Fully Physical Coupled Electro-Thermal Modelling of Transient and Steady-State Behaviour in Microwave Semiconductor Devices

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ABSTRACT

Results are presented for the first fully physical, time-dependent, coupled electro-thermal simulations of microwave power FETs and MMICs, on timescales suitable for CAD. This is achieved by combining an original, analytical thermal resistance matrix model of time-dependent heat flow in a power FET or MMIC, with a fully physical electrical CAD model for transistors.

I. INTRODUCTION

The constant reduction in size of microwave semiconductor devices has led to new challenges in the way these components are designed. In order to be used at microwave frequencies, the gate length of FETs has to be reduced below the micron scale. The heat capacity of these devices is equally reduced, whereas their power dissipation remains unchanged. As well as this, the compound materials used in the manufacturing of these transistors have a major impact on their thermal behaviour. The smaller thermal conductivity of GaAs for example, means that the heat will be more concentrated in such a device, compared to a traditional silicon transistor. The effects of temperature on the device performance are also increased by thermal interaction in MMICs, where several transistors can be implemented on the same substrate. Thermal simulation and characterisation of microwave transistors and circuits is well described in the literature, e.g. Bonani et al [1], Dorkel et al [2], and the authors have presented previously a fully physical electro-thermal CAD model for the thermal time-independent case, Batty et al [3]. However, a fully physical, coupled electro-thermal model of transient behaviour, solving on timescales suitable for CAD, has yet to be presented. This paper presents a fully physical thermal model suitable for describing transient thermal effects and mutual thermal interaction between active devices in MMICs. A description of complex MMIC structure is possible by solving for the fluxes at interfaces between rectangular subvolumes. This allows for the implementation of surface metalisation, air bridges and via holes. The thermal model is coupled self-consistently to the Leeds Physical Model (LPM) which provides a fully physical description of the electrical behaviour of MESFETs and HEMTs.

II. THERMAL MODELLING

The transient thermal problem requires the solution of the time-dependent heat diffusion equation. This equation, governing the conduction of heat in an isotropic solid, is

$$\nabla^2 T + \frac{\dot{q}}{K} = \frac{1}{\kappa}\frac{\partial T}{\partial t}, \tag{1}$$

where T is temperature, t is time, $\dot{q}$ represents generated heat and K is thermal conductivity. Diffusivity $\kappa = \frac{K}{\rho c}$, where c is specific heat and ρ is density.

The heat is generated in the active channels of a MESFET or HEMT. These can be assumed to be simple rectangular volumes placed at or near the top surface of a MMIC. The heatsink mounted MMIC can itself be represented by a rectangular volume, $0 \leq x \leq L, 0 \leq y \leq D, 0 \leq z \leq H$. Its bottom surface is kept at a constant temperature T_0 to model a perfect heat sink, and adiabatic boundary conditions are assumed on the other surfaces. To generate a rapid coupled electro-thermal solution, the thermal resistance matrix (TRM) approach is adopted. This has many advantages over more computationally expensive methods, such as finite volume, finite element, finite difference, boundary element, or transmission line methods, generally used to solve heat diffusion problems. In particular, it allows rapid thermal updates by small matrix multiplications, in the necessarily interative coupled simulation.

The TRM is calculated by analytical solution of Eq. (1). This avoids volume discretisation, as typically required by numerical methods, and returns temperature rise as a function of power dissipation only in the vicinity of active devices. The TRM does not have to describe a mesh extended over the complete volume of the MMIC as is the case for thermal networks generated by finite element discretisation. However, the mathematical solution used to build the TRM represents a full solution of the time-dependent heat diffusion equation, taking full account of the boundaries and initial conditions. The order of the TRM is determined only by the number of heat dissipating volumes, assuming a constant power dissipation across these sources. Importantly, the TRM is fully evaluated at all required time points by precomputation prior to the coupled electro-thermal solution. This greatly reduces the computational cost when coupling the thermal and electrical models.

When implemented in conjunction with the Kirchhoff transformation, Joyce [4], to treat the non linearity inherent in the temperature dependence of the thermal conductivity, the analytically generated TRM is independent of power dissipation and hence independent of bias. The thermal model can therefore be updated by simple matrix multiplication when solved self-consistently with the electrical model.

It can be shown from Duhamel's theorem, or by a simple convolution of the power dissipation and the thermal impulse response of the MMIC, that temperature rise above heatsink temperature, $\Delta T_i(t)$, of heating element i, is described in terms of the TRM and the power dissipations $P_j(t)$ of active devices $j = 1, ..., i, ..., N$, as

$$\Delta T_i(t) = \sum_j \int_0^t P_j(t-\tau) \frac{\partial Rth_{i,j}(\tau)}{\partial \tau} d\tau = \sum_p P_{p,j} \left[U(t-t_{p-1}) Rth_{i,j}(t-t_{p-1}) - U(t-t_p) Rth_{i,j}(t-t_p) \right], \quad (2)$$

where the power dissipation $P_j(t)$ has been assumed to take the form of a succession of constant power steps, and $U(t)$ is the unit step function.

The TRM was implemented using a Green's function approach to solution of the heat diffusion equation, as described in Beck et al [5] giving,

$$Rth_{i,j}(t) = \frac{2}{KLDH} \times \sum_{mnl=0}^{\infty} \frac{4}{(1+\delta_{m0})(1+\delta_{n0})} \frac{I^i_{mnl} I^j_{mnl}}{V_i} \left(1 - \exp(-\kappa t(\frac{n^2\pi^2}{L^2} + \frac{m^2\pi^2}{D^2} + \frac{\beta_l^2}{H^2}))\right) \left[\frac{n^2\pi^2}{L^2} + \frac{m^2\pi^2}{D^2} + \frac{\beta_l^2}{H^2}\right]^{-1}. \quad (3)$$

Here, V_i is the volume of the i^{th} heat source, L, D and H are the dimensions of the MMIC, δ_{mn} is the Kronecker delta function, $\beta_l = \pi(l + 1/2)$ and I^i_{mnl} is the volume integral over heating element V_i

$$\int_{V_i} \cos\frac{n\pi x}{L} \cos\frac{m\pi y}{D} \cos\frac{\beta_l z}{H} dxdydz. \quad (4)$$

The $3-$dimensional Green's function for the problem is immediately obtained by multiplying three 1-dimensional functions. All expressions for the temperature rise are obtained fully analytically. It should be noted that different expressions for the solution are obtained when considering small or large time applications. These expressions are equivalent, but their rate of convergence is different according to the time range. It can be seen that a solution to the steady-state problem is immediately generated by taking the limit as $t \to \infty$.

The analytical solution, Eq. (3), represents an exact description of 3-dimensional, time-dependent heat flow between an arbitrary distribution of heat generating elements in the body of a MMIC. It takes full account of finite length and end effects of the active device elements, and describes fully the finite extent of the GaAs die. The heating elements can have any position and grouping within the MMIC.

The Green's function solution also provides the first step towards the description of more complex structures. Via holes and surface metallisation can be described using the unsteady surface element (USE) method of Beck et al [5]. Assuming a perfect contact beween the MMIC and the metallic surface elements, then equating temperatures and fluxes at the interfaces, Eq.(5) is obtained,

$$\sum_k \int_0^t q_k(\tau) \frac{\partial \Phi_{ki}(t-\tau)}{\partial t} d\tau = -\sum_j \sum_p P_{p,j} \left[U(t-t_{p-1}) Rth_{i,j}(t-t_{p-1}) - U(t-t_p) Rth_{i,j}(t-t_p) \right], \quad (5)$$

with $\Phi_{ki} = \varphi^s_{ki} + \varphi^e_{ki}$. Here, q_k is the flux across MMIC/element interface, k, φ^s_{ki} is the influence function of the MMIC, and φ^e_{ki} is the influence function of the surface element. The influence functions are simple generalisations of the TRM, Eq. (3).

Eq. (5) is a Volterra equation of the first kind, which can be solved by time discretisation (over P time steps), assuming a constant flux over each time step. By using the same time discretisation as in Eq.(2), and after some algebraic manipulation including a further summation over p, this equation can be reformulated in matrix form as,

$$\bar{q}_P = \bar{\bar{\Phi}}_1^{-1} \sum_{p=1}^{P} \left[\bar{H}_P - \bar{G}_P \right] - \bar{\bar{\Phi}}_1^{-1} \bar{F}_P, \tag{6}$$

where,

$$\Delta\Phi_{ki,P-p} = \Phi_{ki,P+1-p} - \Phi_{ki,P-p}, \;\; q_{kp} = q_k(t_p) \;\text{ and }\; \Phi_{ki,p} = \Phi_{ki}(t_p), \tag{7}$$

and the vectors $\bar{F}_P$, $\bar{G}_P$ and $\bar{H}_P$ have been defined as follows,

$$\bar{F}_P = \sum_{p=1}^{P-1} \bar{\bar{\Phi}}_{P+1-p} \bar{q}_p, \;\; \bar{G}_P = \sum_{p=1}^{P} \bar{P}_p U(P-p+1) \bar{\bar{Rth}}_{P-p+1}, \;\; \bar{H}_P = \sum_{p=1}^{P} \bar{P}_p U(P-p) \bar{\bar{Rth}}_{P-p}. \tag{8}$$

The expression for the influence function of the via hole, based on the approximate equivalence principle of Bonani et al [1], is similar to that for the MMIC, as it shares similar boundary conditions.

This thermal model was validated using a MESFET based LMA116 MMIC fabricated by Filtronic Solid State. The computed results are presented in Table 1, along with the measured temperatures. The results are for the heat sources, as shown on the circuit outline, Fig. 1.

III. THE ELECTRO-THERMAL MODEL

The electro-thermal model combines both electrical and thermal models in a self consistent solution. The LPM provides the fully physical electrical solution. Description of transient behaviour requires the time range to be discretised as in Eq. (2). The electrical model is solved at each of those time steps self-consistently with the thermal model described by the thermal resistance matrix. It is found that a simple relaxation method can be used, although the inclusion of time derivative information in a Newton solution is expected to improve solution robustness.

Fig. 2 presents a time-dependent simulation of a multi-gate MESFET at turn-on. The MESFET is fabricated on a $300\mu m$ cubic die and has finger dimensions $0.25\mu m$ by $30\mu m$. Initial device temperature is 300 K throughout. Device bias is kept constant at $(V_{DS}, V_{GS}) = (3V, 0V)$. The DC drain-source current I_{DS} is seen to gradually reduce as the temperature increases due to the dissipated power. Temperature rise is calculated self-consistently with I_{DS}. It should be noted that this fully self-consistent electro-thermal calculation is plotted on a logarithmic scale and spans 7 orders of magnitude in time. The coupled electro-thermal calculation took 8 minutes on an HP 712/60 workstation after precomputation of the thermal resistance matrices. The jagged appearance of the curve is due to the time discretisation (250 points over 7 decades) and the details of the implementation of the coupled electro-thermal relaxation algorithm. This description can be improved by a linear or polynomial interpolation. It was assumed that the temperature was uniform under all the gate fingers. Improved results can be obtained by modelling the gate areas as multiple elements. Fig. 3 shows a set of DC I-V curves for a multi-finger power FET generated fully self-consistently using the steady-state form of the thermal resistance matrix. The coupled electro-thermal calculation for the full set of curves took ~30 minutes on a 500 MHz Pentium using a non optimised relaxation algorithm. The thermal resistance matrix included the effects of surface metallisation and vias, and the observed thermal droop was predicted fully physically, based on the analytical construction of the thermal resistance matrix.

IV. DISCUSSION

Fig. 2 represents the first reported time-dependent, coupled electro-thermal simulation of a power FET on CAD timescales, that is based on fully physical electrical and thermal models. The figure shows that important current variations occur on time scales between 0.1 μs and 1 ms. This means that the intermodulation distortion characteristics of the device will be affected if input signals with a typical frequency difference of ~1 KHz to ~10 MHz are used [6]. The figure also shows that temperature rises on timescales corresponding to inverse ~GHz frequencies are negligible due to thermal inertia. This means that the electrical and thermal problems will decouple for RF operation, with thermal variation at (inverse) μs to ms timescales acting as a slowly varying background on microwave electrical signals. This has important implications for the implementation of coupled electro-thermal RF simulations.

Fig. 3 shows that the thermal resistance matrix, describing self-heating and mutual thermal interaction between active device elements in a power FET or MMIC, can be fully obtained by analytical construction, taking into account all effects required for accurate description of thermal response.

V. CONCLUSION

A fully physical, analytical thermal resistance matrix model has been presented that describes transient thermal effects and allows modelling of complex MMIC structures. The model includes the effects of non linearity due to temperature dependent conductivity, and has the potential to include any level of device detail necessary for accurate predication of temperature variation. This thermal model has been coupled to a physical electrical model of a transistor, to produce the first fully physical, time-dependent, coupled electro-thermal simulation of a power FET on timescales suitable for CAD. Calculated results show that the physical thermal resistance matrix approach can ultimately be used to make fully predictive studies of the impact of thermal effects on intermodulation distortion, and to study thermal transient effects on electrical behaviour. This is particularly interesting for the modelling of MMIC based power amplifiers where significant temperature variation is observed, affecting the optimum performance of the devices.

VI. ACKNOWLEDGEMENTS

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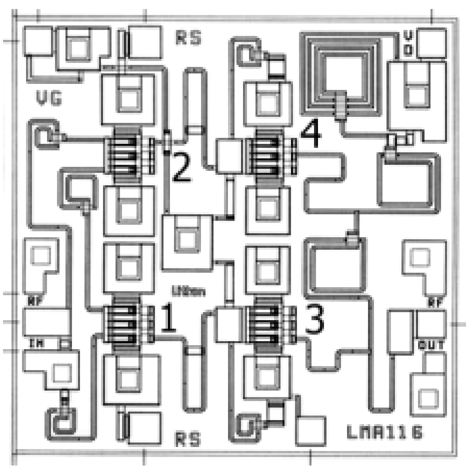

Fig. 1: Layout of the Filtronic MESFET based MMIC.

Device	Power (W)	Computed	Measured
1	0.12	7.8	8
2	0.12	6.4	7
3	0.2	13.2	12
4	0.2	10.7	12

Table 1: Measured and computed temperature rise for the four devices (K).

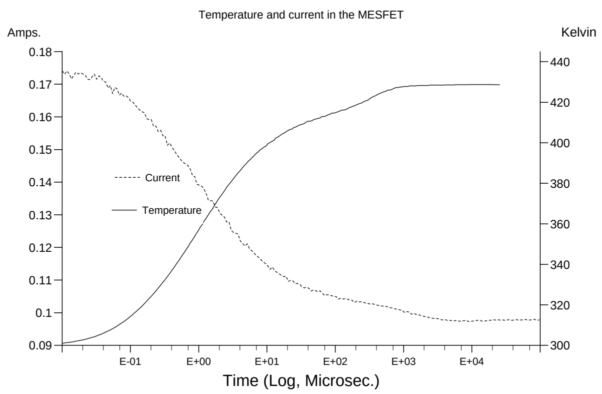

Fig. 2: Coupled electro-thermal transient simulation of a multi-finger power FET at turn-on.

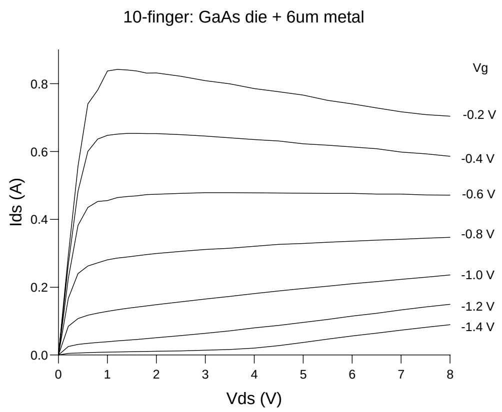

Fig. 3: Coupled electro-thermal simulation of DC I-V characteristics for a multi-finger power FET.