

# MODELLING OF RESPONSIVITY OF INP-PIN PHOTODIODE FOR STUDYING OPTICAL-MICROWAVE FREQUENCY CONVERSION PROCESSES<sup>1</sup>

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## ABSTRACT

The paper presents investigations of optical-microwave frequency conversion processes for InP-PIN photodiode. The planar InGaAsP/InGaAs/InP heterostructure PIN photodiode operating at 1.3  $\mu\text{m}$  has been used in an optoelectronic mixer configuration. The nonlinear model of the PIN photodiode responsivity has been assumed. The mathematical analysis of frequency conversion process and results of simulation are presented. The obtained results of calculation and results of measurements are compared and discussed.

## INTRODUCTION

PIN photodiodes are widely used in optical fiber link receivers. In traditional solutions a photodiode converts an optical signal into an electrical one. Then it is amplified and up- or down- converted. The use of optical-microwave mixing phenomena allows to simplify the process of electrical signal recovering. This solution offers cost-effective, simple, and potentially high-performance new applications in microwave sub-carrier multiplexing systems - Paoletta et al. (1), Berceci et al. (2), Galwas et al. (3).

In this case the photodetection process takes place in the presence of alternate voltage, which is a superposition of DC bias voltage  $V_O$  and AC voltage with high amplitude  $V_H$  and frequency  $f_H$  directed from the output side. When the optical power is intensity modulated by frequency  $f_S$  the mixing process takes place. The results of optical-microwave frequency conversion are photodiode current components at frequencies  $|f_H \pm f_S|$ , next harmonic components of heterodyne frequency  $nf_H$ , as well as components of subharmonic mixing  $|nf_H \pm f_S|$  for  $n = 2,3,\dots$  Berceci and Jaro (4), Dawidczyk et al. (5), Dawidczyk and Galwas (6).

## MODELLING OF PIN RESPONSIVITY

To perform the analysis of this process an equivalent circuit of the InP-PIN photodiode was assumed - Fig.1. This circuit contains a few nonlinear elements. These elements play different roles in the frequency conversion process. For purposes of this analysis the following assumptions were made:

- The responsivity  $R_D(V_D)$  of photodiode PD is a dominant nonlinear element in the whole circuit - Wiliams et al. (7).
- The effect of nonlinear capacitance  $C_j(V_D)$  can be neglected if frequency  $f_H$  is small enough, and the photodetector operates with a broadband matched load.

In the simplest case of the photodetection the current source in the PD equivalent circuit can be described by:

$$i_D = R_D(V_D)P_{\text{OPT}}; \quad (1)$$

where  $R_D(V_D)$  is the responsivity of the photodiode PD.

Assuming that:  $P_{\text{OPT}} = P_0 [1 + m \cos(\omega_S t)]$  the  $i_D(t)$  relation is given as:

$$i_D(t) = I_0 + I_1 \cos(\omega_S t); \quad (2)$$

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In this case the photodetector responsivity  $R(V_D)$  is the function of the bias voltage  $V_D$  and the modulation frequency  $f_s$ . This relationship is shown on Fig.2.  $R(V_D)$  can be simply measured:

$$R_D(f_s = 0) = \frac{I_0}{P_0}; \quad (3)$$

$$R_D(f_s) = \frac{I_1}{mP_0}; \quad (4)$$

The responsivity  $R(V_D)$  may be modelled by analytical formula (5), with 4 parameters:  $R_0$ ,  $\alpha$  and  $V_C$ :

$$R(V_D) = \frac{R_0}{1 + e^{\alpha(V_C + V_D)}}; \quad (5)$$

On the basis of measured characteristics  $I_0(V_D)$  and  $I_1(V_D, f_s)$  the family of characteristics  $R(V_D, f_s)$  have been calculated, and next the values of parameters  $R_0$ ,  $\alpha$  and  $V_C$  have been extracted. The results are presented in TABLE 1.

## SIMULATION OF OTICAL-MICROWAVE MIXING

The model described by equation (3) has been used to determine the mechanism of optical-microwave mixing process. Let's assume that the bias voltage contains a so-called heterodyne component and can be given as:

$$V_D(t) = V_0 + V_H \cos(\omega_H t); \quad (6)$$

Then we can write:

$$R[V_D(t)] = \frac{R_0}{1 + \exp\{\alpha[V_C + V_0 + V_H \cos(\omega_H t)]\}}; \quad (7)$$

$R[V_D(t)]$  can be expanded in Fourier series:

$$R[V_D(t)] = R_0[b_0 + b_1 \cos(\omega_H t) + b_2 \cos(2\omega_H t) + \dots + b_n \cos(n\omega_H t) + \dots]; \quad (8)$$

After some transformations one can obtain:

$$b_0 = \frac{1}{1 + e^{\alpha V_0'} I_0(\alpha V_H)}; \quad (9)$$

$$b_n = \frac{(-1)^n 2e^{\alpha V_0'} I_n(\alpha V_H)}{[1 + e^{\alpha V_0'} I_0(\alpha V_H)]^2}; \quad (10)$$

where:  $V_0' = V_0 + V_C$ ;  $I_0(\alpha V_H)$  and  $I_n(\alpha V_H)$  are the modified Bessel functions.

The results of simulations for  $b_0$ ,  $b_1$  and  $b_2$  are shown in Fig.3 and in Fig.4.

When the optical power is intensity modulated ( $m \neq 0$ ), the photodiode current spectrum consists many components: harmonic components  $I_{1,0}$ ,  $I_{2,0}$ ... of heterodyne frequency  $n f_H$ , next components  $I_{1,1}$ ,  $I_{1,-1}$  at frequencies  $|f_H \pm f_s|$  and components of subharmonic mixing  $|n f_H \pm f_s|$  for  $n = 2, 3, \dots$ . The final relation can be described as follow:

$$i_D(t) = I_0 + I_{1,0} \cos \omega_H t + I_{0,1} \cos \omega_S t + I_{1,-1} \cos[(\omega_H - \omega_S)t] + I_{1,1} \cos[(\omega_H + \omega_S)t] + I_{2,0} \cos 2\omega_H t + I_{2,-1} \cos[(2\omega_H - \omega_S)t] + I_{2,1} \cos[(2\omega_H + \omega_S)t] + \dots; \quad (11)$$

$$I_{n,\pm 1} = R_0 b_n m P_0; \quad (12)$$

On the basis of formulas (8) and (11) we can construct the equivalent circuit of PIN photodiode, what has been shown in Fig.5.

## MEASUREMENT RESULTS OF OPTICAL-MICROWAVE MIXING

The conversion products at different local oscillator power levels were measured:  $P_{1,-1}$ ,  $P_{1,1}$ ,  $P_{2,-1}$ ,  $P_{2,1}$ . The maximal detected signal power  $P_{Smax}$  was also measured ( $P_{Smax}$  was measured at bias voltage  $V_D = -7V$ ). The relative conversion loss was defined as:

$$L_R = \frac{P_{n,\pm 1}}{P_{Smax}}; \quad (13)$$

where  $P_{n,m}$  is the conversion product at frequency  $n \cdot f_{LO} \pm f_S$ . The chosen measurement results shown in Fig.6 and Fig.7 are in good agreement with results of simulation.

## CONCLUSIONS

Optical-microwave mixing using PIN photodiode has been investigated. There are two main elements in the PIN photodiode equivalent circuit responsible for the mixing process: the nonlinear current source, represented by hear by responsivity  $R(V_D)$  and the nonlinear capacitance  $C_j$ . The influence of the capacitance can be neglected if the frequency is small enough. The measurements of the subharmonic components have proved the appropriate chose of the model. The agreement between the measured and simulated results provides credibility to the proposed model and analysis.

TABLE 1: Extracted values of parameters  $R_0$ ,  $a$ ,  $\alpha$  and  $V_C$  for PIN photodiode.

	$f_S = 0$	$f_S = 1 \text{ MHz}$	$f_S = 100 \text{ MHz}$	$f_S = 1 \text{ GHz}$
$R_0$ [A/W]	1,02	1,01	1,01	0,98
$\alpha$	22,8	18,3	16,6	4,78
$V_C$ [V]	-0,408	-0,284	-0,128	0,0182

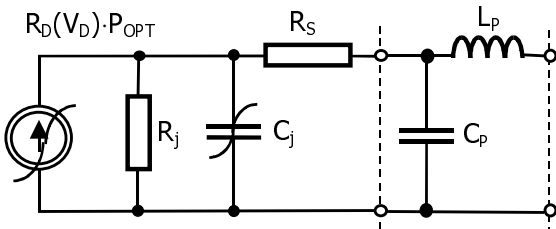


Fig. 1 Equivalent circuit of PIN photodiode

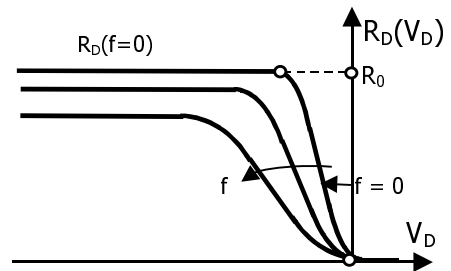


Fig.2. Photodetector responsivity as the function of bias voltage

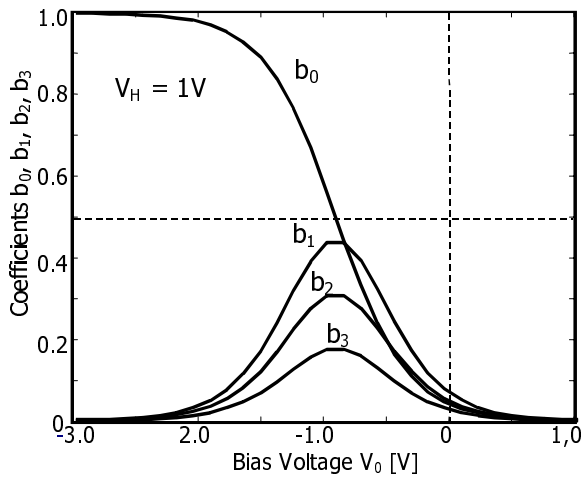


Fig.3 Coefficients  $b_0$ ,  $b_1$ ,  $b_2$  and  $b_3$  calculated according to formulas (9) and (10).

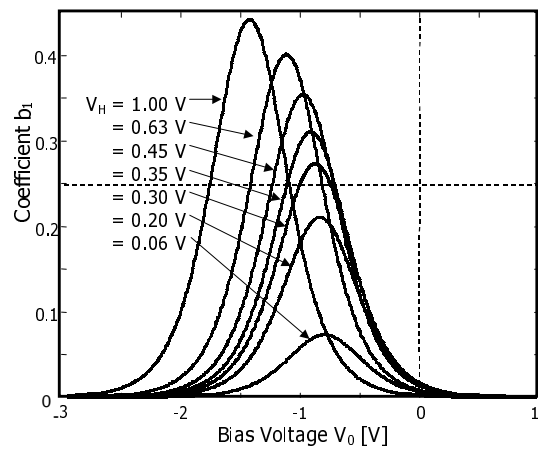


Fig.4 Coefficient  $b_1$  calculated according to formula (10), as a function of heterodyne voltage

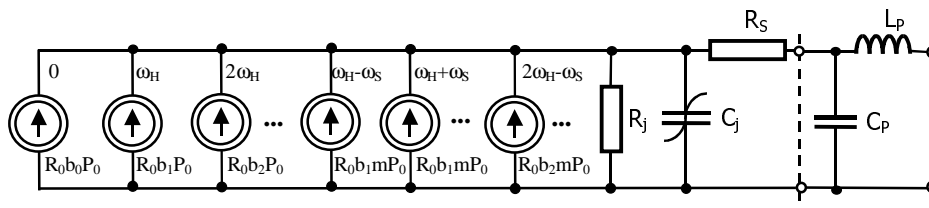


Fig.5 Equivalent circuit of PIN photodiode built according to (8) and (11) formulas.

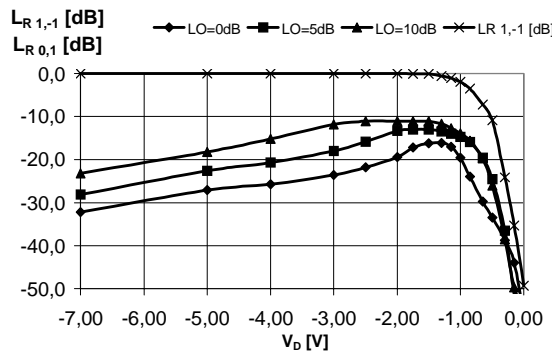


Fig.6 Relative conversion loss of component  $P_{1,-1}$  vs. bias voltage.

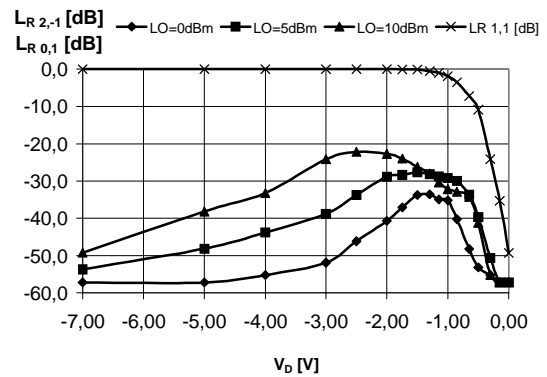


Fig.7 Relative conversion loss of component  $P_{2,-1}$  vs. bias voltage.

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