PRICING EQUITY AND DEBT TRANCHES OF COLLATERALIZED FUNDS OF HEDGE FUND OBLIGATIONS: AN APPROACH BASED ON STOCHASTIC TIME CHANGE AND ESSCHER TRANSFORMED MARTINGALE MEASURE

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ABSTRACT. Collateralized Funds of Hedge Fund Obligations (CFOs) are relatively recent structured finance products linked to the performance of underlying funds of hedge funds. The capital structure of a CFO is similar to traditional Collateralized Debt Obligations (CDOs), meaning that investors are offered different rated notes and equity interest. CFOs are structured as arbitrage market value CDOs. The fund of funds manager actively manages the fund to maximize total return while limiting price volatility within the guidelines of the structure.
The aim of this paper is to provide a useful framework to evaluate Collateralized Funds of Hedge Fund Obligations, that is pricing the equity and the debt tranches of a CFO.
The basic idea is to evaluate each CFO liability as an option written on the underlying pool of hedge funds. The value of every tranche depends on the evolution of the collateral portfolio during the life of the contract. Care is taken to distinguish between the fund of hedge funds and its underlying hedge funds, each of which is itself a portfolio of various securities, debt instruments and financial contracts.
The proposed model incorporates skewness, excess-kurtosis and is able to capture more complex dependence structures among hedge fund log-returns than the mere correlation matrix. With this model it is possible to describe the impact of an equivalent change of probability measure not only on the marginal processes but also on the underlying dependence structure among hedge funds.

1. Introduction

The problem

Consider a portfolio of financial products (in our case hedge funds) whose log-returns have distributions very far from the Normal. Assume that the value of this portfolio is regularly checked during the time with a fixed frequency, for example at

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the end of every month. Assume also that only historical data are available. Our task is to compute the *fair* price of an asset whose payoff is linked to the value of this portfolio at each date. To reach this purpose we can apply the martingale method: we have to compute the expected discounted payoff of the asset under a risk neutral probability measure.
The general problem is to define the joint distribution of the value of all the assets in the portfolio at each date of control under a suitable risk neutral probability measure.
To face this problem we consider separately two issues:

- the definition of the dependence structure under the physical probability measure;
- the choice of an appropriate equivalent martingale measure allowing to study the impact of the change of measure on the dependence structure.

Finally, since no traded options are available for calibration purpose, we use an approach to change measure that allows to capture the impact of the change of measure on the marginal and joint parameters.

Methodological contribution
The physical dependence among hedge fund log-returns is introduced through a Gamma stochastic time change of a Multivariate Brownian motion with drift, with independent components. The idea is that the economy is driven by only one common factor, whose dynamics is given by a Gamma subordinator [14]. A jump in the time-change leads to a jump in the price processes and so all jumps occur simultaneously. However the size of individual jumps is caused by the independent Brownian Motions.
In the current setting, the market is incomplete. The risk due to jumps cannot be hedged and there is no more a unique risk neutral measure. Among the measures equivalent to the historical one, we choose the Esscher measure for which the discounted Net Asset Value process of each hedge fund is a martingale. In particular, we change the probability measure using the *Multivariate Esscher Transform* and we show that under this new measure the log-return evolution of hedge funds can be expressed again as Multivariate Brownian motion with drift, time changed with a Gamma stochastic clock identical to the physical one and with independent Brownian motions. We find also functional relations between real world and risk neutral parameters. The use of the Multivariate Esscher Transform in our model represents a powerful tool to study the impact of the change of measure not only on the marginal price processes but also on the underlying dependence structure.

The Application
The model is applied to evaluate the equity and the debt tranches of a CFO. The analysis is done starting from a simple CFO structure, which is then progressively complicated with the introduction of the structural features we encounter in typical CFOs. In this way, at each step of the evolution of the structure, the reader can understand the impact on the value, measured with respect to the first four moments of the distribution, and how this value is divided among the different tranches. The result is a useful schema that can provide some help in designing a CFO transaction. The analysis is also helpful for the CFO manager who usually invests in the equity tranche.

The work is organized as follows. In section 2 an introduction to Collateralized Funds of Hedge Funds Obligations is provided. In section 3 statistical properties of Hedge Fund monthly log-returns are analysed. In particular, it is shown that the evolution of hedge funds Net Asset Value cannot be described by a Geometric Brownian motion. Hedge funds monthly log-returns exhibit leptokurtic and usually negatively skewed distributions. In section 4 we present the model applied to describe the physical evolution of hedge fund log-returns. Then we discuss the change of measure and its impact on marginal and joint processes. In section 5 the estimation methodology and the simulation procedure are illustrated. In section 6 we discuss the pricing applications and the results. Finally, in section 7 we report our conclusions and indicate a future line of research.

2. Collateralized Funds of Hedge Fund Obligations

Collateralized Debt Obligations are structured finance products that redistribute credit risk to investors providing them a wide range of rated securities with scheduled interest and principal payments and different risk levels. CDOs are securitized by diversified pools of debt instruments. Collateralized Funds of Hedge Fund Obligations (CFOs) are structured finance products similar to CDOs. A CFO is created by using a standard securitization approach. Often a special purpose vehicle (SPV) issues multiple tranches of senior and subordinated notes that pay interest at fixed or floating rates and an equity tranche, and invests the proceeds in a portfolio of hedge funds. Picture 1 shows a schematic example of a possible CFO structure. The SPV, a new structure with no operating history, is set up as a bankruptcy-remote entity. This allows CFO investors to take only the risk of ownership of the assets but not the bankruptcy risk of the CFO's sponsor. The capital structure of a CFO is similar to that of a CDO. It consists of the collateral pool held in the SPV on the asset side and a group of notes having different priorities and payment obligations on the liability side. The asset-backed notes are expected to be redeemed at the scheduled maturity date with the liquidation proceeds of the collateral portfolio. The priority of payments among the different classes is sequential such that the Class A investors will be redeemed first. Following the full redemption of the Class A notes, the Class B notes will be redeemed. The Class C notes will repaid after the full redemption of the Class B notes. The Equity holders will receive all the residuals after the full redemption of the Class C notes[1]. The most senior tranche is usually rated AAA and is protected by the subordination of the lower tranches. In case of loss the lowest tranche, that is the equity tranche, pays for economic losses first. When the equity tranche is exhausted, the next lowest tranche begins absorbing losses. A CFO may have a AAA rated tranche, an AA rated tranche, a single A rated tranche and an unrated equity tranche. The assets of the special purpose vehicle secure the notes issued under an indenture or deed of trust under which a trustee is appointed. CFOs tend to be structured as *arbitrage market-value* CDOs that invest in hedge funds. CFO assets are actively managed by an investment adviser or manager with fund of funds expertise in order to maximize total return while limitinng price volatility within the guidelines of the structure,

[1] In the picture I, P, D, G indicate respectively interests, principal, dividends and capital gains. Notice that capital gains can be negative. In the worst case scenario G+P=0, i.e. equity holders lose all their invested capital.

in return for management fees and incentive compensation. The leverage (the ratio of debt to equity issued) in a CFO typically ranges from two-to-one to five-to-one while a CDO may have leverage as high as twenty-five-to-one for investment grade assets. A CFO can also be seen as a financial firm with equity holders and lenders in which all liabilities are invested in a diversified portfolio of hedge funds. The bond holders should earn a spread over interest rates and the equity investors earn the total return of the fund of hedge funds minus the financing fees.

CFOs typically have a stated term of three to seven years at the end of which all of the securities must be redeemed. Investors have limited redemption rights prior to maturity. Typically, redemptions before maturity are only possible if some predetermined events happen.

Detailed descriptions of real CFO structures can be found in [28] and in some Moody's pre-sale reports [21, 22, 23, 24] for example. The interested reader can also see [31] which illustrates Moody's approach to rating CFOs and CFOs structural features designed to address the risk due to illiquidity and lack of trasparency typical of the hedge fund investment.[2]

3. Statistical Properties of Hedge Fund Returns

3.1. **The Data.** We got hedge fund index data from Credit Suisse/Tremont Hedge Index.[3] Credit Suisse/Tremont maintains monthly NAV and simple return data for a Global hedge fund index and for the following 13 indices corresponding to different styles: convertible arbitrage, dedicated short bias, emerging markets, equity market neutral, event driven, ED distressed, ED multi-strategy, ED risk arbitrage, fixed income arbitrage, global macro, long/short equity, managed futures and multi-strategy. Contrary to other hedge fund indices, the Credit Suisse/Tremont indices reflect the monthly net of fee NAV on an asset-weighted basket of funds. Large funds therefore have a larger influence on the index than smaller funds. Most indices are affected by some form of survivorship bias. In order to minimize this effect, Credit Suisse/Tremont does not remove hedge funds in the process of liquidation from an index, and therefore captures all of the potential negative performance before a fund ceases to operate.

Our sample covers the period from January 1994 through May 2008, a total of 173 monthly log-return data for each hedge fund index.

3.2. **Summary Statistics.** Descriptive statistics are reported in Table 1. A brief examination of the last two columns of this table indicates that hedge fund returns are not Gaussian. Twelve hedge fund indices out of fourteen exhibit a negative skewness. All index display excess kurtosis. However, the degree of asymmetry and fat tails is quite different among hedge funds. These results are similar to those reported in [1, 2, 13] obtained using different hedge fund indices and in [26] obtained employing CS/T indices on a shorter time period. Negative skewness and excess kurtosis are due to hedge funds' use of derivatives, leverage and short selling.

3.3. **Serial Correlation.** The first three columns of Table 2 report hedge fund autocorrelations up to order three. In particular, we note that all first order autocorrelations are positive and nine of them are significantly different from zero. The

[2]Other useful references on CFOs are [3, 18, 20, 30].

[3]http://hedgeindex.com

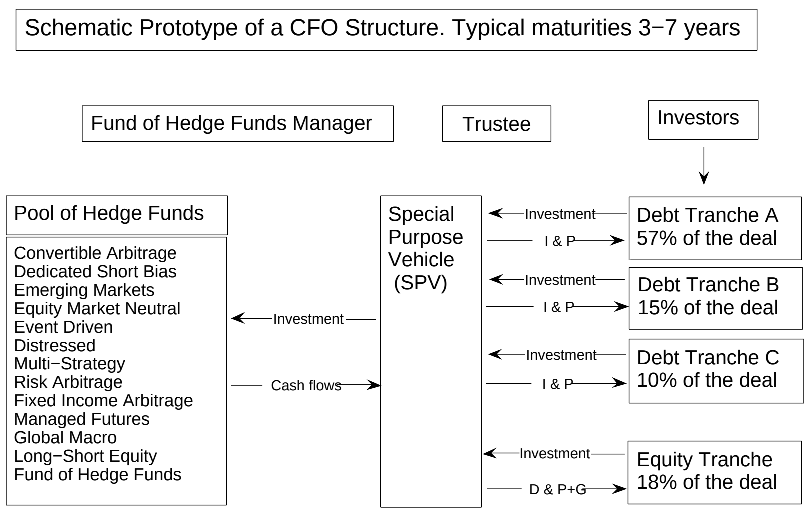

Figure 1. A CFO structure

last column reports P-values of Ljung-Box statistic test for the joint relevance of autocorrelations up to order twelve. As already noted in [5, 13], also in our sample, Convertible Arbitrage and ED Distressed indices seem to be among the most affected by first order and general serial correlation.

3.4. **Unsmoothed Data.** The positive first order autocorrelation is a typical characteristic of empirical hedge fund returns and appears incoherent with the market

efficiency hypothesis. According to [5, 13] an explanation consists in the difficulty for hedge fund managers to get updated valuations of their positions in illiquid over-the-counter securities. To face this problem, hedge fund managers usually employ the last reported transaction value to estimate the current market price. However, this estimation procedure may easily create lags in the evolution of the net asset value. This would explain why the different convertible arbitrage and distressed indices, employed in their work, show the most significant serial correlation.
According to [5, 13] a possible approach to mitigate the effect of this autocorrelation can be taken from the real estate finance literature. The returns of property investment indices are autocorrelated and this is caused by infrequent valuations and smoothing in appraisal. The approach employed in the real estate literature by Geltner *et al.* [7, 8, 9, 10] has been to *unsmooth* the observed returns to build a new series of returns which are more volatile and whose characteristics are believed to more accurately describe the evolution of the underlying property value. Following this literature and [5, 13], the observed value of a hedge fund index each month can be expressed as a weighted average of the underlying true value and the observed value of the hedge fund in the previous month.
Given these assumptions, it is possible to get the unsmoothed series with approximately zero first order autocorrelation:

$$y_t = \frac{y_t^* - \alpha y_{t-1}^*}{1-\alpha} \tag{3.1}$$

where y_t and y_t^* are the true unobservable underlying return and the observed return at time t. The parameter α is set equal to the first order autocorrelation coefficient of the time series.
We apply this procedure to get unsmoothed log-return series for each hedge fund index and repeat the previous statistical analyses with these new data to evaluate the impact of the unsmoothing procedure.

3.5. **Summary Statistics.** Descriptive statistics are reported in Table 3. The most interesting result is shown in the fifth column. All the unsmoothed time series exhibit a greater standard deviation, with a mean increment of 23%. The biggest increment (70%) is reached by Convertible Arbitrage. As evidenced by [13], the unsmoothing procedure has also a relatively small impact on the skewness and kurtosis of each hedge fund, but the direction of these changes is not clear. The last two columns of table 3 clearly evidence that the normality hypothesis for the distribution of hedge fund log-returns is still unlikely.

3.6. **Serial Correlation.** Table 4 shows that these unsmoothed time series are not affected by first order autocorrelation. Notice, however some problems of general serial correlation for two indices. Ljung-Box Statistics LB-Q(12) for ED Risk Arbitrage and Global Macro do not allow to reject the hypothesis that autocorrelations up to order 12 are different from zero at significance level of 5%.
To conclude this section we summarize the main results on statistical properties of hedge funds' log-returns:

- the distributions of monthly hedge funds' log-returns are usually not symmetric and negatively skewed;
- these distributions have fatter tails than the Normal;
- often hedge fund log-returns exhibit first order serial correlation. However, this is very likely a result of the appraisal procedure, and so observed data

do not reflect exactly the true values that are unobserved. In other words, the true generating hedge fund log-returns process can be considered as a process with uncorrelated increments;[4]

- the evolution of hedge funds' Net Asset Value in general cannot be described by a Geometric Brownian motion.

To model the temporal behaviour of hedge funds' log-returns more flexible stochastic processes than Brownian motion are therefore necessary. More general Lévy processes can yield possible solution.

Table 1.
Summary Statistics of Monthly Log-returns for CS/Tremont Indices
Period January 1994-May 2008 (Smoothed Data)

Index	Mean %	Median %	Max %	Min %	Std.Dev. %	Skew.	Kurt.
CS/T Global Index	0,78	0,76	7,94	-7,98	2,12	-0,04	5,73
Convertible Arbitrage	0,58	0,86	3,45	-5,80	1,38	-1,64	7,64
Dedicated Short Bias	-0,21	-0,36	20,2	-9,36	4,75	0,56	4,11
Emerging Markets	0,70	1,38	15,3	-26,2	4,50	-1,18	10,4
Equity Market Neutral	0,71	0,67	3,19	-1,27	0,76	0,36	3,90
Event Driven	0,83	1,02	3,84	-12,6	1,61	-3,58	30,1
ED Distressed	0,93	1,11	4,08	-13,4	1,78	-3,15	26,1
ED Multi-Strategy	0,78	0,86	4,29	-12,3	1,74	-2,65	20,9
ED Risk Arbitrage	0,55	0,55	3,58	-6,48	1,16	-1,29	10,4
Fixed Income Arbitrage	0,40	0,59	2,18	-7,25	1,16	-3,19	19,2
Global Macro	1,02	1,04	10,5	-12,2	2,97	-0,14	6,69
Long/Short Equity	0,86	0,94	12,0	-12,2	2,79	-0,13	7,31
Managed Futures	0,47	0,32	9,30	-9,71	3,39	-0,12	3,15
Multi-Strategy	0,65	0,75	3,57	-4,93	1,22	-1,11	5,89

4. The Model

4.1. **Building Multivariate Lévy Processes through Subordination of Multivariate Brownian motion with drift.** One method to introduce jumps into a multidimensional model is to take a Multivariate Brownian motion with drift and time change it with an independent one-dimensional subordinator [6, 14]. This approach allows constructing multidimensional versions of many popular one-dimensional Lévy processes, including Variance Gamma, Normal Inverse Gaussian, Generalized Hyperbolic, Meixner and Carr-Geman-Madan-Yor process. The principal advantage of this method is its simplicity and analytic tractability. In particular:

- the computation of the Characteristic Function of the process is simple;
- the knowledge of the Characteristic Function allows to find expressions for joint and marginal moments;
- conditional Normality of log-returns simplifies the simulation procedure;

[4] See the previous discussion about smoothed and unsmoothed data.

Table 2.
Autocorrelations up to order 3
Ljung-Box Autocorrelation Tests with lags up to order 12
Period January 1994-May 2008 (Smoothed Data)

Index	AC(1)	AC(2)	AC(3)	Ljung-Box-Q(12) P-Value
CS/T Global Index	0,099	0,014	-0.026	0,661
Convertible Arbitrage	0,484***	0,284	0,113	0,000
Dedicated Short Bias	0,099	-0,037	-0,072	0,248
Emerging Markets	0,275***	0,020	0,002	0,033
Equity Market Neutral	0,227**	0,095	0,031	0,081
Event Driven	0,282***	0,135	-0,001	0,038
ED Distressed	0,282***	0,137	0,019	0,025
ED Multi-Strategy	0,251**	0,142	0,009	0,098
ED Risk Arbitrage	0,220**	-0,090	-0,158	0,002
Fixed Income Arbitrage	0,280***	0,006	0,016	0,048
Global Macro	0,057	0,018	0,088	0,029
Long/Short Equity	0,145*	0,024	-0,083	0,036
Managed Futures	0,057	-0,154	-0,076	0,071
Multi-Strategy	0,041	0,050	0,077	0,958

Table 3.
Summary Statistics of Monthly Log-returns for CS/Tremont Indices
Period January 1994-May 2008 (Unsmoothed Data)

Index	Mean %	Median %	Max %	Min %	Std.Dev. %	Skew.	Kurt.
CS/T Global Index	0,78	0,72	8,28	-8,94	2,35	-0,08	5,52
Convertible Arbitrage	0,58	0,80	8,39	-10,1	2,34	-1,13	8,38
Dedicated Short Bias	-0,20	-0,44	22,2	-10,8	5,25	0,55	4,05
Emerging Markets	0,63	1,59	18,9	-36,2	5,89	-1,47	11,5
Equity Market Neutral	0,72	0,64	3,83	-2,29	0,96	0,27	4,34
Event Driven	0,81	1,05	4,29	-17,6	2,14	-3,86	33,5
ED Distressed	0,91	1,04	5,69	-18,7	2,37	-3,40	29,7
ED Multi-Strategy	0,77	0,99	5,32	-16,38	2,24	-2,68	21,7
ED Risk Arbitrage	0,55	0,66	4,79	-8,17	1,45	-1,17	10,4
Fixed Income Arbitrage	0,40	0,63	5,49	-8,61	1,55	-2,30	15,7
Global Macro	1,03	1,02	11,0	-12,7	3,15	-0,14	6,50
Long/Short Equity	0,86	0,82	12,6	-14,4	3,23	-0,18	6,89
Managed Futures	0,47	0,37	9,94	-10,3	3,59	-0,17	3,18
Multi-Strategy	0,65	0,81	3,87	-5,17	1,28	-1,12	5,98

- the GMM estimator based on the Moment Generating Function or other *ad hoc* technique based on Method of Moments can be used to estimate model parameters;

TABLE 4.
Autocorrelations up to order 3
Ljung-Box Autocorrelation Tests with lags up to order 12
Period January 1994-May 2008 (Unsmoothed Data)

Index	AC(1)	AC(2)	AC(3)	Ljung-Box Q(12)/P-Value
CS/T Global Index	0,003	0,010	-0,017	0.725
Convertible Arbitrage	-0,032	0,085	-0,085	0,810
Dedicated Short Bias	0,005	-0,038	-0,057	0,575
Emerging Markets	0,025	-0,045	0,035	0,634
Equity Market Neutral	-0,012	0,036	0,018	0,718
Event Driven	-0,008	0,085	-0,036	0,978
ED Distressed	-0,008	0,083	-0,023	0,910
ED Multi-Strategy	-0,017	0,100	-0,019	0,880
ED Risk Arbitrage	0,033	-0,116	-0,128	0,023
Fixed Income Arbitrage	0,034	-0,077	-0,007	0,889
Global Macro	-0,003	0,008	0,090	0,013
Long/Short Equity	0,000	0,018	-0,072	0,192
Managed Futures	0,009	-0,154	-0,068	0,115
Multi-Strategy	-0,004	0,044	0,083	0,966

- a parsimonious description of dependence is very important because one typically does not have enough information about the dependence structure to estimate many parameters;
- since no traded option on hedge funds are available we cannot calibrate the model parameters directly in the *risk neutral measure chosen by the market*. The method we use to find an equivalent martingale measure requires the knowledge of the Characteristic Function of the multivariate process.

This method presents also some drawbacks. The range of dependence patterns that one can obtain using this approach is limited (for example, independence is not included), and all components must follow the same parametric model. Finally, building a Multivariate Lévy process time changing a Multivariate Brownian Motion with a one-dimensional subordinator imposes some constraints in the parameters of the marginal processes. Therefore, the greater the number of parameters describing the distribution of the subordinator is, the more similar the moments of the margins are. In the Multivariate Variance Gamma process with linear drift we have only one constrained parameter for every margin.[5]

4.2. **Multivariate Variance Gamma Process.** The evolution of hedge funds' log-returns is described through a Multivariate Variance Gamma process with linear drift. The Multivariate Variance Gamma process is obtained time changing a Multivariate Brownian motion with drift, with independent components, with an independent one-dimensional Gamma process. As mentioned in [14] modelling dependence in this way is like starting from an independent Gaussian World in which all assets are driven by independent Geometric Brownian motions. Then, dependence is introduced time-changing all the asset price processes by a common

[5]For details on the Univariate Variance Gamma process see [15, 16, 17].

Gamma subordinator. A jump in the time-change produces a jump in the price processes and all jumps happen simultaneously. However, the jump sizes are caused by the individual Brownian motions. In this way a new business time in which the general market operates is introduced [11, 14].
The NAV at time t of each hedge fund is given by the product of the initial NAV times the exponential of a Variance Gamma process with linear drift:

$$F_t^j = F_0^j \exp(Y_t^j) \tag{4.1}$$

where F_t^j and F_0^j is the NAV of the hedge fund j at times t and 0, while Y_t^j is the log-return of the j-th hedge fund over the period $[0;\ t]$ for every $j = 1, \ldots, n$. The log-return of the j-th hedge fund is

$$Y_t^j = \mu_j t + X_t^j = \mu_j t + \theta_j G_t + \sigma_j W_{G_t}^j \tag{4.2}$$

where $G = \{G_t,\ t \geq 0\}$ is the common Gamma stochastic time change process such that $G_t \sim Gamma(t/\nu,\ 1/\nu)$ and $\nu > 0$, $W^j = \{W_t^j, t \geq 0\}$ and $W^k = \{W_t^k,\ t \geq 0\}$ are independent Wiener processes for all $j \neq k$, W_t^j and W_t^k are $Gaussian(0,\ t)$, $W_G^j = \{W_{G_t}^j,\ t \geq 0\}$ are n independent Wiener processes subordinated by the common Gamma process $G = \{G_t,\ t \geq 0\}$, θ_j, μ_j and $\sigma_j > 0$ are constants.
If we set $t = 1$, we get the yearly log-return for asset j, that is

$$Y_1^j = \mu_j + X_1^j = \mu_j + \theta_j G_1 + \sigma_j W_{G_1}^j \tag{4.3}$$

The above assumptions lead to the following simple expression for hedge funds' j and k yearly log-returns covariance:

$$\sigma\left(Y_1^j; Y_1^k\right) = \theta_j \theta_k \nu \tag{4.4}$$

and for correlation:

$$\rho\left(Y_1^j; Y_1^k\right) = \frac{\theta_j \theta_k \nu}{\sqrt{\sigma_j^2 + \nu\theta_j^2}\sqrt{\sigma_k^2 + \nu\theta_k^2}} \tag{4.5}$$

Since ν is strictly positive, the j-th and k-th hedge funds are positively correlated if and only if θ_j and θ_k have the same sign. In other words, this model implies a positive correlation for all the assets having the same sign of skewness. Pairs of negatively skewed or pairs of positively skewed hedge funds have a positive correlation coefficient. If a hedge fund has a symmetric VG distribution then it will be uncorrelated with all other hedge funds. However, by construction, it is clear that this hedge fund cannot be independent of the others. Negative correlation between pairs of hedge funds is only possible if their distributions exhibit skewness of opposite sign.
The Characteristic Function of Y_1^j for $j = 1, \ldots, n$ is

$$\Psi_{Y_1^j}(u) = \exp\left(iu\mu_j\right)\left(1 - iu\theta_j\nu + \frac{1}{2}u^2\sigma_j^2\nu\right)^{-1/\nu} \tag{4.6}$$

Conditional normality allows to compute the joint Characteristic Function of the Multivariate Variance Gamma distribution (with $t = 1$). The computations required are easy but tedious. Yet, we can compute this function in a more efficient way by using *Theorem 4.2* [6]. In order to apply this theorem we need to know:

- the Laplace Exponent $l(u)$ of the Gamma subordinator;

- the Charactheristic Exponent $c(\mathbf{u})$ of a Multivariate Brownian motion, with independent components.

The Laplace Exponent of a generic subordinator is defined in the following way:

$$E\left[\exp\left(uG_t\right)\right] = \exp\left[tl\left(u\right)\right] \quad \forall u \leq 0 \tag{4.7}$$

$$l\left(u\right) = \ln \frac{E\left[\exp\left(uG_t\right)\right]}{t} \tag{4.8}$$

The Moment Generating Function of $G_t \sim Gamma(t/\nu, 1/\nu)$ is

$$E\left[\exp\left(uG_t\right)\right] = \left(1 - u\nu\right)^{-t/\nu} \quad u < \frac{1}{\nu} \tag{4.9}$$

and consequently its Laplace Exponent is simply

$$l\left(u\right) = -\frac{\ln\left(1 - u\nu\right)}{\nu} \tag{4.10}$$

The joint Characteristic Function of the Multivariate Brownian motion with independent components with $t = 1$ is

$$\begin{aligned} \Psi(\mathbf{W})_1(\mathbf{u}) &= E\left[\exp\left(i\sum_{j=1}^{n}(u_j\theta_j + u_j\sigma_j W_1^j)\right)\right] \\ &= \exp\left[\sum_{j=1}^{n}\left(iu_j\theta_j - \frac{1}{2}u_j^2\sigma_j^2\right)\right] \end{aligned} \tag{4.11}$$

and therefore its Characteristic Exponent is

$$c\left(\mathbf{u}\right) = \sum_{j=1}^{n}\left(iu_j\theta_j - \frac{1}{2}u_j^2\sigma_j^2\right) \tag{4.12}$$

for every $\mathbf{u} \in R^n$.
Now we have all the elements necessary to compute the Characteristic Function of the Multivariate Variance Gamma distribution. Using *Theorem 4.2* [6] we get

$$\begin{aligned} \Psi(\mathbf{Y})_1\left(\mathbf{u}\right) = \exp\left(i\sum_{j=1}^{n}u_j\mu_j\right) \times \\ \left[1 - \nu\sum_{j=1}^{n}(iu_j\theta_j - \frac{1}{2}u_j^2\sigma_j^2)\right]^{(-1/\nu)} \end{aligned} \tag{4.13}$$

From this function it is immediate to derive the joint Moment Generating Function[6] of $\mathbf{Y}_1$, whose existence requires that the argument between the square brackets is

[6]Contrary to the Characteristic Function, which is always well-defined (as the Fourier transform of a probability measure), the Moment Generating Function is not always defined: the integral (4.14) may not converge for some values of $\mathbf{u}$. When it is well-defined, it can be formally related to the Characteristic Function (4.13) by: $M(\mathbf{Y})_1\left(\mathbf{u}\right) = \Psi(\mathbf{Y})_1\left(-i\mathbf{u}\right)$. However, we can use this relation to find the formal expression for the Moment Generating Function for the set of values of $\mathbf{u}$ such that the expectation (4.14) is finite. See [6] Paragraph 2.2.4.

positive:

$$M(\mathbf{Y})_1(\mathbf{u}) = \exp\left(\sum_{j=1}^{n} u_j \mu_j\right) \times \left[1 - \nu \sum_{j=1}^{n} (u_j \theta_j + \frac{1}{2} u_j^2 \sigma_j^2)\right]^{(-1/\nu)} \tag{4.14}$$

This function will play a crucial role in the section devoted to the change of measure.

4.3. **The Change of Measure.** The evolution of the Net Asset Value of each hedge fund is described as an Exponential Variance Gamma process under the real world probability measure. This model is arbitrage free since the price process of every asset has both positive and negative jumps [6]. Consequently, there exists an equivalent martingale measure. However, the model belongs to the class of incomplete market models: the equivalent martingale measure is not unique. Among the possible candidates we select the Esscher Equivalent Martingale Measure [6, 12, 27].

4.4. **Multivariate Esscher Transform.** In this section we explain how to use the Esscher Transform in a multivariate context in order to find the Esscher Equivalent Martingale Measure.
Consider a market with n risky assets and a bank account which provides a risk free interest rate r constant over the time period $[0, T]$. The value of the bank account at time t is $A_t = A_0 \exp(rt)$. Suppose that the price of every risky asset at time $t \in [0, T]$ can be described by a Geometric Lévy model, say $F_t^j = F_0^j \exp(Y_t^j)$ for $j = 1, \ldots, n$.
Let $\mathbf{Y} = \{\mathbf{Y}_t,\ t \geq 0\}$ be the n-dimensional Lévy process describing the multivariate log-returns process, then the $Q_{\mathbf{h}}$ Esscher measure associated with the risk process $\mathbf{Y}$ is defined by the following Radon-Nikodym derivative

$$\frac{dQ_{\mathbf{h}}}{dP}|\Im_t = \frac{\exp(\sum_{j=1}^{n} h_j Y_t^j)}{E\left[\exp(\sum_{j=1}^{n} h_j Y_t^j)\right]} \tag{4.15}$$

In order to find the Esscher risk neutral dynamic of $\mathbf{Y} = \{\mathbf{Y}_t,\ t \geq 0\}$ two steps are necessary:

- find a vector $\hat{\mathbf{h}}$ such that the discounted price process of every asset is a martingale under the new probability measure $Q_{\mathbf{h}}$;
- find the joint Characteristic Function of the multivariate process $\mathbf{Y} = \{\mathbf{Y}_t,\ t \geq 0\}$ under $Q_{\mathbf{h}}$.

Any transformation of the Lévy measure satisfying some integrability constraints (see [6] Section 4.2.3) leads to a new Lévy process. In particular, the Esscher Transform corresponds to an *exponential tilting* of the P Lévy measure. If there exists a vector $\hat{\mathbf{h}}$ such that

$$\int_{|\mathbf{y}|\geq 1} \upsilon^{Q_h}(d\mathbf{y}) = \int_{|\mathbf{y}|\geq 1} \exp(\mathbf{h}^T \mathbf{y}) \upsilon(d\mathbf{y}) < \infty \tag{4.16}$$

then the process is a Lévy process under this new probability measure. However, if it is possible to find a vector $\hat{\mathbf{h}}$ such that the discounted price process of each asset is a martingale under the measure $Q_{\mathbf{h}}$, then the existence of the Esscher Martingale Measure is ensured.

In the following sections we apply these steps to our multivariate model. Actually, we cannot be sure that such an equivalent martingale measure exists.
Notice that in our case the choice of this risk neutral measure (if it exists) seems to be the best for at least two reasons:

- Each step requires the knowledge of the joint P Characteristic Function. Usually, it is not easy to find this function explicitly. However, building a multidimensional Lévy process by a stochastic time change of a multivariate Brownian motion makes easy the computation of the Characteristic Function of the process.
- If this equivalent martingale measure exists usually it is possible to find the link between the physical and the risk neutral parameters. This is very useful when no option prices are available to calibrate the model. Since no traded options on hedge funds are available we cannot apply the *improperly* called Mean Correcting Martingale method, a change of measure which is the easiest and most frequently encountered in financial applications.

Finally, it should be emphasized that even if the ESMM exists, we cannot be sure that marginal and joint processes remain of the same type.

4.5. **MVG and ESMM.** The first step of the procedure described in the previous section requires the solution of the following system of n equations

$$\begin{cases} E\left[\exp(\sum_{j=1}^n h_j Y_t^j + Y_t^1)\right]/E\left[\exp(\sum_{j=1}^n h_j Y_t^j)\right] & = \exp(rt) \\ & \vdots \\ E\left[\exp(\sum_{j=1}^n h_j Y_t^j + Y_t^n)\right]/E\left[\exp(\sum_{j=1}^n h_j Y_t^j)\right] & = \exp(rt) \end{cases} \tag{4.17}$$

To solve this system we need the P Moment Generating function of the model introduced in section 4.2:

$$M(\mathbf{Y})_t(\mathbf{u}) = \exp\left(\sum_{j=1}^n u_j\mu_j t\right)\left[1-\nu\sum_{j=1}^n (u_j\theta_j + \frac{1}{2}u_j^2\sigma_j^2)\right]^{(-t/\nu)}$$

Thanks to the infinitely divisibility property of hedge funds' log-returns distributions, the solution of the *Esscher* system does not depend on t. The previous system after some computation leads to the next one:

$$\begin{cases} \ln\left[1-(\nu(\theta_1+h_1\sigma_1^2+0.5\sigma_1^2))/(1-\nu\sum_j^n(h_j\theta_j+0.5h_j^2\sigma_j^2))\right] & = (\mu_1-r)\nu \\ & \vdots \\ \ln\left[1-(\nu(\theta_n+h_n\sigma_n^2+0.5\sigma_n^2))/(1-\nu\sum_j^n(h_j\theta_j+0.5h_j^2\sigma_j^2))\right] & = (\mu_n-r)\nu \end{cases} \tag{4.18}$$

with the following constraints

$$[1-\nu\sum_j^n(h_j\theta_j+0.5h_j^2\sigma_j^2)]>0 \tag{4.19}$$

and

$$[1-\nu(\sum_{j\neq k}^n(h_j\theta_j+0.5h_j^2\sigma_j^2)+((h_k+1)\theta_k+0.5(h_k+1)^2\sigma_k^2))]>0,\ \ k=1,\dots,n. \tag{4.20}$$

From (4.18) we readily obtain:

$$\begin{cases} (a_1 + b_1 h_1)/A_1 = 1 - \sum_j^n (c_j h_j + d_j h_j^2) \\ \vdots \\ (a_n + b_n h_n)/A_n = 1 - \sum_j^n (c_j h_j + d_j h_j^2) \end{cases} \tag{4.21}$$

where
$a_j = \nu(\theta_j + 0.5\sigma_j^2)$, $b_j = \nu\sigma_j^2$, $c_j = \nu\theta_j$, $d_j = 0.5\nu\sigma_j^2$, $A_j = 1 - \exp[\nu(\mu_j - r)]$, $j = 1, ..., n$.
The term on the right hand side is the same in all equations so we can express for example $h_2, h_3, \dots, h_n$ as linear functions of h_1:

$$\begin{cases} h_2 = \frac{A_2}{A_1}\frac{b_1}{b_2} h_1 + \frac{1}{b_2}(\frac{A_2}{A_1} a_1 - a_2) \\ \vdots \\ h_n = \frac{A_n}{A_1}\frac{b_1}{b_n} h_1 + \frac{1}{b_n}(\frac{A_n}{A_1} a_1 - a_n) \end{cases} \tag{4.22}$$

To make the notation easier we define

$$D_j = \frac{A_j}{A_1}\frac{b_1}{b_j} \tag{4.23}$$

$$E_j = \frac{1}{b_j}(\frac{A_j}{A_1} a_1 - a_j) \tag{4.24}$$

for $j = 2, 3, \dots, n$, so that (4.22) becomes

$$\begin{cases} h_2 = D_2 h_1 + E_2 \\ \vdots \\ h_n = D_n h_1 + E_n \end{cases} \tag{4.25}$$

Substituting (4.25) in the first equation of the system (4.21), after tedious computations, we get a quadratic equation in only one unknown h_1:

$$p h_1^2 + q h_1 + s = 0 \tag{4.26}$$

where

$$p = A_1 (d_1 + \sum_{j=2}^n d_j D_j^2) \tag{4.27}$$

$$q = A_1 \left[c_1 + \sum_{j=2}^n (c_j D_j + 2 d_j D_j E_j) \right] + b_1 \tag{4.28}$$

$$s = A_1 \left[\sum_{j=2}^n (c_j E_j + d_j E_j^2) - 1 \right] + a_1 \tag{4.29}$$

The analysis of the existence of solutions of equation (4.26), although simple in principle, is a very hard task in practice. However, in all our experiments we found that equation (4.26) possesses a unique solution and therefore a unique vector

$$\hat{\mathbf{h}} = \left[\hat{h}_1; D_2 \hat{h}_1 + E_2; \dots; D_n \hat{h}_1 + E_n \right] \tag{4.30}$$

exists satisfying the constraints (4.19) and (4.20), where

$$\hat{h}_1 = \frac{-q + \sqrt{q^2 - 4ps}}{2p} \tag{4.31}$$

This ensures the existence and the uniqueness of the EEMM. For this reason in the sequel we indicate the *Esscher Vector* as $\hat{\mathbf{h}}$.
The joint Moment Generating Function for $t = 1$ under the ESMM can be computed as follows:

$$M^{Q_{\mathbf{h}}}(\mathbf{Y})_1(\mathbf{u}) = E^{Q_{\mathbf{h}}}\left[\exp \sum_{j=1}^{n} u_j Y_1^j\right] = \frac{E\left[\exp \sum_{j=1}^{n} (\hat{h}_j + u_j) Y_1^j\right]}{E\left[\exp \sum_{j=1}^{n} \hat{h}_j Y_1^j\right]} \tag{4.32}$$

where $\hat{h}_j$ is the j-th component of the vector $\hat{\mathbf{h}}$. The computation of the Esscher risk neutral joint Moment Generating Function requires the knowledge of the Moment Generating Function of the Multivariate Variance Gamma process for $t = 1$ under the statistical measure (see (4.14)). We substitute the following expressions

(4.33)

$$E\left[\exp \sum_{j=1}^{n} (\hat{h}_j + u_j) Y_1^j\right] = \exp\left(\sum_{j=1}^{n} (\hat{h}_j + u_j)\mu_j\right) \times \left[1 - \nu \sum_{j=1}^{n} ((\hat{h}_j + u_j)\theta_j + \frac{1}{2}(\hat{h}_j + u_j)^2 \sigma_j^2)\right]^{(-1/\nu)}$$

$$E\left[\exp \sum_{j=1}^{n} \hat{h}_j Y_1^j\right] = \exp\left(\sum_{j=1}^{n} \hat{h}_j \mu_j\right) \times \left[1 - \nu \sum_{j=1}^{n} (\hat{h}_j \theta_j + \frac{1}{2}\hat{h}_j^2 \sigma_j^2)\right]^{(-1/\nu)} \tag{4.34}$$

into equation (4.32) and after tedious computations and rearrangements we get

$$M^{Q_{\mathbf{h}}}(\mathbf{Y})_1(\mathbf{u}) = \exp\left(\sum_{j=1}^{n} u_j \mu_j\right) \times \left[1 - \nu \sum_{j=1}^{n} \frac{(u_j(\theta_j + \hat{h}_j \sigma_j^2) + \frac{1}{2}u_j^2 \sigma_j^2)}{1 - \nu \sum_{j=1}^{n} (\hat{h}_j \theta_j + \frac{1}{2}\hat{h}_j^2 \sigma_j^2)}\right]^{-1/\nu} \tag{4.35}$$

The joint $Q_{\mathbf{h}}$ Moment Generating Function can be written in the following more compact form:
(4.36)

$$M^{Q_{\mathbf{h}}}(\mathbf{Y})_1(\mathbf{u}) = \exp\left(\sum_{j=1}^{n} u_j \mu_j^{Q_{\mathbf{h}}}\right) \left[1 - \nu^{Q_{\mathbf{h}}} \sum_{j=1}^{n} (u_j \theta_j^{Q_{\mathbf{h}}} + \frac{1}{2} u_j^2 \sigma_j^{Q_{\mathbf{h}} 2})\right]^{(-1/\nu^{Q_{\mathbf{h}}})}$$

where relations among physical and Esscher risk neutral parameters are

$$\mu_j^{Q_{\mathbf{h}}} = \mu_j \tag{4.37}$$

$$\nu_j^{Q_{\mathbf{h}}} = \nu \tag{4.38}$$

$$\theta_j^{Q_{\mathbf{h}}} = \frac{\theta_j + \hat{h}_j \sigma_j^2}{1 - \nu \sum_{j=1}^{n} (\hat{h}_j \theta_j + \frac{1}{2} \hat{h}_j^2 \sigma_j^2)} \tag{4.39}$$

$$\sigma_j^{Q_{\mathbf{h}}\,2} = \frac{\sigma_j^2}{1 - \nu \sum_{j=1}^{n} (\hat{h}_j \theta_j + \frac{1}{2} \hat{h}_j^2 \sigma_j^2)} \tag{4.40}$$

$$\rho_{jk}^{Q_{\mathbf{h}}} = \rho_{jk} = 0 \tag{4.41}$$

From (4.36) it is easy to get the joint $Q_{\mathbf{h}}$ Characteristic Function

$$\Psi^{Q_{\mathbf{h}}}(\mathbf{Y})_1\left(\mathbf{u}\right) = \exp\left(i \sum_{j=1}^{n} u_j \mu_j\right) \left[\ 1 - \nu \sum_{j=1}^{n} (i u_j \theta_j^{Q_{\mathbf{h}}} - \frac{1}{2} u_j^2 \sigma_j^{Q_{\mathbf{h}}\,2})\right]^{(-1/\nu)} \tag{4.42}$$

The j-th $Q_{\mathbf{h}}$ marginal Characteristic Function (for $j = 1, \ldots,\ n$) is given by:

$$\Psi_{Y_1{}^j}^{Q_{\mathbf{h}}}\left(u_j\right) = \exp\left(i u_j \mu_j\right) \left[\ 1 - \nu(i u_j \theta_j^{Q_{\mathbf{h}}} - \frac{1}{2} u_j^2 \sigma_j^{Q_{\mathbf{h}}\,2})\right]^{(-1/\nu)} \tag{4.43}$$

Comparing (4.42) and (4.43) with (4.13) and (4.6) it is readily seen that the joint and marginal Characteristic Functions under the P and $Q_{\mathbf{h}}$ measures are of the same type. For each marginal process, only two parameters change. Under the ESMM the multivariate log-returns process can be expressed again as a Multivariate Brownian motion with independent components, time-changed by an independent Gamma process, identical to the physical one (plus a linear drift). In other words, the underlying dependence structure remains unchanged. However, covariances, correlations, and marginal moments change. In particular, the log-return of the j-th hedge fund over the period $[0;\ t]$ under $Q_{\mathbf{h}}$ is

$$Y_t^j = \mu_j t + \theta_j^{Q_{\mathbf{h}}} G_t + \sigma_j^{Q_{\mathbf{h}}} W_{G_t}^j \tag{4.44}$$

where $G = \{G_t,\ t \geq 0\}$ is the common Gamma stochastic process with $G_t \sim Gamma(t/\nu,\ 1/\nu)$, $\nu > 0$, $W^j = \{W_t^j,\ t \geq 0\}$ and $W^k = \{W_t^k,\ t \geq 0\}$ are independent Wiener processes for all $j \neq k$, $W_G^j = \{W_{G_t}^j,\ t \geq 0\}$ are n independent Wiener processes subordinated by the common Gamma process $G = \{G_t,\ t \geq 0\}$, $\theta_j^{Q_{\mathbf{h}}}$, μ_j and $\sigma_j^{Q_{\mathbf{h}}} > 0$ are constants.

5. Estimation and Simulation

5.1. **Real World and Risk Neutral parameters estimation.** To estimate real world parameters we use a constrained version of the method of moments.
First, we select a value for the common parameter ν. Then, we estimate marginal parameters requiring the equality among the first three empirical moments of log-returns and their theoretical VG counterparts. By so doing, we get Variance Gamma distributions able to replicate empirical means, variances and skewnesses. Then, we compute the mean of the resulting kurtoses and we compare this value with the empirical one of hedge funds in the collateral portfolio.[7] Varying the value of parameter ν we estimate again the model replicating the first three moments. The resulting mean kurtosis depends of course on the value of ν. After several trials

[7] See next section for the composition of the fund of hedge funds.

we choose $\nu = 0,33333$. This value leads to a mean fitted kurtosis similar to the mean empirical one. Annual marginal parameter estimates are reported in Table 8 and Table 10.[8]
Then, using the vectors $\hat{\mathbf{h}}$, the estimates of physical parameters and their relations with the risk neutral ones, we get the following tables reporting the Esscher risk neutral parameters:

- table 9 (smoothed data)
- table 11 (unsmoothed data)

As an example, with real world processes estimated with observed data, the solutions of the system (4.18) is:
$\hat{\mathbf{h}} = [-3,3385; 0,3396; -0,5281; -38,3631; -2,2028; -5,2944; -3,3096; -5,5798]$.
Finally, we have shown that our change of measure does not modify the underlying dependence structure among Brownian motions. Consequently, the underlying Brownian motions are still independent. Figure 2 exhibits a comparison between real world and risk neutral Kernel density for the Convertible Arbitrage Index.

5.2. **Simulation.** To simulate the paths of n dependent hedge fund NAVs under the Esscher Equivalent Martingale Measure we can proceed as follows.
Let $F^j_{t_0}$ the NAV of hedge fund j at time 0 for $j = 1, \ldots, n$.
Divide the time-interval $[0, T]$ into N equally spaced intervals $\Delta t = T/N$ and set $t_k = k\Delta t$, for $k = 0, ..., N$.
For every hedge fund repeat the following steps for k from 1 to N:

- sample a random number g_k out of the Gamma$(\Delta t/\nu,\ 1/\nu)$ distribution;
- sample for each $j = 1, \ldots, n$ an independent standard Normal random number $w^j_{t_k}$.
- compute

$$F^j_{t_k} = F^j_{t_{k-1}} \exp\left[\mu_j \Delta t + \theta_j^{Q_h} g_k + \sigma_j^{Q_h} \sqrt{g_k} w^j_{t_k}\right] \tag{5.1}$$

To simulate a simple trajectory of the NAV of the collateral fund of hedge funds it is sufficient to compute for k from 1 to N

$$F_{t_k} = \sum_{j=1}^{n} F^j_{t_k}. \tag{5.2}$$

In the applications, we will also take into account the impact of CFO structural features such as coupon payments, equity distribution rules, Over Collateralization tests, liquidity profile and management fees to describe the temporal evolution of the NAV of the collateral portfolio.

6. Pricing CFOs equity and debt tranches

As we have already explained in the Introduction of this work, our aim is to provide a useful framework to evaluate Collateralized Fund of Hedge Funds Obligations, that is pricing the equity and the debt tranches of a CFO. The fair price of each tranche is computed as its expected discounted payoff under a suitable risk neutral probability measure.
The payoff of every tranche is linked to

[8] We used the above estimation procedure, although far from being rigorous, mainly for its simplicity.

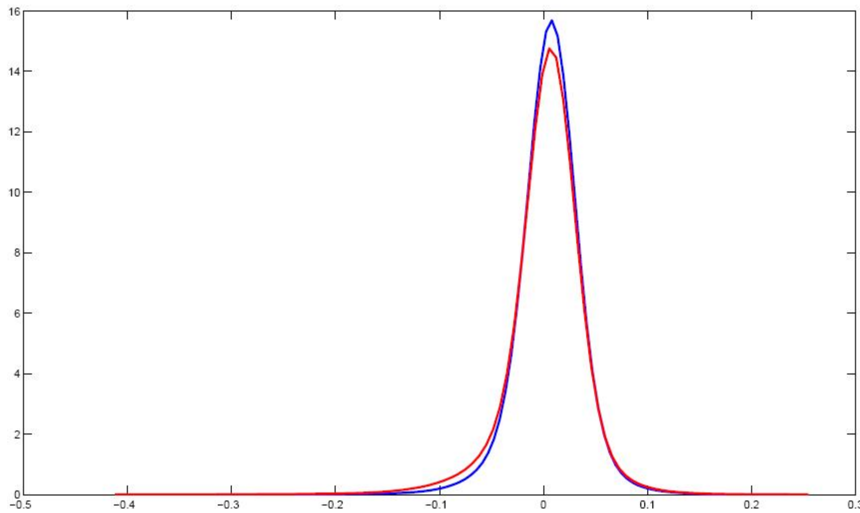

FIGURE 2. Risk Neutral (red) and Real World (blue) Kernel Densities for Convertible Arbitrage (Unsmoothed case)

- the risk neutral evolution of the CFO portfolio NAV, which depends on the temporal behaviour of all its underlying hedge funds;
- the structural features of the CFO such as Over-Collateralization test, priority of payment waterfall (which includes all coupon payments), equity distribution rules, liquidity profile and so on.

To compute the collateral portfolio NAV under the selected risk neutral probability measure at each time of control t, it is necessary to model the joint risk neutral evolution of the underlying hedge funds. At the same time we have to consider any anticipated payments, Over-Collateralization test and the CFO liquidity profile. As we have already mentioned, CFOs equity securities and notes are different types of investment in the underlying pool of hedge funds. In this section we price debt and equity securities of a CFO as options written on a basket of hedge funds. In particular, we price equity and debt tranches for a theoretical CFO using a sort of *structural firm value approach*. In fact, CFOs can be seen as firms with a fixed maturity (if we do not consider default). We use a Merton-type model [19] and a Black-Cox-type model [4], where we assume that the hedge fund NAV processes are described by dependent Geometric Variance Gamma processes under the real world probability measure. Default can be triggered either by the fact that the CFO Net Asset Value at maturity is too low to cover the promised debt payment, as in the traditional Merton's model, or by the violation of an Over Collateralization Test, which represents a barrier, as in the traditional Black-Cox's model. In the second case default before the scheduled maturity is possible.
While the CFO collateral is the same in all our applications, the covenants of the CFO constitutive document change. First, we price a very simple CFO, in which its liability side is represented only by zero coupon bonds with different priorities and an equity tranche. Second, we consider a CFO structure in which liabilities are represented by different coupon bonds and a paying dividend equity tranche. In both cases, we assume that default can happen only at maturity. Then, we

introduce the possibility of default before the CFO maturity and consider the CFO liquidity profile.

6.1. **Applications and Results.** Consider the following theoretical Collateralized Fund of Hedge Funds Obligations structure with a scheduled maturity $T = 5$ years.

- Asset Side: fund of hedge funds with a current price of 1000 monetary units;
- Liability Side: debt and equity with total initial investment of 1000 monetary units;
  1. Debt tranche A with nominal value of 570 monetary units;
  2. Debt tranche B with nominal value of 150 monetary units;
  3. Debt tranche C with nominal value of 100 monetary units;
  4. Equity tranche with nominal value of 180 monetary units.

The most senior tranche in this CFO is tranche A and is credit enhanced due to the subordination of the lower tranches. This means that the lowest tranche, the equity tranche, absorbs losses first. When this tranche is exhausted, the next lowest tranche, i.e tranche C, begins absorbing losses. If tranche C is consumed, tranche B starts absorbing losses. Finally, only if tranche B is completely dissipated then tranche A is exposed to losses.

Now, we report the collateral portfolio composition and the amounts invested at time 0 in each underlying hedge funds:

1. Convertible Arbitrage: 175 monetary units;
2. Dedicated Short Bias: 50 monetary units;
3. Emerging Markets: 50 monetary units;
4. Equity Market Neutral: 250 monetary units;
5. Event Driven: 100 monetary units;
6. ED Distressed: 50 monetary units
7. ED Multi-Strategy: 100 monetary units;
8. ED Risk Arbitrage: 225 monetary units.

Finally, assume the existence of a risk free asset with a constant annual log-return $r = 4\%$.

These features are common to all the CFOs we price.

6.2. **First CFO: pricing and sensitivity analyses.** The distinctive characteristics of this CFO are the followings:

1. Debt tranche A is a zero coupon bond with a promised maturity payment $D_T^A = 696,20$, with an implicit promised annual log-return of $r = 4\%$;
2. Debt tranche B is a zero coupon bond with a promised final payment of $D_T^B = 183,67$, with $r = 4,05\%$;
3. Debt tranche C is a zero coupon bond with a promised payment $D_T^C = 125,23$, with $r = 4,5\%$;
4. Equity tranche is a stock that pays no dividends.

Default is only possible at the CFO scheduled maturity date if the value of the collateral pool of hedge funds is not sufficient to pay the liabilities. Note that it is not very precise to talk of default at the maturity for a CFO. In fact, even if CFOs have a capital structure similar to firms, at the fixed maturity date, they always cease to exist. Instead, it makes sense to talk of CFOs default prior to their scheduled maturity date.

In this simple case, each tranche can be expressed as a European option on the collateral portfolio. In particular, the equity tranche is a European call option on the pool of hedge funds with strike price $D_T = D_T^A + D_T^B + D_T^C$ and maturity T. Its intrinsic value is given by:

$$E_0 = \exp(-rT)E^{Q_h}\left[\max(F_T - D_T;\ 0)\right] \tag{6.1}$$

where $F_T = \sum_{j=1}^{8} F_T^j$ is the value of the collateral portfolio at time T.
Current fair prices of tranches $A,\ B,\ C$ are given by the following expressions:

$$A_0 = \exp(-rT)E^{Q_h}\left[D_T^A - \max(D_T^A - F_T;\ 0)\right] \tag{6.2}$$

$$B_0 = \exp(-rT)E^{Q_h}\left[\max(F_T - D_T^A;\ 0) - \max(F_T - (D_T^A + D_T^B);\ 0)\right] \tag{6.3}$$

$$C_0 = \exp(-rT)E^{Q_h}\left[\max(F_T - (D_T^A + D_T^B);\ 0) - \max(F_T - D_T;\ 0)\right] \tag{6.4}$$

To compute fair prices we perform the following steps:

- we simulate several times (50000) the Nav of the collateral portfolio at the maturity $T = 5$, under the Esscher risk neutral probability measure;
- we compute the mean payoff of every tranche;
- we discount these values with the risk free rate.

Tables 12, 13 report CFOs notes and equity fair prices. These tables also show some sensitivity analyses. All the results in Table 12 are based on a value of the common parameter ν equal to 0,3333, while those in Table 13 on $\nu = 0,5833$. In the Benchmark case of Table 12 prices are based on the risk neutral parameters reported in Table 9. Half variances is the scenario under which, preserving the common Gamma parameter value $\nu = 0,3333$ (or $\nu = 0,5833$), all other real parameters are estimated using method of moments, with all the empirical variances divided by two. In the case called Double Variances all empirical variances are multiplied by two, other empirical moments and the common parameter ν are unchanged. Then real world parameters are again estimated by constrained method of moments. Similar considerations hold for the other cases reported on the tables. In each scenario, risk neutral parameters are then computed as previously explained.
Finally, all tables report the minimum value of each thanche and the number of losses based on a simulated sample of 50000 values. These details are not very useful, but we report them only to show how the risk can change under different hypotheses concerning empirical marginal moments.
We conclude that, ceteris paribus,

- if variance increases the equity tranche becomes a more attractive investment opportunity, while the debt becomes riskier and its valuation diminishes. On the contrary, a reduction of the variance results in a decline of the equity fair price, while the debt tranches become more appreciated;
- if negative skewness increase in absolute value the equity tranche is more valued while the price of the debt tranches decreases;
- building a collateral portfolio with a positive skewness is the best thing a CFO manager can try to do for debt holders but the worst for equity investors;
- if kurtosis increases the theoretical value of the equity tranche increases, while the prices of the notes decrease;

- tranche A is very protected by the structure and only in some extreme and rare scenarios (trajectories) can suffer medium losses. Its fair price is almost always equal by the amount invested. A triple AAA rating for this tranche seems very plausible;
- tranche A has a fair price less than its initial invested amount if and only if two risks are high at the same time. Examples are high kurtosis and big variance, high variance and high negative skewness, or high negative skewness and big kurtosis;
- tranche B has also a good protection, but not at the same level of tranche A;
- tranche C is the most risky among debt tranches. Its fair price is often less then the initial investment;
- notice that the model can be used to infer final promised payments, i.e. the promised rates of return, to make the price of each debt tranche fair;
- to sum up: as risk increases equity holders take advantage over debt investors.

Our main results are summarized in Table 5.

6.3. **Second CFO: pricing and sensitivity analysis.** The distinctive features of this CFO are the following:

(1) Debt tranche A is a coupon bond with an annual cash flow of $23,26$, i.e. the coupon rate is $c = 4\%$;
(2) Debt tranche B is a coupon bond with an annual cash flow of $6,20$, i.e. the coupon rate is $c = 4,05\%$;
(3) Debt tranche C is a coupon bond with an annual cash flow of $4,60$, i.e. the coupon rate is $c = 4,5\%$;
(4) Equity tranche is a stock that pays dividends computed as a given percentage of the annual net profit. Notice that the dividend payment at the end of a year is not sure. Only if the NAV of the collateral portfolio at the end of a year is greater than 1000 after the payment of coupons to bondholders, a portion of the profits is distributed. In particular, we consider three different hypotheses concerning the equity distribution rule: 0%, 50%, 100% of annual net profit.

These differences influence the simulation procedure. In the previous case, it was sufficient to simulate directly the value of the collateral portfolio at the CFO maturity. Now, we have to simulate the NAV at the end of every year until time T, to take into account jumps due to coupon payments and possible dividend payments. It is assumed that every payment is made through the liquidation of a part of the collateral portfolio. In particular, we suppose that a part of each hedge fund, proportional to its NAV at the payment date, is sold. Implicitly, we assume that the CFO has enough liquidity to pay coupons and dividends.
Tables 14 and 15 show fair prices and some sensitivity analyses. All previous observations still hold. However, in this case we can analyse the impact on fair prices of different equity distribution rules. In particular, the following observations can be made:

- if the dividend increases then equity fair price increases of an amount approximately equal to the value lost by lower debt tranches. Especially, the dividend policy has a direct impact on the price of equity and C tranches.

Tranche A is unaffected by a change in the equity distribution rule. Tranche B is only marginally influenced in extremely risky situations;
- dividend policy relevance is strictly linked to the degree of risk of the collateral portfolio. Specifically, the greater the risk is more relevant the impact of a change on the portion of net profits distributed on fair prices is. On the contrary, when the collateral pool is made up by positively skewed hedge funds, the dividend policy seems to be irrelevant.

The above results are summarized in Table 6.

6.4. **Third CFO: pricing and sensitivity analysis.** The third and the second CFO have the same liability structure. However, now we take into account the possibility of default prior to maturity and CFO liquidity profile. Tables 16 and 17 show the price of each CFO tranche computed under different equity distribution rules and using two different models to describe the physical evolution of hedge funds log-returns:

(1) Multivariate Brownian Motion;
(2) Multivariate Variance Gamma Process with independent underlying Brownian Motions.

Tables 16 and 17 report prices based respectively on observed and unsmoothed data.
In the simulation procedure we consider a barrier equal to 1,05 times the total nominal value of the debt tranches. If the NAV of the fund of hedge funds falls below this level, when its value is checked by the CFO manager, then the collateral portfolio will be sold in order to redeem the rated notes. In the event of default, we model the sale of the assets by assuming this simple liquidity profile:

- 30% after three months;
- 30% after six months;
- all the residual collateral portfolio value after nine months.

If default happens six months before CFO legal maturity the liquidity profile will be the following:

- 30% after three months;
- all the residual collateral portfolio value at the maturity.

If default occurs three months before CFO legal maturity, the liquidity profile will be 100% of the NAV at maturity. For simplicity, we assume that hedge funds are liquidated proportionally to their NAV. In the default event, tranche A is redeemed first. In particular, we assume that both capital and current coupon have to be paid. Then, tranche B has to be repaid in the same way and so on. The CFO manager usually makes the Over Collateralization test on a monthly basis. However, it can happen that to do all the necessary operations, more time is needed. For pratical reasons, we simulate portfolio NAV and make Over collateralization test every three months. Finally, we assume the existence of an initial lock out period of two years. This implies that redemptions before two years are not admitted.
Fair prices reported in these tables allow us to make the following observations:

- barriers destroy value for all tranches;
- the intrinsic value of the equity tranche is slightly influenced by CFO dividend policy. Notice that the introduction of a barrier modifies the sign

of the relation between equity price and the percentage of net profits distributed;
- when the percentage of net profits distributed increases debt tranches values decrease; this effect is especially relevant for tranche C;
- debt tranche prices are strongly affected by the type of data (smoothed or unsmoothed) used to estimate the model and by the choice of the model.

Tables 18 and 19 report some sensitivity analyses. The main results can be summarized as follows:

- the higher the barrier is the greater the value destroyed is in terms of fair prices;
- the higher the risk is the bigger the negative impact on debt tranches theoretical prices is;
- as the barrier increases equity price tends to become independent with respect to risk. Without barrier, the value of equity tranche increases when risk increases;
- the introduction of a barrier can protect apparently the capital invested by debt holders. Early redemptions force to sell the assets when the price is low and bondholders loses one ore more promised coupon payments;
- as the level of the barrier decreases all tranches becomes more valued and converge to the prices obtained in the case of the second CFO;
- the less risky the collateral portfolio is the faster the speed of convergence of prices towards prices without barrier is. As an example, look at the case opposite skewnesses in all these tables. The barrier seems to be irrelevant.

Finally, Tables 20 and 21 presents fair prices with an annual management fee of 0,5% of the total nominal amount of CFO tranches. The main results of this section are summarized in Table 7.

7. Conclusion and Future Developments

The analysis was performed starting from a simple CFO structure, which was then progressively complicated with the introduction of the structural features we encounter in typical CFOs. In this way, at each step of the evolution of the structure, the reader can understand the impact on the value, measured with respect to the first four moments of the distribution of log-returns, and how this value is divided among the different tranches. The result is a useful scheme that can provide some help in designing a CFO transaction. In particular, we believe the model can be useful for rating agencies as well as for deal structurers, to efficiently evaluate various capital structures, test levels, liquidity profiles, coupons and equity distribution rules. The analysis is also helpful for the CFO manager who usually invests in the equity tranche, because gives him some suggestions on how to increase the value of his investment. In this paper we built a multivariate Lévy process by time changing a Multivariate Brownian motion with independent components with a Gamma subordinator. The main limitation of the model is the lack of flexibility. In particular, we cannot replicate the correlation observed in the market and fit perfectly all the first four moments of the marginal distributions but only three. An investigation in this area is in progress and the results obtained will be documented in some future contributions.

References

[1] J. Alexiev: *The Impact of Higher Moments on Hedge Fund Risk Exposure*, The Journal of Alternative Investments, Spring (2005), pp. 50-64.

[2] G. Amin, H. Kat: *Welcome to the Dark Side: Hedge Fund Attrition and Survivorship Bias 1994-2001*, Working Paper ISMA Centre, University of Reading (2001).

[3] U. Ansejo, M. Escobar, A. Bergara, L. Seco: *Correlation Breakdown in the Valuation of Collateralized Fund Obligations*, The Journal of Alternative Investments, **9(3)** (2006), pp. 77-88.

[4] F. Black, J. Cox: *Valuing Corporate Securities: Some Effects on Bond Indenture Provisions*, Journal of Finance,**31** (1976), pp. 351-367.

[5] C. Brooks, H. M. Kat: *The Statistical Properties of Hedge Fund Index Returns and Their Implications For Investors*, Journal of Alternative Investments, **5** (2002), pp. 26-44.

[6] R. Cont, P. Tankov: *Financial Modeling with Jump Processes*, Chapman&Hall/CRC, London(2004).

[7] D. Geltner: *Smoothing in Appraisal-Based Returns*, Journal of Real Estate Finance and Economics, **4 (3)** (1991), pp. 327-345.

[8] D. Geltner: *Estimating Market Values from Appraised Values without Assuming an Efficient Market*, Journal of Real Estate Research, **8 (3)** (1993), pp. 325-345.

[9] J. Clayton, D. Geltner, S. W. Hamilton: *Smoothing in Commercial Property Valuations: Evidence from Individual Appraisals*, Real Estate Economics, **29 (3)** (2001), pp. 337-360.

[10] D. Geltner: *Appraisal Smoothing: The Other Side of the Story. A Comment*, Working Paper, Department of Finance, University of Cincinnati, (1998).

[11] H. Geman, D. Madan, M. Yor: *Time Changes for Lévy Processes*, Mathematical Finance, 11 (1) (2001), pp. 79-96.

[12] H. U. Gerber, E. S. W. Shiu: *Option Pricing by Esscher Transforms*, Transactions of the Society of Actuaries , **46** (1994), pp. 99-144.

[13] H. M. Kat, S. Lu: *An Excursion into the Statistical Properties of Hedge Fund Returns*, Working Paper Alternative Investment Research Centre n.16, (2002).

[14] E. Luciano, W. Shoutens: *A Multivariate Jump-Driven Asset Pricing Model*, Quantitative Finance, **6 (5)** (2006), pp. 385-402.

[15] D. Madan, E. Seneta: *The Variance Gamma Model for Share Market Returns*, Journal of Business, **63 (4)** (1990), pp. 511-524.

[16] D. Madan, F. Milne: *Option Pricing with VG Martingale Components*, Mathematical Finance, **1** (1991), pp. 39-55.

[17] D. Madan, P. Carr, E. Chang: *The Variance Gamma Process and Option Pricing*, European Finance Review, **2** (1998), pp. 79-105.

[18] S. Mahadevan, D. Schwartz: *Hedge Fund Collateralized Fund Obligations*, The Journal of Alternative Investment, **5 (2)** (2002), pp. 45-62.

[19] R. Merton: *On the Pricing of Corporate Debt: the Risk Structure of Interest Rates*, Journal of Finance **29**, (1974), pp. 449-470.

[20] J. Missinhoun, L. Chacowry: *Collateralied Fund Obligations: The Value of Investing in the Equity Tranche*, The Journal of Structured Finance, **10 (4)** (2005), pp. 32-36

[21] Moody's Investors Service: *RMF Four Seasons CFO Ltd. Collateralized Fund of Hedge Fund Obligations*, International Stuctured Finance (Pre-Sale Report), 31 August (2006).

[22] Moody's Investors Service: *CFO Premium Limited. Collateralized Fund of Hedge Fund Obligations*, International Stuctured Finance (Pre-Sale Report), 16 October (2006).

[23] Moody's Investors Service: *Sciens CFO I Limited. Collateralized Fund of Hedge Fund Obligations*, International Stuctured Finance (New Issue Report), 5 January (2007).

[24] Moody's Investors Service: *Syracuse Funding EUR Limited Collateralised Fund of Hedge Funds Obligations*, International Stuctured Finance (Pre-Sale Report), 22 February (2008).

[25] Moody's Investors Service: *Assigning Unsecured Credit Rating To Hedge Funds*, Special Comment, April (2007).

[26] D. P. Morton, E. Papova, I. Papova: *Efficient Fund of Hedge Funds Construction Under Downside Risk Measures*, Journal of Banking & Finance, **30** (2006), pp. 503-518.

[27] K. Prause: *The Generalized Hyperbolic Model: Estimation, Financial Derivatives, and Risk Measures*, Phd Dissertation, University of Freiburg (1999).

[28] C. A. Stone, A. Zissu: *Fund of Fund Securitization*, Journal of Derivatives, **11 (4)** (2004), pp. 62-68.

[29] G. L. Tassinari: *Pricing Equity and Debt Tranches of Collateralized Funds of Hedge Fund Obligations*, Phd Dissertation, University of Bergamo, (2009).

[30] J. Tavakoli: *Ultimate Leverage. Collateralized Fund Obligations*, HedgeWord's Accredited Investor, June (2004).

[31] G. Witt, Y. Fu, J. Leahy: *Moody's Approach to Rating Collateralized Funds of Hedge Fund Obligations*, Moody's Investors Service, 10 June (2003).

Table 5.
Results for CFO 1

Hedge Funds	EQUITY TRANCHE	ZCB A TRANCHE	ZCB B TRANCHE	ZCB C TRANCHE
Variance $\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Variance $\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Skewness $\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Skewness $\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Kurtosis $\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Kurtosis $\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$

Table 6.
Results for CFO 2

Hedge Funds	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
Variance $\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Variance $\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Skewness $\uparrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Skewness $\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Kurtosis $\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Kurtosis $\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Dividend $\uparrow$	$\uparrow$	$\downarrow or \perp$	$\downarrow$	$\downarrow$
Dividend $\downarrow$	$\downarrow$	$\uparrow or \perp$	$\uparrow$	$\uparrow$

Table 7.
Results for CFO 3

Hedge Funds	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
Variance $\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Variance $\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Skewness $\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
Skewness $\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Kurtosis $\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$
Kurtosis $\downarrow$	$\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$
Barrier $\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Barrier $\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
Dividend $\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
Dividend $\downarrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
Fees $\uparrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$

TABLE 8.
(Smoothed) Real World Parameters

Index	μ_j	θ_j	σ_j	ν
Convertible Arbitrage	0,09318	-0,02330	0,04590	0,33333
Dedicated Short Bias	-0,05208	0,02691	0,16397	0,33333
Emerging Markets	0,13886	-0,05419	0,15268	0,33333
Equity Market Neutral	0,08316	0,00281	0,02647	0,33333
Event Driven	0,17030	-0,07013	0,03866	0,33333
ED Distressed	0,17588	-0,06401	0,04969	0,33333
ED Multi-Strategy	0,14482	-0,05025	0,05321	0,33333
ED Risk Arbitrage	0,08215	-0,01534	0,03925	0,33333

TABLE 9.
(Smoothed) Risk Neutral Parameters

Index	μ_j	$\theta_j^{Q_h}$	$\sigma_j^{Q_h}$	ν
Convertible Arbitrage	0,09318	-0,05559	0,06214	0,33333
Dedicated Short Bias	-0,05208	0,06605	0,22197	0,33333
Emerging Markets	0,13886	-0,12187	0,20668	0,33333
Equity Market Neutral	0,08316	-0,04412	0,03584	0,33333
Event Driven	0,17030	-0,13454	0,05233	0,33333
ED Distressed	0,17588	-0,14126	0,06726	0,33333
ED Multi-Strategy	0,14482	-0,10927	0,07204	0,33333
ED Risk Arbitrage	0,08215	-0,04386	0,05313	0,33333

TABLE 10.
(Unsmoothed) Real World Parameters

Index	μ_j	θ_j	σ_j	ν
Convertible Arbitrage	0,09668	-0,02685	0,07974	0,33333
Dedicated Short Bias	-0,05341	0,02913	0,18126	0,33333
Emerging Markets	0,16393	-0,08836	0,19764	0,33333
Equity Market Neutral	0,08424	0,00257	0,03326	0,33333
Event Driven	0,20534	-0,10811	0,03994	0,33333
ED Distressed	0,20328	-0,09448	0,06129	0,33333
ED Multi-Strategy	0,15701	-0,06522	0,06800	0,33333
ED Risk Arbitrage	0,08382	-0,01723	0,04935	0,33333

Table 11.
(Unsmoothed) Risk Neutral Parameters

Index	μ_j	$\theta_j^{Q_h}$	$\sigma_j^{Q_h}$	ν
Convertible Arbitrage	0,09668	-0,06227	0,10046	0,33333
Dedicated Short Bias	-0,05341	0,06589	0,22837	0,33333
Emerging Markets	0,16393	-0,15753	0,24900	0,33333
Equity Market Neutral	0,08424	-0,04544	0,04190	0,33333
Event Driven	0,20534	-0,17125	0,05032	0,33333
ED Distressed	0,20328	-0,17079	0,07722	0,33333
ED Multi-Strategy	0,15701	-0,12300	0,08567	0,33333
ED Risk Arbitrage	0,08382	-0,04608	0,06218	0,33333

Table 12.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Equity and Three Zero Coupon Bonds
(Smoothed Data)

Fund of Hedge funds	EQUITY TRANCHE	ZCB A TRANCHE	ZCB B TRANCHE	ZCB C TRANCHE
Benchmark $\nu = 0,3333$				
Prices	178,641	570	150,281	101,078
Minimum	0	551,879	0	0
Num. Losses	25106	1	138	1749
Half Variances				
Prices	177,754	570	150,353	101,894
Minimum	0	570	33,488	0
Num. Losses	25044	0	43	905
Double Variances				
Prices	180,778	569,995	149,973	99,254
Minimum	0	478,205	0	0
Num. Losses	25427	7	509	3348
Double Skewnesses				
Prices	179,373	569,999	150,196	100,433
Minimum	0	532,191	0	0
Num. Losses	25206	3	240	2361
Opposite Skewnesses				
Prices	177,093	570	150,375	102,531
Minimum	0	570	150,375	68,888
Num. Losses	25958	0	0	2

TABLE 13.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Equity and Three Zero Coupon Bonds
(Smoothed Data)

Fund of Hedge funds	EQUITY TRANCHE	ZCB A TRANCHE	ZCB B TRANCHE	ZCB C TRANCHE
Benchmark $\nu = 0,5833$				
Prices	179,968	569,998	150,102	99,932
Minimum	0	507,777	0	0
Num. Losses	24659	6	356	2734
Half Variances				
Prices	178,668	570	150,273	101,059
Minimum	0	570	30,955	0
Num. Losses	24544	0	157	1709
Double Variances				
Prices	182,514	569,984	149,556	97,945
Minimum	0	496,739	0	0
Num. Losses	24930	32	854	4237
Double Skewnesses				
Prices	181,261	569,993	149,851	98,895
Minimum	0	527,943	0	0
Num. Losses	24737	15	600	3545
Opposite Skewnesses				
Prices	177,336	570	150,374	102,29
Minimum	0	570	106,936	0
Num. Losses	24830	0	4	386

Table 14.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Paying Dividend Equity and Three Coupon Bonds
(Smoothed Data)

Fund of Hedge funds	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
Benchmark $\nu = 0,3333$				
Prices (0% Div.)	178,339	570	150,284	101,304
Prices (50% Div.)	178,439	570	150,276	101,165
Prices (100% Div.)	178,623	570	150,264	100,978
Half Variances				
Prices (0% Div.)	177,685	570	150,336	101,914
Prices (50% Div.)	177,707	570	150,334	101,86
Prices (100% Div.)	177,773	570	150,332	101,785
Double Variances				
Prices (0% Div.)	180,058	569,997	150,008	99,852
Prices (50% Div.)	180,395	569,997	149,969	99,491
Prices (100% Div.)	180,914	569,997	149,915	99,004
Double Skewnesses				
Prices (0% Div.)	178,906	570	150,212	100,799
Prices (50% Div.)	179,078	570	150,196	100,587
Prices (100% Div.)	179,386	570	150,173	100,285
Opposite Skewnesses				
Prices (0% Div.)	177,308	570	150,347	102,317
Prices (50% Div.)	177,298	570	150,347	102,317
Prices (100% Div.)	177,295	570	150,347	102,317

TABLE 15.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Paying Dividend Equity and Three Coupon Bonds
(Smoothed Data)

Fund of Hedge funds	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
Benchmark $\nu = 0,5833$				
Prices (0% Div.)	179,249	569,998	150,117	100,537
Prices (50% Div.)	179,545	569,998	150,095	100,239
Prices (100% Div.)	179,927	569,998	150,061	99,881
Half Variances				
Prices (0% Div.)	178,296	570	150,265	101,352
Prices (50% Div.)	178,439	570	150,259	101,196
Prices (100% Div.)	178,644	570	150,250	100,993
Double Variances				
Prices (0% Div.)	181,294	569,980	149,691	98,921
Prices (50% Div.)	181,924	569,978	149,609	98,345
Prices (100% Div.)	182,729	569,977	149,498	97,637
Double Skewnesses				
Prices (0% Div.)	180,328	569,994	149,907	99,655
Prices (50% Div.)	180,821	569,994	149,856	99,187
Prices (100% Div.)	181,433	569,994	149,786	98,634
Opposite Skewnesses				
Prices (0% Div.)	177,413	570	150,345	102,188
Prices (50% Div.)	177,415	570	150,345	102,174
Prices (100% Div.)	177,432	570	150,345	102,153

TABLE 16.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Paying Dividend Equity and Three Coupon Bonds
Models with barrier 105% (Smoothed Data)

Fund of Hedge funds	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
M.G.B.M.				
Prices (0% Div.)	177,265	569,974	150,229	102,022
Prices (50% Div.)	177,282	569,974	150,226	102,014
Prices (100% Div.)	177,274	569,972	150,212	101,979
Model 1				
Prices (0% Div.)	176,547	568,242	148,111	96,218
Prices (50% Div.)	176,443	568,073	147,725	95,291
Prices (100% Div.)	176,309	567,800	147,131	93,927

Table 17.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Paying Dividend Equity and Three Coupon Bonds
Models with barrier 105% (Unsmoothed Data)

Fund of Hedge funds	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
M.G.B.M.				
Prices (0% Div.)	176,700	569,529	149,266	99,487
Prices (50% Div.)	176,576	569,475	148,910	98,769
Prices (100% Div.)	176,504	569,069	148,877	98,459
Model 1				
Prices (0% Div.)	176,673	566,882	146,735	92,738
Prices (50% Div.)	176,557	565,982	146,010	91,083
Prices (100% Div.)	176,395	565,982	145,053	88,835

Table 18.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Paying (50%) Dividend Equity and Three Coupon Bonds
CFO tranche prices with barriers and liquidity profile
(Smoothed Data)

Collateral NAV and Debt Ratio	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
Benchmark $\nu = 0,3333$				
(105%)	176,443	568,073	147,725	95,291
(100%)	178,141	569,357	148,410	99,280
(95%)	178,338	569,806	149,351	100,854
Half Variances				
(105%)	176,564	569,054	148,952	98,461
(100%)	177,581	569,769	149,523	101,052
(95%)	177,654	569,943	149,996	101,774
Double Variances				
(105%)	176,637	566,035	145,380	89,697
(100%)	179,654	568,250	145,899	95,418
(95%)	180,201	569,318	147,379	98,400
Double Skewnesses				
(105%)	176,364	567,251	146,836	93,037
(100%)	178,572	568,967	147,479	97,770
(95%)	178,920	569,650	148,690	100,004
Opposite Skewnesses				
(105%)	177,289	569,996	150,330	102,279
(100%)	177,296	570,000	150,347	102,317
(95%)	177,296	570,000	150,347	102,317

Table 19.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Paying (50%) Dividend Equity and Three Coupon Bonds
CFO tranche prices with barriers and liquidity profile
(Smoothed Data)

Collateral NAV and Debt Ratio	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
Benchmark $\nu = 0,5833$				
(105%)	177,099	567,153	146,950	93,704
(100%)	179,021	568,746	147,292	97,667
(95%)	179,352	569,481	148,270	99,699
Half Variances				
(105%)	176,785	568,004	147,641	95,388
(100%)	178,376	569,270	148,302	99,175
(95%)	178,557	569,747	149,153	100,676
Double Variances				
(105%)	178,000	564,900	143,755	87,171
(100%)	181,243	567,363	144,095	93,077
(95%)	182,021	568,679	145,582	96,579
Double Skewnesses				
(105%)	177,462	565,697	144,801	89,175
(100%)	180,335	567,921	145,285	94,742
(95%)	180,905	569,035	146,700	97,787
Opposite Skewnesses				
(105%)	177,007	569,584	149,671	100,506
(100%)	177,442	569,920	149,987	101,792
(95%)	177,460	569,984	150,220	102,104

Table 20.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Paying (50%) Dividend Equity and Three Coupon Bonds
CFO tranche prices with barrier (105%) and management fees
(Smoothed Data)

MODEL	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
M.G.B.Motion				
Prices with fees	154,977	569,912	149,994	101,439
(Prices with no fees)	(177,282)	(569,974)	(150,226)	(102,014)
Model 1 $\nu = 0,333$				
Prices with fees	154,894	567,517	146,788	92,873
(Prices with no fees)	(176,443)	(568,073)	(147,725)	(95,291)

Table 21.
Asset Side 1000: Fund of Hedge Funds
Liability Side 1000: Paying (50%) Dividend Equity and Three Coupon Bonds
CFO tranche prices with barrier (105%) and management fees
(Unsmoothed Data)

MODEL	EQUITY TRANCHE	CB A TRANCHE	CB B TRANCHE	CB C TRANCHE
M.G.B.Motion				
Prices with fees	154,317	569,185	148,217	97,010
(Prices with no fees)	(176,700)	(569,529)	(149,266)	(99,487)
Model 1 $\nu = 0,333$				
Prices with fees	155,638	565,833	144,838	88,181
(Prices with no fees)	(176,557)	(566,552)	(146,010)	(91,083)