Corner Solutions in a Model of Military Alliances

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Abstract

In this paper I study corner solutions in a Model of Military Alliances. In particular I analyze how corner solutions affect the level of defence expenditure. I find that accounting for corner solutions, the so called 'Exploitation Hypothesis' may no longer be reversed and can indeed be re – inforced as one moves from the Cournot to the Stackelberg equilibrium.

Keywords: ***military alliances, defence, public goods, free-riding, corner solutions.***

JEL Classification**: A1 – General Economics**

1. Introduction

Starting with Olson and Zeckhauser (1966), the economics literature on the theory of military alliances has studied the behavior of two allied countries, which interact strategically in order to identify the level of provision of defence. Defence is viewed as a pure public good, such that the benefits from defence are non rival and non excludable. The two countries have been assumed to act in a Cournot or in a Stackelberg fashion and the following conclusions have been reached:

1. In a Cournot set up, "...the large rich ally shoulders the defence burdens of the small, poor allies by providing the latter with a relatively free ride. This proposition is known as the 'exploitation hypothesis' " (Sandler and Hartley, 2001).

2. The resulting level of defence expenditure is inefficient, "since (differently from the private goods case) benefits conferred on others by an agent's action are not taken into account" (Sandler, 1992).

3. In a Stackelberg set up the 'exploitation hypothesis' is however reversed in the sense that the poorer country contributes to the defence expenditure: "the leader country chooses a lower level of expenditure on guns than in the (Cournot) Nash equilibrium while the ally is forced to choose a higher level of guns spending. Total spending on guns by the alliance falls" (Bruce, 1990).

At second sight, however, it appears that so far the analysis has disregarded corners solutions. Filling this gap may be important because corner solutions may arise with well-bahaved preferences, and because there are various historical examples where corners solutions seem relevant. The military alliance between US and Japan is a case in point (Carpenter, 1996). Fearing the possibility of a conflict in East Asia and the fallout of consequences, the US decided to reaffirm the already existing military alliance with Japan. However Japan did not contribute to the related defence expenditure (at the time of the events the US had several military bases in the Japanese soil). Thus the game between US and Japan appears to had been at a corner solution, where Japan had given to the US the permission to use its soil for military purposes, enjoying the benefits of defence without sharing the costs.

In what follows, we provide a Cobb-Douglas example where:

1. in the Cournot scenario, the strategic interaction of two countries may result either in an interior or in a corner equilibrium;

2. in a Stackelberg scenario, the leader might be the only one to contribute to the provision of defence, if the level of income of the follower falls below a certain threshold; in this case the 'exploitation hypothesis' is not reversed under the Stackelberg assumption.

2. Cournot equilibria

We assume that both countries have a Cobb - Douglas utility function (i = 1,2):

$$\boldsymbol{U^i = ln y^i + ln Q} \tag{1}$$

where:

- y^i is the country's consumption of the private good;
- Q is the total amount of the pure public good and it is equal to the sum of the nations' contributions ($Q = q_i + q_j$). This provision technology implies that the public good is perfectly substitutable between the two countries.

Each country is constrained by a linear budget constraint $I^i = y^i + pq_i$, where the price of the private good is normalized to 1 and the price of the public good is denoted by p. The basic problem for country 'i' can be written as:

$$\begin{aligned} &Max_{q_i} U^i = \ln(I^i - pq_i) + \ln(q_i + q_j) \\ &s.to\ q_i \geq 0 \quad with\ q_j\ given \end{aligned} \tag{2}$$

and the first-order condition for a maximum is:

$$\frac{\partial U^i}{\partial q_i} = -\frac{p}{I^i - pq_i} + \frac{1}{q_i + q_j} = 0 \qquad \textbf{\textit{(3)}}$$

The reaction curve for country 'i' is as follows:

$$\begin{cases} q_i^{CN} = \dfrac{I^i}{2p} - \dfrac{1}{2} q_j \quad if \quad \dfrac{I^i}{p} \geq q_j \\ q_i^{CN} = 0 \quad if \quad \dfrac{I^i}{p} < q_j \end{cases} \qquad \textbf{\textit{(4)}}$$

Inverting the indices gives the reaction curve for the other country.

Inspection of ***(4)*** reveals that the Cournot equilibrium can be of three types:

- If $(I^i/2) \leq I^j \leq 2I^i$, we have the standard Cournot equilibrium with interior solution that is usually considered in the literature:

$$\begin{aligned} q_i^{CN*} &= \frac{2I^i}{3p} - \frac{I^j}{3p} \\ q_j^{CN*} &= \frac{2I^j}{3p} - \frac{I^i}{3p} \end{aligned} \qquad \textbf{(5)}$$

- If $I^j > 2I^i$, however, the Cournot equilibrium is attained at a point that corresponds to a corner solution of country i's maximization problem:

$$\begin{aligned} q_i^{CN*} &= 0 \\ q_j^{CN*} &= \frac{I^j}{2p} \end{aligned} \qquad \textbf{(6)}$$

- Finally, if $I^j < I^i/2$ the equilibrium is again a corner solution equilibrium, but now it is country j's contribution that vanishes:

$$q_i^{CN*} = \frac{I^j}{2p}$$
$$q_j^{CN*} = 0 \tag{7}$$

3. Stackelberg equilibria

In this section we suppose that the first country is the leader, while the second one is the follower. Following the literature, leadership is defined on the basis of the level of income. To fix ideas, suppose that $I^i > I^j$ so that country 'i' leads. To determine the Stackelberg equilibrium, we first compute the follower's reaction curve, as in (4), and then we solve the leader's maximization problem.

If $I^j/p \geq q^i \geq 0$, country 'i' solves:

$$Max_{\frac{I^j}{p} \geq q_i \geq 0} \; U_i = \ln(I^i - pq_i) + \ln(I^j + pq_i) - \ln(2p) \tag{8}$$

The first-order condition for a maximum is:

$$\frac{\partial U^i}{\partial q_i} = -\frac{p}{I^i - pq_i} + \frac{1}{I^J + pq_i} = 0 \tag{9}$$

The equilibrium level of provision for the leader therefore is:

$$q_i^{S*} = \frac{I^i - I^j}{2p} \tag{10}$$

and is always strictly positive. In order to obtain the equilibrium provision level for the follower, we substitute q_i^{S*} into the follower's reaction curve obtaining:

$$q_j^{S*} = \frac{3I^J}{4p} - \frac{I^i}{4p} \qquad \textbf{\textit{(11)}}$$

When ***I^j > (3/5)I^i***, this solution delivers the well known reversal of the 'exploitation hypothesis'. Note, however, that the 'exploitation hypothesis' is not reversed when the smaller country is sufficiently small, i.e. ***I^j < (3/5)I^i***; in particular, if the small country is very small (i.e. ***I^j < (1/3)I^i***) the Stackelberg equilibrium is attained at a corner solution where ***$q_j^{S*} = 0$*** and ***$q_i^{S*} = (I^i/2p)$.***
If instead ***$I^j/p < q^i$*** , then the leader's maximization problem is:

$$Max_{\frac{I^j}{p} < q_i} \quad U_i = \ln(I^i - pq_i) + \ln q_i \qquad \textbf{\textit{(12)}}$$

The first-order condition for a maximum is:

$$\frac{\partial U^i}{\partial q_i} = -\frac{p}{I^i - pq_i} + \frac{1}{q_i} = 0 \qquad \textbf{\textit{(13)}}$$

whence the equilibrium provisions of two countries can be immediately calculated as:

$$q_i^{S*} = \frac{I^i}{2p} \quad and \quad q_j^{S*} = 0 \qquad \textbf{\textit{(14)}}$$

As it turns out, when the Cournot equilibrium entails a corner solution for the smaller country, the Stackelberg equilibrium is also characterized by a corner solution. In this case, the 'exploitation hypothesis' is not reversed as we move from the Cournot to the Stackelberg scenario.

Summarizing, if in the Cournot set-up the equilibrium is interior, then in a Stackelberg set-up we may end up with another interior solution, where the leader still provides more defence than the follower, or we may jump to a corner solution where only the leader provides defence. If instead in the Cournot set-up the equilibrium is attained at a corner, then the Stackelberg equilibrium is also necessarily at a corner.

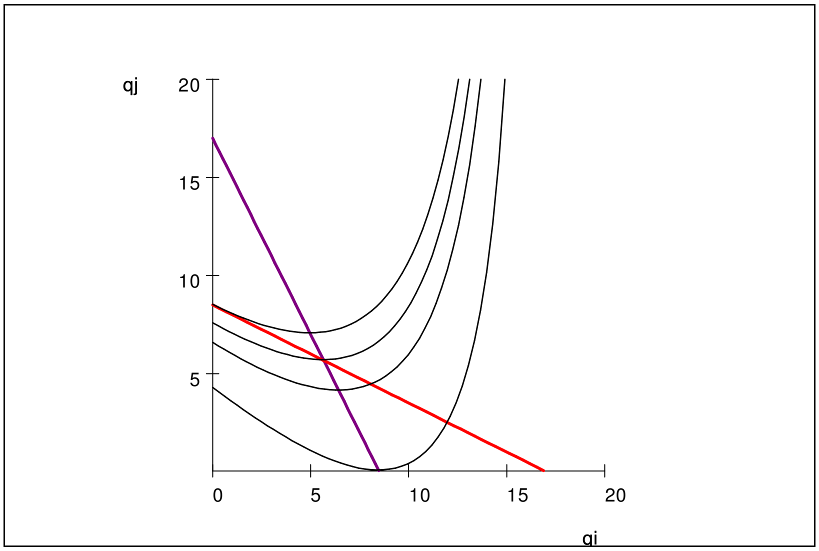

Figure 1: Interior Solution

Figure 1 depicts the indifference curves of the country ‘i’ (the small country) and the two countries' reaction curves. It illustrates the case where the equilibrium jumps from an interior solution to a corner solution as we move from Cournot to Stackelberg. In this case the ‘exploitation hypothesis’ is not reversed; in a sense it is reinforced in that the ratio between the smaller country's provision of defences and that of the larger countries falls as we move from the Cournot to the Stackelberg equilibrium.

More generally, the ‘exploitation hypothesis’ is reversed only if the difference in the size of the two countries is not too large. When the countries are highly asymmetric in size, the defence provision game has a corner solution. The occurrence of a corner solution in which the follower free rides and the leader bear the burden of defence fully is even more likely in a Stackelberg set-up than in a Cournot set-up.

4. Concluding remarks

In this paper we have presented a model of military alliance where the strategic interaction between two countries is studied both in a Cournot and in a Stackelberg framework. Corner solutions were considered, giving rise to the possibility of situations where only one of the two countries bears the burden of defence. When countries are highly asymmetric, not only is the `exploitation hypothesis' not reversed in a Stackelberg equilibrium; it can actually be reinforced. Our results regarding corner solutions in military alliance games may help explain certain historical cases, as the military alliance between US and Japan.

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