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International dynamic risk sharing

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Abstract

In this paper we examine the formal implications of international risk sharing among a set of countries in the presence of market frictions and forward-looking behaviour. We show that if frictions prevent consumption to adjust instantaneously to its optimal long run level, consumption streams in the countries belonging to the risk sharing pool change over time according to a dynamic disequilibrium model which can be nested within an error-correcting vector autoregressive process. Econometric methods for testing the restrictions imposed by the theory at both short and long horizons are proposed and discussed. The empirical analysis of a set of core European countries suggest that consumption data do not seem to contrast neither with the existence of risk sharing against permanent income fluctuations and integrated capital markets, nor with a gradual and interrelated process of adjustment towards the equilibrium. The apparent lack of risk sharing in Europe documented in earlier works might depend not only on the misspecification of the short run dynamics of consumption, but also on the relatively low speed of adjustment toward the equilibrium.

Keywords: Adjustment costs, Consumption risk sharing, Cointegrated VAR models, Financial market integration, Market frictions.

J.E.L. Classification: C32, E21.

1 Introduction

Common wisdom contends that under complete markets changes to country per capita consumption should be related to changes in the consumption streams of the partner countries in the risk sharing pool or to changes in aggregate (world) per capita consumption only. Conventional risk sharing tests and/or techniques aimed to measure the different channels of consumption insurance are based on this requirement, see e.g. Asdrubali et al. (1996), Asdrubali and Kim (2004) and references therein. However, several empirical tests have shown substantial departures from this proposition, the so-called 'full risk sharing hypothesis' (FRS), both on individual and aggregated data, Lewis (1999).

Standard risk sharing tests are usually based on the idea that changes to individual consumptions, once corrected for changes in aggregate consumption or consumption of the 'leader country', and possibly for real exchange rates, are not predictable on the basis of the available information set, see e.g. Canova and Ravn (1996). These implications arise from the well known condition that under complete markets the ex-post nominal marginal rate of substitution equalize across countries. However, the empirical evidence based on consumption data and power utility functions suggests that risks are poorly shared internationally. Also the correlations between the ratio of domestic to foreign consumption and real exchange rates appear sharply below one or even negative, as found in Backus and Smith (1993), Kollmann (1995) and Ravn (2001).

The most recurrent explanations for the observed 'consumption correlations puzzle' in the international business cycle literature hinges on the idea that some components of utility are not separable and internationally tradeable such as leisure (Backus et al., 1992) and nontradeable goods (Backus and Smith, 1993, Stockman and Tesar, 1995)[1]. However, as argued (and shown empirically) by Lewis (1999), risk sharing tests that simply correct for the presence of nontradeables do not seem to be sufficient alone

[1]These explanations of the lack of international risk sharing can be potentially reconciled with the "equity home bias" puzzle, see e.g. Lewis (1999). The fact that "consumption home bias" is somehow related to "equity home bias" can be understood, intuitively, by observing that countries that bias their equity holdings away from foreign assets will not diversify all of their home output risk .

to explain the lack of consumption risk sharing. A further explanation is that international financial markets are not developed enough, i.e. markets incompleteness, see e.g. Kollmann (1996). Also within this perspective, however, theory offers convincing arguments to doubt that incomplete asset markets alone can account for the observed low international consumption correlations. Recently, Obstfeld and Rogoff (2000) provide a unified explanation of the major puzzles of international macroeconomics, including the violation of FRS and purchasing power parity (PPP), in terms of costs in goods markets (transport costs, tariffs, nontariff barriers) that impede trade, see also Ravn and Mazzega (2004) and Brandt et al. (2005).

Although full risk sharing requires frictionless markets, in practice individuals face the (dis)utility costs implied by restrictions on factory mobility as well as on trade in international goods markets. For instance, if there is a positive shock in one country, asset holdings by the other countries should in principle lead to an outflow of goods; if on the one hand, restrains in capital markets can be considered negligible, it can be costly in goods markets to ship goods and these costs might increase with the volume being shipped. Nevertheless, if these frictions and the related costs are not large enough to keep consumers far from the 'frictionless' first order conditions, the ex-post nominal marginal rate of substitution will not equalize instantaneously across countries but after a gradual process of adjustment. Consequently departures from the FRS hypothesis may depend on the lack of dynamic structure characterizing the models which are traditionally implemented to test the FRS hypothesis. Starting from these observations in this paper we show that amending standard intertemporal risk sharing models with simple exogenous costs which impede instantaneous adjustment to the optimal risk sharing position entails a dynamic structure for countries consumption changes that is not accounted in the traditional empirical analyses. Omitting such dynamics flaws standard measures of the extent of risk sharing.

Since market frictions are difficult to quantify, our model describes temporary deviations from the FRS hypothesis without being specific about the nature of the rigidities. We sketch a dynamic disequilibrium model where representative agents in each countries are forward-looking and attempt to minimize intertemporally the costs of being away from the 'frictionless' first order conditions. The model we formalize hinges on the idea that countries attempt to minimize the costs of deviations from FRS. Specifically, the model has the following features and (testable) implications. First, consumption changes of a given country other than depending on contemporaneous consumption changes of the partners, display an error-correcting structure involving lagged deviations from the optimal risk sharing position of (potentially) all countries in the risk sharing pool. Thus the model predicts that adjustment is interrelated across countries, i.e. shocks affecting one country in the risk sharing pool produce adjustment in the other countries. This mechanism of interrelated adjustment reflects the idea that countries in the risk sharing pool aim in the long run at the maximization of collective utility. Second, as agents are forward-looking in an environment characterized by impediments to trade and factor (especially labour) mobility, beliefs on the evolution of expected future consumption changes of the leader country and of real exchange rates of all countries affect the risk sharing allocation. Third, consistently with recent findings (see Bacchiocchi and Fanelli, 2005, and references therein), our model do not require that PPP holds among the countries in the risk sharing pool; this feature contrasts with the large majority of papers on international risk sharing tests where PPP is assumed to hold and the FRS proposition tends to be rejected. As a result, correcting the tests of the FRS hypothesis for the dynamics implied by costs of adjustment and expectations on future market developments helps to explain why according to traditional analyses – where a dynamic structure is omitted - international risks are poorly shared.

We set out maximum likelihood (ML) and regression-based procedures for the model which allows to assess the existence of international risk sharing as an equilibrium relation and to analyze the dynamic adjustment of consumption towards optimal levels. The proposed framework provides an alternative way, compared to e.g. Obstfeld (1989, 1994), Kollmann (1995), Canova and Ravn (1996) and Ravn (2001), to tackle the empirical analysis of risk sharing and assess the degree of integration in international capital markets.

The dynamic risk sharing model is applied to a set of 'core' European countries that joined the European Monetary Union (EMU) in 1999. Economic intuition suggests that countries with closer economic ties, as the ones we consider in the paper, might have more efficient risk sharing mechanisms at work; in fact, by using data over a forty year period, our results suggest that if preference parameters are allowed to vary across countries European consumption data do not seem to contrast with the existence of risk sharing as a long run phenomenon. Moreover a gradual and interrelated dynamic process of adjustment towards the equilibrium is detected. These results contrast with the findings in e.g. Canova and Ravn (1996) and Sørensen and Yosha (1998), obtained through different estimation methods and overlooking the role of dynamic adjustment.

The plan of the paper is the following. Section 2 introduces the standard international consumption risk sharing model and discusses its main implications. Section 3 provides a dynamic extension under the assumption of frictions that prevent instantaneous adjustment to optimal risk sharing. Section 4 discusses estimation issues and in Section 5 the proposed risk sharing model is applied to investigate the extent of risk sharing among a set of 'core' European countries which joined the European Monetary Union in 1999. Some final remarks may be found in Section 6.

2 Model and implications

As in Canova and Ravn (1996) and Kolmann (1995) we consider a standard international business cycle model. It is assumed that a world of N countries (indexed by $i = 1, ..., N$) exists, each country being inhabited by a infinitely lived representative agent. His/her expected lifetime utility is given by $V^i = E_t\left(\sum_{t=0}^{\infty}(\rho^i)^t U^i\left(C_t^i, b_t^i\right)\right)$, where C_t^i denotes the i-th country consumption good at time t, while $b_t^i > 0$ represents a country-specific stochastic taste shock; ρ^i $(0 < \rho^i < 1)$ denotes country i-th discount factor. Note that the goods consumed by the different countries are allowed to differ. As usual, $E_t\left(\cdot\right)$ denotes expectations conditional on all information available up to time t, Ω_t. As is standard in the literature, we further assume that the utility function $U^i\left(x_1, x_2\right)$ of country i-th representative agent is an isoelastic instantaneous period utility function, i.e. $U^i\left(x_1, x_2\right) = x_2\left(1/\sigma^i\right)x_1^{\sigma^i}$ $(\sigma^i < 1)$, where $1 - \sigma^i$ is a CRRA coefficient.

Without loss of generality we suppose that the N-th country of the risk sharing pool can be considered as the leader of the arrangement; we denote this country with the superscript '0'. Within this set-up the allocation which maximize the expected average utility of consumption (under standard budget constraints), gives rise, for each pair of countries $i, j = 1, ..., N$ to the relation

$$(\rho^i)^t U_c^i \, P_t^i = (\rho^j)^t U_c^j \, P_t^j \, e_t^{i/j} \tag{1}$$

where for a generic country h, U_c^h is the marginal utility of consumption, P_t^h is the price level of the country and $e_t^{i/j}$ is the nominal exchange rate between the currencies of country i and j; the equilibrium condition (1) simply establishes that nominal rates of substitution are equalized across countries under FRS.

From (1) it follows that the optimal consumption streams, relative to the leader country's consumption, are restricted as follows (Kollmann, 1995; Ravn, 2001)

$$c_t^{*i} = \theta^i c_t^0 + \delta^i r_t^{i/0} + \phi^i t + \eta_t^i, \text{ all } t \text{ and } i = 1, ..., N-1 \tag{2}$$

where c_t^{*i} is the optimal level of (logged) consumption in country i, c_t^0 is the (logged) consumption in the leader country, $\theta^i = (1-\sigma^0)(1-\sigma^i)^{-1}$ is leader country CRRA coefficient relative to country i-th CRRA coefficient, $\delta^i = (1-\sigma^i)^{-1}$ corresponds to the intertemporal elasticity of substitution of country i, $\phi^i = \log\left(\rho^0/\rho^i\right) / \left(\sigma^i - 1\right)$, $r_t^{i/0} = \log(e_t^{i/0}) + \log(P_t^0) - \log(P_t^i)$ is the (logged) price of one unit of the leader country's consumption good in terms of country i's consumption good, i.e. the logged bilateral real exchange rate between country i and the leader country[2]. Finally, η_t^i in (2) depends on the stochastic terms which enter the utility functions of the countries i and 0, i.e. the x_2 variable in $U^i\,(x_1, x_2)$; these terms may represent preference shocks, factors which are beyond the control of agents, or variables which interact non-separably with consumption but which have not explicitly modelled, e.g. hours worked/leisure, government spending, real money balances and so on. Throughout it will be assumed, except where indicated, that η_t^i embodies the preference shocks of the countries i and 0 (Kollmann, 1995).

The model has strong implications on the optimal level of consumption that each country should achieve, although large part of the literature de-

[2]The formal derivation of (2) can be directly obtained from Kollmann (1995) and it is therefore omitted for brevity. A full proof is available from the authors upon request.

votes attention to the growth rate version of (2) ignoring the information in the levels. If the terms η_t^i, $i = 1, ..., N-1$, are stationary and if consumption can be well represented by means of integrated processes of order one, I(1) hereafter, then equation (2) can be seen as a cointegrating relation involving the optimal consumption level, the leader country consumption level and the real exchange rate. Therefore net of the preference shock, η_t^i, the linear combination

$$\left(1\ ,\ -\theta^i\ ,\ -\delta^i\ ,\ -\phi^i\right)\begin{pmatrix} c_t^i \\ c_t^0 \\ r_t^{i/0} \\ t \end{pmatrix} = c_t^i - \theta^i c_t^0 - \delta^i r_t^{i/0} - \phi^i t \qquad (3)$$

must be stationary, see also Backus and Smith (1993), Kollmann (1995) and Apte et al. (2004). Furthermore, if the real exchange rate is stationary (that is, PPP holds in the long run), then the equilibrium relation involves country i-th optimal consumption level and the leader country optimal consumption and can be analyzed and estimated as a cointegration relation between c_t^i and c_t^0.

Empirical tests based on consumption levels are usually carried out by assuming that c_t^i equalize the corresponding optimal levels c_t^{i*}, described thorough the FRS proposition (2)[3]. For instance, both both Kollmann (1995) and Ravn (2001) assume that adjustment is instantaneous, i.e. $c_t^i = c_t^{i*}$, all i, implying therefore that $\left(c_t^i - c_t^{i*}\right)$ is unpredictable given information available at time t, which further requires preference shock, η_t^i to be unpredictable given past information. On the other hand, Canova and Ravn (1996) employ, among others, nonparametric tests for cointegration among the consumption of pairs of G7 countries, hence setting δ^i to 0 in (2) and finding little support of the FRS proposition.[4]

[3]The proposition that relative consumptions and real exchange rates should be positively (and higly) correlated under FRS can be derived directly from (2) under the assumption that the i-th and the leader country have the same risk aversion coefficients and intertemporal discount rates. Indeed, with $\theta^i = 1$ and $\phi^i = 0$ and ignoring η_t^i it follows that $c_t^i - c_t^0 = \delta^i ex_t^{i/0}$, where $\delta^i > 0$.

[4]Implicitly, setting $\delta^i = 0$ is equivalent to assume that the real exchange rate is stationary; i.e., that PPP holds in the long run. This condition is not needed in the following.

3 Dynamic adjustment

In this section we derive a dynamic disequilibrium model which formalizes the process of adjustment toward optimal risk sharing. In what follows it is convenient to adopt the following matrix notation. Let $\mathbf{c}_t = (c_t^1$, ..., $c_t^{N-1})'$ be the $(N-1) \times 1$ vector containing per capita consumption of the N countries except the N-th, i.e. that of leader one, and let $\mathbf{c}_t^*$ be the vector containing the corresponding equilibrium quantities given by (2). Finally, let $\mathbf{w}_t$ be the $N \times 1$ vector defined as $\mathbf{w}_t = (c_t^0, r_t^{1/0}, \ r_t^{2/0}$, ..., $r_t^{N-1/0})'$, where $c_t^0 \equiv c_t^N$ and $r_t^{j/0}$ is defined as above. In the light of the FRS equilibrium relation (2) and its empirical counterpart (3), the vector of deviations of actual per capita consumption from optimal consumption, $\mathbf{c}_t - \mathbf{c}_t^*$, can be written as

$$\mathbf{c}_t - \mathbf{c}_t^* = \mathbf{c}_t - \boldsymbol{\Upsilon}\mathbf{w}_t - \phi t - \eta_t \tag{4}$$

where $\boldsymbol{\Upsilon}$ is a $(N-1) \times N$ matrix depending on the preference parameters θ^i and δ^i, $i = 1, .., N-1$. Specifically, $\boldsymbol{\Upsilon} = (\theta \vdots diag\,(\delta))$, $\theta = \left(\theta^1, ..., \theta^{N-1}\right)'$, $\delta = \left(\delta^1, ..., \delta^{N-1}\right)'$, $\phi = \left(\phi^1, ..., \phi^{N-1}\right)'$ and $\eta_t = (\eta_t^1, ...,$ $\eta_t^{N-1})'$. For instance, with $N = 3$, $\mathbf{c}_t = (c_t^1 \ , c_t^2)'$, $c_t^0 \equiv c_t^3$ is the consumption stream of the leader country, $\mathbf{w}_t = (c_t^0 \ , r_t^{1/0} \ , r_t^{2/0})'$ and the matrix $\boldsymbol{\Upsilon}$ and the vector ϕ take the form:

$$\boldsymbol{\Upsilon} = \begin{bmatrix} \theta^1 & \delta^1 & 0 \\ \theta^2 & 0 & \delta^2 \end{bmatrix}, \phi = \begin{bmatrix} \phi^1 \\ \phi^2 \end{bmatrix}. \tag{5}$$

If the adjustment of $\mathbf{c}_t$ to $\mathbf{c}_t^*$ would be instantaneous, the vector $\mathbf{u}_t = \mathbf{c}_t - \mathbf{c}_t^*$ should be unpredictable given information available at time t, meaning that countries benefit the ‘frictionless’ first order conditions (1) without utility losses. On the other hand, if market imperfections impede the instantaneous adjustment but are not large enough to generate permanent deviations from (1), one would expects $\mathbf{u}_t$ to be a persistent, though mean-reverting, process.

Assume that at each point in time being away from the equilibrium and varying consumption to achieve it is costly and that costs can be also generated by deviations from FRS of the other countries comprising the risk sharing agreement. A convenient way to describe the above mentioned disequilibrium dynamic mechanism is to posit that the representative agent of country i faces the following intertemporal optimization problem:

$$\min_{\{c^i_{t+h}\}} E_t \sum_{h=0}^{\infty} (\rho^i)^h [\psi_{1,ii}(c^i_{t+h} - c^{*i}_{t+h})^2$$
$$+_{j \neq i}\, \psi_{1,ij}(c^i_{t+h} - c^{*i}_{t+h})(c^j_{t+h} - c^{*j}_{t+h})$$
$$+ \psi_{2,ii}(c^i_{t+h} - c^i_{t+h-1})^2] \qquad (6)$$

where $\psi_{1,ij}$, $\psi_{2,ii}$, $i, j = 1, ..., N-1$ are (positive) adjustment parameters. There are two types of costs embedded in the present value cost function minimized in (6): the terms in the first line measure respectively the cost for country i of being away from its own FRS consumption level ($\psi_{1,ii}$) and the costs implied by deviations from FRS of the other countries (provided $\psi_{1,ji} \neq 0$); the term in the second line measures the cost ($\psi_{2,ii}$) for country i of changing consumption levels to restore equilibrium.[5]

It can be shown that considering the problem (6) for all $i = 1, ..., N-1$ countries in the risk sharing pool corresponds to solve the problem

$$\min_{\{\mathbf{c}_{t+h}\}} E_t \sum_{h=0}^{\infty} \rho_h [(\mathbf{c}_{t+h} - \mathbf{c}^*_{t+h})' \mathbf{\Psi}_1 (\mathbf{c}_{t+h} - \mathbf{c}^*_{t+h}) \qquad (7)$$
$$+ (\mathbf{c}_{t+h} - \mathbf{c}_{t+h-1})' \mathbf{\Psi}_2 (\mathbf{c}_{t+h} - \mathbf{c}_{t+h-1})]$$

where ρ_h is a matrix defined as $\rho_h = diag((\rho^1)^h, ..., (\rho^{N-1})^h)$, $\Psi_1 = [\psi_{1,ij}]$ is a $(N-1) \times (N-1)$ symmetric positive definite matrix, and $\Psi_2 = diag(\psi_{2,11}, ..., \psi_{2,(N-1)(N-1)})$. Assuming for simplicity that in this second stage countries discount future costs at the same (average) rate, ρ, then $\rho_h = \rho I_{N-1}$ and the solution to (7) can be computed by using

[5] Actually agents might face also the costs of adjusting the speed with which changes in consumption streams are put into effect; a third cost term of the form $\psi_{3,ii}(\Delta c^i_{t+h} - \Delta c^i_{t+h-1})^2$ could be thus included in (6), see e.g. Binder and Pesaran (1995) and Fanelli (2005). Observe that quadratic costs of adjustment are here solely used for mathematical convenience; in principle, there is no reason or motivation to argue that positive and negative deviations from optimal risk sharing have the same effect on the process of adjustment. Thus the cost function in (6) has to be regarded as a mere approximation of the "true" adjustment process.

techniques as in Hansen and Sargent (1981) and Binder and Pesaran (1995). The first-order conditions read as the system of Euler equations

$$\Delta \mathbf{c}_t = \rho E_t \Delta \mathbf{c}_{t+1} - \mathbf{\Psi}(\mathbf{c}_t - \mathbf{c}_t^*) \tag{8}$$

where $\Delta \mathbf{c}_t = \mathbf{c}_t - \mathbf{c}_{t-1}$ and $\mathbf{\Psi} = \mathbf{\Psi}_2^{-1}\mathbf{\Psi}_1$. The elements of the $\mathbf{\Psi}$ matrix, which is neither diagonal if $\psi_{1,ji} \neq 0$, nor symmetric, measure the relative importance of disequilibrium, adjustment and cross-adjustment costs. The i-th equation of (8) (i.e. that relative to country i) is then given by

$$\Delta c_t^i = \rho E_t \Delta c_{t+1}^i - \psi_i'(\mathbf{c}_t - \mathbf{\Upsilon}\mathbf{w}_t - \phi t) + \widetilde{\eta}_t^i \tag{9}$$

where $\widetilde{\eta}_t^i$ is the i-th element of $\widetilde{\eta}_t$, $\widetilde{\eta}_t = \mathbf{\Psi}\eta_t$, and ψ_i' is the i-th row of $\mathbf{\Psi}$; observe that

$$\begin{aligned} \psi_i'(\mathbf{c}_t - \mathbf{\Upsilon}\mathbf{w}_t - \phi t) \quad = \quad & \psi_{ii}(c_t^i - \theta^i c_t^0 - \delta^i r_t^{i/0} - \phi^i t) \\ & + \sum_{j=1, j\neq i}^{N-1} \psi_{ij}(c_t^j - \theta^j c_t^0 - \delta^j r_t^{j/0} - \phi^j t) + \widetilde{\eta}_t^i \end{aligned}$$

where ψ_{ii} is the i-th element on the principal diagonal of $\mathbf{\Psi}$ and the ψ_{ij}, for $i \neq j$, are the corresponding off-diagonal elements; unless $\psi_{ij} = 0$ (i.e. $\mathbf{\Psi}$ is diagonal), consumption changes in country i depend not only on their own future expected changes but also on the extent of the deviation of country i and (potentially) all the other countries from the optimal risk sharing position. Within this set-up if $\psi_{ij} \neq 0$, and a given country faces temporary departures from its equilibrium, all the other countries in the risk sharing pool experience next-time period consumption variations.

The system of Euler equations (9) apparently hides the role of the variables in $\mathbf{w}_t$, i.e. the consumption stream of the leader country and real exchange rates. Upon imposing a proper transversality condition the level version of (8) can be solved forward (Binder and Pesaran, 1995) as:

$$\mathbf{c}_t = \mathbf{K}\mathbf{c}_{t-1} + \sum_{h=0}^{\infty}(\rho\mathbf{K})^h(\mathbf{I}_{(N-1)} - \rho\mathbf{K})(\mathbf{I}_{(N-1)} - \mathbf{K})E_t\mathbf{c}_{t+h}^* \tag{10}$$

where $\mathbf{K}$ is a $(N-1) \times (N-1)$ matrix with stable eigenvalues obtained as the (unique) solution to the second-order matrix equation

$$\rho\mathbf{K}^2 - [(1+\rho)\mathbf{I}_{(N-1)} + \mathbf{\Psi}]\mathbf{K} + \mathbf{I}_N = \mathbf{0}_{(N-1),(N-1)}.$$

The representation (10) highlights that for country j consumption at time t is a weighted average of consumption at time $t-1$ of all countries in the risk sharing pool and expected future values of optimal consumption which in turn depends on the variables in $\mathbf{w}_t$, with weights declining geometrically over time.

By using the equality $\sum_{h=0}^{\infty} (\rho\mathbf{K})^h \left(\mathbf{I}_{(N-1)} - \rho\mathbf{K}\right) = \sum_{h=0}^{\infty} (\rho\mathbf{K})^h - \sum_{h=0}^{\infty} (\rho\mathbf{K})^{h+1}$, adding $(-\mathbf{c}_{t-1})$ to both sides and $\pm \left(\mathbf{I}_{(N-1)} - \mathbf{K}\right) \boldsymbol{\Upsilon}\mathbf{w}_t$ to the right hand side, after rearranging terms and assuming that that $E_t\eta_{t+h} = \mathbf{0}$ for $h = 1, 2, ...$, the model can be reparameterized in the error-correcting format

$$\begin{aligned} \Delta\mathbf{c}_t &= (\mathbf{K} - \mathbf{I}_{(N-1)})[\mathbf{c}_{t-1} - \boldsymbol{\Upsilon}\mathbf{w}_{t-1} - \phi t] \\ &\quad + \sum_{h=0}^{\infty} (\rho\mathbf{K})^h (\mathbf{I}_{(N-1)} - \mathbf{K})\boldsymbol{\Upsilon} E_t \Delta\mathbf{w}_{t+h} + \mathbf{a} + \mathbf{v}_t \end{aligned} \quad (11)$$

where $\mathbf{v}_t = (\mathbf{I}_N - \mathbf{K})\eta_t$ and $\mathbf{a} = (\rho\mathbf{K})(\mathbf{I}_{(N-1)} - \rho\mathbf{K})^{-1}\phi$ is a constant. The model (11) shows that the dynamics of consumption of the countries in the risk sharing pool depends on past deviations from the optimal risk sharing position (of potentially all countries), and future expected changes of the bilateral real exchange rates and growth consumption of the reference (leader) country. The $(\mathbf{K} - \mathbf{I}_{(N-1)})$ matrix in (11) plays a role similar to that of the adjustment matrix $\boldsymbol{\Psi}$ in the system (9); indeed, the elements of $\mathbf{K}$ are function of $\boldsymbol{\Psi}$ and, in general, if $\boldsymbol{\Psi}$ is non-diagonal, $\mathbf{K}$ will be non-diagonal too.

3.1 Implications under VAR dynamics

Under precise conditions the model (11) can be solved for future expected values of $\Delta\mathbf{w}_t$. Assume, for example, that the process generating $\Delta\mathbf{w}_t$ can be described as a stable VAR($p-1$) (for simplicity and without loss of generality we omit deterministic terms), which written in companion form reads as

$$\Delta\widetilde{\mathbf{w}}_t = \boldsymbol{\Phi}\Delta\widetilde{\mathbf{w}}_{t-1} + \widetilde{\mathbf{u}}_t \quad (12)$$

where $\Delta\widetilde{\mathbf{w}}_t = (\Delta\mathbf{w}_t', \Delta\mathbf{w}_{t-1}', ..., \Delta\mathbf{w}_{t-p+2}')'$ and $\widetilde{\mathbf{u}}_t = (\mathbf{u}_t', \mathbf{0}')'$ are $g \times 1$ ($g = N(p-1)$), $\mathbf{u}_t \sim WN(0, \boldsymbol{\Sigma}_{uu})$ with covariance matrix $\boldsymbol{\Sigma}_{uu}$ positive definite and the matrix $\mathbf{\Phi}$ is defined as

$$\underset{g\times g}{\mathbf{\Phi}} = \begin{bmatrix} \mathbf{\Phi}_1 & \mathbf{\Phi}_2 & \cdots & \mathbf{\Phi}_{p-1} \\ \mathbf{I}_N & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \ddots & \cdots & \vdots \\ \mathbf{0} & \cdots & \mathbf{I}_N & \mathbf{0} \end{bmatrix} \tag{13}$$

and has eigenvalues inside the unit circle in the complex plane. Let $\mathbf{H}_w$ denote a selection matrix such that $\mathbf{H}_w \Delta\widetilde{\mathbf{w}}_t = \Delta\mathbf{w}_t$; then from (12) and after conditioning with respect to the 'smaller' information set $\mathcal{F}_t = \{\mathbf{c}_t, \mathbf{w}_t, \mathbf{c}_{t-1}, \mathbf{w}_{t-1}, ...\}$ and applying the law of iterated expectation, the quantity $E_t\Delta\mathbf{w}_{t+h}$ can be computed as $E_t\Delta\mathbf{w}_{t+h} = \mathbf{H}_w E(\Delta\widetilde{\mathbf{w}}_{t+h} \mid \mathcal{F}_t) = \mathbf{H}_w \mathbf{\Phi}^h \Delta\widetilde{\mathbf{w}}_t$. By substituting into (11), after some algebra the model simplifies in the expression

$$\begin{aligned} \Delta\mathbf{c}_t &= (\mathbf{K} - \mathbf{I}_{(N-1)})[\mathbf{c}_{t-1} - \mathbf{\Upsilon}\mathbf{w}_{t-1} - \phi t] \\ &\quad + \mathbf{\Gamma}_0\Delta\mathbf{w}_t + \mathbf{\Gamma}_1\Delta\mathbf{w}_{t-1} + ... + \mathbf{\Gamma}_{p-2}\Delta\mathbf{w}_{t-p+2} + \mathbf{a} + \mathbf{v}_t \end{aligned} \tag{14}$$

where the $(N-1) \times g$ matrix of parameters $\mathbf{\Gamma} = [\mathbf{\Gamma}_0, \mathbf{\Gamma}_1, ..., \mathbf{\Gamma}_{p-2}]$ is subject to the cross-equation restrictions[6]

$$vec(\mathbf{\Gamma}) = [\mathbf{I}_{(N-1)g} - \mathbf{\Phi} \otimes (\rho\mathbf{K})]^{-1}[\mathbf{H}_w' \otimes (\mathbf{I}_{N-1} - \mathbf{K})]vec(\mathbf{\Upsilon}) \tag{15}$$

and it has been assumed that $E(\mathbf{v}_t \mid \mathcal{F}_t) = \mathbf{v}_t$.

The i-th equation of (14) (i.e. that relative to country i) reads as

$$\begin{aligned} \Delta c_t^i &= (k_{ii} - 1)(c_{t-1}^i - \theta^i c_{t-1}^0 - \delta^i r_{t-1}^{i/0} - \phi^i t) \\ &\quad + \sum_{j=1, j\neq i}^{N} k_{ij}(c_{t-1}^j - \theta^j c_{t-1}^0 - \delta^j r_{t-1}^{j/0} - \phi^j t) \\ &\quad + \gamma_{0,ic}\Delta c_t^0 + \gamma_{0,ii}\Delta r_t^{i/0} + \sum_{j=1, j\neq i}^{N} \gamma_{0,ij}\Delta r_t^{j/0} \\ &\quad + \gamma_{1,ic}\Delta c_{t-1}^0 + \gamma_{1,ii}\Delta r_{t-1}^{i/0} + \sum_{j=1, j\neq i}^{N} \gamma_{1,ij}\Delta r_{t-1}^{j/0} + v_t^i \end{aligned} \tag{16}$$

[6] A formal proof of this result is available from the authors upon request.

where $(\gamma_{0,ic}, \gamma_{0,i1}, ..., \gamma_{0,iN})$ are the (opportunely restricted) parameters of the i-th row of $\mathbf{\Gamma}_0$, $(\gamma_{1,ic}, \gamma_{1,i1}, ..., \gamma_{1,iN})$ are the (opportunely restricted) parameters of the i-th row of $\mathbf{\Gamma}_1$, $(k_{ii}-1)$ is the i-th element on the principal diagonal of $(\mathbf{K} - \mathbf{I}_{(N-1)})$, k_{ij} $(j \neq i)$ are the corresponding off-diagonal elements and v_t^i corresponds to the i-th elements of $\mathbf{v}_t$. This equation shows that consumption changes in country i not only depend on contemporaneous changes of consumption of the leader country and of the real exchange rate of all countries in the risk sharing pool and possibly their lags, but also on past deviations from the optimal risk sharing levels in country i as well as in all the other countries (provided $k_{ij} \neq 0$, $i \neq j$, i.e. $\mathbf{K}$ non-diagonal). Moreover, in general the number of lags in (16) depends on the lags characterizing the process (12)-(13) for $\Delta \mathbf{w}_t$.

The error-correcting dynamic structure of the system (14) and its equations (16) also allow to explain the failure of conventional risk sharing tests. By referring to the differenced version of (2), risk sharing tests are typically aimed at establishing the orthogonality of Δc_t^j, corrected for Δc_t^0 and $\Delta r_t^{j/0}$, to the information set $\mathcal{F}_t$; the equation (16) suggests that if consumers compute and update expectations through a dynamic model similar to (12), then Δc_t^j must be orthogonal with respect to the information set $\mathcal{F}_t$ only after correcting for Δc_t^0, $\Delta r_t^{1/0}$, ..., $\Delta r_t^{j/0}$, ..., $\Delta r_t^{N/0}$, Δc_{t-1}^0, $\Delta r_{t-1}^{1/0}$, ..., $\Delta r_{t-1}^{j/0}$, ... and all the elements in $(\mathbf{c}_{t-1} - \mathbf{\Upsilon} \mathbf{w}_{t-1})$.

3.2 Further implications

The system (14) has been derived under precise assumptions on the process used by agents to compute expectations on exchange rates changes and on the growth rate of consumption of the leader country. It is indeed assumed that the data generating process for $\Delta \mathbf{w}_t$ belongs to the class of VAR processes (12). Abstracting from the fact that the exogeneity restrictions implied by (12) can be easily tested (see Section 4), the hypothesis that expectations on the short term fluctuations of exchange rates are not driven by future developments in 'fundamentals' may be viewed as restrictive, see e.g. Engel and West (2005).

When the process (12) is misspecified because of feedbacks from $\Delta \mathbf{c}_t$ to $\Delta \mathbf{w}_t$, the system (14)-(15) can be no more intended as the approximate solution to the proposed risk sharing model. An alternative approach can be pursued to derive testable implications of the model without being specific about the expectations generating system; along the lines of Engsted and Haldrup (1994) the idea is to interpret the system (11) as a special case of a present value (PV) model and apply suitable econometric techniques.

Let $\mathbf{e}_t = \mathbf{c}_t - \boldsymbol{\Upsilon}\mathbf{w}_t - \phi(t+1)$ be the $(N-1)\times 1$ vector containing deviations of actual consumption from the optimal risk sharing position, and define the (stationary) linear combination

$$\mathbf{S}_t = \mathbf{e}_t - \mathbf{K}\mathbf{e}_{t-1} + \mathbf{K}\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_t.$$

By simply extending Engsted and Haldrup (1994) to the case of multiple decision variables, standard algebraic manipulations of the model (11) imply that the stationary 'spread'

$$\xi_t = \mathbf{S}_t - (\rho\mathbf{K})^{-1}\mathbf{S}_{t-1} + (\mathbf{I}_{(N-1)} - \mathbf{K})\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_t \tag{17}$$

must be unpredictable given information available at time $t-2$. This property of ξ_t is derived under the assumption that the disturbance term $\mathbf{v}_t$ in (11) is such that $E(\mathbf{v}_t \mid \mathcal{F}_t) = \mathbf{v}_t$, that means that $\mathbf{v}_t$ belongs to the information set at time t; on the contrary, if it is assumed that $\mathbf{v}_t = 0$ then it can be proved that the spread in (17) is unpredictable given information dated $t-1$ and earlier.

As it will be detailed in Section 4.2, the property of the spread with respect to the observable information may be used to set out a test of the dynamic risk sharing models in the situations where consumption changes and relative consumption help to predict real exchange rates and to estimate the parameters of interest.

4 Estimation and testing

The cointegration implications (3) of the risk sharing model introduced in the present paper have been already discussed in Section 2. Thus it will assumed, except where explicitly indicated, that the preference parameters in $\boldsymbol{\Upsilon}$ and ϕ are known or fixed at their super-consistent estimates obtained in a first stage by means of cointegration techniques (see below).

Given $\mathbf{\Upsilon}$ and ϕ and after fixing the average discount factor ρ to an economically plausible value, the empirical analysis of the dynamic risk sharing model introduced in Section 3 requires the estimation of the structural parameters $\mathbf{K}$ in (11) and a test of consistency with the data. To this purpose we propose two approaches discussed respectively in Section 4.1 and Section 4.2. In the first case we deal with the maximum likelihood (ML) estimation of the dynamic risk sharing model (12)-(14), i.e. the model obtained under the assumption agents compute expectations on the consumption growth rate of the leader country and on the changes of real exchange rates by assuming that $\Delta\mathbf{w}_t = (\Delta c_t^0,\ \Delta r_t^{1/0},\ \ \Delta r_t^{2/0},\ ...,\ \Delta r_t^{N-1/0})'$ is strongly exogenous with respect to $\mathbf{\Upsilon}$ (see Section 3.1). In the second case we set out a regression-based method which hinges on the PV nature of model (11) regardless the expectations generating system.

4.1 Maximum likelihood approach

Define the $(2N-1)\times 1$ vector $\mathbf{y}_t = (\mathbf{c}_t'\ ,\ \mathbf{w}_t')'$ and assume that the DGP is generated by the I(1) cointegrated model (Johansen, 1996)

$$\Delta\mathbf{y}_t = \alpha\beta'\mathbf{y}_{t-1} + \mathbf{\Pi}_1\Delta\mathbf{y}_{t-1} + ... + \mathbf{\Pi}_{p-1}\Delta\mathbf{y}_{t-p+1} + \mu_0 + \mu_1 t + \varepsilon_t \quad (18)$$

where α is the $(2N-1)\times r$ matrix of adjustment parameters, r the number of cointegrating relations among the elements of $\mathbf{y}_t$, β is the $(2N-1)\times(N-1)$ matrix containing the cointegrating vectors, $\widetilde{\mathbf{\Pi}}_i$, $i = 1,...,p-1$ are $(2N-1)\times(2N-1)$ matrices, $\varepsilon_t = (\varepsilon_{ct}'\ ,\ \varepsilon_{wt}')'$ is a MDS with respect to $\mathcal{F}_t$ with Gaussian distribution and covariance matrix partitioned as follows

$$\mathbf{V}_\varepsilon = \left[\begin{array}{cc} \mathbf{V}_{cc} & \mathbf{V}_{cw} \\ \mathbf{V}_{wc} & \mathbf{V}_{ww} \end{array}\right].$$

Given $\mathbf{y}_t = (\mathbf{c}_t'\ ,\ \mathbf{w}_t')'$ and $\varepsilon_t = (\varepsilon_{ct}'\ ,\ \varepsilon_{wt}')'$, we can consider the corresponding partitions of the parameters of the VECM:

$$\begin{aligned} \alpha &= \begin{pmatrix} \alpha_c \\ \alpha_w \end{pmatrix},\ \mu_0 = \begin{pmatrix} \mu_{0,c} \\ \mu_{0,w} \end{pmatrix},\ \mu_1 = \begin{pmatrix} \mu_{1,c} \\ \mu_{1,w} \end{pmatrix}. \\ \mathbf{\Pi}_i &= \left[\begin{array}{c} \mathbf{\Pi}_{c,i} \\ \mathbf{\Pi}_{w,i} \end{array}\right] = \left[\begin{array}{cc} \mathbf{\Pi}_{cc,i} & \mathbf{\Pi}_{cw,i} \\ \mathbf{\Pi}_{wc,i} & \mathbf{\Pi}_{ww,i} \end{array}\right],\ i = 1,...,p-1 \end{aligned}$$

which allows to express the partial system for $\Delta \mathbf{c}_t$ and the partial system for $\Delta \mathbf{w}_t$ given the past information $\mathcal{F}_{t-1}$ as

$$\begin{aligned} \Delta \mathbf{c}_t &= \alpha_c(\beta' \mathbf{y}_{t-1}) + \mathbf{\Pi}_{c,1}\Delta \mathbf{w}_{t-1} + ... + \mathbf{\Pi}_{c,p-1}\Delta \mathbf{w}_{t-p+1} \\ &\quad + \mu_{0c} + \mu_{1c}t + \varepsilon_{ct} \end{aligned} \tag{19}$$

$$\begin{aligned} \Delta \mathbf{w}_t &= \alpha_w(\beta' \mathbf{y}_{t-1}) + \mathbf{\Pi}_{w,1}\Delta \mathbf{w}_{t-1} + ... + \mathbf{\Pi}_{w,p-1}\Delta \mathbf{w}_{t-p+1} \\ &\quad + \mu_{0w} + \mu_{1w}t + \varepsilon_{wt} \end{aligned} \tag{20}$$

As ε_t is Gaussian, the partial system for $\Delta \mathbf{c}_t$ conditioned with respect to $\Delta \mathbf{w}_t$ and past information $\mathcal{F}_{t-1}$ is then given by

$$\begin{aligned} \Delta \mathbf{c}_t &= \omega \Delta \mathbf{w}_t + \delta(\beta' \mathbf{y}_{t-1}) + \widetilde{\mathbf{\Pi}}_1 \Delta \mathbf{w}_{t-1} + ... + \widetilde{\mathbf{\Pi}}_{p-1}\Delta \mathbf{w}_{t-p+1} \\ &\quad + \widetilde{\mu}_0 + \widetilde{\mu}_1 t + \widetilde{\varepsilon}_{ct} \end{aligned} \tag{21}$$

where $\omega = \mathbf{V}_{cw}\mathbf{V}_{ww}^{-1}$, $\delta = \alpha_c - \omega\alpha_w$, $\widetilde{\mathbf{\Pi}}_i = \mathbf{\Pi}_{c,i} - \omega \mathbf{\Pi}_{w,,i}$, $\widetilde{\mu}_h = \mu_{h,c} - \omega\mu_{h,w}$, $h = 0, 1$, and $\widetilde{\varepsilon}_{ct} = \varepsilon_{ct} - \omega\varepsilon_{wt}$ (Johansen, 1996) with $E(\widetilde{\varepsilon}_{ct}\varepsilon'_{wt}) = \mathbf{0}_{(N-1)\times N}$ by construction.

It can be now seen that the dynamic risk sharing model (12)-(14) represents a special case of (20)-(21). Indeed, the former can be obtained from the latter under a precise set of restrictions on both the long run and short run parameters.

As regards the long run, the vector $\mathbf{e}_t = \mathbf{c}_t - \mathbf{\Upsilon}\mathbf{w}_t - \phi\,(t+1)$ should be stationary for the risk sharing model to hold (Section 2); therefore, the number r of cointegrating relations involving $\mathbf{y}_t$ should not be lower that $N-1$. Furthermore, by partitioning β of (18) as $\beta = (\beta_c\ ,\ \beta_w)$, it must hold:

$$\beta_c = \begin{pmatrix} \mathbf{I} \\ -\mathbf{\Upsilon}' \end{pmatrix} \quad , \quad \mu_{1c} = \alpha_c\phi \tag{22}$$

so that given $\delta = (\delta_c\ ,\ \delta_w)$, the quantity $\delta(\beta'\mathbf{y}_{t-1}) + \widetilde{\mu}_1 t$ corresponds to $\delta_c\mathbf{e}_{t-1} + \delta_w\beta'_w\mathbf{y}_{t-1}$ in (21), where the second term cancels out when $r = N-1$. It is clear that a number of cointegrating relations lower than $N-1$ implies that in the long run consumption streams and real exchange rates do not conform to (3)[7].

[7]For the sake of simplicity we do not consider the case $r > (N-1)$.

As regards the short run, once the cointegration rank is fixed to $r = N-1$ and β is fixed to (22) (in this case $\beta_w = \mathbf{0}$, $\delta_w = \mathbf{0}$), the structure of dynamic adjustment implied by (12)-(14) is obtained if the following exclusion (testable) restrictions on (21) and (20) hold jointly:

$$\widetilde{\mathbf{\Pi}}_{cc,i} = \mathbf{0} \ \ , i = 1, ..., \ p-1 \ , \ \ \widetilde{\mathbf{\Pi}}_{cw,p-1} = \mathbf{0} \tag{23}$$

$$\alpha_w = \mathbf{0} \ \ , \ \mathbf{\Pi}_{wc,i} = \mathbf{0} \ \ , i = 1, ..., \ p-1 \ \ , \tag{24}$$

where $\widetilde{\mathbf{\Pi}}_i = \left[\begin{array}{cc} \widetilde{\mathbf{\Pi}}_{cc,i} & \widetilde{\mathbf{\Pi}}_{cw,i} \end{array} \right]$. It can be easily recognized that the latter constraints correspond to the strong exogeneity of $\Delta \mathbf{w}_t$ with respect to β.

Under (23)-(24) the dynamic risk sharing model is equivalent to the unrestricted conditional VAR model (21) under the constraints

$$\delta = \mathbf{K} - \mathbf{I}_{(N-1)} \ \ , \ \omega = \mathbf{\Gamma}_0 \ , \ \widetilde{\mathbf{\Pi}}_{cw,i} = \mathbf{\Gamma}_i \ \ , \ i = 1, ..., p-2$$

and therefore constrained estimation provides the ML estimates of $\mathbf{K}$, $\mathbf{\Gamma}_0$, $\mathbf{\Gamma}_i$, $i = 1, ..., p-2$. Furthermore, for fixed values of β ($\mathbf{\Upsilon}$ and ϕ) and of the intertemporal discount factor ρ, a likelihood ratio (LR) or Wald test can be carried out for the cross-equation rational expectations restrictions (15), where $\mathbf{\Phi}$ is defined exactly as in (13) (with $\mathbf{\Phi}_i = \mathbf{\Pi}_{ww,i}$, $i = 1, ..., p-1$).

Summing up, the VEqCM (18) can be used to estimate the parameters of the dynamic risk sharing model through standard likelihood methods. In principle, the number of cointegrating relation could be determined by Johansen's (1996) procedure. After the cointegration rank is determined, the constraints on the cointegration space as well as those on the deterministic drift terms as in (22) could be tested through standard LR or Wald-type statistics. However, since the model involves both a time-series and spatial dimension, it can be easily recognized that even with a relatively long span of data, it is sufficient a small number of countries, N, to the make the analysis based on $\mathbf{y}_t = (\mathbf{c}_t' \ , \mathbf{w}_t')'$ cumbersome, due to the huge number of parameters to be estimated.[8]

The estimation approach is sketched in the steps that follow:

[8]On the other hand, the approach proposed by Pesaran et al. (2004) for modelling regional interdependencies through a "Global" VAR (GVAR) model, though appealing, is in this case not carried out; indeed, by construction in Pesaran et al. (2004) each country-specific VAR contains a set of foreign variables constructed as weighted averages of the

1 Estimate the preference parameters $\boldsymbol{\Upsilon}$ and ϕ of (4) by specifying, for each country $i = 1, ..., N-1$, VAR models of the form $\mathbf{x}_t = (c_t^i, c_t^0, r_t^{i/0})'$ (with a linear trend in the deterministic part) and investigating the number and structure of cointegration relations by following Johansen's (1996) procedure. This allows, for $i = 1, ..., N-1$, to recover super-consistent estimates of the preference parameters in $\boldsymbol{\Upsilon}$ and ϕ.

2 Given the (super-consistent) estimates of $\boldsymbol{\Upsilon}$ and ϕ and having fixed β and μ_{1c} as in (22), estimate the VECM (18) (hence (21)-(20)) and test the validity of the zero restrictions (23)-(24)[9]; if these are not supported by the data reject the form (14)-(12) of the dynamic risk sharing model, otherwise go to the step 3 below. Observe that the rejection of (12)-(14) implies that the dynamic risk sharing model obtained under the assumption of strong exogeneity of $\Delta\mathbf{w}_t$ is not supported by the data; it could happen, in fact, that the more general implications the implications described in Section 4.2

3 Estimate the system (21) subject to (23) and recover the ML estimates of $\mathbf{K}$, $\boldsymbol{\Gamma}_0$, $\boldsymbol{\Gamma}_i$, $i = 1, ..., p-2$. For fixed values of the intertemporal discount factor ρ, check the validity of the rational expectations constraints (15).

4.2 Present value test

As observed in Section 3.2, regardless the 'true' structure of the expectations generating feedbacks and the presence of feedbacks from $\Delta\mathbf{c}_t$ (and $\mathbf{e}_t$) to $\Delta\mathbf{w}_t$, a general implication of the PV model (11) is that the spread

$$\xi_t = \mathbf{S}_t - (\rho\mathbf{K})^{-1}\mathbf{S}_{t-1} + (\mathbf{I}_{(N-1)} - \mathbf{K})\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_t \tag{25}$$

country specific variables with weight given by trade shares (or similar weighting schemes), whereas in our analysis each country-specific VAR involves per capita consumption of the leader country (see below).

[9]Observe that as long as the number of countries involved is around $N = 5$ or $N = 6$, the estimation of the stationary model (21)-(20) with cointegrating relations fixed is still feasible if the number of time-series observations T, is sufficiently high, see the results that follows.

is unpredictable given the information available at time $t-2$ if the disturbance term $\mathbf{v}_t = (\mathbf{I}_N - \mathbf{K})\eta_t$ can be argued to belong to the observable information set $\mathcal{F}_t$, and is unpredictable given the information available at time $t-1$ if $\mathbf{v}_t$ is set to zero. This consideration suggests that if the structural parameters $\boldsymbol{\Upsilon}$, ρ, $\mathbf{K}$ of the model were known, a simple (weaker) test of the dynamic risk sharing model might be constructed by regressing ξ_t on information variables dated $t-2$ (or $t-1$) and testing for their joint significance.

More specifically, consider the standard VAR(q) approximation for the spread ξ_t, q being sufficiently large:

$$\xi_t = \mathbf{b} + \sum_{i=1}^{q} \mathbf{G}_i \xi_{t-i} + \varphi_t \quad ; \quad \varphi_t \sim WN(\mathbf{0}\,, \boldsymbol{\Lambda}). \tag{26}$$

where $\mathbf{b}$ is a $(N-1)\times 1$ constant and $\mathbf{G}_i$ are $(N-1)\times(N-1)$ matrices of parameters. Given the above VAR representation a necessary condition for the spread to be unpredictable given information available at time $t-2$ is that

$$\mathbf{G}_j = \mathbf{0}_{(N-1),(N-1)}\ ,\ j = 2, ..., p \tag{27}$$

whereas the additional constraint

$$\mathbf{G}_1 = \mathbf{0}_{(N-1),(N-1)} \tag{28}$$

must hold if the spread is unpredictable given information available at time $t-1$.

In principle, the above zero restrictions can be easily tested in the context of the stationary VAR(q) model for ξ_t. In practice, however, although the a priori knowledge on the average intertemporal discount factor ρ is high and the parameters in $\boldsymbol{\Upsilon}$ can be replaced by their super-consistent estimates without affecting the asymptotics, the adjustment matrix $\mathbf{K}$ entering the spread equation is unknown and must be estimated.

The simple regression-based approach of Section 3.2 can be followed. Indeed by using the definition of ξ_t and $\mathbf{S}_t$ and imposing the zero restrictions (27), the VAR model (26) reads, after rearranging terms as,

$$\begin{aligned}
\mathbf{e}_t - \boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_t &= \mathbf{b}+[\mathbf{K} + (\rho\mathbf{K})^{-1} + \mathbf{G}_1]\mathbf{e}_{t-1} \\
&\quad +[\mathbf{G}_1 + \rho^{-1}\mathbf{I}_{N-1}]\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_{t-1} \\
&\quad +[\rho^{-1}\mathbf{I}_{N-1} - \mathbf{K} - \mathbf{G}_1(\rho\mathbf{K})^{-1}]\mathbf{e}_{t-2} \\
&\quad -\rho_1^{-1}\mathbf{G}_1\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_{t-2} + \rho_1^{-1}\mathbf{G}\mathbf{e}_{t-3}+\varphi_t \qquad (29)
\end{aligned}$$

where we recall that ρ and $\boldsymbol{\Upsilon}$ can be treated as known. The structure of the above multiple regression model suggests that, abstracting from the moment from the non-linear (over-identifying) restrictions characterizing the parameters $\mathbf{G}_1$ and $\mathbf{K}$, a test of the necessary condition for the dynamic risk sharing model (11) to hold can be carried out by regressing $\mathbf{e}_t - \boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_t$ over a constant, $\mathbf{e}_{t-1}$, $\mathbf{e}_{t-2}$, $\mathbf{e}_{t-3}$, $\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_{t-1}$ and $\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_{t-2}$ and testing whether the residuals $\widehat{\varphi}_t$ are generated by an uncorrelated process. If the residuals of such multiple regressions are 'well-behaved' it is possible to proceed by re-estimating (29) by ML subject to the non-linear restrictions

$$\begin{aligned}
\mathbf{A}_1 &= \mathbf{K} + (\rho\mathbf{K})^{-1} + \boldsymbol{\Psi}_1 \\
\mathbf{A}_2 &= (\mathbf{G}_1 + \rho^{-1}\mathbf{I}_{N-1}) \\
\mathbf{A}_3 &= \rho^{-1}\mathbf{I}_{N-1} - \mathbf{K} - \mathbf{G}_1(\rho\mathbf{K})^{-1} \\
\mathbf{A}_4 &= -\rho_1^{-1}\mathbf{G} \\
\mathbf{A}_5 &= \rho_1^{-1}\mathbf{G}
\end{aligned}$$

where we denoted respectively by $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, $\mathbf{A}_4$ and $\mathbf{A}_5$ the parameters associated with the regression.

Estimation simplifies remarkably if ξ_t is assumed to be unpredictable given information available at time $t-1$, see the discussion in Section 3.2. Now, in addition to (27) also (28) holds in the VAR (26) and the regression model (29) collapses to

$$\begin{aligned}
\mathbf{e}_t - \boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_t - \rho^{-1}\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_{t-1} &= \mathbf{b}+[\mathbf{K} + (\rho\mathbf{K})^{-1}]\mathbf{e}_{t-1} \qquad (30) \\
&\quad +[\rho^{-1}\mathbf{I}_{N-1} - \mathbf{K}]\mathbf{e}_{t-2}+\varphi_t.
\end{aligned}$$

and the necessary condition for the spread to be unpredictable is that the residuals $\widehat{\varphi}_t$ of the OLS regression of $\mathbf{e}_t - \boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_t - \rho^{-1}\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_{t-1}$ over a constant, $\mathbf{e}_{t-1}$ and $\mathbf{e}_{t-2}$ are uncorrelated. Moreover, given the equation above the matrices of parameters $\mathbf{B}_1$ and $\mathbf{B}_2$ of the multiple regression

$$\mathbf{e}_t - \boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_t - \rho^{\mathbf{-1}}\boldsymbol{\Upsilon}\boldsymbol{\Delta}\mathbf{w}_{t-1} = \mathbf{b} + \mathbf{B}_1\mathbf{e}_{\mathbf{t-1}} + \mathbf{B}_2\mathbf{e}_{t-2} + \varphi_t \tag{31}$$

satisfy, under the null (27)-(28)

$$\mathbf{B}_1 + \mathbf{B}_2 = \rho^{-1}(\mathbf{K}^{-1} + \mathbf{I}_{N-1}) \tag{32}$$

$$\mathbf{B}_2 = \rho^{-1}\mathbf{I}_{N-1} - \mathbf{K} \tag{33}$$

so that $\mathbf{B}_1$ and $\mathbf{B}_2$ are non-linearly restricted with $(N-1)^2$ over-identifying constraints. The estimation of (31) through ML subject to (32)-(33) delivers a LR test for the model and, if the over-identifying restrictions are supported by the data, a consistent estimate of $\mathbf{K}$.

5 Empirical results

In this Section we apply the proposed international risk sharing model to a set of 'core' European countries that joined the European Monetary Union (EMU) in 1999. Specifically, some of the major (in terms of population and GDP level) countries are considered, i.e. Germany, France, Italy, Spain, Netherlands, Belgium, Portugal and Austria; we also included the U.K. as the most important 'non-EMU joining' country of the European Union (EU). As is standard in the risk sharing literature (Sørensen and Yosha, 1998, Asbrubali and Kim, 2004) we use annual data over a relatively long period from 1963 to 2003 and consider Germany as the 'leader' country. Data are collected from several international sources. Data on private final consumption expenditure at current prices, total population, and price deflators for final consumption expenditure are taken from National Accounts and Eurostat. Nominal exchange rates are constructed by using conversion rates between Euro and former Euro-zone national currencies (source: Eurostat). The real exchange rates, $rex_t^{i/0}$, are defined as $rex_t^{i/0} = ex_t^{i/0} + p_t^0 - p_t^i$, where $ex_t^{i/0}$ is the (logged) nominal exchange rate between the currency of the i-th country and the DM, p_t^0 is the (logged) private consumption deflator of Germany and p_t^i is the (logged) private consumption deflator of country i. After 1999 variations in real exchange rates are due to relative prices only, except for the U.K..

Some descriptive evidence is reported in Table 1, where for each country i =France, Italy, Spain, Netherlands, Belgium, Portugal, Austria and U.K., and given 0 =Germany, the correlation between real (per capita) consumption growth (Δc_t^i) relative to Germany (Δc_t^0) and the corresponding changes of the bilateral real exchange rate ($\Delta r_t^{i/0}$), $Corr(\Delta c_t^i - \Delta c_t^0$, $\Delta r_t^{i/0})$ are computed. Consistent with other studies, Table 1 presents *prima facie* evidence at odds with open economy models with FRS.

The apparent lack of risk sharing among the major European countries resulting from Table 1 could be explained, in the light of the theoretical reference equation (1), by observing that each pair of investigated countries might not share the same relative risk aversion parameters, i.e. $\theta^i \neq 1$ so that the actual correlation is between $\Delta c_t^i - \theta^i \Delta c_t^0$ and $\Delta r_t^{i/0}$.[10] Another argument is that the poor correlations of Table 1 can be due to the effect of non-negligible stochastic terms η_t^i, which in the light of the model specified in Section 2 may reflect: the preference shocks of the two countries, or arguments of the utility function that interact non-separably with consumption but which have not explicitly modelled, e.g. hours worked/leisure, government spending, real money balances, see e.g. Ravn (2001). Our alternative explanation is that the empirical counterpart of the model (1)-(2) should be regarded as a long run relation between the variables in levels c_t^i, c_t^0 and $r_t^{i/0}$.

RISK SHARING IN THE LONG RUN. Following the procedure outlined in Section 4.1 we proceeded by estimating for i=France, Italy, Spain, Netherlands, Belgium, Portugal, Austria and U.K., separate VARs of the form $\mathbf{x}_t = (c_t^i, c_t^0, r_t^{i/0})'$ over the 1963-2003 period, with a linear trend in the deterministic part and restricted to belong to the cointegration space. Although one could reasonably expect the presence of a structural break in

[10]Moreover, as argued in Section 3, if countries face adjustment costs even the correlation between $\Delta c_t^i - \theta^i \Delta c_t^0$ and $\Delta r_t^{i/0}$ might be flawed by the omission of the lagged quantities Δc_{t-1}^0, $\Delta r_{t-1}^{j/0}$, ... and the error-correcting deviations $(\mathbf{c}_{t-1} - \boldsymbol{\Upsilon}\mathbf{w}_{t-1})$.

1991 due to the German unification in all estimated VARs, we did not find clear evidence in favor of a structural change.

Before estimating the VARs we preliminarily carried out standard unit-roots tests on the time-series involved; results are reported in Table 2 and indicate that all variables appear driven by I(1) stochastic trends. This result suggests that 'strong form' PPP, i.e. the stationarity of real exchange rates can be sharply rejected in our framework. This result also shows that an additional cointegrating relation given by the trivial linear combination $(0, 0, 1)\mathbf{x}_t$ should not be expected to hold in $\mathbf{x}_t = (c_t^i, c_t^0, r_t^{i/0})'$.

After selecting the optimal number of lags ensuring 'well-behaved' VAR residuals, we tested for the number of cointegrating relations and estimated, where possible, the relation (3). Results are reported in Table 3 and Table 4. Table 3 summarizes the 'lambda-max' and 'trace' LR tests for cointegration rank (Johansen, 1996) and Table 4 the estimated equation (3) for the country pairs where a theoretically consistent cointegrating relation between c_t^i, c_t^0 and $r_t^{i/0}$ was found.[11] It can be noticed that for the Netherlands-Germany pair we reported the long run risk sharing equation (3) even though the tests for cointegration rank did not support its existence; indeed, differently from the Spain-Germany and UK-Germany pairs, in that case we obtained, by forcing the cointegration rank to one, a theoretically consistent (stationary) relation in levels. On the other hand, the cointegration rank tests shed some doubt on the cointegration rank characterizing the VAR model for Portugal-Germany; nevertheless, in this case we were not able to identify any cointegrating relation consistent with the theory.

In summary, from the results of Table 3 and Table 4 it is possible to identify six 'core' EMU countries: Germany, France, Netherlands, Belgium, Austria and to some extent Italy, which seem to share risks, over the 1963-2003 period, consistently with the predictions of the model (1)-(2). For easy of reference henceforth we shall refer these countries as 'Group 1'. On the other hand, the two Iberian countries and the U.K. seem to depart from the implications of the theory. We shall refer to Spain and Portugal as

[11] We carried out the empirical analysis also over sub-samples. In particular, we devoted attention to the 1975-2003 and 1987-2003 sub-periods and in both cases we did not find significant differences with respect to the results reported in the tables that follow.

'Group 2'.[12]

Referring to the countries of Group 1, the estimated relations in Table 4 suggest that German real per capita consumption plays the role of a driving factor; moreover for these countries we do find a significant role for the real exchange rate in the risk sharing relations. Except for the Italy-Germany country pair, all estimated coefficients exhibit signs consistent with the theory and reveal that countries do not have identical attitudes towards risk. The estimated ratio of the German coefficient relative to country i-th relative risk aversion coefficient, θ^i, ranges from 0.57 to 1.31, whereas the estimated intertemporal rate of substitution of consumption, δ^i, ranges from 0.22 to 2.81 (excluding the case of Italy-Germany). The unexpected sign of the estimated δ^i in the Italy-Germany VAR model seems to indicate a violation of concavity in the CRRA utility function, albeit the χ^2 test for the hypothesis $\delta^i = 0$ leads to a p-value slightly above 0.05. The magnitude of the estimated δ^is in the Netherlands-Germany and Belgium-Germany pairs also points that albeit useful, the assumed CRRA specification for utility functions can be too restrictive, a common finding in the consumption literature. For all five country-pairs a linear trend enters significantly the cointegrating relations (see the parameter ϕ^i in (3)), highlight valuable (although small in magnitude) differences between countries intertemporal discount rates. Moreover, for the five European country-pairs reported in Table 4 the η_t^i stochastic terms appearing in the risk sharing relation (2) can be modelled as a stationary processes; this means that either preference shocks are stationary, or (less likely) that variables omitted from the utility function that interact non-separably with consumption are stationary.

By interpreting the results of Table 4 in the spirit of Obstfeld (1989, 1994), Backus and Smith (1993), Canova and Ravn (1996) and Ravn (2001), it can be concluded that the countries of Group 1 tend to insure risks against permanent income shocks. These findings contrast with Sørensen and Yosha (1998) where, using methods that ignore dynamic adjustment, poor risk sharing is detected in Europe.

On the other hand, the countries of Group 2, including the UK, do not seem to be part of this process. This evidence, nevertheless, does not rule

[12]Notice that Spain and Portugal joined the EU only in 1986. Among the countries of Group 1 Austria joined the EU in 1995.

out the occurrence of international insurance mechanisms shielding against short term income fluctuations, an issue which is investigated below.

DYNAMIC ADJUSTMENT. Having established that international risk sharing as a long run phenomenon seem to characterize the EMU countries of Group 1, we investigated the short run dynamic properties of the real per capita consumptions in the light of the theoretical implications of the model outlined in Section 3. Recall that the proposed dynamic risk sharing model implies that adjustments should be interrelated, in the sense that also foreign countries' lagged temporary deviations from FRS are important for explaining future home consumption changes. One of our primary interests is to evaluate whether this prediction of the model finds support from the data.

We start by estimating an unrestricted VECM model for $\mathbf{y}_t = (\mathbf{c}_t'$, $\mathbf{w}_t')'$, $\mathbf{c}_t = (c_t^{fr}, c_t^{it}, c_t^{ne}, c_t^{be}, c_t^{au})'$, $\mathbf{w}_t = (c_t^{ge}, r_t^{fr}, r_t^{it}, r_t^{ne}, r_t^{be}, r_t^{au})'$ over the 1963-2003 period with the cointegration equations (22) fixed at the super-consistent estimates obtained in Table 4. The number of lags of the VECM is set to 1 (i.e. 2 lags in the corresponding VAR in levels); standard residual-based specification tests, reported in Table 5, do not show any particular departure from the model assumptions. Furthermore, the stability of the model, for fixed cointegration relations, has also been checked through recursive techniques, showing that the short run parameters appear to be substantially stable over the considered time span.[13]

According to (4) the relation between $\mathbf{c}_t - \mathbf{c}_t^* = \mathbf{u}_t$ and disequilibrium error terms, $\mathbf{e}_t = \mathbf{c}_t - \boldsymbol{\Upsilon}\mathbf{w}_t - \phi(t+1)$, is given by $\mathbf{e}_t = \mathbf{u}_t + \eta_t$. Thus the dynamic behavior of the estimated deviations from long run risk sharing of Table 4 summarized in the vector $\widehat{\mathbf{e}}_t = (\widehat{e}_t^{fr}, \widehat{e}_t^{it}, \widehat{e}_t^{ne}, \widehat{e}_t^{be}, \widehat{e}_t^{au})'$ provide insights about the process of adjustment. Consider a VAR representation for the stationary vector $\widehat{\mathbf{e}}_t$; by computing generalized impulse response functions (Pesaran and Shin, 1996, 1998) it is possible to figure out not only the speed of adjustment to FRS of each country of Group 1, but also whether the adjustment to equilibrium is interrelated across coun-

[13]We did not find clear-cut evidence of the presence of structural breaks in correspondence of the German unification. Indeed, the unification might have lead to an increase of the amount of risk sharing.

tries. Figure 1 plots the generalized impulse response functions calculated with respect to one standard error shock to respectively each equation of the VAR over an horizon of ten years; the estimated VAR includes one lag and an unrestricted constant. The graphs support the idea that convergence is slow in the countries of Group 1 and that temporary deviations from risk sharing in one of these have remarkable effect in the other countries. From the generalized impulse response functions it turns out that the average half-life[14] of deviations from FRS due to country specific shocks is about 3 years. The average half-life of deviations from FRS due to generalized shocks in the other countries is about 3.9 years for France, 4.7 years for Italy, 2.9 years for Netherlands, 3.4 years for Belgium and 3.9 years for Austria. This evidence suggests that the dynamics of consumption streams might be captured by the model sketched in Section 3.

We thus proceeded, along the line summarized in Section 4.1 by investigating whether the restrictions that the system (14) entails on the short run parameters of the VECM are consistent with the empirical evidence. The unrestricted VECM model for $\mathbf{y}_t = (\mathbf{c}_t' \, , \, \mathbf{w}_t')'$ clearly appears to be overparametrized. In order to find a suitable reduction of the model, in Table 6 a number of sets of zero restrictions are tested (through standard likelihood ratio tests) separately for each equation as well as at the system level. In the first row of the table, we test for each equation whether the lagged first differences of non-country specific variables (with the exception of the German consumption) can be excluded. Similarly, in the second row of the table, we test for each equation whether the lagged first differences of all variables with the exception of the German consumption and the country-specific real exchange rate can be excluded (recall that, according to (14), Δc_{t-1}^j should not appear among the regressors of the j-th equation).[15] For France, Belgium and Austria, the exclusion restrictions are not rejected, either with or without lagged home consumption growth among the regressors. For the Netherlands, the restrictions are not rejected

[14] The half-life can be defined in this context as the expected number of years for a deviation to FRS to decay by 50%.

[15] The dynamic adjustment structure obtained in our model under quadratic adjustment costs is tight: the inclusion of lagged consumption growth might help to catch possible habit persistence effects (or higher-order adjustment costs) which are implicitly not accounted in the solution of the model.

provided that lagged consumption growth appears among the regressors. Only in the case of Italy the restrictions are strongly rejected, showing that Italian consumption growth seems to be characterized by a more involved dynamic structure than the other countries (which may indicate imperfection in the risk sharing mechanisms at shorter horizons, as documented by Cavaliere et al., 2005). Turning to the issue of the impact of disequilibrium terms, i.e. the impact on the short-run consumption dynamics of the deviations of actual consumption streams from optimal risk sharing levels, in Table 6, rows 3 to 5, we test for each country (as well as at the system level) whether changes to home consumption do not depend (i) on the lagged disequilibria in all countries (row 3), (ii) on the lagged home disequilibrium (row 4) and (iii) on the lagged foreign disequilibria (row 5). Overall, the picture is quite clear: consumption changes *do* depend on past deviations from the optimal consumption level and cross-adjustment terms seem to be important for most countries.

Detailed estimates of the VECM model with the exclusion restrictions tested in Table 6 imposed are reported in Table 7. For France we find strong adjustment with respect to home past deviations from the optimal risk sharing position. For the Netherlands, home disequilibrium is partially significant (the associated p-value is just above 10%), and we also find evidence of interrelated disequilibrium correction mechanisms. Note that lagged consumption changes matters, stressing habit persistence phenomena even stronger than those accounted by the adjustment cost structure implied by the minimization cost problem (7). In the case of Belgium, we observe a significant effect of the home disequilibrium term; interestingly, we also observe a significant effect of the disequilibrium terms associated with neighbouring countries. In the case of Austria, only its own disequilibrium from FRS matters in explaining future home consumption changes. Finally, as anticipated above, changes to consumption in Italy display a quite complicate dynamic structure, which tend to be strongly affected by lagged consumption disequilibrium of most countries. In summary, these results shows that risk sharing, when present, is not likely to take place instantaneously; adjustment costs appear a possible explanation of the observed short run dynamic patterns of consumption.[16] In general,

[16]As previously noticed, a by-product implication is that traditional risk sharing tests

the presence of cross-adjustment terms in the country specific consumption equations points that idiosyncratic shocks tend to propagate across the 'core' European countries engaged in the risk sharing arrangement.

Even though labour mobility can be hardly regarded as a risk sharing channel in Europe (Puhani, 2001) and a permanent tax-transfer system ruled by a central fiscal institution is far from being fully at work, the above results show that short run consumption fluctuations of Group 1 countries appear consistent with a gradual process of adjustment towards the outcome that would prevail under 'frictionless' markets.

An interesting question is whether the Group 2 countries, which are not found to share risk with the other countries in the long run, are characterized by short term movements in their consumption which are similar to those observed for the Group 1 countries. To this purpose, in Table 8 we report the estimates of a VAR model for Spain and Portugal, including exogenous regressors given by the lagged error correcting terms of the Group 1 countries estimated in Table 4. Interestingly, while for the case of Portugal changes in real per capita consumption are found to depend only on their lagged value, for the case of Spain changes in consumption are correlated to contemporaneous changes in German consumption as well as to the deviations from the optimal FRS consumption levels of all the Group 1 countries. Hence, although failing to share risks in the long run with the Group 1 countries, Spain (but not Portugal) seems to be characterized by a marked process of adjustment of consumption which resembles – to some extent – that predicted by the proposed dynamic risk sharing model. Finally, as regards the UK we did not find a connection with consumption growth rates of Group 1 countries and the corresponding bilateral real exchange rate changes.

PRESENT VALUE TEST OF DYNAMIC RISK SHARING. According to the estimates of the VECM reported in Table 7, the zero restrictions (23) implied by the dynamic risk sharing model on the parameters of the estimated VECM can be rejected. The zero constraints in (23) result in 55 exclusion restrictions on the system (21) and lead to a LR statistic equal to 126.49.

based on growth rate specifications are usually misspecified in that they overlook the role of disequilibrium dynamics.

This result may depend, however, on the assumption of exogeneity of real exchange rates (and of German consumption), which underlie the derivation of the restrictions. The results in Table 9 shows that the exogeneity assumption (24) is clearly rejected[17], hence making formal tests of the cross-equation restriction derived in equation (15) unreliable. Hence we focused on the (testable) further implications of the risk sharing model discussed in the sections 3.2 and 4.2.

The risk sharing model (11) entails that the spread variable defined in (25) must be unpredictable given the information observable at time $t-2$ or $t-1$, depending on the assumptions made on preference shocks. Given the annual frequency of the data it is reasonable to investigate predictability of the vector of spreads relative the Group 1 countries with respect to the information available at time $t-1$. This can be done by checking whether the estimation of the regression model (31) delivers residuals which are not correlated. In this multiple regression model the vector of deviations of the observed consumption streams from the optimal FRS positions is defined as $\widehat{\mathbf{e}}_t = \mathbf{c}_t - \widehat{\boldsymbol{\Upsilon}}\mathbf{w}_t - \widehat{\phi}\,(t+1)$, where the preference parameters $\boldsymbol{\Upsilon}$ and ϕ are fixed at their super-consistent ML estimates of Table 4. The average intertemporal discount factor ρ can be selected within a given grid.

Table 10 reports standard residual correlation tests associated with the unrestricted OLS regression of $\widehat{\mathbf{e}}_t - \widehat{\boldsymbol{\Upsilon}}\boldsymbol{\Delta}\mathbf{w}_t - \rho^{-1}\widehat{\boldsymbol{\Upsilon}}\boldsymbol{\Delta}\mathbf{w}_{t-1}$ (with ρ fixed at 0.96) over a constant, $\mathbf{e}_{t-1}$ and $\mathbf{e}_{t-2}$. Residuals should be uncorrelated for the spread to be unpredictable given information at time $t-1$. Although at the system level the test seems to reject the hypothesis of uncorrelated residuals, at the single-equation level support for the null can be observed. Thus in its weakest form, i.e. without explicit reference to the implied non linear parametric constraints, the risk sharing model (11) appears consistent with the data. On the other hand, by regressing $\widehat{\mathbf{e}}_t - \widehat{\boldsymbol{\Upsilon}}\boldsymbol{\Delta}\mathbf{w}_t - \rho^{-1}\widehat{\boldsymbol{\Upsilon}}\boldsymbol{\Delta}\mathbf{w}_{t-1}$ over a constant, $\mathbf{e}_{t-1}$ and $\mathbf{e}_{t-2}$ subject to (32)-(33) the $(N-1)^2$=25 over-identification restrictions lead to a LR statistic of 126.8.[18]

[17]It is worth noting that the result that consumption helps forecasting real exchange rates confirms the finding in Apte et al. (2004).

[18]Due to the relatively high order (5×5) of the matrices of parameters involved, the estimation of the model (31) subject to (32)-(33) was simplified by replacing the constraints in (33) by a first-order Taylor series expansion around a 5×5 diagonal matrix; more precisely, assuming for easy of exposition that $\rho \approx 1$, the function $f(K) = K^{-1} + I$ (see

6 Final remarks

Risk sharing can be achieved through private markets or central fiscal institutions. Actual markets, however, are characterized by frictions that impede the instantaneous adjustment to the equilibrium and generate costs to countries. However, investment barriers, transport costs, tariffs and other barriers might not be large enough to impede the equalization of ex-post nominal marginal rate of substitution after a gradual process of adjustment.

This paper formalizes a stylized process of adjustment towards optimal risk sharing where countries attempt to minimize the costs of deviations from FRS. The idea is that intertemporal deviations from the first order conditions of one country may imply dynamic adjustment in all countries of the risk sharing pool. The resulting dynamic risk sharing equations give a precise role to expectations on future market developments and generate a slow and interrelated process of adjustment. One of the main implications is that positive (negative) shocks affecting one country produce adjustments in the consumption streams of the other countries belonging to the agreement. Omitting such kind of dynamics as in e.g. Kollmann (1995) and Canova and Ravn (1996) or by simply differentiating optimal risk sharing relations may flaw both tests and measures of the extent of international risk sharing.

Our dynamic risk sharing model is applied to a set of 'core' European countries which joined the EMU in 1999 over forty years span of data; in the absence of mobility in European labour markets one would expect integrated capital markets acting as shielding devices against permanent income fluctuations for a single currency to have economic grounds (Eichengreen, 1993). Our results suggest that once we allow preference parameters to vary across countries, European consumption streams tend to conform to the implications of risk sharing as a long run phenomenon and the PPP is violated. Moreover, our estimates highlight a marked process of adjustment towards equilibrium which, to some extent, can be attributed to intertemporal (dis)utility costs. Therefore the lack of European risk sharing found

(32)) was approximated linearly as $f(K) \approx f(K_0) + (-K_0)^{-1}(K - K_0)(K_0)^{-1}$, with K_0 containing the diagonal elements of the estimated adjustment coefficients $\widehat{\alpha}_c$ of (21), see Table 8. Given the statistical rejection of the over-identifying restrictions (32)-(33) we did not report in Table 10 the estimated $(I_5 - \widehat{K})$ matrix.

in previous studies, see e.g. Sørensen and Yosha (1998), can be potentially explained by the rich dynamic structure underlying European consumption streams.

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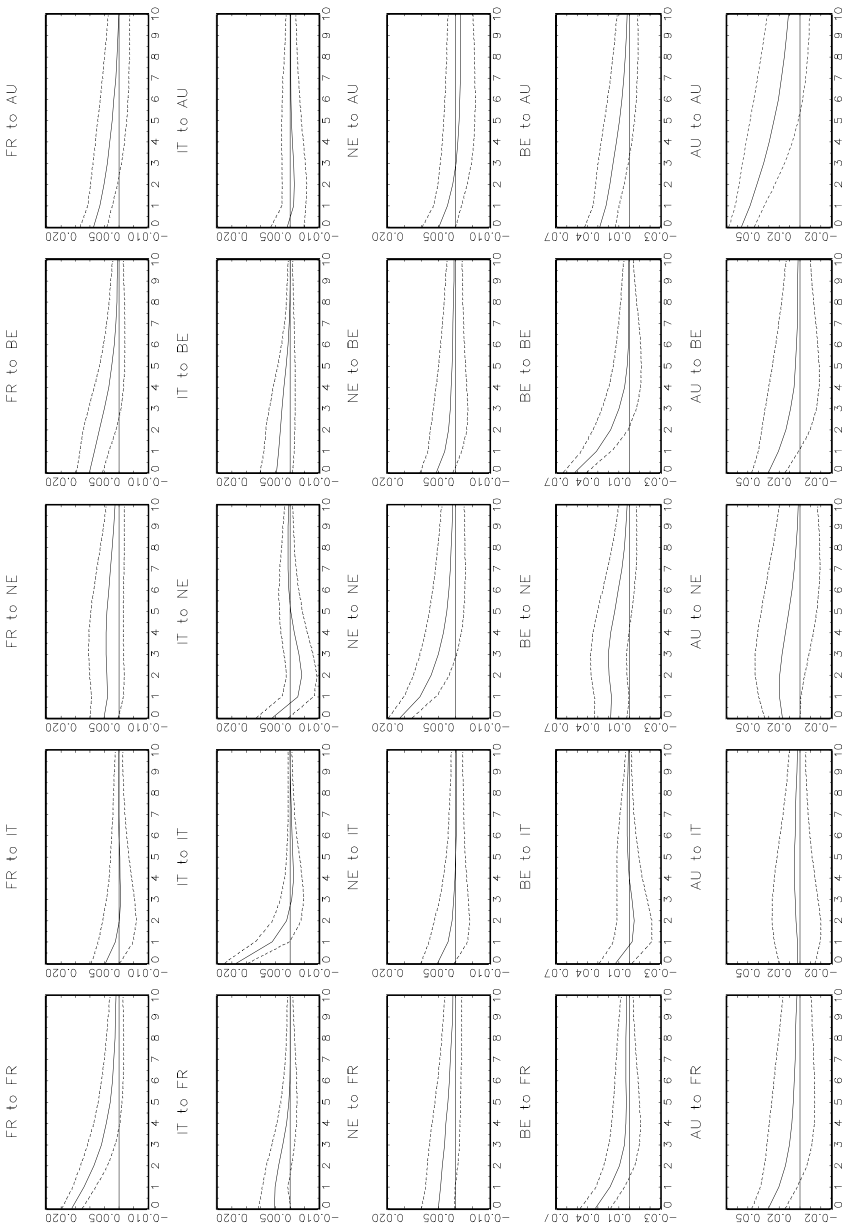

Figure 1: Generalized impulse–response functions computed from the VAR relative to consumption disequilibria terms $\widehat{\mathbf{e}}_t = (\widehat{e}_t^{fr}, \widehat{e}_t^{it}, \widehat{e}_t^{ne}, \widehat{e}_t^{be}, \widehat{e}_t^{au})'$.

FR	IT	SP	NE	BE	PO	AU	UK
−0.06	−0.29	−0.19	−0.06	0.32	−0.22	−0.08	−0.03

Table 1: Pearson correlation coefficients between real consumption growth (relative to Germany) and the real exchange rates growth.

Notes: correlations are computed as $Corr(\Delta c_t^i - \Delta c_t^0\ ,\ \Delta r_t^{i/0})$, $i =$ FRA,ITA,SPA,NET,BEL, POR,AUS,UK; $0 =$ GER.

	GE	FR	IT	SP	NE	BE	PO	AU	UK
c_t^i	−1.24	−1.52	−1.60	−2.50	−1.95	−1.94	−0.70	−0.95	−1.78
Δc_t^i	−1.40	−2.20	−2.10	−3.14	−2.47	−2.80	−1.32	−1.23	−4.28
$r_t^{i/0}$	−2.66	−2.00	−1.57	−2.62	−2.00	−1.42	−1.44	−1.96	−1.94
$\Delta r_t^{i/0}$	−7.36	−5.66	−3.59	−5.33	−2.88	−4.54	−4.75	−3.17	−4.48

Table 2: Augumented Dickey-Fuller tests on real per capita consumptions and real exchange rates.

Notes: The tests are based on univariate $AR(p)$ models with the number of lags p selected according to the MAIC criterion of Ng and Perron (2001) under the constraint $p \leq [12(T/100)^{1/4}]$. The ADF regression for the variables in levels include a linear time trend. The 5% critical value of the ADF statistics is −3.43 for regressions with trends, −2.86 for regressions without trends.

1963-2003									Critical values	
	FR	IT	SP	NE	BE	PO	AU	UK	5%	10%
VAR lags	2	2	2	2	2	2	1	2		
Eigenvalues										
1	0.50	0.45	0.38	0.33	0.47	0.43	0.47	0.36		
2	0.23	0.24	0.19	0.21	0.17	0.38	0.32	0.28		
3	0.18	0.15	0.18	0.14	0.16	0.19	0.14	0.16		
Maximum eigenvalue statistics										
$r = 0$	28.53	24.70	19.33	16.59	26.08	22.88	25.77	18.68	25.43	23.11
$r \leq 1$	10.54	11.22	8.84	9.85	7.83	19.43	15.69	13.79	19.18	16.85
$r \leq 2$	8.02	6.83	8.08	6.36	7.17	8.41	6.11	7.16	12.36	10.49
Trace statistics										
$r = 0$	47.10	42.74	36.25	32.80	41.07	50.71	47.57	39.63	42.20	39.08
$r \leq 1$	18.56	18.05	16.92	16.21	14.99	27.84	21.80	20.95	25.47	22.95
$r \leq 2$	8.02	6.83	8.08	6.36	7.17	8.41	6.11	7.16	12.39	10.56

Table 3: Cointegration rank tests in the country-specific VAR models

Notes: Each VAR is based on $x_t^i = (c_t^i, c_t^0, r_t^{i/0})'$, i = FRA,ITA,SPA,NET,BEL,POR,AUS,UK; 0 = GER. The estimated model contains restricted linear trend coefficients and unrestricted intercepts. Critical values of the Trace LR statistics are from Johansen (1996), Table 15.4; critical values for the maximum eigenvalue tests are from Osterwald Lenun (1992)

1963-2003	$(c_t^j - \theta^j c_t^0 - \delta^j r_t^{j/0} - \phi^j t)$
France-Germany	$c_t^{fr} - 0.57c_t^0 - 0.37r_t^{fr/0} - 0.0054t$
Italy - Germany	$c_t^{it} - 0.88c_t^0 + 0.11r_t^{it/0} - 0.009t$
Netherlands - Germany	$c_t^{ne} - 0.72c_t^0 - 2.81r_t^{ne/0} - 0.007t$
Belgium - Germany	$c_t^{be} - 1.31c_t^0 - 2.15r_t^{be/0} - 0.009t$
Austria - Germany	$c_t^{au} - 0.59c_t^0 - 0.22r_t^{au/0} - 0.015t$

Table 4: ESTIMATED RISK SHARING RELATIONS.
Notes: Estimates are based on the country-specific VAR models with cointegration rank set to 1. All coefficients are significant at the 5% level except δ^{ita}.

	FR	IT	NE	BE	AU	System
Serial Correlation, $F(2,22)$	0.30	0.72	0.00	1.79	0.25	0.86^a
Normality, $\chi^2(2)$	2.28	0.53	0.73	1.14	5.04^*	9.26^b
Heteroskedasticity, $\chi^2(32)$	30.74	35.26	35.13	37.22	34.15	485.9^c
Heteroskedasticity(conditional), $F(1,22)$	0.00	0.01	0.70	0.09	0.21	–

Table 5: MISSPECIFICATION TESTS FOR THE UNRESTRICTED REDUCED FORM SYSTEM IN THE VECM FORMAT (19)□
Notes: The number of lags p is set to 2; the cointegration matrix is fixed at the estimates of Table 4. * denotes significance at the 10% level. The null asymptotic distributions of the tests are as given in the table except: ^{a} F(50,48), ^{b} $\chi^2(10)$, ^{c} $\chi^2(480)$.

	FR	IT	NE	BE	AU	System
Exclusion of lagged home consumption and all foreign variables, $\chi^2(9)$	10.84	22.47***	26.11***	13.66	10.60	71.60^{a***}
Exclusion of all foreign variables, $\chi^2(8)$	7.67	20.05**	12.29	13.66*	10.36	68.11^{b***}
Esclusion of all ECM terms, $\chi^2(5)$	12.18**	8.32*	18.66**	23.95***	16.84***	88.88^{c***}
Esclusion of home ECM terms, $\chi^2(1)$	3.83**	8.79***	1.41	3.37*	8.30***	30.83^{d***}
Esclusion of cross ECM terms, $\chi^2(4)$	7.27	14.39**	13.18**	14.88***	2.56	59.22^{e***}

Table 6: Tests of some zero restrictions on the unrestricted reduced form system in the VECM format (19)

Notes: The number of lags p is set to 2; the cointegration relations are fixed at the estimates of Table 4. First row: tests of exclusion of all foreign variables and of the lagged home consumption from each equation; Second row: tests of exclusion of all foreign variables and of the lagged home consumption from each equation; Third row: tests of exclusion of all (home and foreign) lagged deviations (ECMs) from the optimal consumption levels under full risk sharing; Fourth row: tests of exclusion of the home (lagged) ECM; Fifth row: tests of exclusion of all foreign (lagged) ECMs. * denotes significance at the 10% level; ** denotes significance at the 5% level; *** at the 1% level. The null asymptotic distributions of the tests are as given in the table except: ^{a} $\chi^2(45)$; ^{b} $\chi^2(40)$; ^{c} $\chi^2(25)$; ^{d} $\chi^2(5)$; ^{e} $\chi^{10}(4)$.

	FR	IT	NE	BE	AU
Δc^{fr}_{t-1}		-0.60^{**} $_{0.23}$			
Δc^{it}_{t-1}		0.19 $_{0.14}$			
Δc^{ne}_{t}		0.59^{***} $_{0.16}$	0.30^{**} $_{0.11}$		
Δc^{be}_{t-1}		-0.35^{*} $_{0.18}$			
Δc^{au}_{t-1}		0.29^{*} $_{0.16}$			
Δc^{ge}_{t-1}	0.16 $_{0.12}$	0.17 $_{0.17}$	0.47^{**} $_{0.18}$	-0.00 $_{0.13}$	-0.03 $_{0.13}$
$\Delta r^{fr/0}_{t-1}$	-0.12^{**} $_{0.05}$	-0.17^{**} $_{0.07}$			
$\Delta r^{it/0}_{t-1}$		0.02 $_{0.04}$			
$\Delta r^{ne/0}_{t-1}$			-0.39^{***} $_{0.10}$		
$\Delta r^{be/0}_{t-1}$				-0.15^{**} $_{0.07}$	
$\Delta r^{au/0}_{t-1}$					-0.09 $_{0.12}$
ECM^{fr}_{t-1}	-0.30^{**} $_{0.11}$	0.12 $_{0.16}$	-0.12 $_{0.16}$	0.22^{*} $_{0.12}$	-0.04 $_{0.13}$
ECM^{it}_{t-1}	0.15 $_{0.10}$	-0.43^{***} $_{0.14}$	0.29^{*} $_{0.14}$	0.10 $_{0.11}$	0.09 $_{0.12}$
ECM^{ne}_{t-1}	0.02 $_{0.02}$	0.01 $_{0.03}$	-0.05 $_{0.03}$	-0.06^{***} $_{0.02}$	0.01 $_{0.02}$
ECM^{be}_{t-1}	0.02 $_{0.03}$	-0.01 $_{0.05}$	-0.01 $_{0.05}$	-0.08^{*} $_{0.04}$	-0.02 $_{0.04}$
ECM^{au}_{t-1}	-0.18^{*} $_{0.10}$	-0.50^{**} $_{0.15}$	0.16 $_{0.15}$	-0.14 $_{0.11}$	-0.48^{***} $_{0.13}$
R^2	0.66	0.72	0.73	0.61	0.60

Table 7: Estimation of the reduced form system in the VECM format (19) under a set of zero restrictions

Notes: The cointegration relations are fixed at the estimates of Table 4. The likelihood ratio test for the $m = 38$ zero restrictions is $LR = 51.604$ (p-value: 0.07).

	SP	PO
Δc_t^{ge}	0.41^{***} 0.11	0.19 0.33
Δc_{t-1}^{ge}	-0.24^{*} 0.14	
Δc_{t-1}^{sp}	-0.25^{**} 0.11	
Δc_{t-1}^{po}		0.61^{***} 0.13
$\Delta r_t^{sp/0}$	0.05 0.05	
$\Delta r_{t-1}^{sp/0}$		
$\Delta r_t^{po/0}$		-0.03 0.14
$\Delta r_{t-1}^{po/0}$		0.23 0.14
ECM_{t-1}^{fr}	-0.27^{**} 0.13	0.35 0.30
ECM_{t-1}^{it}	-0.58^{***} 0.14	-0.53 0.33
ECM_{t-1}^{ne}	-0.09^{***} 0.03	
ECM_{t-1}^{be}	-0.07^{*} 0.04	
ECM_{t-1}^{au}	-0.49^{***} 0.12	
R^2	0.81	0.53

Table 8: Estimation of a conditional VECM model for the Iberian countries under a set of zero restrictions

Notes: The cointegration relations are fixed at the estimates of Table 4. The likelihood ratio test for the $m = 11$ zero restrictions is 10.53 (p-value: 0.48).

	FR	IT	NE	BE	AU	System
$r_t^{i/0}$ (Weak)	18.28***	12.97**	7.67	34.41***	21.25***	74.83^{a}***
$r_t^{i/0}$ (Strong)	25.71***	19.15**	17.97*	42.54***	30.52***	121.33^{b}***

Table 9: Exogeneity tests for real exchange rates. Notes: The tests are based on the reduced form system (19). * denotes significance at the 10% level; ** denotes significance at the 5% level; *** at the 1% level. All weak (strong) exogeneity tests have $\chi^2(5)$ ($\chi^2(10)$) null asymptotic distribution except: ^{a} $\chi^2(25)$; ^{b} $\chi^2(50)$.

	FR	IT	NE	BE	AU	System
Serial Correlation, $F(2,28)$	1.30	1.56	0.70	1.08	0.52	2.49^{a}***

Table 10: Tests of the present value restrictions.
Notes: *** detones significance at the 1% level. The null asymptotic distributions of the tests are as given in the table except: ^{a} F(50,76).