Chapter 14

Stock market valuation of patent portfolios

1. Introduction

As extensively discussed in this book, patents may have a significant impact on a firm's economic performance. We would then expect that they represent an important share of a firm value (see Chapter 5). Therefore, if a firm is going public through an Initial Public Offer (IPO) or is already publicly traded on a stock market, the value of its patents should be reflected to some extent into stock prices.

However, this book has also discussed the difficulties of assessing patent values. Given the uncertainty on patent returns and the specificity of and idiosyncrasy of each patent, a proper valuation could require a set of private information not available to external investors. The existence of information asymmetries between insiders (managers) and outsiders (financial investors) could bring the latter to not adequately evaluate a firm's patent portfolio.

Several empirical studies taking different methodological approaches have tried to assess whether and how stock market investors evaluate a firm's patent portfolio. Some of these studies have looked in particular at the relationship between patent indicators (see Chapter 7) and market values. Other studies in the accounting literature, instead, have tried to assess whether the information disclosed on patents usefully complements the data from financial reports (see Chapter 13) for stock market investors.

The question of how the stock markets evaluate firms' patent portfolios is important for several reasons. First, a better understanding of whether and how stock prices reflect patent value can provide interesting insights for the application of business valuation methods to firms holding strong patent portfolios. Second, stock market valuation of patents provides a signal to managers about their expected economic returns or their potential value. Third, if investors positively evaluate patents, it will be easier for firms with better patent portfolios to raise new financial resources on the stock market. This is particularly important at the IPO stage, where patents can provide an important signal on the quality of the firm to the potential investors. Fourth, if firms with better patents also exhibit better financial returns, patents become an important aspect that investors should account for in their investment decision.

In this chapter, we will first explain theoretically why patents should have an impact on firm market value. We will then analyze the empirical results on the relationship between patent indicators and stock market values at different stages of a firm's financing cycle. We will first discuss the dole of patents for firms going public through an IPO and then for firms already traded on a stock market. For the latter case, we will also distinguish between the studies that have analyzed the relationship between patent indicators and firm market value and those that that have investigated the value relevance of the patent information for stock market investors. We will conclude by discussing the main implications of the results reviewed in the next sections for the academic literature and the professional practice.

2. Patents and market value of the firm: A stylized model

The use of stock market-based measures to assess patent value clearly requires some assumptions on the way financial markets work. In particular, the eventual finding of a positive and significant relationship between patents and market values would imply some statement of stock market informational efficiency (i.e., stock prices reflect all available information). Fama (1970) makes a well known distinction between three degrees of market efficiency corresponding to different information subsets included in stock prices: the weak form, in which the information set consists only of historical prices; the semi-strong form, in which prices adjust to other information that is obviously public available, such as public announcements; the strong form, in which all the investors have access to any information relevant for price determination. Even though the debate is still open, there exists robust empirical evidence supporting the efficiency hypothesis, at least in the first two forms (see Fama, 1991, for a discussion). There is much more scepticism about the strong form of efficiency, above all because of the existence of information asymmetries between insiders and outsiders (i.e., managers have private information that investors do not have). These information asymmetries could be particularly important in the case of patents because we have seen in the previous chapters that patent returns are highly uncertain and the protected technology is very idiosyncratic, so that insiders could have private information with respect to financial investors. Indeed, Aboody and Lev (2000) have proved that higher R&D intensity is associated with greater information asymmetries between managers and stock market investors.

The assumption of market efficiency, even in the semi-strong form, has several potential important implications for the relationship between patents and market value:

- the market capitalization of the firm can be considered a reasonable proxy of its underlying value;
- this value will change if and only if the stock market receives new general or firm-specific information that modifies investors' expectations about the expected cash flows of the firm (Pakes, 1985; Woolridge and Snow, 1990);
- if patents are associated with the capacity of generating cash flows in the future (see Chapter 5), information about a firm's patents will affect the market valuation of the firm.

Pakes and Griliches (1984) have elaborated a model that very clearly expresses the theoretical relationship between R&D investments, patents and the stock market value of the firm (Figure 1). This diagram relates the unobservable ΔK, which is the net addition to knowledge capital K during a particular time period, to a set of observable inputs (*R&D*) and outputs (patents, *PATS*), random disturbances (ν, ω), and several indicators of performance (Z), including the stock market value of the firm. Firm performance is also assumed to be influenced by other observable variables such as investment and labor inputs (X) and unobservable factors (ε). The disturbance ω reflects the effects of informal R&D activities and the inherent randomness of inventive success, whereas v represents noise in the relationship between the patents a firm is granted and the associated increment to knowledge capital.

As concerns the issue investigated in this chapter, the model represented in Figure 1 states that if stock markets are efficient, the firm market value (one of the possible indicators of Z) should be affected by patents (*Pats*) because they represent a proxy of the new knowledge available to the firm (ΔK), which ultimately affect the expected cash flows. The empirical studies that we will review in the next sections are implicitly or explicitly based on this basic idea.

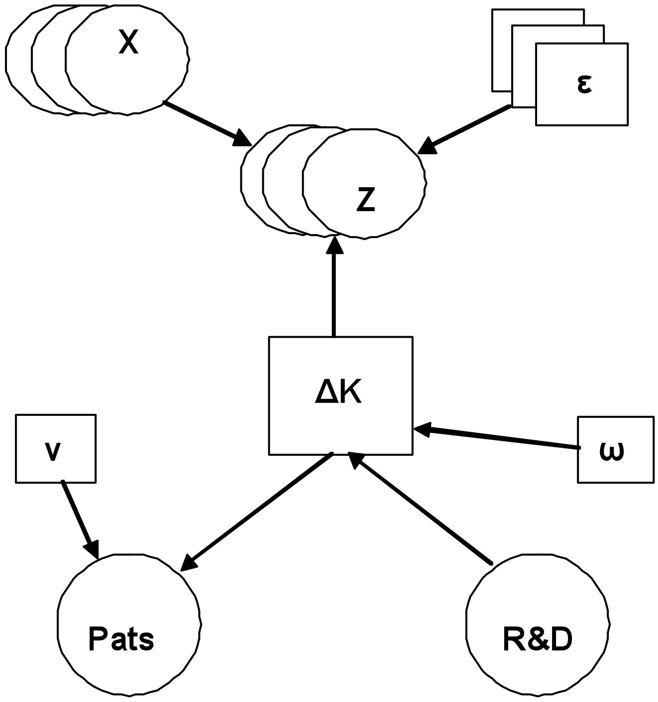

Figure 1: The measurement of knowledge capital (from Pakes and Griliches 1984)

3. Stock market valuation of patents at the IPO

A broad stream of literature has analyzed the impact that patents and patent-based indicators have on the stock market value of the firm. In this section, we focus on the impact of patents on the pricing of the stocks at the moment of the IPO, whereas in the next section we will analyze the relationship between patent indicators and stock market values for firms that are already publicly traded. We make this distinction because patents can have a different role for firms going public as compared to firms that are already publicly traded. In the case of IPO, in fact, they provide an important quality signal, that can reduce information asymmetries and the problem of undervaluation ('underpricing') at the moment of the offer of the firm going public. For more mature firms, instead, the market valuation of patents provides an indication of whether and to what extent stock market investors may want to provide new financial resources to firms having patent portfolios.

A central question for both stages of financing, however, is whether the stock market investors recognize patent value. The main problems in this respect could be information asymmetries and the uncertainty over patent returns. In fact, whereas information on patents and patent-based indicators is public and can be easily and cheaply obtained from existing databases, the knowledge about their quality and potential use in firms' activities is highly idiosyncratic.

Firms that go public undertake an initial public offering (IPO) of shares to public investors. This event provides access to public equity capital and financial resources that firms can use to finance growth or to reconcile current obligations (Certo, Holcomb and Holmes, 2009).
Most of empirical studies that focus on IPOs have documented two main evidences. First, substantial empirical evidence suggests that a typical pattern associated with the process of going public is the frequent incidence of "underpricing", as measured by the high initial returns of an IPO which occur when the IPO offer price is below the closing price at the end of the first day of trading (Ritter 1998). For instance, Loughran and Ritter (2002) have shown that first-day returns averaged 24.5% in the period from 1998 to 2007. Second, empirical evidence also indicates a long-term underperformace in the period following an IPO (Ritter, 1991). Loughran and Ritter (1995) have documented that, on average, IPO stocks underperformed seasoned firms of the same size by approximately 7% during a five-year period post IPO.
As concerns IPO underpricing, its empirical regularity has motivated a large theoretical literature trying to explain its causes (see Ritter and Welch, 2002, for a review of the empirical evidence). The most established explanation for underpricing resides in the model based on information asymmetries, which assumes that firms and underwriters will discount the initial offer price in order to induce external investors to buy stock in absence of full information on firm's value (Rock 1986). In particular, the initial underpricing should increase with the ex ante uncertainty about the value of the IPO firm (Ritter, 1984, Beatty & Ritter, 1986). In order to control for ex ante uncertainty, empirical studies used various proxies related to company characteristics, offering characteristics, prospectus disclosure, and aftermarket variables (see Ljungqvist 2005 for a review).[i]
Some recent studies have focused on different innovation measures as proxies of ex ante uncertainty about the value of the IPO firm. In particular, some scholars (Guo et al., 2004; Guo et al., 2006) documented a positive relation between R&D intensity and initial IPO underpricing, thus singling out R&D as a major contributor to information asymmetries. Other studies examined patent-related measures as a signal of firm's value. Chin, Lee, Kleinman and Chen (2006) analyzed a cross-sectional sample of 623 Taiwanese IPO firms and found that official monthly reports of newly developed patents released to the public and the frequency of patent citations significantly increase IPO underpricing. Chin and colleagues (2006) concluded that, because of high uncertain value of patents, greater level of innovation capital leads to greater uncertainty in firm's value. Bessler and Bittelmeyer (2008) investigated the relation of innovation and performance of German firms that went public at the "Neuer Markt" during the period from 1997 to 2002. The authors found that

mean underpricing for IPOs with patents is lower relative to the group of IPOs without patents in hot issue periods, but in contrast it is higher in cold issue markets. However, these results are far to be conclusive as they are based on descriptive statistics and not tested with multivariate analyses.
Heeley, Matusik and Jain (2007) conducted a cross-sectional study on a sample of 1,413 US firms conducting initial public offerings in the period 1981–98. They showed the potential interpretable information brought to investors by patents depend on the industry where the firm operates. In particular, patents reduce information asymmetries (and therefore underpricing) in industries where the link between patents and inventive returns is transparent. This occurs when the technology is discrete and the motive for patenting is the generation of monopoly rents (e.g. pharmaceuticals) Conversely, in industries where the link is not transparent (e.g., electronics), patents reflect increased information asymmetries and underpricing.
Finally, Morricone, Munari and Oriani (2010) analyzed the patent commercialization strategy on a sample of 130 semiconductor companies that went public in the United States in the period 1996-2007. Their findings documented that IPO underpricing might not be influenced only by the possession of patents, but also by the strategic use which is made of them to create and capture value. In particular, they show that when firms rely more on licensing-based patent commercialization strategies, the underpricing at their IPO is higher. Moreover, a firm's stock of patents can reduce underpricing only when the patent commercialization strategy is more based on licenses.
As concerns long-term performance, previous studies have also motivated the underperformance of IPO firms based on information asymmetries. In particular, empirical studies documented a positive relation between patents and long-term performance suggesting that investors are not overly optimistic about the prospects of firms at the time they go public. In particular, Chin, Lee, Kleinman and Chen (2006) shown that firms granted more patents have better aftermarket stock performance relative to firms granted fewer patents. Bessler and Bittelmeyer (2008) focused on the relation between long-term performance and different patent variables such as the number of patents, the family size, the number of IPC-classes, the number of backward and forward citations, and the frequency of cited articles. They found that the abnormal performance of IPOs with patents is positive and significantly higher than that of IPOs without patented technology when measured after 6, 12 and 24 months of trading.

The results of the studies reported in this section suggest that patents may represent an important signal for the investors buying shares at the moment of an IPO. However, the most recent works have recognized that the ability of the patents to provide valuable information to external investors is affected by some contextual factors that make the information provided by patents more or less relevant. In particular, patents are evaluated more positively by external investors (i.e., they reduce underpricing) when the link between patents and firm performance is clearer or when patents are more central to a firm's strategy.

4. The effect of patents and patent indicators on the stock market value of traded firms

Several authors have tested the relationship between different types of innovation inputs or outputs with firm-level performance measures derived from stock market data. Here we distinguish two main approaches, which we treat separately as they have different objectives and adopt different methodologies. First, we review those studies that have analyzed the relationship between patents and market value to analyze firms' innovation activities. Second, we report the results of those studies that, in the accounting literature, have analyzed whether patent data are 'value relevant', in the sense that they complement accounting information within stock price determination.

4.1. Stock market valuation of innovation

The studies that have analyzed the relationship between patents, taken as a proxy of a firm's innovation output, and firm market value have implicitly or explicitly assumed that stock market investors value the firm as a bundle of tangible and intangible assets (Griliches, 1981). In equilibrium, the market valuation of any asset results from the interaction between the capitalization of the firm's expected rate of return from investment in that asset and the market supply of capital for that type of asset (Hall, 1993). Using this idea, it is possible to represent the market value V of firm i at time t as a function of its assets:

$$V_{it} = V(A_{it}, K_{it}, I_{it}^{1}, \ldots, I_{it}^{n}) \quad [1]$$

where A_{it} is the book value of tangible assets, K_{it} is the replacement value of the firm's knowledge capital generated by its innovation activities and I_{it}^{j} is the replacement value of the j^{th} other intangible asset.

Empirical testing of this formulation requires an observable proxy for knowledge capital and the literature suggests using patent indicators for this purpose. However, because R&D and patents are highly correlated in the cross section, it is necessary to be careful about the choice of specification when both variables are entered into the same market value equation. The two possibilities are either to include a measure of the stock of patents held by the firm in place of the stock of R&D, or to include a patents-per-R&D yield variable in addition to the R&D variable. Following the latter approach, the empirical equation can be written as follows (see Appendix 1 for its derivation):

$$\log\left(V_{it}/A_{it}\right) = \log b + \gamma_1 K_{it}/A_{it} + \gamma_2 P_{it}/K_{it} \qquad [2]$$

In the above equation, the ratio V_{it}/A_{it} is a proxy of Tobin's Q, b is the market valuation coefficient of firm's total assets (see Appendix 1 for more details), K_{it} is a measure of knowledge capital based on the capitalization of past and present R&D investments (R%D capital) and P_{it} is a measure of the patent stock (see Appendix 2 for a detailed explanation of how the R&D and the patent stock are calculated in the literature). In equation [2], the coefficient γ_1 measures the impact of the R&D capital on firm market value, while the coefficient γ_2 measures the contribution to market value of acquiring an additional patent per unit of R&D stock.

By including patents in the market value equation in addition to R&D capital, a number of studies have shown that in general patents are positively evaluated (the coefficient γ_2 is positive), although they add a small amount of information above and beyond that obtained from R&D. Table 1 shows results from various studies using US and UK data. The coefficients are not directly comparable as the patent variable is often measured on a different scale, but they show that, with few exceptions (e.g., Toivanen et al., 2002) for the UK and the US stock market investors normally pose a positive value on them.

Study	**Patent Coefficient (Std. err)**	**Sample characteristics (country, no. of firms, years, data sources)**
Griliches (1981)	Patents: 10-25	US, 157 firms, 1968-1974, Compustat and USPTO
Ben-Zion (1984)	Patents/assets: 0.065 (0.055)	US, 93 firms, 1969-1977, Compustat and USPTO
Cockburn and Griliches (1988)	Patent stock/assets: 0.11 (.09)	US, 722 firms, 1980, Compustat and USPTO
Megna and Klock	Patent stock: 0.38 (0.2)	US semiconductor, 11 firms, 1972-1990, Compustat and

(1993)		USPTO
Blundell *et al.* (1995)	Patent stock/R&D stock: 1.93 (0.93)	UK, 340 firms, 1972-1982, LBS Share Price database and Datastream, NBER patent database
Shane and Klock (1987)	Patents/assets: -0.41 (.25) Cites/assets: .012 (.005)	US semiconductor, 11 firms, 1977-1990, Compustat and CHI Research
Haneda and Odagiri (1998)	Patent stock elasticity: ~0.3	Japan, 90 firms, 1981-1991, NEEDS database
Bloom and Van Reenen (2002)	Patent stock/assets: 1.221 (0.492) Cite stock/Assets: 0.345 (0.140)	UK, 172 firms, 1969-1994, Datastream and NBER patent database
Toivanen *et al.* (2002)	Patents/assets: insignificant	UK, 877 firms, 1989-1995, Extel financial company analysis
Hall *et al.* (2005)	Patent stock/R&D capital: .030 (.042) Cite stock/Patent stock : .052 (.004)	US, 4800 firms, 1965-1995, Compustat, NBER patent database
Hall *et al.* (2007)	Patent stock/R&D capital: .330 (.066) Value indicator/Patent stock : .032 (.0080)	33 European countries, 10218 firms, Amadeus, Datastream, Patstat

Table 1. Main results of previous studies on patents and market value (adapted from Czarnitzki, Hall and Oriani, 2006)

One disadvantage of using patents is the large variance in the of value of individual patents, rendering patent counts an extremely noisy indicator of R&D success. One way to account for patent heterogeneity is by means of citation-weighted patent counts, that is, a firm's patent counts are supplemented with the number of subsequent citations that patents receive to get a better measure of R&D success (see also Chapter 7). A number of researchers have demonstrated that measures of innovation output or profitability are related to the number of times a patent on the relevant invention is cited by other later patents (e. g., Trajtenberg 1990; Harhoff *et al.* 1999).

Hall, Jaffe, and Trajtenberg (2005) extend the market value equation [2] by including the average citations received by these patents (i.e., the ratio of citations to patent stocks). Thus equation [2] is modified as following:

$$\log\left(V_{it}/A_{it}\right) = \log b + \log\left(1 + \gamma_1 \frac{R\&D_{it}}{A_{it}} + \gamma_2 \frac{PAT_{it}}{R\&D_{it}} + \gamma_3 \frac{CITES_{it}}{PAT_{it}}\right), \qquad [3]$$

where *CITES* stands for the stock citations received by the patents held by a firm.
Hall and colleagues (2005) have found that each of the ratios in expression [3] has a statistically and economically significant impact on market value. In particular, they shows the following effects:

- a 1% increase of in the R&D intensity of the firm generate a 0.8% increase in market value;
- an extra patent per million dollars of R&D boosts market value by about 2%;
- an extra citation per patent increase market value of about 2%.

These results highlight that not only does the market value R&D inputs and R&D outputs as measured by patent counts, but also it values "high-quality" R&D output as measured by citation intensity.

The authors also show that the valuation coefficients change by industry. They identify six sectors: Drugs and Medical Instrumentation (henceforth just “Drugs”); Chemicals; Computers and Communications (“Computers”); Electrical; Metals and Machinery; and miscellaneous (low-tech industries). The impact of the patent ratio (*PAT/R&D*) for Drugs is three times the average effect (.10 versus .031), and that of Computers twice as high; similarly but not as pronounced, the impact of Cites/Patents for Drugs is over 50% higher than the average effect, while that for Computers is small, and lower than that for the other sectors except for the low-tech sector. This contrast is consistent with the differing roles played by patents in the two sectors. Drugs is characterized by discrete product technologies where patents serve their traditional role of exclusion, and some of them are therefore valuable on an individual basis, as measured by citations. Computers and Communications is a group of complex product industries where any particular product may rely on various technologies embodied in several patents held by different firms. In this industry patents are largely valued for negotiating cross-licensing agreements, so their individual quality is not as important, although having them is.[ii] These results are also consistent with those reported by Heeley and colleagues (2009) with respect to the IPO.

The recent work of Hall, Thoma and Torrisi (2007) reports some additional interesting insights. They estimate equation [3] on a broad sample of European countries, finding that the effects of the three ratios of equation [3] on firm market value are lower on average than those that Hall and colleagues (2005) find for the US sample[iii]. However, the effect critically depends on where the patents are released. In particular they show that:

- European patents have practically no effect on firm market value;
- an extra US patent per million Euros increases market value of 1.1%;
- an extra European patent with US equivalent for million Euros increases market value of 3.4% (a similar effect is found for US patents with an European equivalent).

These results highlight that patent families and geographical extensions (see Chapter 2) matter for stock market investors.

Finally, it is worth reporting the results of the study of Bloom and Van Reenen (2002). The authors find that while the stock of patents has an immediate effect on market value, it has a slower effect on productivity, especially in presence of higher uncertainty. This result is considered as a support to a real options view of patents (see Chapter 6). In particular, the authors claim that under uncertainty, the firm prefers holding an option to wait by not using the patent for production activities.

4.2. Value relevance of patent indicators

The studies reviewed in the previous section relate patents and patent indicators to the market value of the firm. Their main interest is in whether inputs (R&D) or outputs (patents) of the innovation process affect the market valuation of publicly traded firms. In this section, we present an alternative empirical model based on the value relevance of patent information, which focuses on the information content of patents. The studies on value relevance use stock market values as informative benchmark in order to understand whether patent information is reflected into firm value effectively and timely (Barth, 2000). The main assumption of value relevance studies is that patents data contain information useful to investors above and beyond accounting values. Accordingly, patent data are 'value relevant' - i.e., have a predicted significant relation with share prices - only if these values convey information relevant to investors in evaluating the firm that is reflected in share prices (Beaver, 1998; Barth, 2000; Barth et al., 2001; Holthausen & Watts, 2001).
Most studies on the value relevance of patents stem their motivations from the problems related to the accounting valuation criteria discussed in Chapter 13. In particular, value relevance studies use patent data as nonfinancial indicators to prove the information deficiency of the financial statements. For high-tech firms, in particular, investors may need to supplement accounting information with non-accounting data that helps them form better estimates of the future profits of

the firm (Hirschey & Richardson, 2001a). These studies suggest that a consistent footnote disclosure on patents would increase the informativeness of the financial statements for high-tech firms.

The studies that have analyzed the association between patent data and stock market could be distinguished on the basis of the period of analysis and the methodology (Lev and Sougiannis, 1996; Holthausen & Watts, 2001; see Appendix 3 for a detailed description of the different methodologies). *Contemporaneous studies* or price models analyze the association between patent information and current stock prices and indicate the extent of information content of patents for investors. *Intertemporal studies* or return models assess the relationship between patent information and future stock returns in order to investigate whether patent data are able to predict future firm performance. *Marginal information content studies* or event studies investigate whether patents add to the information set available to investors.

Contemporaneous studies (price models)

As concerns contemporaneous studies, the empirical findings document that patent indicators represent on average relevant information for investors. The work of Hirschey, Richardson and Scholz (2001) is one of the first contributions seeking evidence on the value relevance of patent data. By using different citation-based indicators as measures of patent quality, Hirschey and colleagues (2001) have shown that patent quality is value relevant. More specifically, they have analysed the value relevance of patent quality through two indicators: the Current Impact Index is an indicator of patent portfolio quality and is measured by the number of times a company's previous five years of patents are cited in the current year, relative to all patents in the U.S. patent system; the indicator Technology Cycle Time indicates the speed of innovation and is defined as the median age in years of the U.S. patent references cited on the front page of company patents. Their findings have shown that patent quality provides investors with a more useful basis upon which to judge the economic merit of the firm's R&D effort. In particular, patent portfolio quality has a robust positive influence on market value, incremental to the effect of the R&D investments. Conversely, there is a negative influence of Technology Cycle Time on the market value effects of R&D expenditures.

In a related study, Hirschey and Richardson (2001a) have studied the association between stock prices and patent quality for firms with different sizes and growth opportunities.[iv] They have found that the effect of patents and patent quality on market value is stronger in the case of small cap stocks and relatively low P/E high tech firms. This finding indicates that patent quality represents useful information especially in the case of small high-tech firms that often have not reliable historical financial information.

Cheng and Chang (2009) have also confirmed the value relevance of patent information by examining the relationships between firm market value and different patent quality indicators (relative patent position, Herfindahl–Hirschman Index of patents, and patent citations) in the US pharmaceutical industry. In particular, the indicator of relative patent position of a company in a given technological field measures its degree of leadership in that field. The authors have shown that the stock market positively values this indicator, probably because of the potential first mover advantage. The Herfindahl–Hirschman index of patents measures the degree of technological concentration of a patent portfolio. Since firms having different technologies can take advantage of new technological opportunities, the risk of missing new technological opportunities is lower. Consistently, Cheng and Chang (2009) have proved that the degree of a firm's technological diversity (concentration) is positively (negatively) associated with its market value.

The value relevance research has also shown the existence of country-specific effects on the valuation of a firm's patents. Hirschey and Richardson (2001b) have evaluated the relative quality of U.S. patenting activity by Japanese and U.S. firms as a means for gaining insight on the growing importance of global competitiveness in the high-technology sector. Their findings demonstrate positive influences of the number of U.S. patents on the market values of U.S firms, but no discernable influence in the case of Japanese firms. These findings support the assertion that patent numbers are useful, but limited economic indicators of the pace of scientific advance. As concerns patent quality, Hirschey and Richardson (2001b) have shown that scientific measures of patent quality have robust influences on market values for both Japanese and U.S. firms.

Intertemporal studies (return models)

Intertemporal studies focus on the ability of patent indicators to predict stock returns. Since public information on firms' innovation is generally scant and not timely (see chapter 13), Deng, Lev and Narin (1999) have examined how patent-related measures alleviate this information deficiency. They suggest that patent-based measures provide a useful tool for the investment analysis of technology- and science-based firms. In particular, they have focused on a set of measures based on patent application citations that are equivalent to those employed by contemporaneous studies (Hirschey et al., 2001; Hirschey and Richardson, 2001a, 2001b).[v] They have demonstrated that these publicly available patent-related measures reflect firms' potential growth and are significantly associated with subsequent market-to-book ratios and stock returns.

Mariginal information content studies (event studies)

In contrast to the aggregate data used in contemporaneous and intertemporal studies, marginal information content studies allows the analysis of individual patents in order to understand the effect of a patent event on firm market value. For example, Austin (1993) has focused on 565 patents owned by the 20 largest biotechnology firms and has analyzed whether the release of patent-related information, such as patent granting, is associated with value changes and price reactions. His findings suggest that the release of patent granting information represents on average a "good news" for investors, which positively react to information release. In particular, Austin (1993) has distinguished among different patents characteristics, such as whether a patent is readily identifiable with end products and whether the patent grants were announced in The Wall Street Journal. His findings have shown that stock market evaluate much more the product-linked patent events and patents announced in the press.

Wang, Hsieh and Chuang (2009) have considered the effect of negative patent events and explored how patent infringement litigation impacts the stock price of firms listed in Taiwan from 1998 to 2008. Their empirical results indicates that patent litigation negatively affects stock prices.

Landsford (2009) has analyzed firms' voluntary disclosure of patent-related information in order to understand whether firms strategically manage these announcements. In particular, the author has analyzed the association between negative earnings surprise announcements and voluntary disclosure on positive patent-related events, such as applications, notice of allowance and issuances. Landsford's (2009) findings show that the average market reaction to the patent disclosures is significantly positive suggesting that patent-related information cause investors to upwardly revise future cash flow expectations. Moreover, results have documented that the probability of strategic patents announcements increases as the size of the earnings disappointment increases.

5. Conclusions

The studies reviewed in this chapter suggest that patents are on average positively evaluated by stock market investors, both for firms going public through an IPO and for firms already publicly traded in the stock market. A general result is that patents are relevant for stock price determination because they add information on the quality of a firm's R&D activity. Moreover, they are associated with future stock returns and with price changes when new information is disclosed to the market.

However, it is also important to remark that not always patents are able to convey a signal to the investors. If patents are important for a firm's economic performance and their effect on expected cash flows is clearly identifiable, as it happens for discrete technologies (e.g., pharmaceuticals), they are significantly and positively evaluated by stock market investors. Instead, if patents refer to a complex technology (e.g., telecommunications equipment), the relationship between patents and market value is less clear and can even turn into negative. This is because in the latter context there will still exist information asymmetries between mangers and investors on the use and value of patents.

These results have some important implications for the issues on patent valuation with which this book has dealt so far. First, if stock market investors recognize a value to patents (and stock markets are efficient with respect to patent information at least in the semi-strong form), it makes sense to evaluate patents. The value of a firm, above all if operating in a high-tech industry, critically depends on its patent portfolio and its quality. Patent valuation is then an essential task within the activity of business valuation. Analytic methods that try to assess patent value on the basis of the contribution to a firm's cash flows (see Chapter 5) are particularly needed in this context. In addition, the studies reviewed here also validate some patent indicators that have been introduced in Chapter 7. These indicators might therefore be used to provide a first and simple estimation of the value of the patent portfolio of a firm.

Second, if patents are associated with stock prices and returns, they become useful information for stock market investors' decisions. Following this idea, for example, Ocean Tomo, a company known for patent auctions and consulting, has created a set of patent-based stock indexes[vi]. These indexes include the stocks of these firms that have better patent indicators. The data reported by the company show that if such an index had been created in 1997, in the ten following years it would have strongly outperformed the Standard&Poor's 500 Index.

Third, the results of the studies on patents and market value also provide useful indications for the management of the information flows between the firm and the investors. In Chapter 13 it has been remarked that often financial statements provides only limited information on patent values. In this respect, the studies on value relevance reviewed in this chapter highlight that more voluntary disclosure of patent information in the financial statements would bring to better stock price efficiency and stability.

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Appendix 1. The market value of R&D and patents: The hedonic model

Starting from Griliches (1981), several authors have used and developed an empirical model of the market value of the firm known as hedonic model. We outline the model here, using a treatment that follows Hall (2000), Hall and Oriani (2006) and Czarnitzki, Hall and Oriani (2006). The model is based on the assumption that it is possible to represent the market value V of firm i at time t as a function of its assets:

$$V_{it} = V\,(A_{it}, K_{it}, I_{it}^{1},\ldots, I_{it}^{n}) \qquad [A1.1]$$

where A_{it} is the book value of tangible assets, K_{it} is the replacement value of the firm's technological knowledge capital and I_{it}^{j} is the replacement value of the j^{th} other intangible asset. If single assets are purely additive, and ignoring the other intangible assets for the sake of simplicity, it is possible to express the market value of the firm as follows:

$$V_{it} = b\,(A_{it} + \gamma K_{it})^{\sigma}$$

[A1.2]

where b is the market valuation coefficient of firm's total assets, reflecting its differential risk, overall costs of adjusting its capital, and its monopoly position, γ is the relative shadow value of knowledge capital to tangible assets, and the product $b\gamma$ is the absolute shadow value of the knowledge capital. In practice, $b\gamma$ reflects the investors' expectations about the overall effect of K_{it} on the discounted value of present and future earnings of the corporation, while γ expresses the differential valuation of the knowledge capital relative to tangible assets. By definition, when γ is unity, a currency unit spent in knowledge capital has the same stock market valuation of a currency unit spent in tangible assets. Conversely, values of γ higher (lower) than unity suggest that the stock market evaluates knowledge capital more (less) than tangible capital.

The expression [A1.2] can be interpreted as a version of the model that is known in the economic literature as hedonic pricing model, where the good being priced is the firm and the characteristics of the good are its assets, both tangible and intangible. Taking the natural logs of both the sides in [A1.2], assuming constant returns to scale ($\sigma=1$), and subtracting log A_{it} from both sides, we obtain the following expression:[vii]

$$\log(V_{it}/A_{it}) = \log b + \log(1 + \gamma\, K_{it}/A_{it})$$

[A1.3]

The ratio V/A is a proxy for average Tobin's q, the ratio of the market value of tangible assets to their physical value. The estimation of equation [3] allows one to assess the average impact of a euro or dollar invested in knowledge on the market value of a firm at a particular point in time. Bloom and Van Reenen (2002) and Hall et al. (2005) estimate equation [A1.3] using non-linear least squares (NLLS). Other authors applying the same model have used the approximation $(1+x) \approx x$, obtaining the equation below, which can be estimated by ordinary least squares (Griliches 1981; Jaffe 1986; Cockburn et al. 1988; Hall 1993a, 1993b):[viii]

$$\log(V_{it}/A_{it}) = \log b + \gamma\, K_{it}/A_{it}$$

[A1.4]

Since the output of R&D investments is inherently uncertain, some R&D projects will result in the creation of more valuable knowledge capital than others. If this success can be observed by investors, then the associated R&D should impact market value more than unsuccessful R&D. Empirical testing of this formulation requires an observable proxy for R&D success and the literature suggests using patent indicators for this purpose. Because R&D and patents are highly correlated in the cross section, it is necessary to be careful about the choice of specification when both variables are entered into the same market value equation. The two possibilities are either to include a measure of the stock of patents held by the firm in place of the stock of R&D, or to include a patents-per-R&D yield variable in addition to the R&D variable:

$$\log\left(V_{it}/A_{it}\right) = \log b + \gamma_1 K_{it}/A_{it} + \gamma_2 P_{it} / K_{it} \qquad \text{[A1.5]}$$

Appendix 2. Measures of R&D and patent stocks

The studies on innovation and market value have used two main proxies for the knowledge capital (K): R&D- and patent-based. In the absence of patent data, Griliches (1995) defines the following formal relationship between a firm's stock of technological knowledge and R&D investments:

$$K = G\,[W(B)R,\ \omega] \qquad \text{[A2.1]}$$

where K is the current level of technological knowledge, $W(B)R$ is an index of current and past R&D expenditures and ω is the set of unmeasured influences on the accumulated level of knowledge described above. Accordingly, an R&D-based measure of a firm's technological knowledge has been often computed as the capitalization of present and past R&D expenditures using a perpetual inventory formula like that used for tangible capital (Griliches and Mairesse, 1984; Hall, 1990):

$$K_{it} = (1 - \delta)\, K_{i,t-1} + R_{it} \qquad \text{[A2.2]}$$

where K_{it} is the R&D capital at time t, R_{it} is annual R&D expenditures at time t and δ is the depreciation rate of the R&D capital from year $t-1$ to year t. The use of expression [6] to capitalize R&D investments is needed because the Generally Accepted Accounting Principles (GAAP) in the US and the IAS accounting standards in Europe require R&D costs to be expensed as incurred (with a few exceptions) because of the lack of a clear link between these expenses and subsequent earnings (see Zhao 2002, for details). The use of a depreciation rate is justified by the fact that knowledge tends to decay or become obsolescent over time, losing economic value due to advances in technology.

Most of the studies that have estimated the hedonic model have used a constant annual 15% depreciation rate (Jaffe 1986; Cockburn and Griliches 1988; Hall 1993a, 1993b; Blundell et al.

1999; Hall and Oriani 2006). Other studies have used an estimation procedure that allows one to determine industry- and time-specific economic depreciation rates (e.g. Lev and Sougiannis 1996).[ix] There also exist analyses using annual R&D expenditures as an alternative measure of R&D capital (Cockburn and Griliches 1988; Hall 1993a, 1993b; Munari and Oriani, 2005).

However, the use of R&D-based measures does not definitively resolve the questions related to the measurement of technological knowledge for several reasons, mostly related to the presence of the disturbance ω discussed above, which introduces noise into the relationship between R&D and the underlying increment to knowledge capital. The first problem is that the quality of corporate financial reporting on R&D activity and intangibles in general is often inadequate for economic analysis purposes (Lev, 2001). Therefore R&D investments can be a source of greater information asymmetries between ownership and management and may not be properly valued by the market (Aboody and Lev, 2000). Second, national accounting laws often do not require corporation to disclose the amount of their annual R&D expenditures. For example, in the European Union, the United Kingdom is one of the few countries where quantitative disclosure of R&D investments is obligatory, while in France, Germany, and Italy there exists only an obligation to report qualitative information about R&D (Belcher 1996; KPMG 2001). Because some firms nevertheless do report R&D expenditures, this creates a potential sample selection bias (see the discussion in the next section). Finally, R&D investments are a not an output but an input measure of the innovation process. Since the outcome of R&D is highly uncertain (e.g. Scherer and Harhoff 2000), in some cases the relationship between R&D investments and a firm's knowledge base may be rather imperfect.

In order to solve these problems, some studies have used patent-based measures of technological knowledge. Recently, the wider availability of patent data in an electronic format and the creation of freely usable databases have spurred the adoption of patent-based measures in the studies on innovation and market value.[x] The first analyses were based on patent counts (e.g., Griliches 1981; Cockburn and Griliches 1988), where the number of patents substitutes for R&D investment in expression [6]. However, such a measure often turns out to be barely significant in the presence of R&D. An explanation of this phenomenon was provided by Griliches *et al.* (1987), who showed that under reasonable assumptions about the distribution of patent *values*, patent *counts* are an extremely noisy measure of the underlying economic value of the innovations with which they are associated, because the distribution of the value of patented innovations is known to be extremely skew. A few patents are very valuable, and many are worth almost nothing (Harhoff *et al.* 1999; Scherer *et al.* 2000).

In an effort to improve the patent measure, more recent studies have weighted the patent counts by the number of citations received by each patent from subsequent patents (Bloom and Van Reenen 2002; Hall *et al.* 2005). The number of citations a patent receives can be viewed as an indicator of its quality or importance, which should be reflected in its market value. At the individual patent level, the use of citations as a proxy for value has been justified by Trajtenberg (1990) and Harhoff *et al*. (1999). In this case, the main source of noise is related to the fact that citations can be added for different purposes, so that a citation does not necessarily imply a technological impact of the previous patent (see Jaffe *et al*. 2002 for a survey on the meaning of patent citations). Moreover, all patent-based measures suffer from the limitation that the propensity to patent significantly differs across industries (Levin *et al.* 1987; Cohen *et al.* 2000), which implies that in different industries patents will vary in quality as indicators of technological knowledge.

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[i] Existing studies have demonstrated, for example, that IPO performance is affected, among others, by underwriter prestige (e.g., Carter, Dark, & Singh, 1998), venture capital backing (e.g., Megginson & Weiss, 1991), the presence of one or more founders (Nelson, 2003), board composition (Filatotchev and Bishop, 2004), top management team legitimacy (Cohen & Dean, 2005) and the level of CEO equity (Certo, Daily, Cannella, & Dalton, 2003).

[ii] See Arora *et al*. (2004) for further discussion of this contrast and Hall and Ziedonis (2001) for evidence on semiconductors.

[iii] The authors use a more complex patent quality indicator to substitute the citation stock of Hall et al. (2005)

[iv] Hirschey and Richardson (2001a) measure patent quality using three indicators. The “Citations Index” is the number of citations generated in the current year by patents granted to the company during the most recent 5-year period, relative to the average number of citations for firms in a given International Patent Classification four-digit subclass and year. ”Non-Patent References” is predicated upon how closely a company’s patents in the present year are to the scientific research base in the area. NPR is a simple count of the number of references in a patent application to a wide variety of non-patent publications, including scientific papers and articles, brochures, books, standards, documents, patent disclosure bulletins, and so on. A third measure of patent quality is the “Technology Cycle Time” indicator that

is the median age, computed in years, of the prior art references to earlier US or European patents.

[v] Deng et al. (1999) have employed the following measure: citation impact, science link and technology cycle time. The citation impact is a measure of "forward citation" that shows whether a firm's most current patent received higher or lower citations compared to the past five year's average. Science link is a measure that focuses on the number of citations in a patent application to scientific papers. This measure shows a relationship between a firm's R&D programs and current scientific developments. It is, therefore, a "backward citation" indicator. The technology cycle time is a measure that focuses on the "median age of U.S. patents cited in a patent application" (p.22). In addition to these three citation measures, Deng et al. also include a patent count measure, which shows the number of U.S. patents granted during a particular year.

[vi] See http://www.oceantomo.com/productsandservices/investments.

[vii] The assumption of constant returns to scale (homogeneity of degree one) in the value function has been confirmed repeatedly in the literature, at least for cross sections of firms.

[viii] In order to investigate the appropriateness of equation [3] or [4], Hall and Oriani (2004) explored the use of semi-parametric estimation for the simple Tobin's *q*-R&D capital relationship by means of kernel regression using data for the United States. They found that the relationship resembles a logistic curve, with zero and very small amounts of R&D capital (less than about one per cent of tangible assets) having no effect on Tobin's q, a roughly linear relationship until $K/A=1$, and a flatter relationship thereafter. Above K/A value of one per cent, the relationship is somewhat better described by equation [3] than equation [4].

[ix] More precisely, the authors estimate a regression model in which the dependent variable is the annual operating income and the independent variables are the lagged values of total assets and advertising expenditures and a vector of the past R&D investments.

[x] See Hall *et al. 2002* for a description of the NBER/Case Western patent database