigma^2)$: when $\lambda \in (0, \pi)$, we have $y_t \sim I_\lambda(1)$, since the spectrum is proportional to $(\cos\omega - \cos\lambda)^{-2}$ and thus it is infinite at $\omega = \lambda$. When $\lambda = 0$, $y_t \sim I_0(2)$; finally, when $\lambda = \pi$, $y_t \sim I_\pi(2)$. For a definition encompassing Def. 1, see Joyeux (1992).

Let us now consider a process which is observed s times a year, with s even (typically it will be $s = 4$ for quarterly data and 12 for monthly data); this process is said to be *seasonal* if its spectrum shows concentration of power at the seasonal frequencies $\lambda_j = 2\pi j/s$, $j = 1, \ldots, s/2$.

There are several ways in which seasonal behaviour may arise; we will assume that they are particular cases of the following data generating process:

$$\psi(L)y_t = \mu_t + \epsilon_t, \qquad t = 1, \ldots, T, \tag{1}$$

where $\psi(L)$ is a lag polynomial of degree p and $\epsilon_t \sim WN(0, \sigma^2)$; μ_t is the component responsible for the deterministic behaviour:

$$\mu_t = \sum_{j=1}^{s} \mu_j S_{jt} + \beta t$$

where the S_{jt}'s are seasonal dummies taking value one in season j and zero otherwise; μ_j are the seasonal means. Segmented trends may also be allowed in μ_t.

The process is stationary if $\psi(L)$ has all its roots lying outside the unit circle and is seasonal when some roots are a complex conjugate pair at seasonal frequencies; for instance the process generated by $y_t = \psi y_{t-4} + \epsilon_t$ with $|\psi| < 1$ has a spectrum reaching its maximum value $(2\pi)^{-1}\sigma_\epsilon^2/(1 - \psi)^2$ at frequencies $0, \pi/2$ and π corresponding to the roots $\psi^{-1/4}$, the complex pair $\pm i\psi^{-1/4}$ and $-\psi^{-1/4}$ respectively. As $\psi \to 1$ the spectrum is unbounded at those frequencies.

When some or all the roots of the autoregressive polynomial are on the unit circle, we can factorize the AR polynomial as follows:

$$\varphi(L)\phi(L)y_t = \mu_t + \epsilon_t \tag{2}$$

where $\varphi(L)$ is a nonstationary AR polynomial whose roots are unity in modulus and $\phi(L)$ is a stationary AR polynomial of order q. y_t is an integrated seasonal process if the spectrum is unbounded at seasonal frequencies $\lambda_j = 2\pi j/s$. The leading cases of interest are: (i) $\varphi(L) = S(L) = 1 + L + \cdots + L^{s-1}$, the seasonal summation operator; (ii) $\varphi(L) = \Delta_s = 1 - L^s$, the seasonal difference operator; (iii) $\varphi(L) = \Delta\Delta_s = (1 - L)(1 - L^s)$.

Let us consider process (2) in case (i) with $s = 4$ (quarterly data): the seasonal summation operator can be factorised as $S(L) = (1 + L)(1 + iL)(1 - iL)$ whence it can be seen that the roots are the complex pair $\pm i$ and -1; correspondingly the power spectrum is unbounded at the fundamental frequency $\pi/2$ and at the harmonic frequency π. Two cycles thus combine in a multiplicative fashion and are responsible for the seasonal behaviour: the first is defined at the fundamental frequency, corresponding to a one-year period; the second is defined at the harmonic frequency and has a two-quarters period[3].

The dynamic properties of an integrated seasonal process differ in many respects from those characterising a stationary one: first and foremost all shocks are persistent and may have a permanent impact on the seasonal pattern; furthermore, the variance increases as we move away from the beginning.

The seasonal difference operator $\Delta_4 = \Delta S(L)$ has four unit roots 1, -1 and the complex pair $\pm i$; thus it is also integrated of order 1 at frequency 0. When $\varphi(L) = \Delta\Delta_4 = \Delta^2 S(L)$ the process is $I_0(2), I_{\pi/2}(1)$ and $I_\pi(1)$.

For these leading cases we envisage the need of a more synthetic notation. Hence, we introduce the following definition, also by Engle *et al.* (1989):

Def. 2: (*Seasonal integration*) y_t is seasonally integrated of orders d_0 and d_s, denoted $y_t \sim SI(d_0, d_s)$, if $\Delta^{d_0} S(L)^{d_s} y_t$ is stationary and invertible.

According to this definition the process $\Delta\Delta_s y_t = \mu + \theta(L)\epsilon_t$ is $SI(2,1)$ provided that $\theta(L)$ has all its roots outside the unit circle (i.e. $|\theta(e^{-i\omega})|^2\sigma_\epsilon^2$ is bounded away from zero and infinity $\forall\omega$).

An alternative definition of the order of seasonal integration, corresponding to a different factorization of the $\varphi(L)$ polynomial, has been considered by Osborn *et al.* (1988):

Def. 3: y_t is *integrated* of orders d and D, denoted $y_t \sim I(d, D)$, if $(1-L)^d(1-L^s)^D y_t = \Delta^d \Delta_s^D y_t$ admits a stationary and invertible Wold representation.

Def. 3 is more in line with standard ARIMA terminology. Of course, if $y_t \sim I(d, D)$ then it is also $y_t \sim SI(d+1, D)$ and vice versa; nevertheless, Def. 2 maintains the distinction between integration at seasonal and nonseasonal frequencies and thus will be adopted here.

4. Testing for seasonal integration

The subject of testing for seasonal integration relative to integration at zero frequency has received little attention. This may be related to the availability of seasonally adjusted data. However, it has been argued that the use of such data may unduly influence the inferences on the order of integration at frequency zero:

[3] In general $S(L)$ has $(s-2)/2$ pairs of complex roots at frequencies $\lambda_j = 2\pi j/s$, $j = 1, \ldots, s/2$ together with a root -1 at $2\pi/s$.

due to the nature of the adjustment filters, it usually results in increased evidence supporting the null of integration (see Ghysels, 1990).

This section aims at reviewing the main tests proposed for seasonal integration. Some are straightforward extensions of zero-frequency unit root tests, whereas others are especially tailored to deal with integration at each of the frequencies λ_j, $j = 1, \ldots, s/2$.

4.1. Augmented Dickey-Fuller (ADF) Test

Said and Dickey (1981) addressed the question of testing for a unit root at frequency zero when the data generating process is approximated by the finite autoregression (1); for this purpose they consider the reparametrization:

$$\Delta y_t = \mu_t + \psi^* y_{t-1} + \sum_{j=1}^{p-1} \psi_j^\dagger \Delta y_{t-j} + \epsilon_t,$$

where $\psi^* = -\psi(1)$ and $\psi_j^\dagger = -\sum_{i=j+1}^{p} \psi_i$.

Testing for the presence of root 1 in the autoregressive polynomial is equivalent to testing $H_0 : \psi^* = 0$ which is done considering the studentized statistic associated with the OLS estimate of ψ^*. The distribution of the test statistic is tabulated by Fuller (1979) and varies according to whether $\mu_t = 0$, $\mu_t = \mu$ or $\mu_t = \mu + \beta t$.

An amended version of the ADF test has been advocated by Dickey *et al.* (1986); this is tailored for processes containing a deterministic seasonal component in μ_t and the authors show that removal of seasonal means has no effect on the limit distribution of the test statistics[4].

The testing sequence in Ilmakunnas (1990) considers four ways in which the ADF test, with correction for seasonal means, can be employed for inferences on unit roots:

i. $H_0 : y_t \sim SI(1,0)$ vs. $H_1 : y_t \sim SI(0,0)$: ADF test on y_t.
ii. $H_0 : y_t \sim SI(2,0)$ vs. $H_1 : y_t \sim SI(1,0)$: ADF test on the Δy_t series.
iii. $H_0 : y_t \sim SI(2,1)$ vs. $H_1 : y_t \sim SI(1,1)$: ADF test on the $\Delta_4 y_t$ series.
iv. $H_0 : y_t \sim SI(1,1)$ vs. $H_1 : y_t \sim SI(0,1)$: ADF test on the $S(L)y_t$ series.

These tests are understood to provide valuable information so long as the maintained order of integration at seasonal frequencies d_s is correct.

4.2. Dickey-Hasza-Fuller (DHF) Test

Dickey *et al.* (1984) consider the problem of testing $H_0 : y_t \sim SI(1,1)$ against

[4] The insensitiveness to the presence of deterministic seasonality has also been observed by Osborn *et al.*.

the alternative $SI(0,0)$. Thus the null model would be (2) with $\varphi(L) = \Delta_4$ whereas the alternative model is:

$$(1 - \varphi_s L^s)\phi(L)(y_t - m_t) = \epsilon_t$$

with $|\varphi_s| < 1$ and $m_t = \sum_{j=1}^{s} m_j S_{jt}$ (m_t is related to μ_t by $\mu_t = m_t/\psi(1)$).

The authors propose the following two-step procedure: a) let $Y_t = y_t - \hat{m}_t$ denote the deviations from the estimated seasonal means; estimate the q-th order autoregression $\phi(L)\Delta_s Y_t = \epsilon_t$ to get consistent estimates of the ϕ_j's under the null; b) denote $\theta = [\varphi_s, \phi_1, \ldots, \phi_q]'$, by a first order Taylor expansion around the true parameter value the residuals evaluated at $\hat{\theta} = [1, \hat{\phi}_1, \ldots, \hat{\phi}_q]'$ can be expressed as

$$\epsilon_t(\hat{\theta}) = \epsilon_t(\theta) - \hat{\phi}(L)Y_{t-s}(1 - \varphi_s) - \sum_{j=1}^{q} \Delta_4 Y_{t-j}(\hat{\phi}_j - \phi_j) + r_t$$

where r_t is the remainder. Hence, in terms of the transformed variable $Z_t = Y_t - \hat{\phi}_1 Y_{t-1} - \cdots - \hat{\phi}_q Y_{t-q} = \hat{\phi}(L)Y_t$, the expression suggests regressing the residuals $\Delta_s\hat{\phi}(L)Y_t = \Delta_s Z_t$ on Z_{t-s} and q lagged values of $\Delta_s Y_t$ to get an estimate of the coefficient $\varphi_s - 1$. A test of the hypothesis $\varphi_s = 1$ is provided by the studentised statistic on the OLS estimate of the coefficient associated with Z_{t-s}; its critical values are given in table 7 by Dickey *et al.* (1984). It should be stressed that the seasonal mean model contemplated by the authors does not have a time trend component.

If the DHF test is carried out on the first differences of the original series Δy_t, then we test $SI(2,1)$ versus $SI(1,0)$.

4.3. Hylleberg, Engle, Granger and Yoo (HEGY) test

The testing schemes considered so far ignore the possibility that the series is integrated at some but not all of the seasonal frequencies; this situation is remedied by Hylleberg *et al.* who consider the null $y_t \sim SI(1,1)$ and break it up according to the decomposition of the seasonal difference operator: $\Delta_4 = (1-L)(1+L)(1+iL)(1-iL)$.

Assuming that the data are generated by the finite autoregression (1), then the authors show that by a Lagrange expansion of $\psi(L)$ around the roots 1, -1 and $\pm i$ it is possible to write:

$$\phi(L)\Delta_4 y_t = \pi_1 Z_{1,t-1} + \pi_2 Z_{2,t-1} + \pi_3 Z_{3,t-2} + \pi_4 Z_{3,t-1} + \mu_t + \epsilon_t, \qquad (3)$$

where $Z_{1t} = S(L)y_t$, $Z_{2t} = -(1 - L + L^2 - L^3)y_t$, and $Z_{3t} = -(1 - L^2)y_t$.

The usefulness of this parametrization is related to the connections between the roots of the polynomial $\psi(L)$ and the parameters π_i's ($i = 1,2,3,4$) in (3): $\psi(1) = 0$ implies that $\pi_1 = 0$ so that the null of a zero frequency unit root can be tested

against $\pi_1 < 0$ (corresponding to the stationary alternative $\psi(1) > 0$); similarly $\psi(-1) = 0$ (a unit root at frequency π) implies $\pi_2 = 0$, whereas the stationary alternative $\psi(-1) > 1$ implies that $\pi_2 < 0$. The null hypothesis that the series is $I_{\pi/2}(1)$, i.e. $|\psi(i)| = 0$ corresponds to both π_3 and π_4 being equal to zero; if either π_3 or π_4 is different from zero, then the process is stationary at $\pi/2$.

Equation (3) can be estimated by least squares and the t-statistics associated with the parameters π_i ($i = 1,2,3,4$) can be used to test the presence of unit roots at the corresponding frequencies. Since the null $H_0 : (\pi_3 = 0) \cap (\pi_4 = 0)$ is two-dimensional, when testing for $I_{\pi/2}(1)$, the authors suggest using an F-statistic. Alternatively one may first perform a two sided test of $\pi_4 = 0$ and, if not significant, compute a test of $\pi_3 = 0$ against the alternative $\pi_3 < 0$.

Representation (3) opens the way to testing a number of hypotheses:

- $H_0 : y_t \sim SI(1,1)$ vs. $H_1 : y_t \sim SI(0,0)$, in which case all the π_i's are tested;
- $H_0 : y_t \sim SI(1,1)$ vs. $H_1 : y_t \sim SI(0,1)$: the maintained hypothesis is $\pi_2 = \pi_3 = \pi_4 = 0$ (i.e. the process is integrated at all seasonal frequencies), and π_1 is tested. This is equivalent to carrying out an ADF test on the $S(L)y_t$ series.
- $H_0 : y_t \sim SI(1,1)$ vs. $H_1 : y_t \sim SI(1,0)$: the maintained hypothesis is relative to the presence of a zero frequency unit root ($\pi_0 = 0$), so that π_2, π_3 and π_4 are tested.
- $H_0 : y_t \sim SI(0,1)$ vs. $H_1 : y_t \sim SI(0,0)$: the maintained hypothesis is $\pi_1 \neq 0$;
- $H_0 : y_t \sim SI(1,0)$ vs. $H_1 : y_t \sim SI(0,0)$: the maintained hypothesis is $\pi_2 \neq 0$ and either π_3 or π_4 is different from zero.
- When $\pi_1 = \pi_3 = \pi_4 = 0$, it is possible to write $\phi(L)\Delta Z_{2t} = -\pi_2 Z_{2,t-1} + \epsilon_t$ and the distribution of the test of the hypothesis $\pi_2 = 0$ is the mirror of the Dickey-Fuller distribution.
- Similarly when it is maintained that $\pi_1 = \pi_2 = \pi_4 = 0$ then we have $\phi^*(L)(1 - L^2)Z_{3t} = \pi_3 Z_{3,t-2} + \epsilon_t$ and the test statistic for $H_0 : \pi_3 = 0$ is the mirror of the DHF distribution when $s = 2$.
- The same testing sequence can be adopted on the first differences of y_t: for instance we can test $H_0 : y_t \sim SI(2,1)$ vs. $H_1 : y_t \sim SI(2,0)$; under the maintained hypothesis $\pi_2 = \pi_3 = \pi_4 = 0$ we can test $y_t \sim SI(2,1)$ against $y_t \sim SI(1,1)$ (this is equivalent to the ADF on the transformed series $\Delta_4 y_t$), and so on and so forth.

The distribution of the test statistics for $\pi_i = 0$ ($i = 1,2,3,4$) is invariant whether Z_j, ($j = 1,2,3,4$, $j \neq i$), is included in the regressors set. However, it changes according to the nature of the deterministic component; Hylleberg *et al.*

tabulate critical values for the one-sided t-tests on π_1, π_2 and π_3, for the two-sided t-tests on π_4 and for the F-test of the hypothesis $(\pi_3 = 0) \cap (\pi_4 = 0)$ under five assumptions concerning the deterministic component: i. $\mu_t = 0$, ii. $\mu_t = \mu$, iii. $\mu_t = \sum_{j=1}^{s} \mu_j S_{jt}$, iv. $\mu_t = \mu + \beta_t$, v. $\mu_t = \sum_{j=1}^{s} \mu_j S_{jt} + \beta t$ (see tables 1a and 1b on page 227).

For the series under investigation it is deemed relevant to consider stationary alternatives whose deterministic components are represented by iii. and v.

4.4. The Hasza-Fuller (HF) test

The test proposed by Hasza and Fuller (1982) is based on the regression:

$$y_t = \beta_1 y_{t-1} + \beta_2 \Delta_s y_{t-1} + \beta_3 \Delta y_{t-s} + \sum_{j=1}^{q} \phi_j \Delta\Delta_s y_{t-j} + \epsilon_t.$$

The null is that y_t is $SI(2,1)$ which in terms of the β's is expressed as H_0 : $\beta_1 = 1, \beta_2 = 0$ and $\beta_3 = 1$; in fact under H_0, $\Delta\Delta_s y_t = \sum_{j=1}^{q} \phi_j \Delta\Delta_s y_{t-j} + \epsilon_t$, where the roots of the AR polynomial are outside the unit circle. HF consider an F-type statistic ($\Phi^{(3)}_{n-3}$ in their notation) for testing H_0 against the alternative $H_1 : y_t \sim SI(0,0)$, and tabulate the critical values.

They also consider two more F-statistics based on the regression:

$$y_t = \beta_1 y_{t-1} + \beta_2 \Delta_s y_{t-1} + \beta_3 \Delta y_{t-s} + \sum_{j=1}^{s} \mu_j S_{jt} + \beta t + \epsilon_t.$$

The first is the F-statistic for the hypothesis $H_0 : \{\beta_1 = 1, \beta_2 = 0, \beta_3 = 1, \mu_{jt} = 0, j = 1, \ldots, s\}$ (denoted by $\Phi^{(d+4)}_{n-d-4}$); the second tests $H_0 : \beta_1 = 1, \beta_2 = 0$, $\beta_3 = 1$ (test $\Phi^{(3)}_{n-d-4}$). The alternative is that y_t is stationary with a deterministic seasonal component and a linear time trend. Unfortunately the data generation mechanism considered is far too simple, since it allows only for WN disturbances. Moreover the presence of deterministic seasonality but no time trend is not considered under the alternative hypothesis.

Dickey and Pantula (1987) have argued that such F-type tests result in decreased power since they fail to account for the one sided nature of the alternative hypothesis.

4.5. Osborn, Chui, Smith and Birchenhall (OCSB) Test

In analogy with the DHF test, Osborn *et al.* consider the Taylor series expansion of $(1-\varphi_1 L)(1-\varphi_s L^s)\phi(L)Y_t = \epsilon_t$ about $\varphi_1 = \varphi_s = 1$, where Y_t is the original series after subtraction of seasonal means, and propose the following two-stage testing strategy: a) regress $\Delta\Delta_s Y_t$ on q lagged values to get estimates of the ϕ_j's; b) consider

the regression model:

$$\Delta\Delta_s Y_t = \beta_1 Z_{4,t-1} + \beta_2 Z_{5,t-4} + \sum_{j=1}^{q} \alpha_j \Delta\Delta_s Y_{t-j} + \epsilon_t \qquad (4)$$

where $Z_{4t} = \hat{\phi}(L)\Delta_s Y_t$, $Z_{5t} = \hat{\phi}(L)\Delta Y_t$, $\beta_1 = \varphi_1 - 1$ and $\beta_2 = \varphi_s - 1$.

OCSB consider the F statistic for testing $\beta_1 = \beta_2 = 0$, according to which the process is $SI(2,1)$ against the alternative that the process is stationary with deterministic seasonals (they label this statistic as HF due to the similarity of the testing problem[5]).

They also consider the t statistics on β_1 and β_2: a test of $H_0 : \beta_1 = 0$ is a test for the need of differencing by means of Δ, whereas a test of $H_0 : \beta_2 = 0$ is a test for the need of differencing by Δ_s.

When the maintained hypothesis is $\beta_2 = 0$ then the t statistic on β_1 is a test of $SI(2,1)$ versus $SI(1,1)$; i.e., of the need for differencing the data by Δ after seasonal differences were taken (this is close to an ADF test on $\Delta_4 Y_t$). When it is maintained that $\beta_1 = 0$, then the t statistic on β_2 is equivalent to a DHF test performed on the first order differences ($SI(2,1) vs. SI(1,0)$). The critical values are given in their paper.

A final point should be noted: according to the Taylor series expansion the variable on the right hand side of (4) should be $\Delta\Delta_s\hat{\phi}(L)Y_t$ instead of $\Delta\Delta_s Y_t$; the authors' motivation for their choice of the former is the argument that it does not affect the test statistics, since there are q lagged values of the dependent variable on the left hand side.

5. Illustrations of the Testing Strategy

This section illustrates how the results of the previous section can be assembled so as to produce a testing sequence. Following Dickey and Pantula (1987) it is customary to adopt a global *top down* strategy which involves starting from the highest order of seasonal integration, $SI(2,1)$, and testing down to the lower orders. This is the path followed by Ilmakunnas (1990) and Osborn *et al.* (1988). The rationale is that such testing procedure enables preservation of the nominal size of the test.

Beginning from $SI(2,1)$, according to which the data generating process (d.g.p.) is (2) with $\varphi(L) = \Delta\Delta_4$, several routes may be undertaken:

- $SI(2,1) \longrightarrow SI(2,0)$: under the alternative the d.g.p. is (2) with $\varphi(L) = \Delta^2$; this can be tested by means of the HEGY on the Δ series when the maintained hypothesis is that $\pi_1 = 0$.

[5] The authors stress however that the advantage of their version of the HF test over the original is the consideration of a stationary alternative around seasonal means with no time trend.

- $SI(2,1) \longrightarrow SI(1,1)$: under the alternative the d.g.p. is (2) with $\varphi(L) = \Delta_4$; this is tested by the ADF test on the Δ_4 series, which is equivalent to performing the HEGY test on the Δ series when the maintained hypothesis is $\pi_2 = \pi_3 = \pi_4 = 0$ and by the OCSB 't' ratio on β_1 when $\beta_2 = 0$.

- $SI(2,1) \longrightarrow SI(1,0)$: under the alternative the d.g.p. is (2) with $\varphi(L) = \Delta$ (stationary at seasonal frequencies); this is tested by the DHF test on the Δ series, the HEGY on the Δ series when all π_i's are tested and by the OCSB 't' ratio on β_2 when $\beta_1 = 0$.

- $SI(2,1) \longrightarrow SI(0,0)$: under the alternative the d.g.p. is (2) with $\varphi(L) = 1$ (i.e. the process is stationary at all frequencies); this is tested by the F-test for $\beta_1 = \beta_2 = 0$, the version of the HF test proposed by OCSB.

Supposing $SI(2,0)$ has been accepted, we may wish to test against just one unit root at frequency zero, that is $SI(1,0)$. The ADF on the Δ series and the HEGY on Δ series when $\pi_2, \pi_3, \pi_4 \neq 0$ provide a way of doing so.

Suppose instead that the null $SI(2,1)$ has been rejected in favour of $S(1,1)$. We can now follow three distinct directions, according to whether we reduce by one the order of seasonal integration, the order of integration at frequency zero, or both. In the first case ($SI(1,1) \longrightarrow SI(0,1)$) we can use the ADF test on the $S(L)$ series which is equivalent to the HEGY on π_1 when all the remaining π_i's are set to zero. In the second ($SI(1,1) \longrightarrow SI(1,0)$) we use the HEGY on the original series when $\pi_1 = 0$. Finally, the OCSB test on β_2 with $\beta_1 = 0$, the DHF and the HEGY on y_t, when all the π_i's are tested, provide the means of testing $SI(1,1)$ versus $SI(0,0)$.

To complete the sequence we need to consider $SI(1,0) \longrightarrow SI(0,0)$ and $SI(0,1) \longrightarrow SI(0,0)$. The former is tested by the ADF, the OCSB test on β_1 with $\beta_2 \neq 0$, and by the HEGY when $\pi_2 = \pi_3 = \pi_4 = 0$ is maintained. The latter is tested by the HEGY on π_2, π_3, π_4 when $\pi_1 \neq 0$.

In the interpretation of the results it ought to be kept in mind that some tests rely on a specific maintained hypothesis and are not valid *per se*, but so long as the hypothesis holds true.

Furthermore, some testing schemes involve ample discontinuity between the null and the alternative (i.e. the order of integration is reduced by more than one, as in the DHF and OCSB) and therefore some doubt ought to be cast on their outcome.

6. Test Results

The testing sequence considered in the previous section has been applied to the quarterly series on consumer expenditures at 1985 prices arising from two sources: the National Accounts (CN series) and the Household Expenditure Survey (BF series). For the latter we have also restricted the sample period to 73:1-90:4 in order to eliminate the effect on inferences of the first three years, characterised by a rather

different seasonal pattern; since inferences on seasonal integration underwent only minor changes we omit the presentation of test results.

Let us start by considering the HEGY test on the first order differences. The results are reported in tables 1 to 3, from which it can be seen that the null of integration at frequency zero is accepted only for the first differences of the series $CN1, CN3, CN6, BF2$, $BF6$ and $BF7$. Nothing or very little (as is cases $CN6$ and $BF5$) is changed depending on whether or not the deterministic component contains a time trend. As far as integration at the seasonal frequencies is concerned, the evidence is less homogeneous. Most series are integrated at frequency π (two cycles per year) with a few notable exceptions: $CN5, BF2$ and $BF5$; moreover, the F-tests for the joint hypothesis $\pi_3 = \pi_4 = 0$ is significant in several cases. This evidence on the unit roots at seasonal frequencies may not be conclusive due to the fact that we have, inappropriately in most cases, filtered the series by means of the Δ operator. $CN1, CN3$, $BF6$ and $BF7$ are stationary at frequency $\pi/2$, but are $I_0(2)$ and $I_\pi(1)$. Conversely, $BF2$ is stationary at frequency π, but is $I_0(2)$ and $I_{\pi/2}(1)$.

When we test $H_0 : y_t \sim SI(2,1)$ versus the alternative $H_1 : y_t \sim SI(1,1)$ by means of the ADF on the seasonal differences Δ_4, the null is accepted only for series $CN3$, and $BF2$, $BF6$ and $BF7$, regardless of the sample period (see table 3). This evidence is not completely in line with that emerging from the OCSB test on β_1 with $\beta_2 = 0$, which is significant for series CN2, CN6, and for all BF series (table 7). This is due to the fact that the maintained hypothesis ($\beta_2 = 0$) may not hold.

Based on these results we are tempted to conclude that the presence of a second unit root at zero frequency in most Δ series is not warranted by the data. Thus it makes no sense to look at the HEGY t tests on π_2, π_3, π_4 when $\pi_1 = 0$; also we rule out route $SI(2,1) \longrightarrow SI(2,0)$.

According to the DHF test (table 6) on the Δ series we would reject the null $SI(2,1)$ in favour of the alternative $SI(1,0)$ in all cases but $CN6$; again this may be only a reflection of the absence of a second unit at frequency zero and calls for a more thorough investigation at seasonal frequencies.

Thus it becomes interesting to look at the HEGY on the original series (tables 4-5): the test on π_1 is never significant and therefore we accept the null concerning the presence of one unit root at the zero frequency.

If we then turn our attention to the seasonal frequencies, we find that restricting π_1 to zero makes little difference on the t-statistics on π_2, π_3 and π_4 (as we would expect from the asymptotic independence of the regressors $Z_i, i = 1,2,3,4$). We may attempt a broad taxonomy of our series by distinguishing the following classes of processes:

1. Processes which are $SI(1,1)$: $CN2$,[6] $CN6, CN8$ and $BF4$.

2. Processes which are $I_0(1), I_{\pi/2}(0)$ and $I_\pi(1)$: $CN4, CN7$ and $BF1$.

[6]The attribution of $CN2$ is uncertain because the F-test is not significant while the t-test on π_4 is.

3. Processes which are $I_0(1), I_{\pi/2}(1)$ and $I_\pi(0)$: $CN5, BF3$ and $BF5$.

4. Processes which are $SI(1,0)$: $BF8$.

Note that integration at seasonal frequencies is accepted more often when dealing with CN series rather than BF series; recalling that the former consist originally of yearly data that are distributed across the quarters recurring mainly to the latter, so that we would expect that the order of integration would not differ significantly. A possible explanation is that the distribution technique produces a smoothing effect which induces higher persistence and autocorrelation in the series, thus having undesirable effects on unit root tests.

The consequence of the previous analysis on univariate time series modelling is that the usual transformations by the filters $\Delta_4, \Delta\Delta_4$, and $S(L)$ are in most cases not completely satisfactory as they may give rise to a strictly non-invertible process at the seasonal frequencies. Now, non-invertibility is believed to be less of a problem with respect to non-stationarity (for instance the forecasting performance is unaffected); however, in order to capture the autocorrelation structure, long autoregressions may be required at the expenses of model parsimony.

Our analysis is of course subject to the usual *caveat*: in particular, it is a well known fact that with 84 observations the unit root tests considered have low power against the alternative of a root close to but below unity.

A final point to make concerns the relative merits of the tests proposed for seasonal integration: it is implicit in the above discussion that the HEGY test provides the most valuable information since it looks for integration separately at each frequency of interest; on the other hand both the DHF and the OCSB tests appear to be "overshooting" this analytic target in that they induce acceptance or rejection of integration at all frequencies. Moreover, the filtering of the regressors by means of an estimated lag polynomial, which may be subject to misspecification, has unknown effects on inferences on unit roots.

7. Concluding Remarks

The quarterly series on consumption covering the period 1970:1-1990:4 are characterised by a strong seasonal pattern whose nature is interesting not only in itself, but also for both univariate and multivariate time series modelling (e.g., cointegration analysis).

The paper has dealt with the problem of determining the order of seasonal integration of these series, which have been classified accordingly. It turns out that integration at seasonal frequencies cannot always be taken for granted and that just one unit root exists at frequency zero in most cases. However, the relevance of these findings should be considered in the light of the fundamental skepticism surrounding unit root tests.

Perhaps the most interesting result is that the nature of the seasonal movements of the CN series differs from that of the twin BF series; in particular, the evidence for integration at seasonal frequencies (after allowing for deterministic seasonals) is stronger for the former. This result adds to the many discrepancies found between the two sources of data on consumption; but it bears different implications, in that it cannot always be associated with the operational concepts or to the estimation process; rather they are attributable either to the properties of the technique adopted for the distribution of the CN data over the quarters (for which the BF series play a major - sometimes exclusive - role) or to the nature of the seasonality of the indicator series.

Appendix

Table 1: *HEGY test - CN Δ series*

Variable	Auxiliary Regressors	't':π_1	't':π_2	't':π_3	't':π_4	'F':$\pi_3 \cap \pi_4$
$\Delta \ln CN1$	DS	-2.79	-1.63	-2.81	-2.30*	7.02*
$\Delta \ln CN1$	DS		-1.60	-2.95	-2.58*	
$\Delta \ln CN1$	DS+T	-3.39	-1.56	-2.72	-2.30*	6.76*
$\Delta \ln CN1$	DS+T		-1.39	-2.79	-2.67*	
$\Delta \ln CN2$	DS	-3.60*	-2.62	-1.65	-2.81*	5.69
$\Delta \ln CN2$	DS		-2.55	-1.82	-3.00*	
$\Delta \ln CN2$	DS+T	-3.82*	-2.55	-1.64	-2.78*	5.57
$\Delta \ln CN2$	DS+T		-2.98	-1.17	-2.45*	
$\Delta \ln CN3$	DS	-2.49	-2.96	-1.55	-3.31*	6.67*
$\Delta \ln CN3$	DS		-2.86	-1.44	-3.46*	
$\Delta \ln CN3$	DS+T	-3.26	-2.94	-1.67	-3.40*	7.16*
$\Delta \ln CN3$	DS+T		-2.42	-1.14	-3.59*	
$\Delta \ln CN4$	DS	-3.79*	-2.58	-3.59*	-1.24	7.42*
$\Delta \ln CN4$	DS		-2.47	-3.98*	-1.82	
$\Delta \ln CN4$	DS+T	-3.93*	-2.54	-3.22	-1.16	7.03*
$\Delta \ln CN4$	DS+T		-2.52	-3.55	-1.19	
$\Delta \ln CN5$	DS	-3.77*	-3.33*	0.77	-2.91*	4.64
$\Delta \ln CN5$	DS		-3.01	-0.89	-2.75*	
$\Delta \ln CN5$	DS+T	-4.53*	-3.50*	-0.70	-2.91*	4.58
$\Delta \ln CN5$	DS+T		-3.54*	0.08	-2.62*	
$\Delta \ln CN6$	DS	-3.38*	-1.12	-1.26	1.27	1.37
$\Delta \ln CN6$	DS		-0.95	-1.62	0.84	
$\Delta \ln CN6$	DS+T	-3.36	-1.12	-1.25	1.06	1.33
$\Delta \ln CN6$	DS+T		-1.17	-1.49	1.27	
$\Delta \ln CN7$	DS	-3.85*	-2.14	-3.69*	-2.16*	10.39*
$\Delta \ln CN7$	DS		-2.11	-3.19	-2.55*	
$\Delta \ln CN7$	DS+T	-3.75*	-2.12	-3.70*	-2.17*	10.37*
$\Delta \ln CN7$	DS+T		-2.26	-3.28	-2.29*	
$\Delta \ln CN8$	DS	-3.53*	-0.18	0.93	-0.09	0.44
$\Delta \ln CN8$	DS		-0.06	0.83	-0.29	
$\Delta \ln CN8$	DS+T	-3.68*	-0.22	0.93	-0.09	0.44
$\Delta \ln CN8$	DS+T		-0.07	0.81	-0.30	

Notes: DS denotes that seasonal dummies $S_{jt}, j = 1,2,3,4$ are included in the regression; DS+T denotes that the deterministic component includes a linear trend. The cells for 't':π_1 are empty when the maintained hypothesis is $\pi_1 = 0$.
* significant at the 5% value.

Table 2: *HEGY test - BF Δ series*

Variable	Auxiliary Regressors	't':π_1	't':π_2	't':π_3	't':π_4	'F':$\pi_3 \cap \pi_4$
$\Delta \ln BF1$	DS	-4.07*	-2.57	-3.82*	0.56	7.62*
$\Delta \ln BF1$	DS		-2.32	-3.76*	0.60	
$\Delta \ln BF1$	DS+T	-4.37*	-2.53	-3.85*	0.59	7.73*
$\Delta \ln BF1$	DS+T		-2.37	-3.73*	0.69	
$\Delta \ln BF2$	DS	-3.37	-4.05*	-3.43	-0.66	6.06
$\Delta \ln BF2$	DS		-4.01*	-3.27	-0.79	
$\Delta \ln BF2$	DS+T	-3.35	-6.00*	-3.40	-0.64	5.95
$\Delta \ln BF2$	DS+T		-3.98*	-3.25	-0.79	
$\Delta \ln BF3$	DS	-4.14*	-2.74	-2.29	1.81	4.57
$\Delta \ln BF3$	DS		-2.41	-2.57	1.64	
$\Delta \ln BF3$	DS+T	-4.09*	-2.71	-2.27	1.83	4.54
$\Delta \ln BF3$	DS+T		-2.39	-2.53	1.66	
$\Delta \ln BF4$	DS	-3.75*	-2.47	-2.58	1.75	5.21
$\Delta \ln BF4$	DS		-2.22	-2.64	1.97	
$\Delta \ln BF4$	DS+T	-3.90*	-2.45	-2.55	1.65	4.92
$\Delta \ln BF4$	DS+T		-2.21	-2.62	1.95	
$\Delta \ln BF5$	DS	-3.46	-3.44*	-1.99	1.85	3.93
$\Delta \ln BF5$	DS		-3.26*	-2.14	1.93	
$\Delta \ln BF5$	DS+T	-3.78*	-3.53*	-1.94	1.75	3.60
$\Delta \ln BF5$	DS+T		-3.21*	-2.12	1.94	
$\Delta \ln BF6$	DS	-3.47	-2.10	-4.06*	0.52	8.45*
$\Delta \ln BF6$	DS		-1.96	-3.87*	0.52	
$\Delta \ln BF6$	DS+T	-3.53	-2.08	-4.01*	0.52	8.23*
$\Delta \ln BF6$	DS+T		-1.94	-3.85*	0.53	
$\Delta \ln BF7$	DS	-2.45	-2.19	-4.89*	1.92	14.56*
$\Delta \ln BF7$	DS		-2.11	-5.33*	1.66	
$\Delta \ln BF7$	DS+T	-2.43	-2.17	-4.83*	1.91	14.30*
$\Delta \ln BF7$	DS+T		-2.09	-5.26*	1.66	
$\Delta \ln BF8$	DS	-4.65*	-3.99*	-4.89*	-2.36*	17.45*
$\Delta \ln BF8$	DS		-4.31*	-5.06*	-1.64	
$\Delta \ln BF8$	DS+T	-4.66*	-3.92*	-4.85*	-2.41*	17.37*
$\Delta \ln BF8$	DS+T		-4.26*	-5.01*	-1.65	

Notes: DS denotes that seasonal dummies $S_{jt}, j = 1,2,3,4$ are included in the regression; DS+T denotes that the deterministic component includes a linear trend. The cells for 't':π_1 are empty when the maintained hypothesis is $\pi_1 = 0$.
* significant at the 5% value.

Table 3: *ADF test on* Δ_4 *series*

Variable	$\hat{\tau}_\mu$	$P \leq \hat{\tau}_\mu$	LM	$\hat{\tau}_\tau$	$\hat{\tau}_\beta$	$P \leq \hat{\tau}_\tau$	LM
$\Delta_4 \ln CN1$	-3.19	0.02	6.56	-3.84	-2.13	0.02	7.75
$\Delta_4 \ln CN2$	-3.88	0.00	1.02	-4.14	-1.39	0.01	1.22
$\Delta_4 \ln CN3$	-2.48	0.13	3.67	-3.09	-1.80	0.11	3.78
$\Delta_4 \ln CN4$	-4.42	0.00	5.21	-4.62	-1.28	0.00	5.45
$\Delta_4 \ln CN5$	-3.47	0.01	4.65	-4.14	2.17	0.01	5.53
$\Delta_4 \ln CN6$	-3.48	0.01	0.11	-3.46	0.21	0.05	0.13
$\Delta_4 \ln CN7$	-3.61	0.01	0.26	-3.54	-0.18	0.04	0.27
$\Delta_4 \ln CN8$	-3.59	0.01	1.38	-3.74	1.05	0.02	1.57
$\Delta_4 \ln BF1$	-3.99	0.00	0.95	-4.26	-1.42	0.01	1.64
$\Delta_4 \ln BF2$	-3.25	0.02	0.15	-3.26	-0.48	0.08	0.17
$\Delta_4 \ln BF3$	-4.07	0.00	0.51	-4.02	-0.47	0.01	0.51
$\Delta_4 \ln BF4$	-3.83	0.00	0.40	-4.04	-1.25	0.01	0.44
$\Delta_4 \ln BF5$	-3.47	0.01	2.32	-3.76	1.54	0.02	1.76
$\Delta_4 \ln BF6$	-3.22	0.02	1.29	-3.32	0.83	0.07	1.61
$\Delta_4 \ln BF7$	-2.77	0.07	0.28	-2.74	-0.32	0.23	0.26
$\Delta_4 \ln BF8$	-4.62	0.00	0.11	-4.61	0.56	0.02	0.19

Notes: $\hat{\tau}_\mu$ is the t-statistic of the ϕ^* parameter in the regression model:

$$\Delta\Delta_4 y_t = \sum_{j=1}^{4} \mu_j S_{jt} + \phi^* \Delta_4 y_{t-1} + \sum_{k=1}^{q} \phi_k^\dagger \Delta\Delta_4 y_{t-k} + \epsilon_t.$$

$q = 2$ for CN series; $q = 4$ for BF series.
$\hat{\tau}_\tau$ and $\hat{\tau}_\beta$ are the t-statistic of the ϕ^* and β parameters in the regression:

$$\Delta\Delta_4 y_t = \sum_{j=1}^{4} \mu_j S_{jt} + \beta t + \phi^* \Delta_4 y_{t-1} + \sum_{k=1}^{q} \phi_k^\dagger \Delta\Delta_4 y_{t-k} + \epsilon_t.$$

The distribution of $\hat{\tau}_\mu$ and $\hat{\tau}_\tau$ is tabulated by Fuller (1976); that of $\hat{\tau}_\beta$ is tabulated by Dickey and Fuller (1981). $q = 2$ for CN series; $q = 4$ for BF series. By $P \leq \hat{\tau}_\mu$ and $P \leq \hat{\tau}_\tau$ we denote the lower tail probability of the relevant Dickey-Fuller distribution; LM denotes the value of the Lagrange Multiplier test statistic for the null of residual autocorrelation up to lag 4, which is distributed as χ^2 with 4 d.f.

Table 4: *HEGY test - CN series*

Variable	Auxiliary Regressors	't':π_1	't':π_2	't':π_3	't':π_4	'F':$\pi_3 \cap \pi_4$
$\ln CN1$	DS	-2.27	-3.00	0.58	-3.08*	4.97
$\ln CN1$	DS		-3.03*	0.74	-3.20*	
$\ln CN1$	DS+T	-0.50	-2.98	0.58	-3.05*	4.90
$\ln CN1$	DS+T		-1.55	-0.29	-3.64*	
$\ln CN2$	DS	-2.68	-1.94	0.56	-3.40*	6.00
$\ln CN2$	DS		-1.88	0.67	-3.45*	
$\ln CN2$	DS+T	-2.44	-1.99	0.56	-3.46*	6.19
$\ln CN2$	DS+T		-2.53	0.75	-3.22*	
$\ln CN3$	DS	-3.19	-4.57*	1.33	-4.09*	9.31*
$\ln CN3$	DS		-4.80*	1.31	-4.30*	
$\ln CN3$	DS+T	-1.41	-4.57*	1.33	-4.05*	9.17*
$\ln CN3$	DS+T		-2.91*	1.11	-3.57*	
$\ln CN4$	DS	-1.77	-2.65	-1.49	-3.21*	6.60*
$\ln CN4$	DS		-2.64	-1.35	-3.21*	
$\ln CN4$	DS+T	-2.25	-2.71	-1.56	-3.21*	6.70*
$\ln CN4$	DS+T		-2.53	-1.62	-3.35*	
$\ln CN5$	DS	-2.41	-4.10*	1.40	-2.56*	4.48
$\ln CN5$	DS		-4.19*	1.60	-2.77*	
$\ln CN5$	DS+T	-2.52	-4.12*	1.33	-2.66*	4.64
$\ln CN5$	DS+T		-3.47*	1.54	-2.60*	
$\ln CN6$	DS	-0.17	-1.43	-1.32	0.24	0.89
$\ln CN6$	DS		-1.44	-1.32	0.24	
$\ln CN6$	DS+T	-2.22	-1.46	-1.17	-0.31	0.73
$\ln CN6$	DS+T		-1.11	-1.61	-0.17	
$\ln CN7$	DS	-0.35	-2.91	-0.38	-4.35*	9.61*
$\ln CN7$	DS		-2.92	-0.41	-4.37*	
$\ln CN7$	DS+T	-1.77	-2.92	-0.52	-4.27*	9.36*
$\ln CN7$	DS+T		-2.10	-1.03	-4.41*	
$\ln CN8$	DS	0.61	0.03	0.66	0.42	0.32
$\ln CN8$	DS		0.03	0.68	0.48	
$\ln CN8$	DS+T	-2.07	-0.05	0.72	0.50	0.40
$\ln CN8$	DS+T		-0.22	0.72	0.59	

Notes: DS denotes that seasonal dummies $S_{jt}, j = 1,2,3,4$ are included in the regression; DS+T denotes that the deterministic component includes a linear trend. The cells for 't':π_1 are empty when the maintained hypothesis is $\pi_1 = 0$.
* significant at the 5% value.

Table 5: *HEGY test - BF series*

Variable	Auxiliary Regressors	't':π_1	't':π_2	't':π_3	't':π_4	'F':$\pi_3 \cap \pi_4$
$\ln BF1$	DS	-2.40	-2.31	-3.40	-1.87	7.87*
$\ln BF1$	DS		-2.31	-3.40	-1.88	
$\ln BF1$	DS+T	-3.17	-2.49	-3.65*	-1.89	8.81*
$\ln BF1$	DS+T		-2.27	-3.35	-1.87	
$\ln BF2$	DS	-2.52	-3.27*	-2.16	-2.11*	4.75
$\ln BF2$	DS		-3.14*	-1.94	-2.08*	
$\ln BF2$	DS+T	-3.21	-3.32*	-2.32	-2.06*	5.00
$\ln BF2$	DS+T		-3.14*	-1.95	-2.08*	
$\ln BF3$	DS	-0.55	-3.14*	-3.12	-0.80	5.30
$\ln BF3$	DS		-3.16*	-3.12	-0.80	
$\ln BF3$	DS+T	-1.83	-3.20*	-3.15	-0.74	5.32
$\ln BF3$	DS+T		-3.14*	-3.11	-0.80	
$\ln BF4$	DS	-2.53	-2.46	-2.99	-0.45	4.61
$\ln BF4$	DS		-2.53	-3.28	-0.65	
$\ln BF4$	DS+T	-2.80	-2.49	-3.02	-0.39	4.66
$\ln BF4$	DS+T		-2.47	-3.13	-0.59	
$\ln BF5$	DS	-0.11	-3.48*	-2.82	0.14	4.00
$\ln BF5$	DS		-3.50*	-2.84	0.14	
$\ln BF5$	DS+T	-3.06	-3.25*	-2.69	0.56	3.81
$\ln BF5$	DS+T		-3.40*	-2.75	0.24	
$\ln BF6$	DS	0.18	-2.15	-3.43	-2.60*	10.41*
$\ln BF6$	DS		-2.17	-3.47	-2.64*	
$\ln BF6$	DS+T	-2.03	-2.22	-3.69*	-2.45*	11.05*
$\ln BF6$	DS+T		-2.16	-3.44	-2.55*	
$\ln BF7$	DS	-0.91	-3.53*	-4.45*	-1.92	13.21*
$\ln BF7$	DS		-3.56*	-4.40*	-1.93	
$\ln BF7$	DS+T	-1.98	-3.59*	-4.52*	-1.84	13.35*
$\ln BF7$	DS+T		-3.53*	-4.39*	-1.92	
$\ln BF8$	DS	0.31	-4.22*	-4.54*	-1.24	16.33*
$\ln BF8$	DS		-4.24*	-5.58*	-1.22	
$\ln BF8$	DS+T	-1.95	-4.07*	-5.56*	-0.94	16.01*
$\ln BF8$	DS+T		-4.26*	-5.59*	-1.25	

Notes: DS denotes that seasonal dummies $S_{jt}, j = 1,2,3,4$ are included in the regression; DS+T denotes that the deterministic component includes a linear trend.
* significant at the 5% value.

Table 6: *DHF test*

Variable	DHF	Variable	DHF
$\ln CN1$	-2.63	$\ln BF1$	-3.82
$\Delta \ln CN1$	-4.34*	$\Delta \ln BF1$	-5.76*
$\ln CN2$	-4.40*	$\ln BF2$	-5.02*
$\Delta \ln CN2$	-4.50*	$\Delta \ln BF2$	-5.29*
$\ln CN3$	-3.05	$\ln BF3$	-2.32
$\Delta \ln CN3$	-5.90*	$\Delta \ln BF3$	-4.82*
$\ln CN4$	-2.92	$\ln BF4$	-4.49*
$\Delta \ln CN4$	-5.33*	$\Delta \ln BF4$	-4.95*
$\ln CN5$	-0.31	$\ln BF5$	-2.52
$\Delta \ln CN5$	-5.03*	$\Delta \ln BF5$	-5.24*
$\ln CN6$	-0.48	$\ln BF6$	-2.20
$\Delta \ln CN6$	-1.12	$\Delta \ln BF6$	-5.54*
$\ln CN7$	-4.26*	$\ln BF7$	-3.53
$\Delta \ln CN7$	-5.89*	$\Delta \ln BF7$	-6.34*
$\ln CN8$	-0.31	$\ln BF8$	-4.29*
$\Delta \ln CN8$	-5.03*	$\Delta \ln BF8$	-7.60*

Notes: *DHF* is the t-statistic of the α parameter in the regression models:

1.

$$\Delta_4 Z_t = \alpha Z_{t-4} + \sum_{k=1}^{q} \alpha_k \Delta_4 Y_{t-j} + \epsilon_t,$$

where $Y_t = y_t - \sum_j^4 \hat{\mu}_j S_{jt}$ and $Z_t = Y_t - \hat{\phi}_1 Y_{t-1} - \cdots - \hat{\phi}_q Y_{t-q}$. $\hat{\phi}_k$ is estimated by a regression of $\Delta_4 Y_t$ on its $q = 4$ lagged values.

2.

$$\Delta\Delta_4 Z_t = \alpha Z^*_{t-4} + \sum_{k=1}^{q} \alpha_k \Delta\Delta_4 Y_{t-j} + \epsilon_t,$$

where $Z^*_t = \Delta Y_t - \hat{\phi}_1 \Delta Y_{t-1} - \cdots - \hat{\phi}_q \Delta Y_{t-q}$; $\hat{\phi}_k$ is estimated by a regression of $\Delta\Delta_4 Y_t$ on its $q = 4$ lagged values.

* significant at the 5% value.

Table 7: *OCSB test*

Variable	't': β_1	't': β_2	't': β_1 $\beta_2 = 0$	't': β_2 $\beta_1 = 0$	'F': $\beta_1 \cap \beta_2$
$\ln CN1$	1.11	-2.27*	-1.60	-2.57*	3.94*
$\ln CN2$	-0.91	-5.11*	-2.13*	-5.61*	16.12*
$\ln CN3$	-0.52	-2.76*	-1.50	-3.15*	5.05*
$\ln CN4$	-0.24	-2.89*	-1.54	-3.32*	5.47*
$\ln CN5$	-1.38	-0.37	1.48	-0.64	1.16
$\ln CN6$	-2.11*	-0.39	-2.24*	-0.79	2.55
$\ln CN7$	0.33	-4.69*	-1.37	-4.70*	12.24*
$\ln CN8$	-1.11	-0.26	-1.18	-0.26	0.71
$\ln BF1$	-1.37	-4.54*	-3.05*	-5.50*	16.29*
$\ln BF2$	-2.10*	-4.72*	-3.23*	-5.48*	17.93*
$\ln BF3$	-0.71	-3.60*	-2.45*	-4.61*	10.80*
$\ln BF4$	-1.89*	-3.53*	-3.30*	-4.56*	12.56*
$\ln BF5$	-1.38	-3.93*	-3.34*	-5.17*	14.57*
$\ln BF6$	-0.82	-3.98*	-2.60*	-4.84*	11.99*
$\ln BF7$	-0.92	-5.79*	-1.85*	-6.14*	19.25*
$\ln BF8$	-2.08*	-5.97*	-3.91*	-7.19*	29.24 *

* significant at the 5% value.

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