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non-renewable to renewable resources:
dirty vs clean energy sources and the
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On the optimal timing of switching from non-renewable to renewable resources: dirty vs clean energy sources and the relative efficiency of generators

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Abstract

We develop a model on the optimal timing of switching from non-renewable to renewable energy sources with endogenous extraction choices under emission taxes and abatement costs. We assume that non-renewable resources are "dirty" inputs and create environmental degradation, while renewable resources are more environmentally friendly, although they may be more or less productive than the exhaustible resources. The value of the switching option from non-renewable to renewable resources is characterized. Numerical applications show that an increase in emission taxes, abatement costs or demand elasticity slows down the adoption of substitutable renewable resources, while an increase in the natural rate of resource regeneration, the stock of renewable resources or the relative productivity parameter speeds up the investment in the green technology.

Key words: Non-renewable resources; Renewable resources; Environmentally friendly technologies; Abatement costs; Taxes; Optimal switching time; Real options

JEL Classification: D81; H23; Q28; Q38; Q40; Q50;

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1 Introduction

Today's primary energy sources are mainly non-renewable: natural gas, oil, coal and conventional nuclear power. There are also renewable sources, including biomass, moving water, geothermal, solar, tidal, wind, and wave energy. In general, all the various energy sources can contribute to the future energy mix worldwide. But each has its own direct influence on health, welfare, food security, climate change and other factors in the improvement of living conditions and the quality of life for the world's population.

In terms of pollution risks, natural gas is by far the cleanest of all the fossil fuels, with oil next, and coal is the most polluting one. But they all pose three interrelated pollution problems: global warming, urban industrial air pollution, and acidification of the environment. Some of the wealthier industrial countries may possess the economic capacity to cope with such threats. Most developing countries do not.

On the other side, modern renewable energy systems are estimated to have the technical potential to provide all global energy services in sustainable ways and with low or virtually zero GHG emissions. For example, the total solar energy absorbed by earth's atmosphere, oceans and land masses is approximately 4 Million EJ (exajoules) per year. The amount of solar energy reaching the surface of the planet is so vast that in one year it is about twice as much as will ever be obtained from all of the earth's non-renewable resources of oil, natural gas, and mined coal combined. There are some disadvantages, which are linked to the physical properties of natural resources and to the process of converting natural resources into electricity. One limit to the solar energy production is that solar energy is not available at night. Other limits include the reliance on weather patterns to generate electricity and a lack of space for solar cells in areas of high demand, such as cities. Like the production with non-renewable resources, the activity of extracting and converting renewable energies into electricity can be subject to capacity constraints. Indeed, there is a maximum rate at which energy can be extracted from the flowing water and this rate is related to the kinetic energy flux in the water turbines. In wind energy production, the Betz Limit states that no wind turbine can convert more than about 60% of the kinetic energy of the wind into mechanical energy turning a rotor¹.

And yet there is an increasing demand for sustainable energy services. The planners of the "2012 International Year of Sustainable Energy for All"² have set three main objectives to be achieved in the next twenty years: (i) to ensure access to everyone to sustainable energy (also known as modern energy) services; (ii) to double the rate of improvement in energy efficiency; and (iii) to make 30% of the world's energy renewable. The goal is the development and promotion of strategies, commitments and activities to encourage a programme of switching to alternative renewable and non-polluting energy sources.

¹The effects of capacity constraints on the optimal rates of extraction of natural (but exhaustible) resources is studied in Ghoddusi (2010) and Mason (2001).

²See the United Nations General Assembly Resolution A/RES/65/151, http://www.un.org/ga/search/view_doc.asp?symbol=A/RES/65/151.

The focus of this paper is on the optimal timing of switching from non-renewable to renewable resources and the role of emission taxes and abatement costs. We assume that non-renewable resources are "dirty" inputs and create environmental degradation, while renewable resources are more environmentally friendly, although they may be more or less productive than the exhaustible resources. The setup is a real option model where the optimal extraction rates of renewable and non-renewable natural resources are determined by solving the firm's profit maximization under emission taxes and abatement costs. Our work endogenously takes into account the level of emissions before and after the adoption of the renewable resource. The firm solves an optimal stopping problem in order to find the critical threshold of switching from an exhaustible polluting input to a low carbon renewable resource. Closed form solutions for the optimal switching timing and the value of the option to switch, are found. In the numerical section we test our theoretical model using real data. The numerical applications show that an increase in emission taxes, abatement costs or demand elasticity slows down the adoption of alternative renewable resources, while an increase in the natural rate of resource regeneration, the stock of the renewable resource or the relative efficiency parameter speeds up the investment in the green firm.

Our paper improves on the current literature in several dimensions. To the best of our knowledge, this is the first paper where the emission tax rate and the cost of carbon abatement are embedded in a real option framework about the timing of switching to renewable resources. Moreover, in our model the renewable resource can be more or less efficient than the non-renewable resource. This feature is most relevant in practical applications. In electricity generation an electric generator is a device that converts energy inputs into electrical energy. Converting fossil fuels into electricity can be more or less efficient than converting renewable inputs into electricity. For example, converting biomass into electricity is less efficient than converting oil into electricity (see Mosiño, 2012), while depending on the water turbines converting hydro energies into electricity can be more or less efficient than converting fossil fuels into electricity. In general, large hydro power stations are by far the most efficient method of large scale electric power generation. Our main results suggest that the option to switch will be exercised, depending on values of the unit abatement cost and on the relative productivity parameter. The data used for the numerical applications are in keeping with the empirical evidence and show that our results are robust to the modelling and to various relevant parameters.

The paper is organized as follows. Section 2 provides a literature review. Section 3 describes the setup of the model and characterizes the case when the energy producer uses a non-renewable resource (Proposition 1) or a renewable resource (Proposition 2). Section 4 solves the optimal stopping problem of the energy producer and finds the value of the switching option and the optimal switching time (Proposition 3). Numerical results are presented in Section 5. In particular, a detailed sensitivity analysis is shown as to deepen our understanding of the effects of emission taxes, unit abatement costs, natural resource regeneration rates and the stock of renewable resources on the optimal timing

of switching from non-renewable to renewable resources and on the firm's value. Section 6 concludes the paper.

2 Literature review

There have been many studies on effective policy solutions to environmental degradation based on approaches that differ from the present paper. Acemoglu et al. (2012) develop a two-sector model of directed technical change to study the effects of environmental policies on different types of technologies. The unique final good is produced by combining the inputs produced by the two sectors (clean and dirty inputs). They characterize the structure of equilibria and the dynamic tax/subsidy policies that achieve sustainable growth or maximize intertemporal welfare. Their main results focus on the types of policies that can prevent environmental disasters, the structure of optimal environmental regulation and its long-run growth implications and the costs of delay in implementing environmental regulation. In particular, they show that without intervention, the economy would rapidly head towards an environmental disaster, while the use of carbon taxes or research subsidies would be sufficient to (re)direct technical change toward the clean sector and avoid environmental disasters. Chakravorty et al. (2012) study how environmental policy in the form of a cap on aggregate emissions from a fossil fuel interacts with the arrival of a clean substitute (e.g., solar energy). They show that the price of energy under such a target may exhibit cyclical trends driven by scarcity of the non-renewable resource, the effect of environmental constraints and potential cost reductions in the clean technology. The seminal work by Dasgupta and Heal (1974, 1979) and Dasgupta and Stiglitz (1981) examine the effects of economic and technological uncertainties on the dynamics of non-renewable resource depletion. Pindyck (1980) studies the effects of two sources of uncertainty (of the future demand and the resource reserves) on the market-price evolution, the optimality of the competitive market and the value of exploration. Pindyck (1984) examines the implication of ecological uncertainty on the optimal extraction rate of a renewable resource. In Pindyck (2000, 2002) the effects of irreversibilities and uncertainties on the optimal timing of adoption of policies for emission reductions are analyzed.

Our paper employs real option methodology. There are important reasons to use real options when dealing with the choice of policy options for emission reductions. Some papers have employed a cost-benefit analysis, where the social planner maximizes expected benefits from some policy to calculate the optimal level of current abatement of greenhouse gases, and hence the current social cost of carbon. However, as pointed out by Pindyck (2000, 2002), standard cost-benefit analysis fails to simultaneously capture the irreversibilities and uncertainties about climate change and environmental policy intervention. Real options are also the appropriate methodology to incorporate the role of timing in adoption of alternative renewable energies. Recently, a number of studies in the real options literature have examined the implications of irreversibility and

uncertainty for the optimal timing problem in environmental economics in different contexts (see, for example, Conrad, 1997, 2000; Saphores and Carr, 2000; Xepapadeas, 2001; Insley, 2003; Requate, 2005; Van Soest, 2005; Wirl, 2006; Ohyama and Tsujimura, 2006; Kumbaroğlu et al., 2008; Nishide and Ohyama, 2009; Balıkcıoğlu et al., 2011; Travaglini and Saltari, 2011; Agliardi and Sereno, 2011, 2012). However, none of them deal with the case of non-renewable vs renewable resources.

In our paper we study the effects of emission taxes and abatement costs as an instrument to accelerate the switching to renewable and clean resources. Also other papers have dealt with the impact of environmental taxes, although in a different context. Van Soest (2005) analyzes the impact of environmental taxes and quotas on the timing of adoption of energy-saving technologies under irreversibility and stochastic arrival rate of the new technologies, and shows that: (i) increased environmental stringency (measured in tax and its equivalent in terms of quota) does not necessarily induce early adoption, and (ii) there is no unambiguous ranking of policy instruments in terms of the length of the adoption lag. While Van Soest (2005) studies the investment decisions of a single firm under environmental regulations that impose sunk costs on the firm, we consider two types of firms: one uses a dirty exhaustible resource and the other uses a low carbon renewable resource and we characterize the optimal timing to adopt the renewable resource in a setting with emission taxes and abatement costs. In Van Soest (2005) innovation is driven by an exogenous jump process, while in our model market demand is stochastic. Agliardi and Sereno (2011, 2012) study the effects of alternative environmental policy options for the reduction of pollution emissions. Their models endogenously take into account the level of emissions before and after the adoption of the new environmental policy. The level of emissions is determined by solving the firm's profit maximization problem under taxes, standards and permits. They find rankings for the adoption of environmental policies in a setting characterized by economic and ecological uncertainties and ambiguity over future costs and benefits over adopting environmental policies. Our model is not concerned with the regulator's behavior about the timing of policy for emission reductions but analyzes the response of firm's investment decisions to changes in environmental regulation, cost of abatement, exhaustibility of resources and the relative efficiency of the energy generators.

Our paper is mainly related to Mosiño (2012) who firstly studies the determinants of switching from non-renewable natural resources to renewable resources in a real option framework. In Mosiño (2012) the stock of both resources is stochastic. In particular, the stock of a renewable resource follows a geometric Brownian motion and the resource's self regeneration function (i.e. the drift component of the process) is a modified version of the logistic equation proposed by P.F. Verhulst in 1838. Since there is no closed form solution for the optimal switching time and the value of the option to switch he provides a numerical solution using the projection method. Differently from Mosiño (2012) we develop a model of optimal timing to switch to renewable energy sources with endogenous extraction choices under emission taxes and abatement costs.

In our model the stock of a renewable resource regenerates naturally following the same differential equation used by Acemoglu et al. (2012) for the modelling of the environmental quality. This seems particularly adequate when valuing investment in hydropower plant. Moreover, this allows us to find a closed form solution for the optimal switching timing and the value of the option to switch. In Mosiño (2012) only renewable resources that are less efficient than fossil fuel are considered, while we consider also the case of renewable resources which are more efficient than fossil fuels. Finally, in our model we endogenously determine the critical level of the non-renewable resource at which it should be abandoned in favour of the renewable resource.

Taschini and Urech (2010) develop a model of real options to evaluate the value of a generation system consisting of a coal-fired and a gas-fired power plant in the presence of expected windfall profits under the EU Emission Trading Scheme. They compute options on spark and dark spreads i.e., the difference between the fuel prices per unit of electricity and the revenues from selling that unit at the market price, but are not concerned with the issues related to the timing of innovation and technical change. Finally, our model is also related to Ghoddusi (2010) who studies the option to expand the capacity of a plant under capacity constraints. However, he does not consider the option to switch to a low carbon renewable resource.

3 The model

Suppose that the firm is currently using a non-renewable source to produce energy but has the option to switch to a renewable (perfect substitute) input. Switching is available at a fixed and irreversible cost $I > 0$. We suppose that, once the irreversible investment is undertaken, there is no incentive to turn back to the use of the non-renewable resources. For example, once the oil platform decides to become a wind farm, almost all of the previously installed capital needs to be dismantled, and very few parts of the old facility can be reutilized.³ The renewable resource can be more or less productive than the fossil fuel. However, the non-renewable energy source is "dirty", while the renewable resource is cleaner, or more environmentally friendly.

In the next subsections we first describe the model and develop the programmes to be solved by the firm when using the non-renewable resource as an input without switching (Section 3.1) or assuming that it already switched to the renewable resource (Section 3.2). Then in Section 4 we analyze the problem when the switching opportunity is taken into account.

³ As Mosiño (2012) pointed out this process is very expensive in terms of time and money, and so is the reverse procedure, making it difficult for the firm even to consider the possibility of switching back.

3.1 Non-renewable and dirty energy sources

A risk-neutral firm is endowed with a deposit of exhaustible natural resources. At time t the firm is assumed to extract $q_{1,t}$ units of the natural resource to produce E_t units of energy according to the following production function:

$$E_t = \xi_{1,t}^\eta (a_1 q_{1,t})^{1-\eta}, \quad 0 < \eta < 1,$$

where $\xi_{1,t}$ is the cumulative GHG emissions related to the activity of energy production and a_1 is the parameter that reflects the efficiency of power plants to convert a fuel into electricity.⁴ No uncertainty is assumed regarding the resources and no exploration costs, therefore the dynamics of resource depletion is given by:

$$dR_{1,t} = -q_{1,t}dt, \quad \text{with } R_{1,0} \text{ given,} \quad (1)$$

where $R_{1,0}(> 0)$ is the initial stock of fuel and $R_{1,t}$ is the residual stock at any time t . There is no storage, so all extracted fuel is instantly used to produce energy.

The spot price of energy at time t is a function of the instantaneous rate of energy production, E_t and an exogenously given random process X_t :

$$\begin{aligned} P_t &= P^{-1}(X_t, E_t) \\ \frac{\partial P^{-1}}{\partial E} &< 0. \end{aligned}$$

Extraction costs are assumed to be negligible, which is a realistic assumption (see for example Ghoddusi, 2010 p. 363). Finally, we assume that the level of emission flow $\xi_{1,t}$ can be reduced by setting an emission tax rate (ζ) which must be paid for each unit of carbon emitted. Therefore the instantaneous profit rate is determined by:

$$\pi_1(X_t, E_t) = P^{-1}(X_t, E_t) E_t - \zeta \xi_{1,t}.$$

In order to analyze the behavior of the energy producer, we need to specify a functional form for the inverse demand function and the dynamics of the demand shock. We consider the case where the demand is of a constant elasticity type, $P_t = X_t E_t^{-\gamma}$, ($0 < \gamma < 1$ is the elasticity of market demand) and the stochastic demand parameter X_t follows a geometric Brownian motion with the following dynamics:

$$dX_t = \alpha X_t dt + \sigma X_t dZ_t, \quad (2)$$

where α and σ are the drift and volatility parameters of the demand process and dZ is an increment to a Wiener process. In particular, reasons for demand

⁴Mosiño (2012) used a different definition where $a_1 = \frac{1}{\beta_1}$ and β_1 is the fabrication coefficient for oil, i.e. how many units of q_1 are needed to produce one unit of energy.

fluctuations may be due for example to variations in the price of substitute products, climate changes, technology improvements, etc.⁵

Throughout, we assume that:

$$\mathbb{E}_0 \left[\int_0^\infty |\pi_1(X_t, E_t)| e^{-rt} dt \right] < \infty,$$

where $r > 0$ is the discount rate. This condition guarantees that the problem is bounded.⁶

The objective of the energy producer is to choose the optimal rate of extraction $q_{1,t}$ and emission $\xi_{1,t}$ to produce E_t units of energy such that the expected net present value of the future profit streams from energy production is maximized:

$$V(X_0, R_{1,0}) = \max_{\xi_{1,t}, q_{1,t}} \mathbb{E}_0 \int_0^\infty \pi_1(X_t, E_t) e^{-rt} dt,$$

subject to Eq. (1) for the evolution of the resource depletion and (2) for the evolution of demand shock X_t .

In the Appendix we show the following proposition:

Proposition 1 *When the firm uses the non-renewable resource forever, the optimal resource extraction rate is a linear function of the remaining reserves, that is, $q_1^* = q_{1,t} = R_{1,t} \frac{r\rho - \alpha}{\gamma}$, and the reserves evolve as follows: $R_{1,t} = R_{1,0} e^{-\frac{r\rho - \alpha}{\gamma} t}$.*

Moreover, the optimal level of emissions is $\xi_1^(q_1^*) = \left[\frac{(1-\rho)X(a_1 q_1^*)^{\rho-\gamma}}{\zeta} \right]^{\frac{1}{\rho}}$, while*

$\pi_1(X, E(\xi_1^(q_1^*))) = \gamma X^{\frac{1}{\rho}} \left(a_1 \frac{r\rho - \alpha}{\gamma} R_{1,t} \right)^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}}$ is the operating profit that can be obtained from an additional unit of the non-renewable resource extracted, processed and sold as energy and $\rho = 1 - \eta(1 - \gamma)$.*

Proof. *of Proposition 1: In the Appendix. ■*

It is beneficial to compare our result with the results in the literature on the optimal extraction of non-renewable resources. In this literature the well-known Hotelling rule suggests that in the absence of pollution emissions (i.e.,

⁵It is well-known that the price of electricity is best modelled using complex stochastic processes exhibiting mean-reverting paths with jumps (see for instance Lucia and Schwartz, 2002; Cartea and Figueroa, 2005 and Geman and Roncoroni, 2006) and a geometric Brownian motion is a simplification. However, the important point is to notice that in our framework we obtain a three-dimensional switching option, whose value is driven by three state variables: the current stock of the non-renewable resource, the current stock of the renewable resource and the stochastic process for the demand shock, and it turns out to be hard to add other sources of uncertainty. Our simplification allows us to find closed form solutions for the optimal switching timing and the value of the switching option.

⁶See the discussion in Section 4.

$\eta = 0$), emission taxes (i.e. $\zeta = 0$) and production costs, the marginal revenue of the firm will grow with the interest rate⁷, yielding $q_{1,t} = \frac{r-\alpha}{\gamma} R_{1,t}$, and $R_{1,t} = R_{1,0} e^{-\frac{r-\alpha}{\gamma} t}$. Thus, the optimal extraction rate is a linear function of the remaining stock and declines exponentially.

Pindyck (1980) shows that even with stochastic demand shocks, the basic Hotelling rule for the expectation of marginal revenue holds. Using $dX_t = \alpha X_t dt + \sigma X_t dZ_t$, straightforward calculations yield $q_{1,t} = \frac{r-\alpha}{\gamma} R_{1,t}$, and again the optimal extraction rate is a linear function of the remaining reserves and declines exponentially⁸. Substituting the optimal extraction rate into firm's

⁷In the absence of pollution emissions (i.e., $\eta = 0$), emission taxes (i.e. $\zeta = 0$) and production costs, the marginal revenue of the firm will grow with the interest rate. Thus:

$$\frac{dMR}{MR} = rdt, \quad MR \equiv \frac{\partial \pi_1(X, q_1)}{\partial q_1} = (1-\gamma) X a_1^{1-\gamma} q_1^{-\gamma}$$

$$dMR = (1-\gamma) \left[a_1^{1-\gamma} q_1^{-\gamma} (dX) - \gamma X a_1^{1-\gamma} q_1^{-(1+\gamma)} dq_1 \right].$$

If the demand shift parameter has a deterministic dynamics, $dX_t = \alpha X_t dt$ and the resource depletion $R_{1,t} = -q_{1,t} dt$ we have $\alpha(1-\gamma) a_1^{1-\gamma} q_1^{-\gamma} X dt - \gamma(1-\gamma) X a_1^{1-\gamma} q_1^{-(1+\gamma)} dq_1$. Accordingly,

$$\frac{dMR}{MR} = rdt \implies \alpha dt - \gamma q_1^{-1} dq_1 = rdt,$$

Hence,

$$\frac{dq_1}{q_1} = -\frac{r-\alpha}{\gamma} dt \implies q_{1,t} = q_{1,0} e^{-\frac{r-\alpha}{\gamma} t}.$$

The solution of Eq. (1) is:

$$\int_0^t dR_{1,s} = \int_0^t -q_{1,0} e^{-\frac{r-\alpha}{\gamma} s} ds \implies R_{1,t} = R_{1,0} - \frac{\gamma q_{1,0}}{r-\alpha} + \frac{\gamma q_{1,t}}{r-\alpha}.$$

Since $\int_0^\infty dR_{1,s} = \int_0^\infty -q_{1,0} e^{-\frac{r-\alpha}{\gamma} s} ds \implies R_{1,0} = \frac{\gamma q_{1,0}}{r-\alpha}$, straightforward calculations yield:

$$q_{1,t} = \frac{r-\alpha}{\gamma} R_{1,t}.$$

We can compute the remaining reserves of fossil fuel at any time t

$$\begin{aligned} dR_{1,t} &= -q_{1,t} dt \\ &= -\frac{r-\alpha}{\gamma} R_{1,t} dt \implies R_{1,t} = R_{1,0} e^{-\frac{r-\alpha}{\gamma} t}. \end{aligned}$$

⁸Using $dX_t = \alpha X_t dt + \sigma X_t dZ_t$,

$$\frac{\mathbb{E}(dMR)}{MR} = rdt, \quad \mathbb{E}(dMR) = (1-\gamma) \left[\alpha a_1^{1-\gamma} q_1^{-\gamma} X dt - \gamma X a_1^{1-\gamma} q_1^{-(1+\gamma)} dq_1 \right].$$

Accordingly,

$$\frac{\mathbb{E}(dMR)}{MR} = rdt \implies \alpha dt - \gamma q_1^{-1} dq_1 = rdt \implies q_{1,t} = q_{1,0} e^{-\frac{r-\alpha}{\gamma} t}.$$

profit we obtain, $\pi_1(X_t, E_t) = X_t \left(a_1 \frac{r-\alpha}{\gamma} R_{1,t} \right)^{1-\gamma}$, in the absence of environmental regulation.

Proposition 1 shows that the optimal extraction rate is always a linear function of the remaining reserves and declines exponentially. Initially, at $t = 0$ the extraction rate $q_{1,0} = R_{1,0} \frac{r-\alpha}{\gamma}$ is less than the extraction rate $q_{1,0} = R_{1,0} \frac{r-\alpha}{\gamma}$ that is obtained in the Pindyck's model in the absence of emissions, taxes and extraction cost. The reason is that the environmental regulation affects firm's production through a reduction of firm's emissions and profit flows. However, as t increases the optimal extraction rate $q_{1,t} = R_{1,t} \frac{r-\alpha}{\gamma}$ increases above the extraction rate obtained in the Pindyck's model in the absence of emissions since the discount rate $e^{-\frac{r-\alpha}{\gamma}t}$ is greater than the discount rate $e^{-\frac{r-\alpha}{\gamma}t}$ for any $0 < \rho < 1$. We illustrate our result in the following numerical example.

Numerical Example 1. Using these results, one can calculate the time it takes for a producer with a given reserve $R_{1,0}$ to reach the exhaustion of resources. Let us assume: $R_{1,0} = 100$ Billions (reserves of crude oil in barrels⁹), $r = 0.05$ (risk-free interest rate), $\alpha = 0.01$ (drift-rate of demand shock), $\eta = 0.15$ (energy elasticity of emissions), $\gamma = 0.1$ (elasticity of market demand). We have $\rho \equiv 1 - \eta(1 - \gamma) = 1 - 0.15(1 - 0.1) = 0.865$, hence:

$$R_{1,0} e^{-\frac{r-\alpha}{\gamma}t} = 1 \implies 100 \cdot 10^9 \cdot e^{-\frac{0.05-0.865-0.01}{0.1}t} = 1 \simeq 76 \text{ years},$$

while, when there is no emission tax:

$$R_{1,0} e^{-\frac{r-\alpha}{\gamma}t} = 1 = 100 \cdot 10^9 \cdot e^{-\frac{0.05-0.01}{0.1}t} = 1 \simeq 63 \text{ years}.$$

Hence with no environmental regulation the resource will exhaust 13 years earlier.

3.2 Production with renewable resources

Suppose that the firm has access to a renewable (perfect substitute) input. The renewable resource can be more or less productive than the fossil fuel. Let a_2 be the parameter of efficiency of generators using hydro, solar, or wind energy. We assume that $a_2 = \psi a_1$, where the energy efficiency factor ψ is such that

Straightforward calculations yield:

$$q_{1,t} = \frac{r-\alpha}{\gamma} R_{1,t}.$$

⁹See the numerical section for a discussion about the source of the data.

$0 < \psi < 1$ if the renewable resource is less productive than the fossil fuel, while $\psi > 1$ if the renewable resource is more productive.¹⁰

Denote by $\xi_{2,t}$ the cumulative emissions related to the renewable energy production. Unlike fossil fuel technologies, the vast majority of GHG emissions from renewable energies occur upstream of the plant operation, typically for the production and construction of the technology and its supporting infrastructure. For biomass power plants the majority of emissions can arise during the fuel-cycle depending on the choice of biomass fuel.¹¹ Also for hydroelectric reservoirs a huge amount of gases is released when the water is passing the turbine and the spillway. Hence, the production function is:

$$E_t = \xi_{2,t}^\eta (a_2 q_{2,t})^{1-\eta}.$$

Here, $\xi_{2,t} < \xi_{1,t}$ since renewable energy does not produce toxins or pollutants that are harmful to the environment in the same manner that non-renewable energy does.¹² To manage this emission, the firm installs an abatement technology. The total cost of abatement can be described as $c\xi_{2,t}$ where c is the exogenous unit abatement cost.¹³

As for the non-renewable resource case, we assume that the stock of the renewable natural resource, R_2 , is deterministic, but now evolves according to the differential equation:

$$\dot{R}_{2,t} = \delta R_{2,t} - q_{2,t}, \text{ with } R_{2,0} \text{ given,} \quad (3)$$

where $R_{2,0}(> 0)$ is the initial stock of renewable resources, $R_{2,t}$ is the residual stock at any time t , and $\delta(> 0)$ is the rate of the resource regeneration.¹⁴ Since

¹⁰For the energy efficiency in various generation technologies see the Union of the Electricity Industry—Eurelectric's report: <http://www.eurelectric.org/Download/Download.aspx?DocumentID=13549>. As we can see from the graph at page 13 the efficiency in converting oil into electricity may be up to 44%, while the efficiency of a biomass may be up to 40%.

¹¹For most renewable energy technologies upstream GHG emissions can account for over 90% of cumulative emissions. See the International Atomic Energy Agency's report, available at: http://www.iaea.org/OurWork/ST/NE/Pess/assets/GHG_manuscript_preprint_versionDanielWeisser.pdf.

¹²A comparison of GHG emissions for various energy generating technology (i.e. nuclear, fossil and renewables) can be found in Frans H. Koch (2002) "Hydropower – Internalised costs and externalised benefits", available at: <http://www.oecd-neo.org/ndd/reports/2002/nea3676-externalities.pdf#page=131>. Table 1 shows the emissions produced by 1 kWh of electricity based on life cycle analysis. The amount of noxious emissions (i.e., SO₂, NO_x, etc.) and greenhouse gas emissions are much smaller for hydropower, nuclear, and wind, than they are for fossil fuels.

¹³Sims et al. (2003) conducted a study to compare carbon emissions and mitigation costs between fossil fuel, nuclear and renewable energy resources for electricity generation.

¹⁴A similar process is used by Acemoglu et al. (2012) to describe the dynamics of environmental quality. In particular, let S_t be the quality of environment which evolves according to the difference equation:

$$S_{t+1} = -\xi Y_{dt} + \delta S_t$$

where ξ measures the rate of environmental degradation resulting from the production of dirty

there is no storage in this version of model, the spot price of energy is:

$$P_t = P^{-1}(X_t, E_t),$$

where X_t is the same geometric Brownian motion as Eq. (2). We assume that extraction costs of the renewable resource equal zero (see Mosiño, 2012) and that the firm pays the emission tax rate ζ for each unit of pollutant emitted. Therefore the firm's profit rate is:

$$\pi_2(X_t, E_t) = P^{-1}(X_t, E_t) E_t - (c + \zeta) \xi_{2,t}.$$

As usual:

$$\mathbb{E}_0 \left[\int_0^\infty |\pi_2(X_t, E_t)| e^{-rt} dt \right] < \infty,$$

to guarantee that the problem is bounded.

The objective of the energy producer is to choose the optimal levels of emission ξ_2 and extraction q_2 of the renewable natural resource to produce E_t units of energy such that the expected net present value of the of future profit streams from energy production is maximized:

$$\begin{aligned} W(X_0, R_{2,0}) &= \max_{\xi_{2,t}, q_{2,t}} \mathbb{E}_0 \left\{ \int_0^\infty \pi_2(X_t, E_t) e^{-rt} dt \right\}, \\ \text{s.t.} \quad &: \\ dR_{2,t} &= (\delta R_{2,t} - q_{2,t}) dt, \\ dX_t &= \alpha X_t dt + \sigma X_t dZ_t. \end{aligned}$$

Following the same argument of the proof of Proposition 1, we obtain the following proposition:

input Y_{dt} and δ ($\delta > 1$) is the rate of environmental regeneration. Here, we follow Mosiño (2012) and assume $\delta > 0$. However, in his model a stochastic logistic growth function is proposed:

$$dR = \left[\delta R \left(1 - \frac{R}{K} \right) - q \right] dt + \sigma dZ_t,$$

where K is the carry capacity or the saturation level of the resource. In particular, it is shown that there exists a maximum sustained yield MSY at $q^{MSY} = \max \left[\delta R \left(1 - \frac{R}{K} \right) \right]$ with the property that any larger harvest rate will lead to the depletion of the resource. As noted by the author such processes are very appropriate in the case of biomass energy. In our model we adopt a deterministic dynamics for the renewable resource depletion and assume that the growth/regeneration rate is simply δ , instead of the more complex logistic version. It seems to us that this model is more general and well fits the property of renewable energy inputs such as hydropower where the production of energy depends on moving water into reservoir and where the risk of resource extinction is negligible.

Proposition 2 When the firm uses the renewable resource forever, the optimal resource extraction rate is a linear function of the remaining reserves, that is, $q_2^* = \frac{r\rho - \alpha - \delta(\rho - \gamma)}{\gamma} R_{2,t}$, and the reserves evolve as follows $R_{2,t} = R_{2,0} e^{-\frac{r\rho - \alpha - \delta\rho}{\gamma} t}$.

Moreover, the optimal level of emissions is $\xi_2^*(q_2^*) = \left[\frac{(1-\rho)X(a_2 q_2^*)^{\rho-\gamma}}{c+\zeta} \right]^{\frac{1}{\rho}}$, while $\pi_2(X, E(\xi_2^*(q_2^*))) = \gamma\varphi X^{\frac{1}{\rho}} \left(a_2 \frac{r\rho - \alpha - \delta(\rho - \gamma)}{\gamma} R_{2,t} \right)^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}}$ is the operating profit that can be obtained from an additional unit of the renewable resource extracted, processed and sold as energy and $\rho = 1 - \eta(1 - \gamma)$.

Proof. of Proposition 2: In the Appendix. ■

Notice that in this case the discount factor $e^{-\frac{r\rho - \alpha - \delta\rho}{\gamma} t}$ is greater than the term $e^{-\frac{r\rho - \alpha}{\gamma} t}$ that is obtained when the firm uses a non-renewable input. Since the water on earth is continuously replenished by precipitation, the firm can extract more input than if the resource depletes over time without regenerating naturally.

Renewable resources are continuously available, unlike non-renewable resources which depletes over time. A simple comparison is an oil reserve and a hydroelectric plant. The quantity of water into a reservoir can be used up but, if carefully managed, it represents a continuous source of energy, contrarily to the oil reserve which, once it has been exhausted, is gone. Let us consider the following numerical experiment.

Numerical Example 2. Let us assume: $R_{2,0} = 105$ Billions (reserves of water in a reservoir in barrels),¹⁵ $r = 0.05$ (risk-free interest rate), $\alpha = 0.01$ (drift-rate of demand shock), $\eta = 0.15$ (energy elasticity of emissions), $\gamma = 0.1$ (elasticity of market demand), $\delta = 0.038$ (rate of replenishment of water into reservoir¹⁶). We compute $\rho = 1 - \eta(1 - \gamma) = 1 - 0.15(1 - 0.1) = 0.865$. We obtain the time it takes for a producer with a given reserve $R_{2,0}$ to reach the exhaustion of the resource:

$$R_{2,0} e^{-\frac{r\rho - \alpha - \delta\rho}{\gamma} t} = 1 \implies 105 \cdot 10^9 \cdot e^{-\frac{0.05 \cdot 0.865 - 0.01 - 0.038 \cdot 0.865}{0.1} t} = 1 \simeq 6678.$$

The quantity of water in the reservoir will take about 6678 years to be exhausted given the initial stock level $R_{2,0} = 105$ Billions and $\delta = 0.038$. Moreover, note that if the current replenishment rate would be $\delta = 0.038439306$, the residual stock at time $t = 1.000.000$ (given $t = 0$ i.e., 25/09/2012) is:

$$\begin{aligned} R_{2,1.000.000} &= R_{2,0} e^{-\frac{r\rho - \alpha - \delta\rho}{\gamma} t} \\ &\implies 105 \cdot 10^9 \cdot e^{-\frac{0.05 \cdot 0.865 - 0.01 - 0.038439306 \cdot 0.865}{0.1} \cdot 10^6} \simeq 105 \cdot 10^9. \end{aligned}$$

¹⁵ See the numerical section for a discussion about the source of the data.

¹⁶ This is the difference between the river's inflows and outflows or the reusing rate of water. For further details see: <http://www.usbr.gov/power/edu/pamphlet.pdf> and <http://ga.water.usgs.gov/edu/hyhowworks.html>.

The remaining stock after 1000000 years is about as its initial stock level at 25/09/2012. Wind and solar are other free commodities and are in infinite supply and thus affordable renewable energy resources.

4 The value of the switching option and the optimal switching time

Having found the optimal extraction rates of non-renewable and renewable resources we now turn to the optimal decision to switch from a non-renewable resource to a renewable resource and characterize the determinants of the optimal switching time τ .

We first compute the value of the "dirty firm" at $t = 0$ when the possibility of switching to a lower emission renewable input is not available:

$$\begin{aligned} V_0 &= \mathbb{E}_0 \int_0^\infty \pi_1(X_t, E_t(\xi_1^*(q_1^*))) e^{-rt} dt \\ &= \gamma (a_1 q_{1,0})^{\frac{\rho-\gamma}{\rho}} X_0^{\frac{1}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}} \int_0^\infty e^{(-r+\frac{\alpha}{\rho}+\frac{1-\rho}{2\rho^2}\sigma^2)t} \left(e^{-\frac{(r\rho-\alpha)}{\gamma}t} \right)^{\frac{\rho-\gamma}{\rho}} dt \\ &= \gamma (a_1 q_{1,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}} \frac{X_0^{\frac{1}{\rho}}}{\Theta}. \end{aligned} \quad (4)$$

In the same way, we can compute the value of the "green firm" at $t = 0$ after the switching to a lower emission renewable input:

$$\begin{aligned} W_0 &= \mathbb{E}_0 \int_0^\infty \pi_2(X, E(\xi_2^*(q_2^*))) e^{-rt} dt \\ &= \gamma \varphi(a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \frac{X_0^{\frac{1}{\rho}}}{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}, \end{aligned}$$

where $\Theta - \frac{\delta(\rho-\gamma)}{\gamma} > 0$ and $\Theta = \frac{r\rho-\alpha}{\gamma} - \frac{1-\rho}{2\rho^2}\sigma^2$. Notice that we make two economically natural assumptions which ensures convergence of the above integral: $r\rho - \alpha > \delta\rho$, which guarantees the integrability of the present value of the extraction rates $q_{1,t}$ and $q_{2,t}$, and $r - \frac{\alpha}{\rho} - \frac{1-\rho}{2\rho^2}\sigma^2 > 0$ for $\rho > \rho_2$, ($0 < \rho_2 < 1$)¹⁷, which guarantees the integrability of the expected present value of the shock flow $X_t^{\frac{1}{\rho}}$.

¹⁷The roots of the quadratic $r - \frac{\alpha}{\rho} - \frac{1-\rho}{2\rho^2}\sigma^2$ are: $\rho_1 = \frac{2\alpha-\sigma^2-\sqrt{8r\sigma^2+(\sigma^2-2\alpha)^2}}{4r} < 0$ and $\rho_2 = \frac{2\alpha-\sigma^2+\sqrt{8r\sigma^2+(\sigma^2-2\alpha)^2}}{4r} < 1$. Notice that $0 < \rho \equiv 1 - \eta(1-\gamma) < 1$, hence we require that $\rho > \rho_2$ for convergence of expected present value of the shock flow $X_t^{\frac{1}{\rho}}$.

Since we assume that the level of emissions flow $\xi_2^*(q_2^*)$ is less than $\xi_1^*(q_1^*)$ we require that:

$$c > \zeta \left[\left(\frac{\psi R_2}{\varphi R_1} \right)^{\rho-\gamma} - 1 \right], \quad (5)$$

where $\varphi = \frac{r\rho-\alpha}{r\rho-\alpha-\delta(\rho-\gamma)} > 1$.

4.1 The optimal stopping problem

The objective of the energy producer is to choose the optimal timing τ of switching to a lower emission renewable resource such that the expected net present value function of the difference between the profit stream $\pi(X_t, E_t)$ and the switching cost I , is maximized:

$$J(X_0, R_{1,0}, R_{2,0}) = \sup_{\tau \in \mathcal{T}} \mathbb{E} \left\{ \int_0^\infty e^{-rt} \pi(X_t, E_t) dt - I e^{-r\tau} \middle| \mathcal{F}_0 \right\} \quad (6)$$

subject to Eq. (2) for the evolution of demand shock X_t and to Eqs. (1) and (3) for the evolution of the natural resource depletions. Here, $\mathcal{T}$ is the class of admissible implementation times conditional to the filtration generated by the stochastic process X_t . Here, $\pi(X_t, E_t) = \gamma X_t^{\frac{1}{\rho}} (a_1 q_1^*)^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}}$ for $0 \leq t < \tau$ and $\pi(X_t, E_t) = \gamma \varphi X_t^{\frac{1}{\rho}} (a_2 q_2^*)^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}}$ for $t \geq \tau$.

Applying the Dixit and Pindyck (1994) methodology, we can derive the optimal timing of switching to a lower emission renewable resource. In particular, we can compute the critical threshold X , such that it is optimal to switch for $X > \hat{X}$. We obtain Proposition 3:

Proposition 3 *If the relative efficiency parameter ψ is sufficiently large and for some appropriate values of the unit cost of abatement, the firm will switch from non-renewable resources to renewable resources as soon as $X > \hat{X}$, where*

$$\hat{X} = \left[\frac{I \rho \phi_1 \left(\Theta - \frac{\delta(\rho-\gamma)}{\gamma} \right)}{\varphi \gamma (\rho \phi_1 - 1) (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \left(1 - \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}} \right)} \right]^\rho,$$

where $\Theta - \frac{\delta(\rho-\gamma)}{\gamma} > 0$, $\varphi = \frac{r\rho-\alpha}{r\rho-\alpha-\delta(\rho-\gamma)} > 1$ and $\rho = 1 - \eta(1 - \gamma) < 1$. The switching option is valued SWO, where:

$$SWO = X_0^{\phi_1} \left(\frac{\phi_1 \rho - 1}{I} \right)^{\rho \phi_1 - 1} \left[\frac{\gamma \varphi (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \left(1 - \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}} \right)}{\rho \phi_1 \left(\Theta - \frac{\delta(\rho-\gamma)}{\gamma} \right)} \right]^{\rho \phi_1}.$$

Proof. of Proposition 3: In the Appendix. ■

Note that SWO is positive if $c < c^*$, where:

$$c^* \equiv \zeta \left[\varphi^{\frac{\gamma}{1-\rho}} \psi^{\frac{\rho-\gamma}{1-\rho}} \left(\frac{\Theta}{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}} \right)^{\frac{\rho-\gamma}{1-\rho}} - 1 \right].$$

By condition (5) computed at the level of the non-renewable resource at which the marginal value with a non-renewable resource equals the marginal value function with a renewable resource we must have $c > \underline{c}$, where:

$$\underline{c} \equiv \zeta \left[\psi^{\frac{\rho-\gamma}{1+\gamma-\rho}} \left(\frac{\Theta}{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}} \right)^{\frac{\rho-\gamma}{1+\gamma-\rho}} - 1 \right],$$

If $\psi > \psi^*$, where:

$$\psi^* \equiv \frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta}$$

then $0 < \underline{c} < c < c^*$. These parameter restrictions on c and ψ ensure that the critical switching threshold and the value of the option to switch are positive.

Proposition 3 gives us a policy indication: if the goal of the government is to provide incentives towards switching to alternative renewable and non-polluting energy sources, then it should subsidize the cost of abatement and increase the productivity of the renewable resource (for example, relying on research and innovation subsidies). In the next section we study the impact of the relevant parameters on the critical threshold $\hat{X}$ above which it is optimal to switch to the renewable and environmentally friendly resource.

5 Numerical application

In this section we provide some numerical results and sensitivity analysis for the optimal threshold of switching from a non-renewable resource to a renewable resource. The data for the computation of the optimal switching time and the value of the option to switch are illustrated as follows.

Energy Efficiency of Generators. One measure of the efficiency of power plants that convert a fossil fuel into electricity is the heat rate, which is the amount of energy used by a power plant to generate one kilowatt-hour (kWh) of electricity. The lower the heat rate, the more efficient the plant. A comparison of the efficiency of different types of power plants (i.e. coal, natural gas and oil) is provided by the U.S. Energy Information Administration (EIA).¹⁸ EIA expresses heat rates in British thermal units (Btu) per net kWh generated. To express the efficiency of a generator or power plant as a percentage, EIA divides the equivalent Btu content of a kWh of electricity (which is 3,412 Btu) by the heat rate. For example, if the heat rate is 10,140 Btu, the efficiency is 34%; if the heat rate is 7,500 Btu, the efficiency is 45%.¹⁹ In general, the best fossil fuel plants are only about 60 percent efficient. Unfortunately, the EIA does not have estimates for the efficiency of generators using hydro, solar, and wind energy. Hence, we use estimates from the Union of the Electricity Industry-Eurelectric.²⁰ Hydroelectric power generation is by far the most efficient method of large scale electric power generation. The conversion efficiency of a hydroelectric power plant depends mainly on the type of water turbine employed and can be up to 95% for large plants. Smaller plants with output powers less than 5 MW may have efficiencies between 80 and 85 %. In the following simulations we assume $a_1 = 0.5$ (efficiency of the fossil fuel power plant) and $a_2 = 0.9$ (efficiency of a hydroelectric power plant). Therefore the efficiency of renewables relative to non-renewables, $\psi \equiv \frac{a_2}{a_1} = 1.8$.

Parameters related to the installed capacity of the green firm. As for the previous numerical examples we assume that the reserves of crude oil is $R_{1,0} = 100 \cdot 10^9$ barrels. This is the initial level (as of 1 January 2011) of proven reserves of crude oil in Kuwait.²¹ Moreover, we assume that the reserves of water in a reservoir is $R_{2,0} = 105 \cdot 10^9$ barrels. This is approximately 12.5 km³ of water i.e., the total reservoir storage capacity of the Gordon Dam in

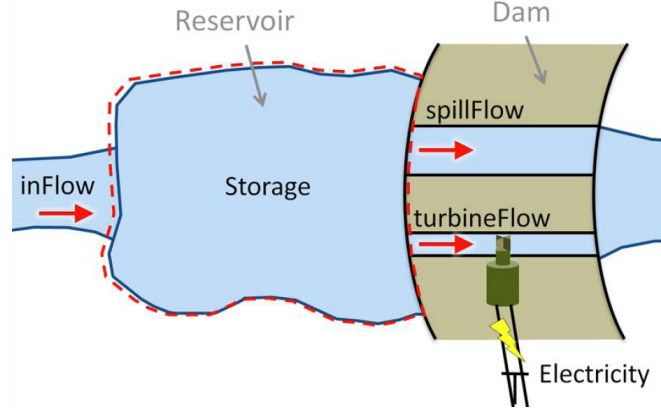
¹⁸Estimations of the historical average annual heat rates for fossil fuel and nuclear power plants can be found here: <http://www.eia.gov/totalenergy/data/annual/showtext.cfm?t=ptb1206>.

¹⁹Source: <http://www.eia.gov/tools/faqs/faq.cfm?id=107&t=3>.

²⁰See also footnote 3.2.

²¹See: <https://www.cia.gov/library/publications/the-world-factbook/rankorder/2178rank.html>.

Tasmania.²²



Hydroelectric power plant diagram.

The Investment costs of large (>10 MWe) hydropower plants range from \$1750/kWe to \$6250/kWe, with an average investment cost of \$4000/kWe US\$. The investment costs of small (1–10 MWe) and very small (≤ 1 MWe) hydro power plants may range from \$2000 to \$7500/kWe and from \$2500 to \$10,000/kWe, respectively, with indicative, average investment costs of \$4500/kWe and \$5000/kWe.²³ Assuming 432 MWe of generating capacity, i.e., the installed capacity of the Gordon Dam which consists of 3×144 MWe turbines, we have $I \cong 2 \cdot 10^9$, that is the cost of installment of the hydropower plant in Euros (we use an exchange rate $\text{€}/\$ \cong 0.77$ as of 10/29/12).

Remaining parameter values. The other parameter values are assumed as follows: $r = 0.05$ (risk-free interest rate), $\alpha = 0.01$ (drift-rate of demand shock), $\eta = 0.25$ (energy elasticity of emissions), $\gamma = 0.3$ (elasticity of market demand), $\delta = 0.037$ (rate of replenishment of water into reservoir), $\zeta = 0.2$ (emission tax rate), $\sigma = 0.3$ (volatility of the demand shock) $X = 1$ (initial level of the shock of the market demand).

We have: $\rho \equiv 1 - \eta(1 - \gamma) = 0.825$, $\varphi \equiv \frac{r\rho - \alpha}{r\rho - \alpha - \delta(\rho - \gamma)} = 2.64271$ and $\psi^* \equiv \frac{\Theta - \frac{\delta(\rho - \gamma)}{\gamma}}{\Theta} = 0.3$. Note that $\psi^* < 1.8$. The unit cost of abatement can be computed easily by using the inequality:

$$\underline{c} < c < c^* \Rightarrow 1.24 < c < 1779.7$$

Let us assume a unit abatement cost of 40€ per tCO₂e (ton of Carbon Dioxide equivalent). It is also in keeping with some empirical evidence, where the cost

²² See <http://www.hydro.com.au/energy/our-power-stations/gordon-pedder>, for some technical details about the Hydro Tasmania's system.

²³ Source International Energy Agency (IEA): <http://www.iea-etsap.org/web/E-TechDS/PDF/E07-hydropower-GS-gct.pdf>.

of abating emissions may rise up to 40 Euros per tCO₂e²⁴. To compare the unit abatement cost with the emission tax rate we express c as a percentage, hence we assume $c = 0.4$ in the following numerical simulations. We can now compute the critical level of the non-renewable resource at which the marginal value function with a non-renewable resource is equal to the marginal value function with a renewable resource:

$$\widehat{R}_1 \equiv R_{2,0} \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}} = 9.90184 \cdot 10^8.$$

Any additional unit of resource extracted, processed and sold as energy will reduce the marginal value function with a non-renewable resource (dashed curve) below the marginal value function with a substitute renewable resource (solid curve) as shown in Figure 1 below.

[Insert Figure 1 here]

Figure 2 shows the relation between the value function, J^N , the the value of reward, $J^S - I$ and the shock of market demand X . The dashed curve refers to the value function in the no-switching region (or value of continuation) while the solid curve refers to the value function in the switching region minus the cost of installment of the hydropower plant (value of reward or value of termination). We consider values of the shock of market demand X ranging from 0 to 350. The critical threshold $\hat{X}$ is found at the point of tangency of J^N with the curve $J^S - I$. We found $\hat{X} \simeq 133.3$. For values of the shock of market demand belonging to the interval $(0, 133.3)$ the renewable resource should not be adopted, while for values of X greater than 133.3 the renewable resource should be immediately adopted. At the optimal adoption threshold, the value of the option to switch, $\Phi_1 X^{\phi_1}$, is about $8.07015 \cdot 10^9 \text{€}$. As R_2 increases (and so the value of the hydropower plant) the value functions before the switch and after the switch increase (the curves shift upwards); the dynamics are illustrated thoroughly in Figure 3 below.

[Insert Figure 2 about here]

Figure 3 shows the relation between the critical switching threshold $\hat{X}$ and the initial stock of the renewable resource R_2 . We consider values of R_2 ranging from 0 to $100 \cdot 10^{10}$ barrels of water. The critical switching threshold is downward sloping with respect to the initial stock of renewable natural resource R_2 . The reason is that a higher R_2 will increase both the firm's extraction rate and the profits rate. Moreover, a higher stock of renewable resource will increase the value of the option to switch and hence speed up the investment in the hydropower plant. Let us consider the following numerical example. Let us assume that the initial stock of resource is $R_2 = 55 \cdot 10^9$ barrels of water. At this level of the stock of renewable resource the critical switching threshold is

²⁴See for example the Mckinsey's report available at: http://www.epa.gov/oar/caaac/coaltech/2007_05_mckinsey.pdf

$\hat{X} \cong 187.2$, the value of option to switch is $2.95284 \cdot 10^6 \text{€}$, the value function before the switch $J^N = 3.12166 \cdot 10^6 \text{€}$ and $J^S - I = -1.9821 \cdot 10^9 \text{€}$. Now, let us assume that the stock of renewable resource R_2 increase from $55 \cdot 10^9$ to $100 \cdot 10^9$ barrels of water. For example let us assume that the energy producer chooses a greater hydropower plant, i.e. a greater dam with a larger reservoir. With this new level of the stock of the renewable resource the critical switching threshold decreases from about 187.2 to about 136.8, the value of the option to switch increases to $4.74678 \cdot 10^6 \text{€}$, also the value functions J^N and $J^S - I$ increase, to $4.99375 \cdot 10^6$ and $-1.97381 \cdot 10^9 \text{€}$, respectively.

[Insert Figure 3 about here]

Figure 4 shows the relation between the critical switching threshold $\hat{X}$ and the unit abatement cost c . We consider values of c ranging from 0 to 40. The critical switching threshold is upward sloping with respect to the unit abatement cost c . The reason is that a higher c reduces firm's profits and thus necessitates a higher X for the investment in the hydropower system to take place. The intuition is as follows. The optimal switching time is defined as that level of X at which the value function before the switch and the value function after the switch are equivalent. Evaluating both values at $\hat{X}$ and increasing the cost of carbon abatement, decreases both the value function before the switch and the value function after the switch (the curves in Figure 2 shift downward). On one hand, a larger c would reduce the profits of the hydropower farm (through a reduction in the level of emission flow), while render the dirty firm more profitable (the value function before the switch increases while the value function after the switch decreases). On the other hand, the value of the option to switch decreases because there is less incentive to switch to renewable resources since the profits of the green firm decrease (the value function before the switch decreases). Thus, the final effect on the timing to switch depends on which effect is the strongest. Overall, the effect on the green firm seems to dominate. That means that when faced with a higher cost of abatement, the value of continuation is greater than the value of termination, and hence the producer will decide to wait longer for adopting the more environmental-friendly renewable technology.

[Insert Figure 4 about here]

Figure 5 shows the relation between the critical switching threshold $\hat{X}$ and the rate of the renewable resource regeneration δ . We consider values of δ ranging from 0 to 0.0378788. This value is chosen so as to have the discount term $e^{-\frac{r\rho-\alpha-\delta\rho}{\gamma}t}$ very close to zero.²⁵ This guarantees that the renewable resource never depletes over time. The critical switching threshold is downward sloping with respect to rate of resource regeneration δ . The intuition is as follows. As δ increases the renewable resource becomes more inexhaustible and thus increases the value of the hydropower plant (through a decrease in the discount term $\Theta - \frac{\delta(\rho-\gamma)}{\gamma}$); also the value of the option to switch to the renewable resource increases

²⁵See for example the computation provided in the numerical example 2.

since there is more incentive to adopt an inexhaustible natural resource. Both the value function before the switch and the value function after the switch increase (the curves in Figure 2 shift upwards). Thus, the final effect on the timing to switch depends on which effect is the strongest. Overall, the effect on the value of the clean firm seems to dominate. That means that when faced with a higher rate of resource regeneration, the value of continuation is lower than the value of termination, and hence the producer will decide to hasten the adoption of the environmental-friendly renewable technology.

[Insert Figure 5 about here]

Figure 6 shows the relation between the critical switching threshold $\hat{X}$ and the emission tax rate ζ . We consider values of the emission tax rate ζ ranging from about 0 to 1. The critical switching thresholds is upward sloping with respect to ζ . The reason is that a higher ζ reduces firm's profits and thus necessitates a higher X for the investment in the hydropower plant to take place. The intuition is as follows. The optimal adoption time is defined as that level of X at which the continuation and the termination values are equal. Evaluating both values at $\hat{X}$ and increasing the level of stringency of environmental policy, decreases both the value function before the switch and the value function after the switch (the curves in Figure 2 shift downwards). Indeed, a larger ζ would reduce the emissions and profits of both types of firms. The value of the option to switch both decreases and increases. On one hand, the value of the option to switch decreases because there is less incentive to switch to renewable resources since the profits of a green firm decrease; on the other hand, there is more incentive to switch to a renewable resource since the profits of a dirty firm decrease. Thus, the final effect on the timing to switch depends on which effect is the strongest. Overall, the effect on the green firm seems to dominate. That means that when faced with a more stringent environmental policy, the value function before the switch is larger than the value function after the switch, and hence the producer will decide to wait longer for adopting the more environmental-friendly renewable technology.

The result that the more stringent the policy, the more the firm delays the adoption of lower emission technologies, was also highlighted in Van Soest (2005), however, using a different model with a different objective function. Also Agliardi and Sereno (2011) obtain the same sensitivity results with respect to the stringency of environmental policy, although they consider a different problem of regulator's behavior with alternative environmental policy options and the finance requirements of the environmental protection.

Figure 7 shows the relation between the critical switching threshold $\hat{X}$ and the relative efficiency parameter ψ . We consider values of the relative efficiency parameter ψ ranging from 0 to 3. The critical switching thresholds is downward sloping with respect to ψ . The reason is that a higher ψ increases the profits of a green firm and thus necessitates a lower X for the investment in the hydropower plant to take place. The intuition is as follows. As ψ increases the generator using a renewable resource becomes more efficient in converting that resource

into electrical energy than the generator converting fuels into electrical energy. Thus, an increase in ψ increases both the value of the hydroelectric plan, through an increase in the firm's profit flow, and the value of the option to switch to the substitute renewable resource. Both the value functions before the switch and after the switch increase (the curves in Figure 2 shift upwards). Thus, the final effect on the timing to switch depends on which effect is the strongest. Overall the effect on firm's profit seems to dominate. That means that when faced with a more efficient renewable resource, the value function before the switch is lower than the value function after the switch, and hence the dirty firm will decide to hasten the adoption of the substitute renewable resource.

Finally, Figure 8 shows the relation between the critical switching threshold $\hat{X}$ and the elasticity of market demand γ . We consider values of the elasticity of market demand ranging from about 0 to 0.6. The critical switching thresholds is upward sloping with respect to γ . The intuition is as follows. The elasticity of demand is the same at every point along a constant elasticity demand curve. A perfectly inelastic demand curve, where $\gamma = 0$ everywhere, is a vertical straight line. A larger γ means that the market demand is flatter (more responsive to price changes) than the inelastic demand curves. As expected, a larger γ would reduce the profits of both the dirty firm and the clean firm, since the market power of the energy producer reduces. The effects on the option to switch is more ambiguous. On one hand, the value of the option to switch decreases because there is less incentive to switch to a renewable resource when the profits of using that resource decrease (both the value function before the switch and the value function after the switch decrease); on the other hand, there is more incentive to switch to a renewable resource since the profits of using a non-renewable resource decrease (hence, the value function before the switch increases). Thus, the final effect on the timing to switch depends on which effect is the strongest. Overall, the effect on the green firm seems to dominate. That means that when faced with a more elastic demand for electricity, the value of continuation is greater than the value of termination, and hence the producer will decide to wait longer for adopting the more environmental friendly renewable technology.

[Insert Figure 6 about here]

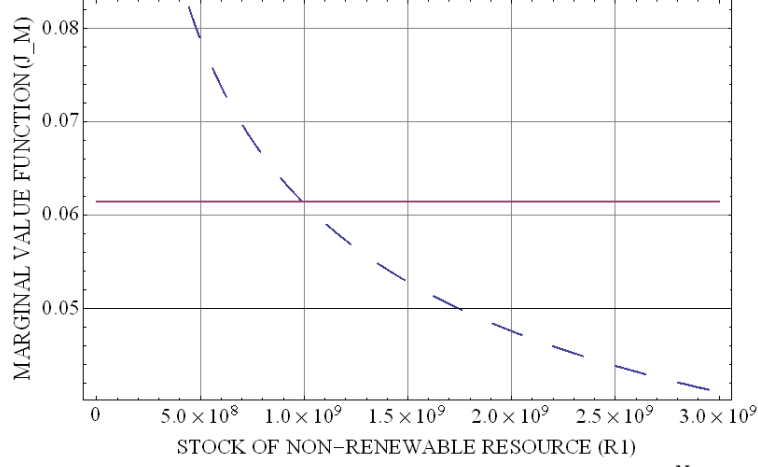


FIGURE 1: Relation between the marginal value function $J_{R_1}^N$ and the initial stock of non-renewable resource (dashed curve). Relation between the value function $J_{R_2}^S$, and the initial stock of non-renewable resource (solid curve). Values of the initial stock of non-renewable resource ranges from 0 to $3 \cdot 10^9$.

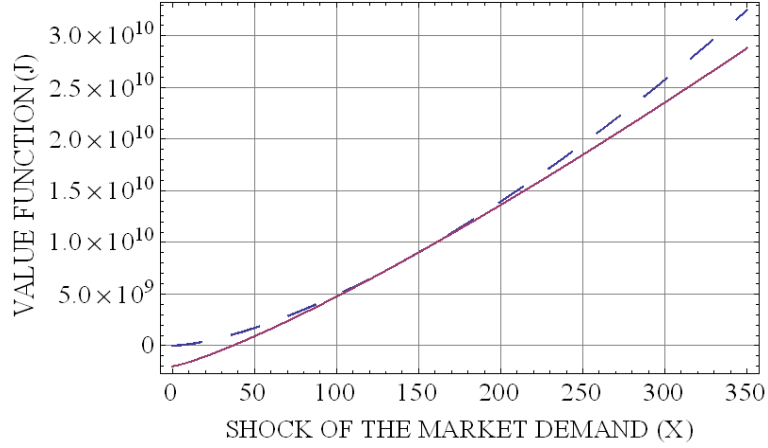


FIGURE 2: Relation between the value function J^N and the shock of market demand X (dashed curve). Relation between the value function J^S minus the cost of installment of the green farm, and the shock of market demand X (solid curve). Values of the stock of shock of market demand ranges from 0 to 350.

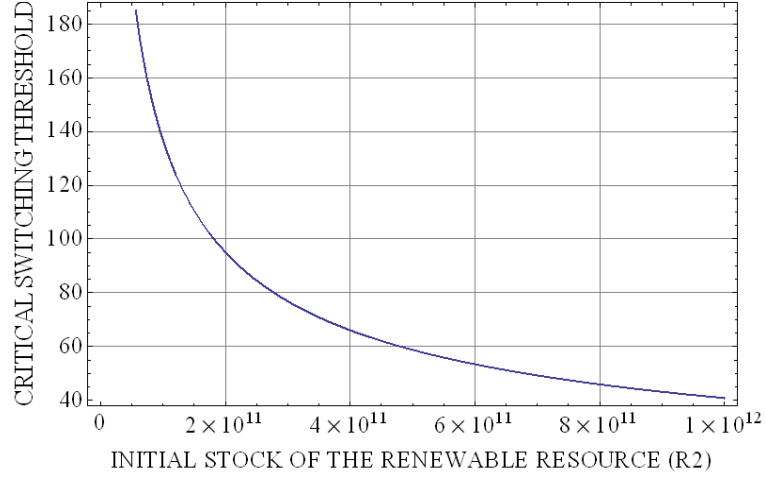


FIGURE 3: Relation between the critical switching threshold $\hat{X}$ and the stock of renewable natural resource R_2 . Values of the stock of renewable resource ranges from 0 to $100 \cdot 10^{10}$.

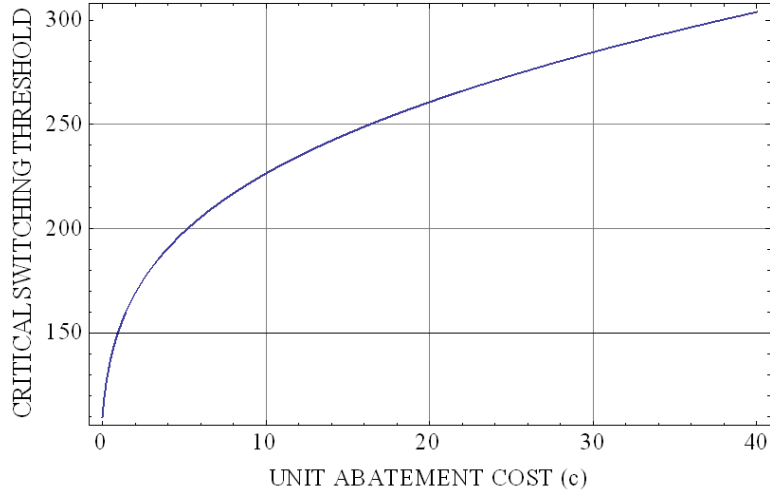


FIGURE 4: Relation between the critical switching threshold $\hat{X}$ and the cost of carbon abatement c . Values of the unit abatement cost c ranges from 0 to 40.

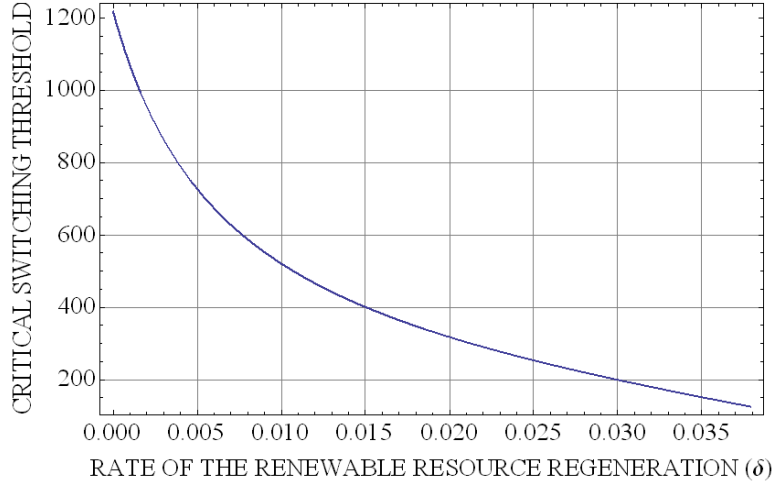


FIGURE 5: Relation between the critical switching threshold $\hat{X}$ and the rate of natural resource regeneration δ . Values of δ ranges from 0 to 0.0378788.

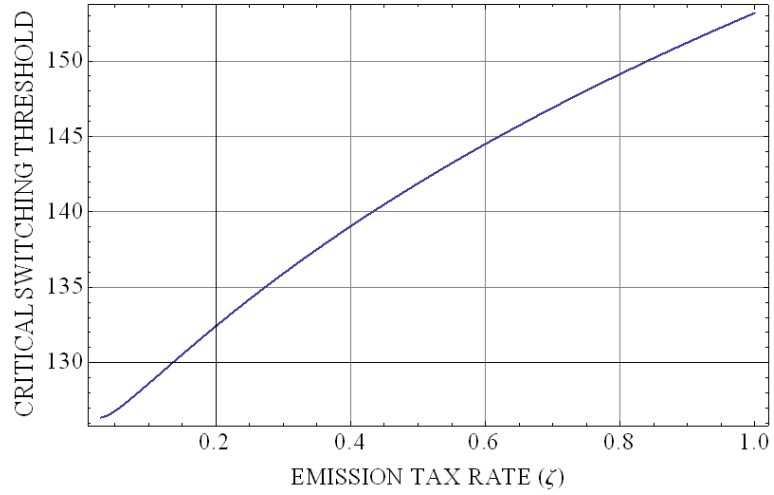


FIGURE 6: Relation between the critical switching threshold $\hat{X}$ and the emission tax rate ζ . Values of the emission tax rate ranges from about 0 to 1.

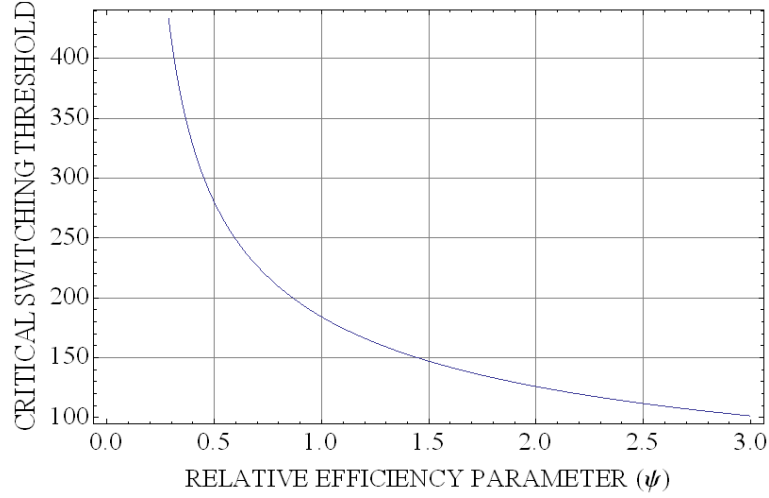


FIGURE 7: Relation between the critical switching threshold $\hat{X}$ and the relative efficiency parameter ψ . Values of the relative efficiency parameter ranges from 0 to 3.

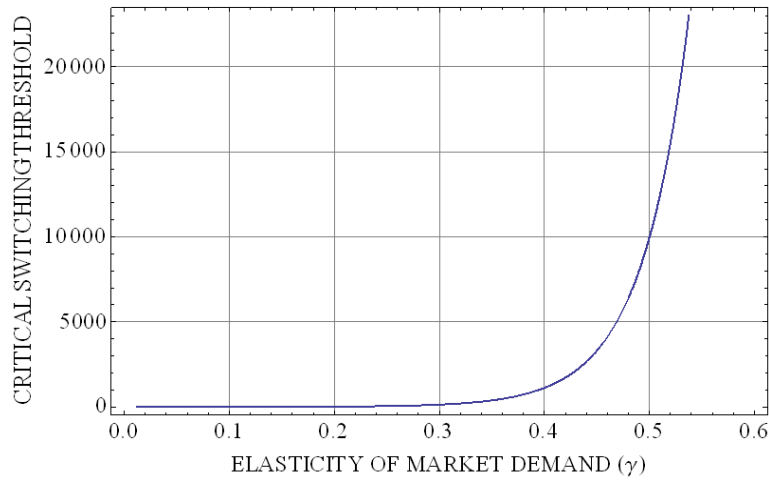


FIGURE 8: Relation between the critical switching threshold $\hat{X}$ and the elasticity of market demand γ . Values of the elasticity of market demand ranges from about 0 to 0.6.

6 Conclusion

It is well known that in the debate about environmental policy intervention either an immediate government action is prescribed (Stern, 2007) or a more gradualist approach is suggested (Nordhaus, 2007) to contrast and control emissions and climate change. Our paper gives some indication on the optimal timing of switching and the types of policies that can implement environmental regulation, encouraging alternative sustainable energy services. We consider a model of switching from non-renewable and dirty resources to renewable and less polluting energy sources. It is found (Proposition 3) that the option to switch will be exercised, depending on values of the unit abatement cost and on the relative productivity parameter. The optimal switching time is sensitive to emission taxes, abatement costs, demand elasticity - whose increases slow down the adoption of substitutable renewable resources - and the natural rate of resource regeneration, the stock of renewable resources, the relative productivity parameter - whose increases speed up the investment in the green technology. These results have some implications for environmental policy. The government may want to accelerate the switching to a less polluting and renewable energy source. Then, it should provide subsidies to decrease the cost of abatement of renewable energies and to increase the productivity of renewable resources. Our framework illustrates also the effects of exhaustibility of resources on the structure of optimal policy. Timing is crucial in policy intervention too: indeed, delaying intervention is costly, not only because of the resulting continued environment degradation, but also because it may widen the gap between dirty and clean technologies.

Some extensions and further directions of research might be fruitful. One is to incorporate environment risk, either modelling uncertainty in the regeneration rate or in the future costs of environmental damage. Deep structural uncertainty could be modelled by fat tailed distributions or multiple priors. Another extension is to study alternative mix of policies, including tax and subsidies, and the impact of different types of environmental regulation on the timing and direction of innovation in the energy sector.

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Appendix

Proof of Proposition 1

We follow Pindyck (1980) and define the optimal value function as:

$$V_0 = V(X_0, R_{1,0}) = \max_{\xi_{1,t}, q_{1,t}} \mathbb{E}_0 \int_0^\infty \pi_1(X_t, E_t) e^{-rt} dt$$

The fundamental equation of optimality for the producer of energy is:²⁶

$$rV(X, R_1) dt = \max_{\xi_1, q_1} [\pi_1(X, E) dt + \mathbb{E}d(V)],$$

where $E = \xi_1^\eta (a_1 q_1)^{1-\eta}$. We use the label "dirty-firm" to indicate the firm producing energy with a non-renewable resource input. The rate of return consists of the instantaneous profit flow $\pi_1(X, E)$ plus the expected change in the value of the dirty firm. Optimality requires that the total expected return of the dirty firm equals the required return r . To calculate the expected change in the value of the dirty firm, $\mathbb{E}d(V)$, we apply Ito's Lemma to obtain:

$$\mathbb{E}d(V) = \left[V_X dX + V_{R_1} dR_1 + \frac{1}{2} V_{XX} (dX)^2 + \frac{1}{2} V_{R_1 R_1} (dR_1)^2 + V_{X R_1} dX dR_1 \right] dt. \quad (7)$$

Substituting (1) and (2) in (7) and recognizing that $\mathbb{E}(dZ) = (dt)^2 = (dt)(dZ) = 0$, we get the expected change in the value of the "dirty firm" over the time interval dt

$$\mathbb{E}d(V) = \left[\alpha X V_X - q_1 V_{R_1} + \frac{1}{2} \sigma^2 X^2 V_{XX} \right] dt.$$

Hence:

$$rV(X, R_1) = \max_{\xi_1, q_1} \left[\pi_1(X, E) - q_1 V_{R_1} + \alpha X V_X + \frac{1}{2} \sigma^2 X^2 V_{XX} \right], \quad (8)$$

where $\pi_1(X, E) = X \left[\xi_1^\eta (a_1 q_1)^{1-\eta} \right]^{1-\gamma} - \zeta \xi_1$ and the subscripts denote partial derivatives i.e., $V_t = \partial V / \partial t$, $V_{R_1} = \partial V / \partial R_1$ etc. This equation contains two state variables X and R_1 and it seems hard to find out a closed form solution. Nonetheless, this expression can be simplified. First, we calculate the amount of emissions used as a function of the extraction rate $\xi_1(q_1)$. Hence:

$$\frac{\partial \pi_1(X_t, E_t)}{\partial \xi_1} = 0 \implies \xi_1(q_1) = \left[\frac{(1-\rho) X (a_1 q_1)^{\rho-\gamma}}{\zeta} \right]^{\frac{1}{\rho}}, \quad (9)$$

²⁶ We suppress time subscripts unless they are needed for clarity.

where $\rho = 1 - \eta(1 - \gamma)$, ($0 < \rho < 1$).

It is useful to rewrite Eq. (8) as:

$$rV(X, R_1) = \max_{q_1} \left[\pi_1(X, E(\xi_1(q_1))) - q_1 V_{R_1} + \alpha X V_X + \frac{1}{2} \sigma^2 X^2 V_{XX} \right], \quad (10)$$

where $\pi_1(X, E(\xi_1(q_1))) = \rho X^{\frac{1}{\rho}} (a_1 q_1)^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}}$.

The first order conditions (FOC) for the problem (10) is:

$$\frac{\partial \pi_1(X, E(\xi_1(q_1)))}{\partial q_1} = V_{R_1}. \quad (11)$$

The marginal value function V_{R_1} equals the incremental profit that can be obtained by extracting, processing and selling one unit of the non-renewable resource as energy.

To get the optimal extraction rate, we eliminate V from the problem (10). First, we differentiate Eq. (10) with respect to R_1 , we obtain:

$$\frac{\partial \pi_1(X, E(\xi_1(q_1)))}{\partial R_1} - q_1 V_{R_1 R_1} + \alpha X V_{R_1 X} + \frac{1}{2} \sigma^2 X^2 V_{R_1 X X} = r V_{R_1}. \quad (12)$$

Note that $\frac{\partial \pi_1(X, E(\xi_1(q_1)))}{\partial R_1} = 0$. Now using Ito's Lemma Eq. (12) can be rewritten as:

$$(1/dt) \mathbb{E} d(V_{R_1}) = r V_{R_1}. \quad (13)$$

Applying the Ito's differential operator $(1/dt) \mathbb{E} d(\bullet)$ to both side of Eq. (11) we obtain:

$$(1/dt) \mathbb{E} d \left[\frac{\partial \pi_1(X, E(\xi_1(q_1)))}{\partial q_1} \right] = (1/dt) \mathbb{E}_t d(V_{R_1}). \quad (14)$$

Hence, we can combine equations (13) and (14) to obtain:

$$(1/dt) \mathbb{E} d \left[\frac{\partial \pi_1(X, E(\xi_1(q_1)))}{\partial q_1} \right] = r \frac{\partial \pi_1(X, E(\xi_1(q_1)))}{\partial q_1},$$

which is the stochastic version of the Euler equation. Substituting the expression for $\pi_1(X, E(\xi_1(q_1)))$, we have:

$$\mathbb{E} d \left[(\rho - \gamma) X^{\frac{1}{\rho}} a_1^{\frac{\rho-\gamma}{\rho}} q_1^{-\frac{\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}} \right] = r (\rho - \gamma) X^{\frac{1}{\rho}} a_1^{\frac{\rho-\gamma}{\rho}} q_1^{-\frac{\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}} dt.$$

Hence, differentiating the left-hand side of the equation above with respect to X and q_1 and after straightforward calculations we finally arrive at:

$$\frac{dq_1}{q_1} = -\frac{r\rho - \alpha}{\gamma} dt \implies q_{1,t} = q_{1,0} e^{-\frac{r\rho - \alpha}{\gamma} t}$$

Since $\int_0^\infty q_{1,0} e^{-\frac{(r\rho - \alpha)}{\gamma} t} dt = R_{1,0} = \frac{\gamma q_{1,0}}{r\rho - \alpha}$, it is easy to show that:

$$q_1^* = q_{1,t} = R_{1,t} \frac{r\rho - \alpha}{\gamma}. \quad (15)$$

Finally, we can compute the remaining reserves of fossil fuel at any time t :

$$dR_{1,t} = -q_1^* dt = -R_{1,t} \frac{r\rho - \alpha}{\gamma} dt \implies R_{1,t} = R_{1,0} e^{-\frac{r\rho - \alpha}{\gamma} t}.$$

Substituting the optimal extraction rate (15) into (9) we find the optimal level of emissions:

$$\xi_1^*(q_1^*) = \left[\frac{(1 - \rho) X (a_1 q_1^*)^{\rho - \gamma}}{\zeta} \right]^{\frac{1}{\rho}},$$

while the firm's profit is $\rho X^{\frac{1}{\rho}} (a_1 q_1^*)^{\frac{\rho - \gamma}{\rho}} \left(\frac{1 - \rho}{\zeta} \right)^{\frac{1 - \rho}{\rho}}$.

By substituting (15) into the FOC (11) we find:

$$(\rho - \gamma) X^{\frac{1}{\rho}} (a_1)^{\frac{\rho - \gamma}{\rho}} (q_1^*)^{-\frac{\gamma}{\rho}} \left(\frac{1 - \rho}{\zeta} \right)^{\frac{1 - \rho}{\rho}} = V_{R_1}.$$

This allows us to rewrite Eq. (10) as:

$$rV(X, R_1) = \pi_1(X, E(\xi_1^*(q_1^*))) + \alpha X V_X + \frac{1}{2} \sigma^2 X^2 V_{XX}. \quad (16)$$

where $\pi_1(X, E(\xi_1^*(q_1^*))) = \gamma X^{\frac{1}{\rho}} \left(a_1 \frac{r\rho - \alpha}{\gamma} R_{1,t} \right)^{\frac{\rho - \gamma}{\rho}} \left(\frac{1 - \rho}{\zeta} \right)^{\frac{1 - \rho}{\rho}}$ is the operating profit that can be obtained from an additional unit of the non-renewable resource extracted, processed and sold as energy.

Proof of Proposition 2

We follow the same argument as in the previous section and define the optimal value function as:

$$W_0 = W(X_0, R_{2,0}) = \max_{\xi_{2,t}, q_{2,t}} \mathbb{E}_0 \int_0^\infty \pi_2(X_t, E_t) e^{-rt} dt$$

The fundamental value of the firm satisfies the following Bellman equation (we suppress time subscripts unless they are needed for clarity):

$$rW(X, R_2) dt = \max_{\xi_2, q_2} [\pi_2(X, E) dt + \mathbb{E}d(W)]$$

where $E = (\xi_2)^\eta (a_2 q_2)^{1-\eta}$. We use the label "green-firm" to indicate the firm producing energy with a renewable resource. As before:

$$\mathbb{E}d(W) = \left[(\delta R_{2,t} - q_{2,t}) W_{R_2} + \alpha X W_X + \sigma^2 X^2 \frac{1}{2} W_{XX} \right] dt$$

is the expected change in the value of the "green-firm" over the time interval dt . Hence, the fundamental equation of optimality is:

$$rW(X, R_2) = \max_{\xi_2, q_2} \left[\pi_2(X_t, E_t) + (\delta R_{2,t} - q_{2,t}) W_{R_2} + \alpha X W_X + \frac{1}{2} \sigma^2 X^2 W_{XX} \right]. \quad (17)$$

First, we calculate the amount of emissions used as a function of the extraction rate. Straightforward calculations yield:

$$\xi_2(q_2) = \left[\frac{(1-\rho) X (a_2 q_2)^{\rho-\gamma}}{c + \zeta} \right]^{\frac{1}{\rho}}. \quad (18)$$

It is useful to rewrite Eq. (17) as:

$$rW(X, R_2) = \max_{q_2} \left[\pi_2(X_t, E_t(\xi_2(q_2))) + (\delta R_{2,t} - q_{2,t}) W_{R_2} + \alpha X W_X + \frac{1}{2} \sigma^2 X^2 W_{XX} \right], \quad (19)$$

where the instantaneous profit flow $\pi_2(X, E(\xi_2(q_2))) = \rho X^{\frac{1}{\rho}} (a_2 q_2)^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}}$.

The FOC for the problem (19) is:

$$\frac{\partial \pi_2(X, E(\xi_2(q_2)))}{\partial q_2} = W_{R_2}. \quad (20)$$

The marginal value function W_{R_2} equals the incremental profit that can be obtained by extracting, processing and selling one unit of the renewable resource as energy.

Differentiating Eq. (17) with respect to R_2 yields:

$$\frac{\partial \pi_2(X, E(\xi_2(q_2)))}{\partial R_2} + \delta W_{R_2} + (\delta R_{2,t} - q_{2,t}) W_{R_2 R_2} + \alpha X W_{R_2 X} + \frac{1}{2} \sigma^2 X^2 W_{R_2 X X} = r W_{R_2}. \quad (21)$$

Again $\frac{\partial \pi_2(X, E(\xi_2(q_2)))}{\partial R_2} = 0$. Now using Ito's Lemma we can rewrite Eq. (21) as follows:

$$(1/dt) \mathbb{E}d(W_{R_2}) = (r - \delta) W_{R_2} \quad (22)$$

Applying the Ito's differential operator $(1/dt) \mathbb{E}_t d(\bullet)$ to both side of Eq. (20) we have:

$$(1/dt) \mathbb{E}d \left[\frac{\partial \pi_2(X, E(\xi_2(q_2)))}{\partial q_2} \right] = (1/dt) \mathbb{E}_t d(W_{R_2}). \quad (23)$$

Hence, we can combine equations (22) and (23) to obtain:

$$(1/dt) \mathbb{E}d \left[\frac{\partial \pi_2(X, E(\xi_2(q_2)))}{\partial q_2} \right] = (r - \delta) W_{R_2}.$$

Since $(\rho - \gamma) X^{\frac{1}{\rho}} a_2^{\frac{\rho-\gamma}{\rho}} q_2^{-\frac{\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} = W_{R_2}$, a simple substitution into the above equation yields:

$$\mathbb{E}d \left[(\rho - \gamma) X^{\frac{1}{\rho}} a_2^{\frac{\rho-\gamma}{\rho}} q_2^{-\frac{\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \right] = (r - \delta) (\rho - \gamma) X^{\frac{1}{\rho}} a_2^{\frac{\rho-\gamma}{\rho}} q_2^{-\frac{\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} dt.$$

Differentiating the left-hand side of the equation above with respect to X and q_2 and after straightforward calculations we obtain:

$$q_2^{-1} dq_2 = -\frac{r\rho - \alpha - \delta\rho}{\gamma} dt \implies q_{2,t} = q_{2,0} e^{-\frac{r\rho - \alpha - \delta\rho}{\gamma} t}.$$

Define the function, $f(R_{2,t}, t) = R_{2,t} e^{-\delta t}$ and differentiate this with respect to $R_{2,t}$ and t . We get:

$$df(R_{2,t}, t) = e^{-\delta t} dR_{2,t} - \delta R_{2,t} e^{-\delta t} dt,$$

by substituting Eq. (1) into the above expression we obtain:

$$df(R_{2,t}, t) = -q_{2,t} e^{-\delta t} dt.$$

Integrating from 0 to t we get:

$$R_{2,t} e^{-\delta t} = R_{2,0} - \int_0^t q_{2,s} e^{-\delta s} ds.$$

where $q_{2,t} = q_{2,0} e^{-\frac{r\rho - \alpha - \delta\rho}{\gamma} t}$, hence:

$$R_{2,t} e^{-\delta t} = R_{2,0} - \frac{q_{2,0}\gamma}{r\rho - \alpha - \delta(\rho - \gamma)} + \frac{q_{2,0}\gamma}{r\rho - \alpha - \delta(\rho - \gamma)} e^{-\frac{r\rho - \alpha - \delta(\rho - \gamma)}{\gamma} t}.$$

Since $R_{2,0} = \int_0^\infty q_{2,t} e^{-\delta t} dt = \frac{q_{2,0}\gamma}{r\rho - \alpha - \delta(\rho - \gamma)}$, substituting this into the above equation we have:

$$R_{2,t} e^{-\delta t} = \frac{q_{2,0}\gamma}{r\rho - \alpha - \delta(\rho - \gamma)} e^{-\frac{r\rho - \alpha - \delta(\rho - \gamma)}{\gamma} t}.$$

A straightforward calculation leads to the following expression of the optimal extraction rate:

$$q_2^* = \frac{r\rho - \alpha - \delta(\rho - \gamma)}{\gamma} R_{2,t}. \quad (24)$$

Finally, we can compute the level of remaining stock of renewable resource at any time t :

$$\begin{aligned} dR_{2,t} &= (\delta R_{2,t} - q_2^*) dt = -\frac{r\rho - \alpha - \delta\rho}{\gamma} R_{2,t} dt \\ \implies R_{2,t} &= R_{2,0} e^{-\frac{r\rho - \alpha - \delta\rho}{\gamma} t}. \end{aligned}$$

Substituting the optimal extraction rate (24) into (18) we find the optimal level of emissions:

$$\xi_2^*(q_2^*) = \left[\frac{(1-\rho) X (a_2 q_2^*)^{\rho-\gamma}}{c + \zeta} \right]^{\frac{1}{\rho}},$$

while the instantaneous profit flow is $\rho X^{\frac{1}{\rho}} (a_2 q_2^*)^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}}$.

Substituting (24) into the FOC (20) we obtain:

$$(\rho - \gamma) X^{\frac{1}{\rho}} (a_2)^{\frac{\rho-\gamma}{\rho}} (q_2^*)^{\frac{-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} = W_{R_2},$$

while Eq. (17) can be rewritten as:

$$rW(X, R_2) = \pi_2(X, E(\xi_2^*(q_2^*))) + \alpha X W_X + \frac{1}{2} \sigma^2 X^2 W_{XX},$$

where $\pi_2(X, E(\xi_2^*(q_2^*))) = \gamma \varphi X^{\frac{1}{\rho}} \left(a_2 \frac{r\rho - \alpha - \delta(\rho - \gamma)}{\gamma} R_2 \right)^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}}$ is the operating profit that can be obtained from an additional unit of the renewable resource extracted, processed and sold as energy.

Proof of Proposition 3

Let $V = J^N(X, R_1)$ denote the value function for the "no-switching" region $0 \leq t < \tau$, in which $\pi(X_t, E_t) = \pi_1(X_t, E_t(\xi_1^*(q_1^*)))$. Substituting $J^N(X, R_1)$ for

V into (16) we find that the corresponding Hamilton-Jacobi-Bellman equation is:

$$rJ^N = \pi_1(X_t, E_t(\xi_1^*(q_1^*))) + \alpha X_t J_X^N + \frac{1}{2} \sigma^2 X_t^2 J_{XX}^N.$$

It has the following general solution:

$$J^N = \Phi_1 X^{\phi_1} + \Phi_2 X^{\phi_2} + \left[\gamma (a_1 q_{1,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}} \frac{X_0^{\frac{1}{\rho}}}{\Theta} \right] \quad (25)$$

where Φ_1 and Φ_2 are unknowns to be determined. Here, ϕ_1 and ϕ_2 are the solutions to the following characteristic equation:

$$\frac{1}{2} \sigma^2 \phi(\phi - 1) + \phi \alpha - r = 0$$

and are given by:

$$\begin{aligned} \phi_1 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > 1 \\ \phi_2 &= \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} < 0. \end{aligned}$$

The term between the squared parentheses in (25) is a particular solution, which captures the expected present value of future stream of profits in the case the energy producer has not switched to the renewable resource and is calculated in (4). Therefore, the parenthesis in (25) represents the fundamental term and the exponential terms account for the perpetual American switching option value.

Next, let $W = J^S(X, R_2)$ denote the value function for the "switching region" $t \geq \tau$, in which $\pi(X_t, E_t) = \pi_2(X, E(\xi_2^*(q_2^*)))$. The corresponding Hamilton-Jacobi-Bellman equation is:

$$rJ^S = \pi_2(X, E(\xi_2^*(q_2^*))) + \alpha X_t J_X^S + \frac{1}{2} \sigma^2 X_t^2 J_{XX}^S.$$

Since it is not worthwhile to switch back to the non-renewable resource, there is no option term after the firm has switched to the renewable resource. So in this case the solution to the above equation is given by:

$$J^S = \mathbb{E}_0 \int_0^\infty \pi_2(X, E(\xi_2^*(q_2^*))) e^{-rt} dt = \gamma \varphi(a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \frac{X_0^{\frac{1}{\rho}}}{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}},$$

The solutions for J^N and J^S must satisfy the following set of boundary conditions:

$$J^N(0, R_1) = 0, \quad (26)$$

$$J^N(\hat{X}, R_1) = J^S(\hat{X}, R_2) - I, \quad (27)$$

$$\frac{\partial J^N(\hat{X}, R_1)}{\partial X} = \frac{\partial J^S(\hat{X}, R_2)}{\partial X}. \quad (28)$$

$$\frac{\partial J^N(\hat{X}, R_1)}{\partial R_1} = \frac{\partial J^S(\hat{X}, R_2)}{\partial R_2} \quad (29)$$

and

$$\lim_{R_1 \rightarrow \infty} [J^N(X, R_1) - J^S(X, R_1)] = 0 \quad (30)$$

Here, $\hat{X}$ is a free boundary, which must be found as part of the solution, and which separates the switching from the no-switching regions. It is also the solution to the stopping problem (6):

$$\tau = \inf \left\{ t > 0, X \geq \hat{X} \right\}$$

The renewable resource should be adopted the first time the process X_t crosses the threshold $\hat{X}$ from below. Boundary condition (26) reflects the fact that if X_t is ever zero, it will remain at zero thereafter. Condition (27) is the value matching condition which says that the value function of the dirty firm at the time of switching is equal to the payoff from adopting the renewable input (which is equal to the value function of the green firm minus the installment cost of the green farm). In addition, to ensure that renewable resource adoption occurs along the optimal path, the value function satisfies the smooth-pasting condition (28) at the endogenous adoption threshold (see Dixit and Pindyck 1994) and the smooth-pasting condition (29) which is the tangency condition of the derivative of the value function with a non-renewable resource with respect to the stock of non-renewable resource (marginal value function) and the derivative of the value function with a renewable resource with respect to the stock of renewable resource. Finally, Eq. (30) is the transversality condition. It implies that if the initial stock of non-renewable resource is infinite, the firm does not care about having the renewable resource as an input substitute in production.

In our problem boundary condition (26) implies that $\Phi_2 = 0$ leaving the solution:

$$J^N = \Phi_1 X^{\phi_1} + \gamma (a_1 q_{1,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}} \frac{X_0^\rho}{\Theta}.$$

The value matching condition (27) can be rearranged in the following manner:

$$\Phi_1 \left(\hat{X} \right)^{\phi_1} + \frac{\gamma \left(\hat{X} \right)^{\frac{1}{\rho}} (a_1 q_{1,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}}}{\Theta} = \frac{\gamma \varphi \left(\hat{X} \right)^{\frac{1}{\rho}} (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}}}{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}} - I \quad (31)$$

The smooth-pasting condition (28) yields:

$$\Phi_1 = \frac{\gamma \left(\hat{X} \right)^{\frac{1}{\rho} - \phi_1}}{\phi_1 \rho} \left[\frac{\varphi (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}}}{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}} - \frac{(a_1 q_{1,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{\zeta} \right)^{\frac{1-\rho}{\rho}}}{\Theta} \right] \quad (32)$$

Moreover, the smooth-pasting condition (29) yields:

$$\widehat{R}_1 = R_{2,0} \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}}. \quad (33)$$

(33) is the critical level of the non-renewable resource at which the marginal value function with a non-renewable resource is equal to the marginal value function with a renewable resource. Any additional units of the non-renewable resource extracted, processed and sold as energy will make the marginal value function of a dirty firm lower than that of a clean firm.²⁷ The reason is that an additional unit of resource extracted, processed and sold as energy will increase the event of exhaustion of the non-renewable resource. Hence, $\widehat{R}_1$ can also be interpreted as the critical level of the stock of non-renewable resource which makes the decision to switch to a substitute renewable resource optimal. Finally, notice that we evaluate the optimal switching time and the option to switch at the critical level $\widehat{R}_1$ which means that their values are driven only by the other economic factors.

Plugging (33) into (32), we get the expression:

$$\Phi_1 = \frac{\gamma \left(\hat{X} \right)^{\frac{1}{\rho} - \phi_1} \varphi (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \left(1 - \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}} \right)}{\rho \phi_1 \left(\Theta - \frac{\delta(\rho-\gamma)}{\gamma} \right)}. \quad (34)$$

Plugging (34) into (31), we get the expression:

$$\frac{(1 - \rho \phi_1) \gamma \left(\hat{X} \right)^{\frac{1}{\rho}} \varphi (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \left(1 - \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}} \right)}{\rho \phi_1 \left(\Theta - \frac{\delta(\rho-\gamma)}{\gamma} \right)} = -I$$

²⁷See Figure 1.

It is easy to show that $\phi_1 \rho > 1^{28}$ so that the critical switching threshold can be written as:

$$\hat{X} = \left[\frac{I \rho \phi_1 \left(\Theta - \frac{\delta(\rho-\gamma)}{\gamma} \right)}{\varphi \gamma (\rho \phi_1 - 1) (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \left(1 - \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}} \right)} \right]^{\rho} \quad (35)$$

where $\Theta - \frac{\delta(\rho-\gamma)}{\gamma} > 0$, $\varphi = \frac{r\rho-\alpha}{r\rho-\alpha-\delta(\rho-\gamma)} > 1$ and $\rho = 1 - \eta(1 - \gamma) < 1$.

Substituting (35) into (34), we get:

$$\Phi_1 = \left(\frac{\rho \phi_1 - 1}{I} \right)^{\rho \phi_1 - 1} \left[\frac{\gamma \varphi (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \left(1 - \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}} \right)}{\rho \phi_1 \left(\Theta - \frac{\delta(\rho-\gamma)}{\gamma} \right)} \right]^{\rho \phi_1} \quad (36)$$

The value of the option to switch is defined by the exponential term $\Phi_1 X_0^{\phi_1}$, so that a simple substitution of (36) allow us to find:

$$SWO = X_0^{\phi_1} \left(\frac{\phi_1 \rho - 1}{I} \right)^{\rho \phi_1 - 1} \left(\frac{\gamma \varphi (a_2 q_{2,0})^{\frac{\rho-\gamma}{\rho}} \left(\frac{1-\rho}{c+\zeta} \right)^{\frac{1-\rho}{\rho}} \left(1 - \frac{1}{\varphi} \left(\frac{1}{\psi} \right)^{\frac{\rho-\gamma}{\gamma}} \left(\frac{c+\zeta}{\zeta} \right)^{\frac{1-\rho}{\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\gamma}} \right)}{\rho \phi_1 \left(\Theta - \frac{\delta(\rho-\gamma)}{\gamma} \right)} \right)^{\rho \phi_1}.$$

Note that SWO should be positive. Hence, $c < c^*$ (and $c^* > 0$ if $\psi > \left(\frac{1}{\varphi} \right)^{\frac{\gamma}{\rho-\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \right)^{\frac{\rho}{\rho-\gamma}}$), where:

$$c^* \equiv \zeta \left[\varphi^{\frac{\gamma}{1-\rho}} \psi^{\frac{\rho-\gamma}{1-\rho}} \left(\frac{\Theta}{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}} \right)^{\frac{\rho}{1-\rho}} - 1 \right].$$

By condition (5) computed at the level of the non-renewable resource at which the marginal value with a non-renewable resource equals the marginal value function with a renewable resource we must have $c > \underline{c}$. If $\psi > \psi^*$ where $\psi^* \equiv \frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta}$, then $\underline{c} > 0$, where:

$$\underline{c} \equiv \zeta \left[\psi^{\frac{\rho-\gamma}{1+\gamma-\rho}} \left(\frac{\Theta}{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}} \right)^{\frac{\rho-\gamma}{1+\gamma-\rho}} - 1 \right]$$

²⁸ Straightforward calculations yield $\frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{1}{2} - \frac{\alpha}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} > \frac{1}{\rho}$. Hence, $\frac{r\rho-\alpha}{\rho} - \frac{1-\rho}{2\rho^2} \sigma^2 > 0$ as assumed in section 4

Note that $c^* > \underline{c}$ if $\psi > \left(\frac{1}{\varphi}\right)^{\frac{1+\gamma-\rho}{\rho-\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta}\right)^{\frac{1}{\rho-\gamma}}$. Since $\psi > \frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta}$ straightforward calculations show that

$$\begin{aligned} \left(\frac{1}{\varphi}\right)^{\frac{1+\gamma-\rho}{\rho-\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta}\right)^{\frac{1}{\rho-\gamma}} &< \frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} \\ \text{if } \varphi &> \frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta}. \end{aligned}$$

By definition $\varphi > 1$ while $\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} < 1$, hence the inequality holds true. Finally, notice that $\left(\frac{1}{\varphi}\right)^{\frac{\gamma}{\rho-\gamma}} \left(\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta}\right)^{\frac{\rho}{\rho-\gamma}} < \frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta}$ since $\frac{\Theta - \frac{\delta(\rho-\gamma)}{\gamma}}{\Theta} < \varphi$.



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