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Monetary Policy Indeterminacy in the U.S.: Results from a Classical Test

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Abstract

We work with a newly developed method to empirically assess whether a specified new-Keynesian business cycle monetary model estimated with U.S. quarterly data is consistent with a unique equilibrium or multiple equilibria under rational expectations. We conduct classical tests to verify if the structural model is correctly specified. Conditional on a positive answer, we formally assess if such model is either consistent with a unique equilibrium or with indeterminacy. Importantly, our full-system approach requires neither the use of prior distributions nor that of non-standard inference. The case of an indeterminate equilibrium in the pre-1984 sample and of a determinate equilibrium in the post-1984 sample is favored by the data. The long-run coefficients on inflation and the output gap in the monetary policy rule are found to be weakly identified. However, our results are further supported by a proposed identification-robust indicator of indeterminacy.

Keywords: GMM, Indeterminacy, Maximum Likelihood, Misspecification, new-Keynesian business cycle model, VAR, Weak identification.

J.E.L.: C31, C22, E31, E52.

1 Introduction

The U.S. inflation and output growth processes have experienced dramatic breaks in the post-WWII. The most evident change regards their volatilities. The 1970s were characterized by high macroeconomic turbulence. Differently, since the mid-1980s, much milder fluctuations in inflation and output growth has been observed, at least before the advent of the recent financial crisis. The marked reduction of the U.S. macroeconomic volatilities has been termed 'Great Moderation' (see, among others, Kim and Nelson, 1999 and McConnell and Perez-Quiros, 2000).

A popular explanation for such phenomenon hinges upon the switch toward an aggressive monetary policy conduct occurred with the appointment of Paul Volcker as Chairman of the Federal Reserve at the end of the 1970s. With his appointment, the argument goes, the Fed moved from a weakly aggressive reaction to inflation to a much stronger one. Such a switch anchored private sector's inflation expectations, therefore leading the U.S. economy to move from an indeterminate equilibrium to determinacy. This story, popularized by Clarida, Galí, and Gertler (2000), has subsequently been supported by Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Benati and Surico (2009), Mavroeidis (2010), and Inoue and Rossi (2011).[1]

All these contributions assume the new-Keynesian model one works with to be correctly specified. If this assumption is false, however, testing for determinacy becomes problematic. Since indeterminacy generally entails a richer correlation structure of the data, the typical risk, as argued by Lubik and Schorfheide (2004), is confounding in the determinate case the possible omission of important propagation mechanisms in the structural equations with indeterminacy. In this respect, different model specifications in the literature appear to have led to different conclusions as for the probability of being in an indeterminate state in the pre-Volcker phase.[2]

[1] A recent contribution by Coibion and Gorodnichenko (2011) estimates a Taylor rule featuring time-varying policy coefficients and trend inflation. They find the reduction of the average inflation level to be a necessary ingredient for the switch to determinacy possibly occurred in the early 1980s. Ascari, Branzoli and Castelnuovo (2011) show that movements in trend inflation may be a proxy for the evolution of labor market frictions in the post-WWII U.S. economy. We postpone the investigation of the role of trend inflation to future research.

[2] Justiniano and Primiceri (2008) test for indeterminacy in the pre-Volcker sample following the strategy adopted by Lubik and Schorfheide (2004) but with a richer, medium scale new-Keynesian model. They find the data to favor the determinate case. Justiniano and Primiceri's (2008) explanation of the

This paper tests for determinacy in the U.S. economy by explicitly recognizing that model misspecification might affect the empirical assessment of determinacy/indeterminacy. From the methodological side, our contribution is twofold. First, we apply a novel, computationally simple test for the hypothesis of determinacy (against the alternative of indeterminacy) that requires neither the use of prior distributions nor that of nonstandard inference. Building on Fanelli (2010), the method hinges on the idea that if the structural new-Keynesian model is not rejected by the data (i.e., it is not misspecified), one can use standard 'misspecification diagnostic analysis' to infer whether the macroeconomic series of interest are better approximated by a determinate vs. indeterminate reduced form solution. In particular, we first estimate the structural Euler equations of the system by GMM and test the model specification at hand through an overidentification restrictions test. Then, we re-estimate the model with maximum likelihood and appeal to a likelihood-based test for the cross-equation restrictions (Hansen and Sargent, 1980, 1981) that the new-Keynesian model entails under determinacy as a tool for disentangling whether the data favour the occurrence of a unique stable solutions or multiple stable solutions. Second, in addition to the above inferential method we introduce an indicator of indeterminacy (determinacy) that is explicitly designed to cope with 'weakly identified' parameters (Stock and Wright, 2000). Recent evidence suggests that the reaction functions coefficients in models involving forward-looking behavior tend to be weakly identified (Mavroeidis, 2010). Clearly, this phenomenon might have negative consequences on the finite sample performance of our test.

Our empirical investigation, conducted with a small scale dynamic stochastic general equilibrium (DSGE) model for the post-WWII U.S. data, leads us to the following findings. First, the standard small scale new-Keynesian monetary policy framework is not rejected by the data over the pre-1984 sample. Conditional on this first step, our test leads us not to reject the hypothesis of indeterminacy. Turning to the post-1984 sample, our joint test does not reject the hypothesis that a determinate equilibrium prevailed. In both samples, we find that the long-run coefficients on inflation and the output gap in the monetary policy rule are weakly identified. We then develop an identification-robust indicator of indeterminacy which exploits the GMM estimates of the structural parameters and the identification-robust confidence sets suggested by Stock and Wright (2000). Our indicator supports our main finding, i.e., multiple equilibria likely occurred in the 1970s, but very

Great Moderation mainly points to the time-variation of structural shocks' volatilities.

unlikely after the appointment of Paul Volcker as Federal Reserve's chairman. Overall, our evidence is in line with the story popularized by Clarida, Galí, and Gertler (2000) on the aggressive monetary policy implemented by the Federal Reserve to engineer a deflation in the early 1980s and maintain inflation to low levels afterwards.

To summarize, our paper apply a novel methodology to formally test for indeterminacy under rational expectations. This enables us to take a stand on the scenario better describing the U.S. economy in different samples. Our conclusions are consistent with the 'good policy' interpretation of the U.S. Great Moderation, in that we find formal support for the switch from indeterminacy to uniqueness occurred in the early 1980s. It must be clear, however, that we do not take any stand on the relative importance of the 'good policy' driver as opposed to its main 'competitor', i.e., the 'good luck' explanation pushed by Sims and Zha (2006), Smets and Wouters (2007), and Justiniano and Primiceri (2008), which points toward a reduction in the volatility of the shocks hitting the economy. An elaboration of our proposal aimed at identifying the relative importance of these two drivers of the Great Moderation is left to future research.

The paper is organized as follows. Section 2 introduces the reference small scale structural model and discusses its reduced form solutions under determinacy and indeterminacy, respectively. Section 3 summarizes the two methods - one based on standard inference and the other on an identification-robust indicator of indeterminacy - we apply for detecting determinacy/indeterminacy. Section 4 presents our empirical results obtained on U.S. quarterly data. Section 5 contains some concluding remarks. Additional methodological details are confined in the Appendix.

2 Model

This Section presents the reference small-scale new-Keynesian business cycle model we focus on in this paper, and discusses its solutions under determinacy and indeterminacy, respectively.

2.1 Structural system

The reference model is taken from Benati and Surico (2009). It features the following three equations:

$$\tilde{y}_t = \gamma E_t \tilde{y}_{t+1} + (1-\gamma)\tilde{y}_{t-1} - \delta(R_t - E_t \pi_{t+1}) + \omega_{\tilde{y},t} \tag{1}$$

$$\pi_t = \frac{\beta}{1+\beta\alpha} E_t \pi_{t+1} + \frac{\alpha}{1+\beta\alpha}\pi_{t-1} + \kappa \tilde{y}_t + \omega_{\pi,t} \tag{2}$$

$$R_t = \rho R_{t-1} + (1-\rho)(\varphi_\pi \pi_t + \varphi_{\tilde{y}} \tilde{y}_t) + \omega_{R,t} \tag{3}$$

where

$$\omega_{x,t} = \rho_x \omega_{x,t-1} + \varepsilon_{x,t} \quad , \text{-1}<\rho_x<1 \quad , \varepsilon_{x,t} \sim \mathrm{WN}(0, \sigma_x^2) \quad , \quad x = \tilde{y}, \pi, R. \tag{4}$$

The variables $\tilde{y}_t$, π_t, and R_t stand for the output gap, inflation, and the nominal interest rate, respectively; γ is the weight of the forward-looking component in the intertemporal IS curve; α is price setters' extent of indexation to past inflation; δ is households' intertemporal elasticity of substitution; κ is the slope of the Phillips curve; ρ, φ_π, and $\varphi_{\tilde{y}}$ are the interest rate smoothing coefficient, the long-run coefficient on inflation, and that on the output gap in the monetary policy rule, respectively; finally, $\omega_{\tilde{y},t}$, $\omega_{\pi,t}$, and $\omega_{R,t}$ are the mutually independent, autoregressive of order one (AR(1)) structural disturbances described by eq. (4).

This or similar small-scale models have successfully been employed to conduct empirical analysis concerning the U.S. economy. Clarida et al. (2000) and Lubik and Schorfheide (2004) have investigated the influence of systematic monetary policy over the U.S. macroeconomic dynamics; Boivin and Giannoni (2006), Benati and Surico (2009), and Lubik and Surico (2010) have replicated the U.S. Great Moderation, Benati (2008) and Benati and Surico (2008) have investigated the drivers of the U.S. inflation persistence; Castelnuovo and Surico (2010) have replicated the VAR dynamics conditional on a monetary policy shock in different sub-samples; Inoue and Rossi (2011) have analyzed the role of parameter instabilities as drivers of the Great Moderation. A more general formulation, allowing for positive trend inflation, has been exploited by Coibion and Gorodnichenko (2011) to assess the relevance of variations in trend inflation for the evolution of the U.S. macroeconomic dynamics.

2.2 Reduced form solutions

We compact the system composed by eqs. (1)-(4) in the representation

$$\Gamma_0 X_t = \Gamma_f E_t X_{t+1} + \Gamma_b X_{t-1} + \omega_t \tag{5}$$

$$\omega_t = \Xi \omega_{t-1} + \varepsilon_t \ , \ \ \varepsilon_t \sim \mathrm{WN}(0, \Sigma_\varepsilon) \tag{6}$$

$$\Xi := dg(\rho_{\tilde{y}}, \rho_\pi, \rho_R) \ \ , \ \ \Sigma_\varepsilon := dg(\sigma^2_{\tilde{y}}, \sigma^2_\pi, \sigma^2_R)$$

where X_t:=$(\tilde{y}_t, \pi_t, R_t)'$, ω_t:=$(\omega_{\tilde{y},t}, \omega_{\pi,t}, \omega_{R,t})'$, ε_t:=$(\varepsilon_{\tilde{y},t}, \varepsilon_{\pi,t}, \varepsilon_{R,t})'$ and

$$\Gamma_0 := \begin{bmatrix} 1 & 0 & \delta \\ -\kappa & 1 & 0 \\ -(1-\rho)\varphi_{\tilde{y}} & -(1-\rho)\varphi_\pi & 1 \end{bmatrix}, \Gamma_f := \begin{bmatrix} \gamma & \delta & 0 \\ 0 & \frac{\beta}{1+\beta\alpha} & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Gamma_b := \begin{bmatrix} 1-\gamma & 0 & 0 \\ 0 & \frac{\alpha}{1+\beta\alpha} & 0 \\ 0 & 0 & \rho \end{bmatrix}. \tag{7}$$

Let $\theta := (\gamma, \delta, \beta, \alpha, \kappa, \rho, \varphi_{\tilde{y}}, \varphi_\pi, \rho_{\tilde{y}}, \rho_\pi, \rho_R)'$ be the $m \times 1$ vector of structural parameters, m:=dim(θ). The elements of the matrices Γ_0, Γ_f, Γ_b and Ξ depend nonlinearly on θ. The space of all theoretically admissible values of θ is denoted by $\mathcal{P}$.

A solution to system (5)-(6) is any stochastic process such that $E_t X_{t+1}$ exists and when substituted into the model (1)-(4), the equations are verified at any time, for given initial conditions. A reduced form solution associated with system (5)-(6) is a model, taken from the solution set, such that X_t depends on its own lags, on ε_t and lags of ε_t and, possibly, other arbitrary stochastic variables (sunspot shocks) independent on ε_t. The solution properties depend on whether θ lies in the determinacy or indeterminacy region of $\mathcal{P}$.

The theoretically admissible parameter space $\mathcal{P}$ is decomposed into two disjoint subspaces, the determinacy region, $\mathcal{P}^D$, and its complement $\mathcal{P}^I$:=$\mathcal{P} \backslash \mathcal{P}^D$. Since we consider only stationary (asymptotically stable) solutions of system (5)-(6), $\mathcal{P}^I$ contains only values of θ that lead to multiple stable solutions. It is assumed that $\forall \theta \in \mathcal{P}$ an asymptotically stationary (stable) reduced form solution to system (5)-(6) exists.

Determinacy/indeterminacy is a system property that depends on all structural parameters in θ. There are cases in which part of the elements of θ are suitably restricted and it becomes relatively simple to identify the region $\mathcal{P}^D$ $(\mathcal{P}^I)$ of the parameter space. For instance, if θ is restricted such that γ:=1, α:=0, ρ:=0, system (1)-(4) collapses to a 'purely forward-looking' model and it can be shown that the inequality

$$\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_{\tilde{y}} > 1 \tag{8}$$

is sufficient and 'generically' necessary (Woodford, 2003, Proposition 4.3, p. 254) for determinacy. In this case $\mathcal{P}^D{:=}\{\theta,\, \varphi_\pi + \frac{1-\beta}{\kappa}\varphi_{\tilde{y}} > 1\}$. Mavroeidis (2010) uses the condition in eq. (8) to address the analysis of determinacy/indeterminacy of U.S. monetary policy by estimating a Taylor-type monetary policy rule in isolation from other structural equations, with the risk of dealing with a 'wrong' determinacy condition if the intertemporal IS curve and NKPC are characterized by non-negligible backward-looking components. Aside from these special cases, however, it is generally difficult to derive from system (5)-(6) a set of (closed-form) inequality constraints that are both necessary and sufficient for determinacy (indeterminacy) and that can potentially be used to test whether θ lies in $\mathcal{P}^D$ or $\mathcal{P}^I$.[3]

It can be proved (Binder and Pesaran, 1995; Fanelli, 2011) that if for a given point $\theta \in \mathcal{P}$ the matrix $G(\theta){:=}(\Gamma_0 - \Gamma_f\Phi_1)^{-1}\Gamma_f$ is stable (i.e. all its eigenvalues lie inside the unit circle in the complex plane) and additional technical conditions hold, then system (5)-(6) has a unique stable reduced form that can be represented as the finite order VAR

$$[I_3 - \Phi_1(\theta)L - \Phi_2(\theta)L^2]X_t = \Upsilon(\theta)^{-1}\varepsilon_t \tag{9}$$

where L is the lag/lead operator ($L^h X_t{:=}X_{t-h}$), X_0 and X_{-1} are fixed initial conditions, $\Phi_1(\theta)$, $\Phi_2(\theta)$ and $\Upsilon(\theta)$ are 3×3 matrices whose elements depend nonlinearly on θ and embody the cross-equation restrictions (Hansen and Sargent, 1980, 1981) implied by the new-Keynesian DSGE model, see the Appendix for details. The matrix $\Upsilon(\theta)$ is defined as $\Upsilon(\theta){:=}(\Psi - \Xi\Gamma_f)$, $\Psi{:=}\Psi(\theta){:=}[\Gamma_0 - \Gamma_f\Phi_1(\theta)]$, and the matrices $\Phi_1(\theta)$ and $\Phi_2(\theta)$ solve a quadratic matrix equation involving Γ_0, Γ_f, Γ_b and Ξ.

Conversely, if for a given point $\theta \in \mathcal{P}$ the matrix $G(\theta)$ has $1 \leq q \leq 3$ eigenvalues that lie outside the unit circle in the complex plane, then the reduced form solutions of the

[3] The following example shows that the condition in eq. (8) is not necessary for determinacy if the structural model (1)-(4) involves lags of the variables, other than leads. Consider the system based on $\beta := 0.99$, $\kappa := 0.085$, $\delta := 0.40$, $\gamma := 0.25$, $\alpha := 0.05$, $\rho := 0.95$, $\varphi_{\tilde{y}} := 2$, $\varphi_\pi := 0.77$, $\rho_{\tilde{y}} := \rho_\pi := \rho_R := 0.9$. In this case $\varphi_\pi + \frac{1-\beta}{\kappa}\varphi_{\tilde{y}} > 1$, but the rational expectation-solution to system (1)-(4), while being stable, is not unique. Recall that we assume the existence of at least a solution under rational expectations.

system can be chosen from the class of 'VARMA(3,1) + VMA(1)' processes:

$$[I_3 - \Pi(\theta)L][I_3 - \Phi_1(\theta)L - \Phi_2(\theta)L^2]X_t = [M(\theta,\mu) - \Pi(\theta)L]\Psi(\theta,\mu)\varepsilon_t + \tau_t \quad (10)$$

$$\tau_t := [M(\theta,\mu) - \Pi(\theta)L]\Psi^{-1}\Xi\Gamma_f V(\theta,\mu)P(\theta)\zeta_t + P(\theta)\zeta_t. \quad (11)$$

In system (10)-(11) the matrices $\Phi_1(\theta)$ and $\Phi_1(\theta)$ are defined as in system (9), $\Pi(\theta)$ and $P(\theta)$ are 3×3 matrices whose elements depend nonlinearly on θ, and $M(\theta,\mu)$, $\Upsilon(\theta,\mu)$ and $V(\theta,\mu)$ are a 3×3 matrices whose elements depend nonlinearly on θ and on the vector μ which may contain up to q^2 arbitrary auxiliary (nuisance) parameters unrelated to θ, where q measures the 'degree of parametric multiplicity' of solutions; finally, the additional moving average term τ_t depends on the 'sunspot shock' ζ_t which is is a 3×1 martingale difference sequence (MDS), $E_t\zeta_{t+1} = 0_{3\times1}$, independent on the fundamental disturbance u_t, with covariance matrix Σ_ζ. Precise details about the structure of the matrices $\Pi(\theta)$, $P(\theta)$, $M(\theta,\mu)$, $\Psi(\theta,\mu)$ and $V(\theta,\mu)$ and on identification issues may be found in Fanelli (2011).

While the determinate equilibrium in eq. (9) depends only on the state variables of the structural system (5)-(6), there are two sources of indeterminacy that characterize the model equilibria in eq. (10). First, the 'parametric indeterminacy' that stems from the presence of the auxiliary parameters in the vector μ: these parameters index solution multiplicity and are not identifiable under determinacy. Secondly, the 'stochastic indeterminacy' that stems from the presence of the sunspot shocks ζ_t; these shocks may arbitrarily alter the dynamics and volatility of the system induced by the fundamental disturbances, see Lubik and Schorfheide (2003, 2004) and Lubik and Surico (2009) for discussions. Interestingly, indeterminacy occurs also if the sunspot shocks are absent, i.e. if $\Sigma_\zeta{:=}0_{3\times3}$ ($\zeta_t{:=}0$ a.s.$\forall t$) implying $\tau_t{:=}0$ a.s. in eq. (11) because of the presence of the auxiliary parameters μ. Lubik and Schorfheide (2004) denote a situation like this as 'indeterminacy without sunspots'.

The reduced form solutions in eq. (9) and eqs. (10)-(11) turn out to be observationally equivalent when in system (10)-(11) the following conditions jointly hold

$$\mu := I_q \quad \Rightarrow M(\theta,\mu) := I_3 \quad (12)$$

$$\tau_t := 0_{3\times1} \text{ a.s. } \forall\, t \quad (\Sigma_\zeta{:=}0_{3\times3}). \quad (13)$$

While equation (12) posits that the matrix of auxiliary parameters is equal to the identity matrix, eq. (13) requires the absence of sunspot shocks in the indeterminate equilibria. Subject to (12)-(13), system (10)-(11) collapses to a Minimum State Variable (MSV) solution that has the same representation as the determinate reduced form in eq. (9). Even admitting the absence of sunspot shocks *a priori*, it must be noticed that the condition in eq. (12) occurs with zero-Lebesgue measure in the space of auxiliary parameters μ, henceforth denoted with the symbol $\mathcal{U}$. This observational equivalence has consequences on the statistical properties of our proposed test for the hypothesis of determinacy, see the next Section.[4]

From our derivation of the reduced form solutions it follows that, provided a set of additional conditions hold, the eigenvalues of the matrix $G(\theta)$ play a crucial role in governing the determinacy/indeterminacy of the system. A useful broad characterization of the determinacy region of the parameter space is therefore given by $\mathcal{P}^D{:=}\{\theta,\ \lambda_{\max}[G(\theta)] < 1\}$, where $\lambda_{\max}[\cdot]$ denotes the largest eigenvalue in absolute value of the matrix in the argument. Similarly, $\mathcal{P}^I{:=}\{\theta,\ \lambda_{\max}[G(\theta)] > 1\}$.[5]

3 Detecting determinacy/indeterminacy

In this Section we first present our inferential method for detecting determinacy against indeterminacy in the reference new-Keynesian system (1)-(4) (Section 3.1). Then, we discuss a simple indicator of determinacy/indeterminacy that can be applied to complement the results of the test when the researcher suspects that some of the structural parameters of the system might be 'weakly identified'.

[4] Observational equivalence between determinate and indeterminate reduced form solutions may be also obtained from system (5) when the vector of structural shocks is absent, i.e. when $\Sigma_\varepsilon{:=}0_{3\times 3}$ ($\varepsilon_t := 0_{3\times 1}$ a.s. $\forall\ t$). In this case, indeed, under a set of restrictions (including $\Xi := 0_{n\times n}$), the structural model can be solved as in eq. (9); see Beyer and Farmer (2007) for a comprehensive discussion. While being interesting from a theoretical standpoint, the case of absence of fundamental disturbances in the structural equations is empirically unpalatable, and will not be considered in our analysis.

[5] The case $\lambda_{\max}[G(\theta)] := 1$ is deliberately ignored because it is associated with non-stationary processes.

3.1 Test

Our inferential approach is based on a simple intuition: provided the structural new-Keynesian model (1)-(4) is correctly specified, it is straightforward to apply standard diagnostic tools to assess whether its reduced form solution obtained under determinacy, given by the econometric model in eq. (9), is supported (determinacy) or rejected (indeterminacy) by the data. Thus, conditional upon the non-rejection of the structural model, if we find that a stationary VAR for $X_t := (\tilde{y}_t, \pi_t, R_t)'$ with two lags, subject to the cross-equation restrictions implied by system (1)-(4) is not rejected by the data, the empirical evidence favours determinacy; otherwise, the data would favour indeterminacy.

This simple argument breaks down when the structural model omits important propagation mechanisms (lags) and is not able to fully address the time series features of the data, i.e., under model misspecification. In these situations, the attempt to determine if the model equilibrium is determinate or indeterminate is likely to lead to misleading conclusions. Therefore, the assessment of the data adequacy of the structural model before one investigates its determinacy/indeterminacy plays a crucial role.

The above considerations inspire the following two-step approach:

Step 1 We estimate θ directly from system (1)-(4) by using a GMM estimator which is robust to determinacy/indeterminacy. Then, we test the data adequacy of the structural model (model misspecification). If the model is rejected, we search for an alternative - possibly dynamically augmented - specification. Otherwise, we move to step 2.

Step 2 If the model estimated in Step 1 is not rejected, we estimate the reduced form VAR in eq. (9) with maximum likelihood (ML) under the cross-equation restrictions implied by rational expectations, and test their validity with a Lagrange multiplier (LM) test. If the cross-equation restrictions are not rejected, the joint evidence favours the hypothesis of determinacy. Otherwise, the joint evidence supports indeterminacy.

As for Step 1, if X_t involves only observable variables (as it can be the case with the class of small-scale new-Keynesian monetary models we deal with)[6], the 'natural' esti-

[6] The output gap, which is included among our observables, is computed by considering the level of potential output as estimated by the Congressional Budget Office. The list of observables exploited to estimate our models is presented in Section 4.

mator of θ is obtained by minimizing the GMM criterion function $Q_{con,T}(\theta)$ (defined in the Appendix); GMM is robust to determinacy/indeterminacy under the maintained assumptions of correct specification and suitably selected instruments (Wickens, 1982; West, 1986; Mavroeidis, 2005; Fanelli, 2011). Accordingly, Hansen's overidentification restrictions test ('J-test') can potentially be used to assess the data adequacy of the structural equations. A well known shortcoming of this test is its finite-sample lack of power against certain types of misspecification (Hall, 2005). However, the researcher's attention may be confined to the case of possible omission of lags in the structural equations and in this case, Mavroeidis (2005) and Jondeau and Le Bihan (2008) have shown that the power of the overidentification restrictions test increases if a limited set of (relevant) instruments is used in conjunction with a parametric estimator of the weighting matrix that accounts for the moving average structure of model disturbances.[7] Another shortcoming of GMM-based inference in system (1)-(4), shared by other estimation methods, is related to the possible occurrence of weakly identified parameters, i.e., parameters with respect to which the criterion $Q_{con,T}(\theta)$ exhibits little curvature. We postpone a discussion on this issue to Section 3.3.

As for Step 2, we estimate θ from the constrained VAR in eq. (9) by ML. Following Hansen and Sargent (1980, 1981), we then test the implied set of cross-equation restrictions via a LM (henceforth LM-CER) test. The mechanics of the ML estimation algorithm and that of the resulting LM-CER test are summarized in our Appendix. The cross-equation restrictions tested on the determinate reduced form solution should not be rejected by the data under determinacy and rejected under indeterminacy (Fanelli, 2010).

By combining these two steps, the overall assessment of the determinacy/indeterminacy (or misspecification) of the new-Keynesian model can be based on the joint test resulting from the sequence: J-test (Step 1) & LM-CER (Step 2). In the next sub-Section we summarize the main properties of this test and show how it can be used in practice; additional technical features are confined in the Appendix.

[7]It is well known that, in finite samples, the power of the J-test may be affected by the type of Heteroscedasticity Autocorrelation Covariance (HAC) estimator used for the weight matrix to account for serial correlation and possible heteroscedasticity in the GMM residuals. Different HAC estimators, albeit asymptotically equivalent, can differ substantially in finite samples, thus imparting substantial distortions to GMM-based inference.

3.2 Properties

Let H_0 be the null hypothesis: '*the reduced form solution of the structural new-Keynesian model* is *given by system (9)*' ($\theta_0 \in \mathcal{P}^D$) and H_1 the alternative: '*the reduced form solution of the structural new-Keynesian model is given by system (10)-(11)*' ($\theta_0 \in \mathcal{P}^I$); both hypotheses are considered under the maintained assumption that the small new-Keynesian DSGE model (1)-(4) is correctly specified.

We denote by J_T the over-identification restriction test statistics obtained from the direct (joint) estimation of the equations of the structural model by GMM (see eq. (24) in the Appendix), and by LM_T the LM-CER test statistic for the cross-equation restrictions that arise under determinacy (see eq. (28) in the Appendix).

From the robustness of the GMM estimator of θ to determinacy/indeterminacy, we have that

$$J_T \longrightarrow_D \chi^2(a_1) \quad , \forall\, \theta \in \mathcal{P} \tag{14}$$

where a_1:=$3r - m$, r is the number of instruments used in estimation, and '$\longrightarrow_D$' denotes converge in distribution for large T. Let $P_\infty^J[\cdot]$ be the probability taken from the asymptotic distribution of J_T; a consequence of eq. (14) is that

$$P_\infty^J[J_T \geq c_{a_1}(\eta_1)] = \eta_1 \ , \forall\, \theta \in \mathcal{P}^D \quad ; \quad P_\infty^J[J_T \geq c_{a_1}(\eta_1)] = \eta_1 \quad , \ \forall \theta \in \mathcal{P}^I$$

where $c_{a_1}(\eta_1)$ is the 100(1-η_1) percentile of the asymptotic distribution of the J_T statistics. On the other hand,

$$LM_T \longrightarrow_D \chi^2(a_2) \ , \ \ \forall\, \theta \in \mathcal{P}^D \tag{15}$$

where a_2:=$18 - m$, while

$$LM_T \sim O_p(T) \ , \ \begin{cases} \forall\, \theta \in \mathcal{P}^I \quad , \forall \mu \in \mathcal{U}^* \text{ if } \Sigma_\varsigma{:=}0_{3\times 3} \\ \forall\, \theta \in \mathcal{P}^I \quad , \forall \mu \in \mathcal{U} \ \text{ if } \Sigma_\varsigma \neq 0_{3\times 3} \end{cases} \tag{16}$$

where $\mathcal{U}^*$:=$\mathcal{U}\backslash \{vec(I_q)\}$. The result in eq. (16) is motivated by the fact that the reduced form solution in system (9) (from which the ML estimation of θ is carried out and the LM-CER test is computed) is misspecified when the data generating process belongs to the class of reduced form solutions described by system (10); moreover, as observed in sub-Section 2.2, when both the conditions in eqs. (12)-(13) hold, system (10)-(11) collapses to a MSV solution that is observationally equivalent to the determinate reduced form solution with the consequence that the LM-CER test is not able to reject the null of determinacy if the data are generated under indeterminacy in the special case $\mu := vec(I_q)$ and, in addition, sunpost shocks are absent. Denoting with $P_{\infty}^{LM}[\cdot]$ the probability taken from the asymptotic distribution of the LM_T test, from eqs. (15)-(16) we derive that

$$\begin{aligned} P_{\infty}^{LM}[LM_T &\geq c_{a_2}(\eta_2)] = \eta_2 \quad , \ \forall\, \theta \in \mathcal{P}^D \ ; \\ P_{\infty}^{LM}[LM_T &\geq c_{a_2}(\eta_2)] = 1 \quad , \left\{ \begin{array}{l} \forall\, \theta \in \mathcal{P}^I \quad , \forall \mu \in \mathcal{U}^* \text{ if } \Sigma_\zeta := 0_{3\times 3} \\ \forall\, \theta \in \mathcal{P}^I \quad , \forall \mu \in \mathcal{U} \ \text{ if } \Sigma_\zeta \neq 0_{3\times 3} \end{array} \right. . \end{aligned}$$

Finally, let $P_{\infty}^{J,LM}[\cdot,\cdot]$ be probability taken from the joint distribution of the J_T and LM_T test statistics. Given these definitions, the asymptotic type I error of the joint test can be specified as

$$\eta := \Pr(\text{ rejecting } H_0 \mid H_0) := P_{\infty}^{J}[J_T \geq c_{a_1}(\eta_1)]$$

$$+ P^{J,LM}[J_T < c_{a_1}(\eta_1) \ , \ LM_T \geq c_{a_2}(\eta_2)] \quad , \quad \forall\, \theta \in \mathcal{P}^D. \tag{17}$$

Eq. (17) formalizes the idea that the rejection of H_0 may occur either indirectly through the rejection of the structural model by the overidentification restrictions test (Step 1), or directly by the LM test for the cross-equation restrictions that arise under determinacy (Step2) when a positive assessment of the structural model by the overidentification restrictions test is obtained. It can be proved that the asymptotic type I error in eq. (17) is bounded by the asymptotic type I errors pre-specified for the two tests, i.e. that (Fanelli, 2010)

$$\eta \leq \eta_1 + \eta_2 \quad , \forall\, \theta \in \mathcal{P}^D. \tag{18}$$

The result in eq. (18) ensures that fixed the levels of significance of the J-test (η_1) and LM-CER test (η_2), respectively, the type I error of the joint test is under control in large samples. On the other hand, if the overall significance level η is pre-fixed before running the test, it is sufficient to set the levels of significance of the individual tests such that the condition in eq. (18) is respected; for instance, a reasonable choice is η:=0.05 and η_1:=0.025=:η_2. In practice, the J-test & LM-CER procedure works as follows: given η (e.g. 5%) and η_1 and η_2 (e.g. 2.5%), if the p-value associated with the J-test is less than η_1 it is necessary to look for an alternative structural specification (since the small DSGE model is not supported by the data); if the p-value associated with the J-test is greater than η_1 one looks at the LM-CER test and selects determinacy (indeterminacy) if the p-value of this test is greater (less) than η_2. As a matter of fact, the computations required to condict out analysis are straightforward. The J-test has become a standard diagnostic for models estimated by GMM and is routinely calculated in most computer packages. Similarly, the LM-CER test can be implemented with any econometric package featuring the maximization of an objective function under nonlinear parametric constraints.

As regards the asymptotic power of the joint test against the alternative of indeterminacy, H_1, one has

$$\eta^p_{ind}{:=}\,\Pr(\text{rejecting } H_0 \mid H_1){:=}P^J_\infty[J_T \geq c_{a_1}(\eta_1)]{+}P^{J,LM}_\infty[J_T < c_{a_1}(\eta_1),\ LM_T \geq c_{a_2}(\eta_2)] \quad , \forall\ \theta \in \mathcal{P}^I \tag{19}$$

because under indeterminacy the null of determinacy can be rejected either because the J-test incorrectly rejects the structural model, or because the LM-CER test correctly rejects the cross-equation restrictions when the J-test validates the structural model. It can be proved that (Fanelli, 2010)

$$\eta^p_{ind}{:=}1 \quad , \quad \begin{cases} \forall\ \theta \in \mathcal{P}^I \quad , \forall \mu \in \mathcal{U}^* \text{ if } \Sigma_\zeta{:=}0_{3\times 3} \\ \forall\ \theta \in \mathcal{P}^I \quad , \forall \mu \in \mathcal{U} \ \text{ if } \Sigma_\zeta \neq 0_{3\times 3} \end{cases}$$

i.e. that the test is consistent 'almost everywhere' in the space of auxiliary parameters if sunpost shocks are absent, and is consistent irrespective of the values assumed by the auxilairy parameters when the indeterminate equilibria feature sunspot shocks.

Notably, since the J-test is consistent against the omission of relevant lags in the specified structural equations (Hall, 2005), the joint test is consistent as well, namely

$$\eta^p_{mis}{:=}\,\Pr(\text{rejecting } H_0 \mid \text{lag-augmented version of system (1)-(4)}) = 1 \ ,$$

see Fanelli (2010) for details. The consistency of our test against the omission of propagation mechnisms in the structural system implies that the risk of incorrectly selecting indeterminacy in misspecified models is under control in large samples.

The advantages of the proposed method are, besides its computational simplicity, that (i) inference is of standard type because no nuisance parameter appears under the null of determinacy;[8] (ii) it is not necessary to identify the parametric inequality restrictions that define the parametric regions $\mathcal{P}^D$ ($\mathcal{P}^I$), with the advantage of circumventing the use

[8] Given our economic representation, even in the absence of sunspot shocks ($\Sigma_\zeta{:=}0_{3\times 3}$ in eq. (11)), a classical likelihood-based test would call for the comparison between the VAR(2) in eq. (9) and the

of nonstandard asymptotic inference; (iii) it is not necessary to specify prior distributions for θ and, notably, for the auxiliary parameters μ governing solution multiplicity. Thus, compared to Lubik and Schorfheide's (2004) approach, the researcher is exempted from choosing a prior distribution for the arbitrary auxiliary parameters μ that index solution multiplicity. As a matter of fact, the use of informative priors for these nuisance parameters is due to the need to simplify computations, more than to a (quite unlikely) prior knowledge about the type of indeterminacy governing the system. With respect to Boivin and Giannoni (2006), our method is based on the direct estimation of the Euler equations in the system (1)-(4). Hence, we need not minimize the distance between some selected impulse responses taken from a VAR for X_t and the structural model-based responses, a strategy which is unfortunately bias-inducing as for expectations-based models like ours (Canova and Sala, 2009). More importantly, we need not assume anything on the solution under indeterminacy, as opposed to the MSV solution assumed by Boivin and Giannoni (2006): while being plausible, such solution is anyhow arbitrary, and may importantly affect the simulated moments of interest (Castelnuovo, 2010). Compared to Mavroeidis (2010), who applies identification-robust methods to investigate the determinacy/indeterminacy of U.S. monetary policy conditional on the estimation of the policy rule in isolation, our full-system analysis other than being based on a 'hybrid' model, provides the estimates of the economic structure as a whole, which we then exploit to assess the determinacy/indeterminacy case.

3.3 Determinacy/indeterminacy indicator

The GMM estimates of the structural parameters θ can be also used to quantify the relative importance of the region $\mathcal{P}^I$ ($\mathcal{P}^D$) of the parameter space such that the possibility of weakly identified parameters is explicitly taken into account. This Section presents an indicator that can be used to evaluate the extend of indeterminacy (determinacy) within a suitably chosen identification-robust confidence set for the parameters of interest.

As detailed in the previous sub-Section, under standard regularity conditions, the J-test is asymptotically $\chi^2(3r-m)$-distributed. There exists little empirical evidence

VARMA(3,1) system in eq. (10). As is known, this would clearly be a non-standard inferential problem involving nuisance parameters, see e.g. Lubik and Schorfheide (2004) and Fanelli (2010). The advantage of our method, along this dimension, is that nonstandard distributions are not required and, notably, the asymptotic type I error of the joint test is under control.

about the finite sample performance of the J_T statistics in presence of weakly identified parameters, especially in the context of new-Keynesian models. It is reasonable to expect, however, that the discrepancy between the empirical size and the nominal size of the test be negatively affected by this phenomenon, see e.g. Hall (2005, Ch. 6). Likewise, the likelihood function of the determinate VAR reduced form might be nearly uninformative ('flat') along several dimensions of the parameter space and this might have consequences on the finite sample performance of our test.

We decompose θ as $\theta = (\theta_{id}', \theta_{un}')'$, where θ_{id} is $m_{id} \times 1$ and contains the structural parameters of the new-Keynesian system which are thought of being 'strongly identified'; θ_{un} is $m_{un} \times 1$; $m_{un}{:=}m - m_{id}$, and contains the structural parameters which are thought of being weakly identified; $\theta_{0,id}$ and $\theta_{0,un}$ are the 'true' values of θ_{id} and θ_{un}, respectively. The available empirical evidence about system (1)-(4) suggests that in samples typically available to macroeconomists, variations in α, κ, $\varphi_{\tilde{y}}$ and φ_π tend to be associated with almost negligible changes in the estimation objective function. Hence, these parameters are potential candidates for being grouped into the sub-vector θ_{un}.[9] Following Stock and Wright (2000), identification-robust, asymptotically 'valid', confidence sets for $\theta_{0,un}$ can be based on the restricted GMM estimation in which the minimization of the objective function is performed over θ_{id}, conditional on fixed values of θ_{un}. More precisely, let $\hat{\theta}_{id,T}(\bar{\theta}_{un})$ denote the GMM estimator of θ_{id} conditional on the choice $\theta_{un}{:=}\bar{\theta}_{un}$. It can be proved that under correct specification the objective function evaluated at the point $(\hat{\theta}_{id,T}'(\theta_{0,un}), \theta_{0,un}')'$, $TQ_{con,T}(\hat{\theta}_{id,T}(\theta_{0,un}), \theta_{0,un})$, is asymptotically $\chi^2(3r - m_{id})$-distributed, hence an asymptotically valid $100(1-\eta)\%$ confidence set for $\theta_{0,un}$ is given by

$$\mathcal{C}_{1-\eta}(\theta_{0,un}) := \left\{ \theta_{un} \ , \ TQ_{con,T}(\hat{\theta}_{id,T}(\theta_{un}), \theta_{un}) < c_{3r-m_{id}}(\eta) \right\} \tag{20}$$

where $c_{3r-m_{id}}(\eta)$ is the 100(1-η) percentile of the $\chi^2(3r - m_{id})$ distribution.

Fixed η and obtained the confidence set $\mathcal{C}_{1-\eta}(\theta_{0,un})$ in Eq. (20), we consider the following algorithm. Given the GMM estimate $\hat{\theta}_T{:=}(\hat{\theta}_{id,T}', \hat{\theta}_{un,T}')'$ obtained with the available sample, we fix the strongly identified parameters θ_{id} at $\theta_{id}{:=}\hat{\theta}_{id,T}$ and then consider a (sufficiently fine) grid of points for the elements of θ_{un}, denoted $\mathcal{G}(\theta_{un})$; for each point $\theta_{un}{:=}\breve{\theta}_{un} \in (\mathcal{C}_{1-\eta}(\theta_{0,un}) \cap \mathcal{G}(\theta_{un}))$, we define the vector $\theta^*{:=}(\hat{\theta}_{id,T}', \breve{\theta}_{un}')'$ and check whether

[9] Given the specific purposes of this paper, the parameters α and κ of the NKPC have been fixed in the empirical analysis of section 4. Hence, only the policy parameters $\varphi_{\tilde{y}}$ and φ_π will be explicitly treated as weakly identified, i.e. $\theta_{un} := (\varphi_{\tilde{y}}, \varphi_\pi)'$.

$\lambda_{\max}[G(\theta^*)]>1$ (indeterminacy) ($\lambda_{\max}[G(\theta^*)] < 1$, indeterminacy), see Section 2. With this procedure we are able to quantify the fraction of points within the identification-robust confidence set $\mathcal{C}_{1-\eta}(\theta_{0,un})$ that belong to the indeterminacy region $\mathcal{P}^I$ (determinacy region $\mathcal{P}^D$) of the parameter space, conditional on fixing the strongly identified parameters at their point GMM estimates.

The resulting indicator, denoted In, varies by construction between 0 (the set $\mathcal{C}_{1-\eta}(\theta_{0,un})$ does not contain points that lead to indeterminacy) and 1 (all points in $\mathcal{C}_{1-\eta}(\theta_{0,un})$ lie in the indeterminacy region of the parameter space). As shown in the next Section, we argue that this indicator of indeterminacy (determinacy) can fruitfully be used to compare the empirical evidence obtained on different samples and to support the outcomes of the test described in sub-Sections 3.1-3.2.

4 Empirical evidence

We employ U.S. quarterly data, sample 1954q3-2008q3. The end of the sample is justified by our intention to avoid dealing with the non-standard 'zero-lower bound' phase began in December 2008, which triggered a series of non-standard policy moves by the Federal Reserve. We employ three observable variables, $X_t:=(\tilde{y}_t, \pi_t, R_t)'$. The output gap $\tilde{y}_t$ is computed as percent log-deviation of the real GDP with respect to the potential output estimated by the Congressional Budget Office. The inflation rate π_t is the quarterly growth rate of the GDP deflator. For the short-term nominal interest rate R_t we consider the effective Federal funds rate expressed in quarterly terms (averages of monthly values). The source of the data is the Federal Reserve Bank of St. Louis' web site.

Following most of the literature on the Great Moderation, we divide the post-WWII U.S. era in two periods, roughly corresponding to the 'Great Inflation' and the 'Great Moderation' samples. We take the advent of Paul Volcker as Chairman of the Federal Reserve to identify our first sub-sample, i.e., 1954q3-1979q2. As for the Great Moderation, we consider the sample 1985q1-2008q3. Our choice is due to the 'credibility build-up' undertaken by the Federal Reserve in the early 1980s, a period during which private agents gradually changed their view on the Fed's ability to deliver low inflation (Goodfriend and King, 2005). Moreover, the first years of Volcker's tenure (until October 1982) were characterized by non-borrowed reserves targeting. Hence, one can hardly expect a good fit of conventional policy rules within this period (Estrella and Fuhrer, 2003; Mavroeidis, 2010), a fact which would carry consequences on the estimates of all parameters of the system.[10]

We first document the estimation of the structural model (1)-(4). Then, we discuss the results of the test of determinacy and the indicator of indeterminacy introduced in the previous Section.

4.1 Estimation and testing results

Classical estimation procedures often lead to unreliable or 'absurd' values as for a subset of the structural parameters characterizing the reference new-Keynesian DSGE model, a typical feature of rational expectation models being nonlinear in the parameters. Our case represents no exception. Accordingly, after some experimentation we fixed a subset of them to plausible values borrowed from the literature to reduce this phenomenon, as well as the extent of weak identification. The discount factor β is set to 0.99, a standard calibration at quarterly frequencies; the intertemporal elasticity of substitution, the slope of the New Keynesian Phillips curve, and the extent of firms' indexation to past prices are calibrated by borrowing the posterior medians in Benati and Surico (2009), i.e., δ=0.125, κ:=0.05, α:=0.05. Given these calibrations, the vector of the 'free' structural parameters $\theta^f:=(\gamma,\rho,\varphi_{\tilde{y}},\varphi_{\pi},\rho_{\tilde{y}},\rho_{\pi},\rho_R)'$ has been estimated by using both GMM and ML under determinacy (some details are summarized in the Appendix).

Our estimates are reported in Table 1.[11] We analyze the results concerning the Great Inflation sample first. Our points estimates turn out to be quite similar to those in a variety of contributions in the literature. In particular, we find the weight of forward looking expectations in the IS curve to be about 0.50, a value supporting the role of habit formation in influencing households' consumption decisions; a fair amount of policy rate smoothing by the Federal Reserve; persistent inflation and policy rate shocks; and a statistically significant policy reaction to inflation. As for this last parameter, our estimates agree with Clarida, Galí, and Gertler (2000) and most of the following contributions, in that (i) the reaction is significant; (ii) it suggests a weak policy reaction, which does not

[10] Our results, however, are robust to the employment of a shorter Great Inflation sample (1966q1-1979q2) and, with qualifications, to a longer Great Moderation sample (1979q4-2008q3). The results obtained on these samples are available upon request to the authors.

[11] Estimation has been carried out over each sub-period of Table 1, taking initial values within each regime and not from the previous regime, see Boivin and Giannoni (2006) for a similar choice

meet the Taylor principle; (iii) a substantial amount of uncertainty surrounds it.

Clearly, relying exclusively on our point estimate of φ_π would not lead us to any reliable conclusion on the presence of multiple equilibria in the 1970s because of the uncertainty surrounding it, and because of the uncertainty surrounding the remaining structural parameters which jointly determine indeterminacy/uniqueness. Differently, our formal test takes, by construction, such uncertainty into account. The result of our testing strategy, which relies in the first step on the J-test, suggests that the small scale model at hand is (statistically) suited to describe our dataset (p-value of 0.31). At first, this result may appear surprising, in light of the popularity of larger scale DSGE models like Smets and Wouters (2007) and a variety of its extensions. While such larger scale models are obviously superior in terms of their ability to describe and predict a set of variables which are left unexplained here (e.g. consumption, investments, and real wages, among others), their superiority as for the variables we focus on in this paper is still to be established. Intriguingly, a recent paper by Herbst and Schorfheide (2011) shows that a small-scale model like ours performs at least as well as Smets and Wouters' (2007) as far as a variety of forecasting exercises is concerned.[12]

Conditional on our model not being misspecified in the sample under scrutiny, we move a step forward and investigate the indeterminacy issue. We estimate the VAR representation in eq. (25) of the new-Keynesian structural model obtained under determinacy with ML, and test the implied set of cross-equation restrictions. Our LM-CER test speaks loud in favor of indeterminacy because the cross-equation restrictions that would arise under determinacy are strongly rejected from the VAR representation of the data (p-value of 0.00). This is possibly the reason why some awkward results arise when taking the model to the data with the VAR-based (and not VARMA-based) likelihood approach, first and foremost the absence of a significant reaction to inflation in the policy rule. Furthermore, the differences between the GMM and ML estimates of the structural parameters reflect

[12] More precisely, Herbst and Schorfheide (2011) contrast a small-scale new-Keynesian AD/AS model with a larger scale model à la Smets and Wouters (2007) in the context of forecasting exercises regarding the U.S. GDP growth rate, inflation, and the federal funds rate during the U.S. Great Moderation. They show that the two models attain very similar root-mean-squared errors. However, the Smets-Wouters model does not lead to a uniform improvement in the quality of the density forecasts and prediction of comovements. In particular, the predictive density for output appears to be poorly calibrated. Moreover, the small-scale model performs better in terms of predicting the sign of the deviations of output growth and inflation with respect to their targets.

the possible misspecification of the VAR representation of the data in eq. (25), along the lines documented in Jondeau and Le Bihan (2008).

Dramatically different results arise when scrutinizing the Great Moderation sample. Our GMM point estimate of the Taylor parameter φ_π is clearly larger than one and significant at standard confidence levels (even if somewhat imprecisely estimated). We also find a slightly lower γ in the IS curve, and a larger degree of policy rate smoothing, this latter result being in line with what found in Mavroeidis (2010). Again, we find shocks to inflation and the policy rate to be autocorrelated. More importantly for our purposes, the J-test does not reject our model specification at conventional significance levels (p-value 0.09).[13] Notably, differently from what obtained on the Great Inflation sample, our LM-CER test for the restricted VAR representation of the data in eq. (9) has a p-value of 0.04; recalling the discussion entertained in sub-Section 3.1 on how to interpret the result of the joint test based on the 'J-test & LM-CER' sequence, our empirical evidence suggests that the null of determinacy is *not* rejected at the overall significance level of 5% (η:=0.05). We record some differences between our GMM and our ML estimates. However, they do not exceed the statistical discrepancies that can be expected in a correctly specified rational expectations model, see West (1986) and Jondeau and Le Bihan (2008). Hence, our results point to an aggressive reaction to inflation and a high degree of interest rate smoothing during the Great Moderation, a policy conduct which - conditional on the economic structure in place - possibly induced a unique equilibrium.

Our conclusions point towards a policy switch in the late 1970s. This result is not new in this literature, as it corroborates the one proposed by Clarida, Galí, and Gertler (2000), Lubik and Schorfheide (2004), Boivin and Giannoni (2006), Benati and Surico (2009), Mavroeidis (2010), and Inoue and Rossi (2011), among others. Importantly, however, this results is obtained in this paper with formal tests on (i) model misspecification, a crucial information when conducting inference, and on (ii) determinacy vs. indeterminacy in a full-system context, without appealing to any a-priori distribution or any calibration of nuisance parameters. Clearly, our prior-free approach maximizes the role attached to the data in determining our results.

[13] The p-value of the J-test conditional on the longer sample 1979q4-2008q2 is even larger, i.e., 0.19.

4.2 Indicator of indeterminacy

As previously discussed, weak-identification might invalidate our testing procedure. The solidity of our results is then checked by appealing to the identification-robust indicator of indeterminacy presented in our sub-Section 3.3.

We proceed with our analysis by constructing, as it is standard in the literature on identification-robust methods, a 90% (η:=0.10) conservative confidence sets for the policy parameters $\varphi_{\tilde{y}}$ and φ_{π}, along the lines discussed in Section 3.3, see eq. (20). We disentangle θ^f as $\theta^f = (\theta_{id}^{f\prime}, \theta_{un}^{f\prime})'$, where θ_{un}^{f}:=$(\varphi_{\tilde{y}}$,$\varphi_{\pi})'$ and θ_{id}^{f}:=$(\gamma, \rho, \rho_{\tilde{y}}, \rho_{\pi}, \rho_R)'$ and construct a grid $\mathcal{G}(\theta_{un}^{f})$ for the weakly identified parameters by letting $\varphi_{\tilde{y}}$ and φ_{π} vary in the intervals 0.05 - 2 and 0.05 - 5.5, respectively. For each sub-sample (pre-Volcker, and post-1984), we collect all points $\breve{\theta}_{un}^{f}$:=$(\breve{\varphi}_{\tilde{y}}$,$\breve{\varphi}_{\pi})' \in \mathcal{G}(\theta_{un}^{f})$ that are not rejected by the $TQ_{con,T}(\hat{\theta}_{id,T}(\breve{\theta}_{un}^{f}), \breve{\theta}_{un}^{f})$ criterion at the 10% level of significance in the identification-robust confidence set $\mathcal{C}_{0.90}(\theta_{0,un}^{f})$. The set $\mathcal{C}_{0.90}(\theta_{0,un}^{f})$ is then projected out (Dufour, 1997) obtaining the identification-robust confidence intervals for $\varphi_{\tilde{y}}$ and φ_{π} reported in Table 2. For comparison purposes, Table 2 also reports the 'conventional' Wald-type 90% confidence intervals for $\varphi_{\tilde{y}}$ and φ_{π}.

It is well known that 'conventional' Wald-type confidence sets tend to look smaller than their identification-robust counterparts when inference is affected by weak identification (see Mavroeidis, 2010 and references therein for details); this phenomenon seems to characterize our projected confidence intervals. We notice that $\varphi_{\tilde{y}}$ and φ_{π} are very imprecisely estimated in the pre-Volcker sample (the identification-robust 90% confidence set $\mathcal{C}_{0.90}(\theta_{0,un}^{f})$ tends to coincide with the whole grid $\mathcal{G}(\theta_{un}^{f})$). Instead, comparatively more informative confidence intervals for the long run responses to output gap and inflation are obtained on the post-1984 sample (despite these intervals remaining still quite 'large'). This result resembles the one in Mavroeidis (2010), who find that, when focusing on the Greenspan tenure (a sample fairly similar to our 'Great Moderation' one), the weak identification issue is less severe than the one arising when also involving data from the 1979 to 1986 in the analysis. Mavroeidis points out that this result can be explained, to a large extent, by the large fall in the variability of the monetary policy shock occurring since the mid-1980s. Some evidence on the 1979q2-2008q3 sample, not reported to save space, confirms that the inclusion of the early 1980s renders the identification issue more severe.[14]

We finally compute the indicator of indeterminacy discussed in Section 3.3. After fixing $\theta^f_{id}:=\hat{\theta}^f_{id,T}$ at the CU-GMM estimates presented in Table 1, for each point $\breve{\theta}^f_{un} \in \mathcal{C}_{0.90}(\theta^f_{0,un})$ we check whether $\lambda_{\max}(G(\theta^*)) > 1$ (indeterminacy), where $\theta^*:=(\breve{\theta}^{f\prime}_{un}, \hat{\theta}^f_{id,T})'$, and obtain the indicator In. Then, we re-compute the value of our In indicator by appealing to the estimates obtained by Benati and Surico (2009), which we employ to calibrate our structural model. In other words, we fix the vector θ^f_{id} at the median of the 90% confidence intervals of Table 1 (last column) in Benati and Surico (2009), while the weakly identified parameters θ^f_{un} are selected from the grid $\mathcal{G}(\theta^f_{un})$.[15] The computation of our In indicator conditional on Benati and Surico's estimates is conducted to ease the interpretation of our results.

Table 2 reports the outcomes of our computations. The indicator In, reported in the fifth row of Table 2, shows that for the Great Inflation sample our identification-robust measure of indeterminacy is as high as 6%. The same measure dramatically drops to zero when considering the alternative Great Moderation sample. Notably, the same values are obtained when conditioning on Benati and Surico's (2009) calibration. Therefore, we draw conclusions in line with those obtained with our formal test, i.e., indeterminacy is very likely to have occurred in the 1970s, but not in the period following Volcker's appointment.

5 Concluding remarks

This paper has proposed and implemented a novel approach to test for monetary policy indeterminacy in the United States. Our approach formally deals with model misspecification, which may render the identification of indeterminacy (vs. determinacy) unreliable. Conditional on a model being correctly specified, we propose a second-stage in which the null hypothesis of equilibrium uniqueness is tested. Importantly, our methodology requires neither the use of prior distributions nor that of nonstandard inference. Therefore, the degree of arbitrariness of our empirical results is substantially reduced.

Our two-step strategy is conducted via a conditional sequence of GMM and ML es-

[14] Results relative to the period 1979q2-2008q3 are available upon request.

[15] The value borrowed from Benati and Surico (2009) read as follows: $\gamma = 0.74$, $\rho = 0.59$, $\rho_{\tilde{y}} = 0.80$, $\rho_\pi = 0.42$, $\rho_R = 0.40$.

timations of a standard small-scale monetary DSGE model for the United States. We find formal support in favor of a switch from indeterminacy to uniqueness roughly corresponding with the advent of Paul Volcker as Chairman of the Federal Reserve. This result, which lines up with a number of previous contributions in the literature, is consistent with, but not necessarily pointing to, the 'good policy' explanation of the U.S. Great Moderation. We plan to elaborate further on our methodology to assess whether other explanations, in addition to the change in conduct of monetary policy documented in this paper, have contributed significantly to the Great Moderation phenomenon.

In light of the recent financial crisis, the uniqueness scenario supported by our analysis as for the period mid-1980s-onwards may very well be over. When enough data become available, our methodology will help shedding light on this issue. We also believe our methodology may be fruitfully applied to understand if other economic realities have experienced changes in their macroeconomic environment of the type documented here. Benati (2008) documents a reduction in inflation persistence in a variety of countries under stable monetary regimes with clearly defined nominal anchors, e.g., official inflation targeters. We plan to apply our formal testing strategy to these countries in future research.

Appendix

This Appendix summarizes the mechanics of the GMM and ML estimators of θ taken form system (1)-(4) and the determinate reduced form in eq. (9), respectively.

GMM estimation

Other than being relevant for the Step 1 of the joint test proposed in Section 3.1, GMM estimation plays a crucial role in the construction of the indicator of determinacy/indeterminacy summarized in Section 3.3.

We start by combining the structural model in the eqs. (5)-(6) in the expression

$$(\Gamma_0 + \Xi\Gamma_f)X_t = \Gamma_f X_{t+1} + (\Gamma_b + \Xi\Gamma_0)X_{t-1} - \Xi\Gamma_b X_{t-2} + \Xi\Gamma_f \xi_t + \varepsilon_t - \Gamma_f \xi_{t+1},$$

where $\xi_t := X_t - E_{t-1}X_t$ is a vector MDS, and then we define the 3×1 vector function

$$h(X_t, \theta) := (\Gamma_0 + \Xi\Gamma_f)X_t - \Gamma_f X_{t+1} - (\Gamma_b + \Xi\Gamma_0)X_{t-1} + \Xi\Gamma_b X_{t-2}. \tag{21}$$

Since both the terms ξ_t and ε_t are MDSs, they can be thought of being linearly connected through the relationship $\xi_t{:=}K\varepsilon_t$, where K is an arbitrary 3×3 matrix; this argument shows that $h(X_t, \theta)$ in eq. (9) has a VMA(1)-type representation.

The 'robust' GMM estimation of θ can be obtained by minimizing the criterion

$$Q_{con,T}(\theta) := T^{-1} \sum_{t=1}^{T} \varphi(X_t, \theta)' \left[S_T(\theta)\right]^{-1} T^{-1} \sum_{t=1}^{T} \varphi(X_t, \theta) \tag{22}$$

where $\varphi(X_t, \theta){:=}h(X_t, \theta) \otimes Z_t$, Z_t is a $r \times 1$ vector of instruments that is discussed below, $r \geq m$, and $S_T(\theta)$ is a consistent estimator of the long run covariance matrix

$$S(\theta) := V_0(\theta) + \left[V_1(\theta) + V_1(\theta)'\right] \quad , \quad V_i(\theta) := E\left[\varphi(X_t, \theta)\varphi(X_{t-i}, \theta)'\right], \, i = 0, 1. \tag{23}$$

The particular structure of the long run covariance matrix in in eq. (23) is motivated by the following argument. Under standard regularity conditions, also the autocorrelation structure of $\varphi(X_t, \theta)$ follows a VMA(1) process. Fanelli (2011) shows that if the data generating process belongs either to system (9) or system (10) (which implies the correct specification of the structural equations), then the $r \times 1$ vector of instruments is given by $Z_t{:=}(X_{t-1}', X_{t-2}', ..., X_{t-\ell}')'$ where ℓ is such that $r{:=}n\ell \geq \dim(\theta)$, is relevant for θ other than valid under both determinacy and indeterminacy.

The overidentification restrictions test (J-test) is obtained as

$$J_T := TQ_{con,T}(\hat{\theta}_T) \tag{24}$$

and is asymptotically $\chi^2(3r - m)$-distributed if the new-Keynesian DSGE model is correctly specified, see sub-Section 3.2.

The minimization of the objective function in eq. (22) in which the dependence of the matrix $S_T(\theta)$ on θ is treated explicitly in the estimation algorithm defines the continuos updating GMM estimator of Hansen et al. (1996) (henceforth CU-GMM). The reason why we prefer this version of GMM compared to the 'iterated' counterpart of GMM (henceforth I-GMM) is twofold. First, in the presence of weak identification, a situation that can be characterized as the objective function $Q_{con,T}(\theta)$ being nearly uninformative about part or all elements of θ, the CU-GMM allows us to construct asymptotic valid confidence sets for θ_0 or part of its elements, the so-called 'S-sets', see Stock and Wright (2000); the indicator of determinacy/indeterminacy discussed in Section 3.3 is based on asymptotic valid confidence set for a subset of elements of θ_0. Second, even if under the correct specification of the structural model the CU-GMM and I-GMM estimators are asymptotically equivalent, the available empirical evidence (e.g. Hall, 2005) shows that the finite sample performance of the overidentification restrictions test based on the CU-GMM is characterized by a smaller discrepancy between the empirical and nominal size of the test.

If we had infinite observations, we might simply replace θ in the matrix $G(\theta){:=}(\Gamma_0 - \Gamma_f\Phi_1)^{-1}\Gamma_f$ defined in Section 2 with its consistent estimate $\hat{\theta}_T$, and evaluate the stability of $G(\hat{\theta}_T)$; as shown in Section 2, the system admits a determinate reduced form if $\lambda_{\max}[G(\hat{\theta}_T)] < 1$, while multiple solutions if $\lambda_{\max}[G(\hat{\theta}_T)] > 1$. In actual samples, however, we can not ignore the variability of the estimator in hand. A 'natural' indicator of indeterminacy (determinacy), therefore, might be based on the evaluation of the instability (stability) of the matrix $G(\theta)$ in correspondence of suitably chosen points within a $100(1-\eta)\%$ confidence sets for θ_0; in particular, we might assess the strength of indeterminacy (determinacy) by computing the fraction of times in which the condition $\lambda_{\max}[G(\theta)] > 1$ ($\lambda_{\max}[G(\theta)] < 1$) is fulfilled within the confidence set. However, 'standard' confidence sets for θ_0 are not reliable when part or all of the structural parameters are weakly identified (Dufour, 1997), hence the indicator described in Section 3.3 relies on an identification-robust confidence set for the elements of θ which are thought of being weakly identified.

ML estimation

Consider the VAR model of lag order two

$$X_t = \Phi Z_t + u_t, \tag{25}$$

where $\Phi{:=}[\Phi_1 : \Phi_2]$, Φ_1 and Φ_2 are unrestricted 3×3 matrices, $Z_t{:=}(X'_{t-1}, X'_{t-2})'$ and u_t is assumed to obey a 3-dimensional Gaussian white noise process with covariance matrix Σ_u. The estimation of the reduced form solution in eq. (9) amounts to estimate a counterpart of system (25) subject to the cross-equation restrictions that the structural system (1)-(4) entails on Φ_1, Φ_2 (and indirectly on Σ_u).

It can be proved (Fanelli, 2010, 2011) that defined the matrices

$$\mathring{\Gamma}_0 := \begin{bmatrix} (\Gamma_0 + \Xi\Gamma_f) & 0_{3\times 3} \\ 0_{3\times 3} & I_3 \end{bmatrix} \ , \ \mathring{\Gamma}_f := \begin{bmatrix} \Gamma_f & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} \end{bmatrix} \ ,$$
$$\mathring{\Gamma}_b := \begin{bmatrix} (\Gamma_b + \Xi\Gamma_0) & -\Xi\Gamma_b \\ I_3 & 0_{3\times 3} \end{bmatrix} \ , \mathring{\Phi} := \begin{bmatrix} \Phi_1 & \Phi_2 \\ I_3 & 0_{3\times 3} \end{bmatrix} \ ,$$

the implicit form version of the cross-equation restrictions is given by the quadratic matrix equation

$$\mathring{\Gamma}_f\mathring{\Phi}^2 - \mathring{\Gamma}_0\mathring{\Phi} + \mathring{\Gamma}_b = 0_{6\times 6}$$

which links the structural parameters θ contained in $\mathring{\Gamma}_f$, $\mathring{\Gamma}_0$ and $\mathring{\Gamma}_b$ to the reduced form parameters contained in $\mathring{\Phi}$. These nonlinear constraints can alternatively be expressed in the form

$$\mathring{\Phi} = (\mathring{\Gamma}_0 - \mathring{\Gamma}_f\mathring{\Phi})^{-1}\mathring{\Gamma}_b \ , \tag{26}$$

which shows that the elements in the upper-block of the matrix $\mathring{\Phi}$ depend on θ, i.e., $\tilde{\Phi}_1 = \Phi_1(\theta)$ and $\tilde{\Phi}_2 = \Phi_2(\theta)$, where $\Phi_1(\theta)$ and $\Phi_1(\theta)$ correspond to the matrices in the reduced form solution in eq. (9).

The log-likelihood of system (9) corresponds to the concentrated log-likelihood of the VAR (25) subject to the restrictions in eq. (26) and is given by

$$\log L(\theta) = c - \frac{T}{2}\log\left[\det\left(\sum_{t=1}^{T}(X_t - \Phi(\theta)Z_t)(X_t - \Phi(\theta)Z_t)'\right)\right] \ , \tag{27}$$

where c is a constant and $\Phi(\theta){:=}[\Phi_1(\theta) : \Phi_2(\theta)]$. The maximization of $\log L(\theta)$ hinges on numerical (iterative) techniques based on various approximations of the restrictions in eq. (26). Departures from the normality assumption imply that the estimator of θ is actually a Quasi-ML estimator. Given the ML estimate of θ, $\hat{\theta}$, and defined the vector of constrained VAR coefficient estimates, $\hat{\phi}_c{:=}vec(\hat{\Phi}_c)$, $\hat{\Phi}_c = \Phi(\hat{\theta}){:=}[\Phi_1(\hat{\theta}) : \Phi_2(\hat{\theta})]$, the efficient score (LM) statistic for the cross-equation restrictions in eq. (26) is given by

$$LM_T := \frac{1}{T}s_T(\hat{\phi}_c)'\left[\hat{\Sigma}_u \otimes (\frac{1}{T}Z'Z)^{-1}\right]s_T(\hat{\phi}_c) \tag{28}$$

where $\hat{\Sigma}_u{:=}\frac{1}{T}\sum_{t=1}^{T}(X_t - \hat{\Phi}_cZ_t)(X_t - \hat{\Phi}_cZ_t)'$, X and Z are the $T\times 1$ and $T\times 2n$ matrices of observations on X_t and Z_t, respectively, and

$$s_T(\hat{\phi}_c) := vec\left(\frac{\partial \log L(\phi)}{\partial \phi'}\Big|_{\phi:=\hat{\phi}_c}\right) = vec\left\{[Z'X - (Z'Z)\hat{\Phi}_c']\left(\hat{\Sigma}_\varepsilon\right)^{-1}\right\}$$

is the score of the unrestricted VAR evaluated at the constrained VAR coefficients. The test statistic LM_T is asymptotically $\chi^2(18 - m)$-distributed under the null that eq. (26) holds, i.e., if the data are generated from the reduced form solution in eq. (9), see sub-Section 3.2.

Joint test

The combined (sequential) use of the J_T test in eq. (24) and the LM_T test in eq. (28) is described in detail in our sub-Sections 3.1-3.2; see also Fanelli (2010).

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Tables

Table 1. CU-GMM and ML estimates of the structural parameters θ^f of system (1)-(4), along with formal tests.

		1954q3-1979q2		1985q1-2008q3	
Parameter	Interpret.	CU-GMM	ML_{det}	CU-GMM	ML_{det}
γ	IS, forward look. term	0.50^* (0.03)	0.73^* (0.02)	0.35^* (0.08)	0.73^* (0.01)
ρ	Rule, smoothing term	0.43^* (0.01)	0.60^* (0.05)	0.87^* (0.04)	0.63^* (0.04)
$\varphi_{\tilde{y}}$	Rule, react. to out. gap	0.06 (0.04)	0.14 (0.51)	0.27^* (0.11)	0.28 (0.41)
φ_{π}	Rule, react. to inflation	0.93^* (0.12)	1.34 (0.75)	2.98^* (1.32)	4.67^* (0.97)
$\rho_{\tilde{y}}$	Out. gap shock, persist.	0.06 (0.15)	0.85^* (0.02)	0.16 (1.22)	0.92^* (0.01)
ρ_{π}	Inflation shock, persist.	0.67^* (0.11)	0.82^* (0.11)	0.70^* (0.12)	0.94^* (0.03)
ρ_R	Pol. rate shock, persist.	0.67^* (0.07)	0.71^* (0.01)	0.60^* (0.08)	0.83^* (0.01)
J-test		12.70 [0.31]	–	17.66 [0.09]	–
LM-CER test		–	45.30 [0.00]	–	20.50 [0.04]

NOTES: The parameters δ, κ and α are fixed at the values 0.125, 0.05 and 0.05, respectively. ML estimates have been obtained as detailed in the Appendix. A numerical grid search was used for the parameters $\gamma, \rho, \varphi_{\tilde{y}}, \varphi_{\pi}$, using the following ranges: [0.55, 0.75] for γ, [0.60, 0.98] for ρ, [0.05, 2] for $\varphi_{\tilde{y}}$, [0.5, 5.5] for φ_{π} and increment 0.03 for all parameters (by discarding all combinations of point that led to $\lambda_{\max}(G(\theta))>1$). CU-GMM system estimates are obtained by using $Z_t{:=}(X'_{t-1}, X'_{t-2})'$ as vector of instruments and the procedure summarized in the Appendix. The starting values of the CU-GMM estimates are I-GMM estimates which in turn have been obtained by using 2SLS initial estimates. J-test is the overidentifications restrictions test; LM-CER is a LM test for the nonlinear cross-equation restrictions that the new-Keynesian model entails on its reduced form solution under determinacy. Asymptotic standard errors in parentheses; asterisks * indicate point estimates whose t-ratio is greater than 1.96 in absolute value; p-values in brackets. Estimation on each sub-period is carried out by considering within-periods initial values and variables are demeaned within each sub-period.

Table 2. Projected 90% asymptotic confidence intervals for the policy parameters $\varphi_{\tilde{y}}$ and φ_{π} and indicator of indeterminacy

	1954q3-1979q2	1985q1-2008q3
Iden-Rob $\varphi_{\tilde{y}}$	[0.05 , 2]	[0.15 , 0.95]
Wald-type $\varphi_{\tilde{y}}$	[0 , 0.129]	[0.09 , 0.45]
Iden-Rob φ_{π}	[0.5 , 5.5]	[1.5 , 5.5]
Wald-type φ_{π}	[0.48 , 0.85]	[0.80 , 5.20]
In (our estimates)	6.00%	0.00%
In (Benati and Surico's)	6.28%	0.00%

NOTES: 'Iden-Rob' 90% confidence intervals are obtained by projecting out the set $\mathcal{C}_{0.90}(\theta_{0,un}^{f})$, $\theta_{un}^{f}:=(\varphi_{\tilde{y}},\varphi_{\pi})'$; 'Wald-type' denotes conventional 90% confidence intervals; In (Indet) is the indicator of indeterminacy discussed in Section 3.3, whose value is computed conditional on either our estimates or Benati and Surico's (2009), Table 1 (last column).