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Abstract

We study the licensing incentives of an independent input producer owning a patented product innovation which allows the downstream firms to improve the quality of their final goods. We consider a general two-part tariff contract for both outside and incumbent innovators. We find that *technology diffusion* critically depends on the nature of market competition (Cournot vs. Bertrand). Moreover, the vertical merger with either downstream firm is always privately profitable and it is welfare improving for large innovations: this implies that not all profitable mergers should be rejected.

Keywords: Patent licensing, two-part tariff, vertical differentiation, vertical integration.

1 Introduction

We study the licensing incentives of an independent input producer owning a patented product innovation which allows the downstream firms to improve the quality of their final goods. We consider a general two-part tariff contract for both outside and incumbent innovators. We endogenize market structure as in Sandonis and Faulì-Oller (2006) allowing the patent holder to vertically integrate with either downstream firm. We also provide the welfare analysis.

More precisely, we consider two downstream firms producing and selling a final output to heterogeneous consumers and two differentiated inputs in the upstream market, a low quality input provided by competitive firms and a high quality patented input provided by an independent input producer. The quality of the final good depends on the quality of the input. Complete technology

*We have benefitted from useful comments from Salvatore Piccolo and Emanuele Tarantino. Any remaining errors are ours.

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diffusion implies a homogeneous final good of high quality, whereas exclusive licensing implies a vertically differentiated market.

We find that *technology diffusion* critically depends on the nature of market competition (Cournot vs. Bertrand). When firms compete in the quantities, the innovation is sold to all firms thus ensuring complete technology diffusion as well as a homogeneous good in the market. In contrast, when firms compete in the prices, the innovator has no incentive for complete technology diffusion, rather he prefers exclusive licensing which implies a vertically differentiated market and in turn positive industry profit to extract. In particular the internal patent holder does not license its innovation to the rival firm; the external patent holder sells only one license via a fixed fee.

As far as the *merger profitability* is concerned, we show that under Cournot competition the vertical integration of the upstream inventor with either downstream firm is always privately profitable. This result is in line with the new market foreclosure theory (see Rey and Tirole 2007) according to which vertical integration allows the monopolist upstream producer to protect its monopoly power. This result is in contrast with Sandonis and Faulì-Oller (2006) that consider non-drastic process innovations in a horizontally differentiated Cournot duopoly and find that the merger is privately profitable only for small innovations. They point out the commitment problem faced by the vertical merger (that is the insider innovator) which has only one instrument, the licensing contract to the rival firm rather than two (a licensing contract to each downstream firm), and cannot credibly restrict its output as the new input is transferred at marginal cost. However this result breaks down in our model as the incentive to diffuse the innovation makes homogeneous the downstream market. As for *social welfare profitability*, we find that under Cournot competition the merger is also welfare improving for large innovations; this implies that not all profitable mergers should be rejected. Indeed on one hand, the merger pushes prices down as it implies the (partial) internalization of the vertical externality; on the other hand, the merger has an anticompetitive effect because the vertically integrated firm is able to (partially) foreclose the rival firm via a positive per-unit royalty. The first effect prevails as long as the quality improvement associated with the innovation is sufficiently large. Under Bertrand competition, we find a result of equivalence between an external and an internal patent holder, both in terms of private and social welfare profitability. Indeed, an external patent holder sells an exclusive license via a fixed fee, so that there is no distortion due to a positive per-unit royalty (as the patent holder were vertically integrated). This way the patent holder maximizes the licensee's profit and extract this profit up to the outside option.

This paper contributes to the literature on licensing a product innovation as well as to the debate on the competitive effects of vertical integration.

From a theoretical viewpoint, most of the literature on optimal licensing focuses on cost-reducing, *process innovations* (see Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien, Oren and Tauman, 1992; Sen and Tauman, 2007; Erutku and Richelle, 2007). To our knowledge, little has been done to investigate the issue of licensing a *product innovation*.[1] In particular, we find that under Cournot competition with homogeneous goods, the external patent holder optimally specifies positive per-unit royalties when the innovation is large. This is in line with the wide prevalence of per-unit royalties over fixed fee in practice (see for instance, Rostoker (1984)). Moreover, Muto (1993) studies optimal licensing of a process innovation and shows that Bertrand competition is a rationale to explain this empirical evidence. We argue that this theoretical result does not hold for a product innovation: in our model the external patent holder prefers fixed fee over royalty licensing.

As for the competitive effects of vertical integration, there are two opposite views. The Chicago School (e.g., Bork, 1978; Posner, 1976) stresses that, in the absence of efficiency gains, vertical integration could not increase the profitability of merging firms (Rey and Tirole, 2007). In contrast the new market foreclosure theory stresses the role played by vertical integration in restricting downstream competition. We show that the social welfare profitability depends on the innovation size.

The remainder of the paper is structured as follows. In Section 2, we set up the ante-innovation model. In Section 3 we introduce the product innovation and we study the licensing incentives of an external innovator. In Section 4 we consider the optimal strategy of an internal innovator. In Section 5, we compare the private and social incentives. In Section 6 we extend the analysis to Bertrand competition.

2 Model

We consider two firms producing a homogeneous good and competing à la Cournot.[2] Final output production requires an essential input provided by a competitive upstream market.

As far as the *demand side* is concerned, we assume that there is a continuum of consumers indexed by θ which is uniformly distributed in the interval $[0, 1]$. Thus, θ is a taste parameter. Each consumer has a unit demand and buys either one unit of a good of quality s at price p or buys nothing at all. Consumer's utility takes the following form:

$$U(\theta) = \begin{cases} \theta s - p, \text{ if consumer type } \theta \text{ buys} \\ 0, \text{ if does not buy} \end{cases}$$

The demand for the good is then

$$Q(p) = 1 - \left(\frac{p}{s}\right) \iff p(Q) = s(1 - Q), \tag{1}$$

[1] However, as pointed out by Kamien et *al.* (1988) "a product innovation can be regarded as a cost reducing innovation by assuming that the new product could have been produced before but with a sufficiently high marginal cost that rendered its production unprofitable."

[2] Given a homogeneous final good, price competition leads to the Bertrand paradox. We extend the analysis to Bertrand competition in Section 6.

where $Q = q_1 + q_2$ and p/s is the fraction of consumers with a taste parameter less than θ, that is the fraction of consumers not buying the good.[3] For future reference we define the consumer surplus as

$$CS(s) = \int_{\frac{p}{s}}^{1} (\theta s - p)\, d\theta = \frac{(p-s)^2}{2s} \tag{2}$$

As for the *supply side*, the essential input of quality s is produced at zero fixed cost $f_L = 0$ and at constant marginal cost $c = 0$ and it is sold at the competitive price $w = 0$. In this framework quality is assimilated to input. The D firm i profit function is: $\pi_i = pq_i$. D firms compete in the quantities, then the Cournot duopoly equilibrium is (superscript C stands for Cournot):

$$q_i^C (c_i = 0, c_j = 0; s, s) = \frac{1}{3} \tag{3}$$

$$\begin{aligned} p^C &= \frac{s}{3} \\ \pi_i^C &= \frac{s}{9} \\ CS^C &= \frac{2}{9}s \end{aligned} \tag{4}$$

The D firms' price and profits depend on the quality of the input s. The D firms sell $Q = q_1 + q_2 = \frac{2}{3}$ which is the demand for the input faced by the upstream market (perfect vertical complementarity).

For future reference (and as a benchmark) consider the monopoly outcome for this market:[4]

$$\begin{aligned} p^m &= \frac{s}{2}, \\ q^m &= \frac{1}{2}, \\ \pi^m &= \frac{s}{4} > 2\pi_i^C. \end{aligned}$$

3 Innovation

Suppose that an independent input producer obtains a patented product innovation which allows the downstream firms to improve the quality of their final goods. In the upstream market there is now a monopolist selling an input that ameliorates final product quality by $\psi > 1$ that measures the innovation size. Assume production cost is $f_H = f > 0$ for the high quality input.

We study the licensing incentives of this patent holder. The U firm can sell the new input either to one or both D firms via a two-part licensing contract

[3] At equilibrium the market is not covered.

[4] The monopolist maximization problem would be $\max_p [p(1 - p/s)]$.

(r,F).[5] Different cases derive:

1. *Complete technology diffusion*: both D firms adopt the new input and we have a homogeneous final good of quality $s\psi > s$. D firms' profits $\pi_i(c_i, c_j; s\psi, s\psi)$ depends on the two part-tariff contracts $c_i = (r_i, F_i)$ with $i = 1,2$ and $i \neq j$.

2. *Exclusive licensing*: only one of the D firms adopt the new input and we have two final goods of different qualities. The non-innovating firm, say firm 1, produces the low quality good thus incurring zero production costs and gains $\pi_1(0, c_2; s, s\psi)$; while the innovating firm 2 produces the high quality good and gets $\pi_2(c_2, 0; s\psi, s)$.

We develop a three-stage game: first, the innovator offers a contract to each D firm on a take-it-or-leave-it basis; second, the potential licensees decide whether to accept or reject the contract; finally the D firms compete. Solving backwards, we find the subgame perfect Nash equilibrium.

3.1 Exclusive licensing

Suppose that only one D firm adopts the new input. Firm 1 does not buy the new input and produces a final good of quality $s_1 = s$ at price p_1; firm 2 adopts the new input and produces a final good of quality $s_2 = \psi s > s$, at price p_2 with $s_2 - s_1 = s(\psi - 1)$. The demands for the goods are:

$$q_1 = \widehat{\theta} - \frac{p_1}{s} \tag{5}$$

$$q_2 = 1 - \widehat{\theta}, \tag{6}$$

where

$$\widehat{\theta} = \frac{p_2 - p_1}{s(\psi - 1)}.$$

The inverse demands are:

$$\begin{aligned} p_1 &= s(1 - q_2 - q_1) \\ p_2 &= s(\psi - \psi q_2 - q_1) \end{aligned}$$

D firms profits are:

$$\begin{aligned} \pi_1 &= p_1 q_1 \\ \pi_2 &= (p_2 - r_2) q_2 - F_2 \end{aligned}$$

[5] A two-part licensing contract covers both the case of the royalty and the case of the fixed fee licensing modes: a royalty can be seen as a two-part tariff with $F = 0$; the fixed fee can be seen as a two-part tariff with $r = 0$.

Cournot competition leads to the following third stage quantity and price equilibrium:

$$\begin{aligned}
q_1(0, r_2; s, s\psi) &= \frac{s\psi + r_2}{s(4\psi - 1)}, \\
q_2(r_2, 0; s\psi, s) &= \frac{(2s\psi - s - 2r_2)}{s(4\psi - 1)}, \\
p_1(0, r_2; s, s\psi) &= \frac{(s\psi + r_2)}{(4\psi - 1)}, \\
p_2(r_2, 0; s\psi, s) &= \frac{(2\psi - 1)(s\psi + r_2)}{(4\psi - 1)},
\end{aligned}$$

with $q_2(r_2, 0; s\psi, s) > 0 \iff \frac{s(2\psi-1)}{2} > r_2$. Firm1 profit is then:

$$\pi_1(0, r_2; s, s\psi) = \frac{(s\psi + r_2)^2}{(4\psi - 1)^2 s} \tag{7}$$

As for firm 2:

$$\pi_2(r_2, 0; s\psi, s) = \frac{\psi(s - 2s\psi + 2r_2)^2}{(4\psi - 1)^2 s} - F_2$$

The U firm chooses the two-part tariff contract for firm 2 (r_2, F_2) such that:

$$\begin{aligned}
&\max_{r_2, F_2} \Pi_U \\
s.t. r_2 &\in \left[0, \frac{s(2\psi - 1)}{2}\right] \\
F_2 &\leq \pi_2(r_2, 0; s\psi, s) - \frac{s}{9}
\end{aligned}$$

with $\Pi_U = \frac{(2s\psi-s-2r_2)}{s(4\psi-1)} r_2 + F_2 - f$. The first constraint comes from the non-negativity of q_2 and the second constraint (binding at equilibrium) ensures that firm 2 has the incentive to get the license rather than the status quo. The solution is a contract such that $r_2^* = 0$ and $F_2^* = \frac{(36\psi^2-16\psi+1)(\psi-1)s}{9(4\psi-1)^2}$. Remaining equilibrium variables are (superscript EL stands for exclusive licensing):

$$\begin{aligned}
q_1^{EL} &= \frac{\psi}{(4\psi - 1)}, q_2^{EL} = \frac{(2\psi - 1)}{(4\psi - 1)}, Q_{EL} = \frac{(3\psi - 1)}{(4\psi - 1)} \\
p_1^{EL} &= \frac{\psi s}{(4\psi - 1)}, p_2^{EL} = \frac{(2\psi - 1)\psi s}{(4\psi - 1)}, \\
\pi_1(s, s\psi) &= \frac{\psi^2 s}{(4\psi - 1)^2} < \pi_2(s\psi, s) = \frac{s}{9} \\
\Pi_U^{EL}(s, s\psi) &= \frac{(36\psi^2-16\psi+1)(\psi-1)s}{9(4\psi-1)^2} - f.
\end{aligned}$$

Π_U^{EL} is the U patent holder equilibrium profit under exclusive licensing, when selling via a two-part tariff, which reduces to a fixed fee, the new input to only one D firm. For completeness we provide equilibrium consumer surplus under exclusive licensing:

$$CS^{EL} = \int_{\frac{p_1}{s}}^{\frac{p_2-p_1}{s(\psi-1)}} (\theta s - p_1)\, d\theta + \int_{\frac{p_2-p_1}{s(\psi-1)}}^{1} (\theta s\psi - p_2)\, d\theta = \frac{\left(\psi+4\psi^2-1\right)s\psi}{2(4\psi-1)^2}.$$

3.2 Complete technology diffusion

Suppose the U firm decides to sell the new input to both D firms via a *two-part tariff* (r, F). The U firm maximization problem is:

$$\max_{F_1,r_1,F_2,r_2} \{r_1 q_1 (r_1, r_2; s\psi, s\psi) + r_2 q_2 (r_1, r_2; s\psi, s\psi) + F_1 + F_2 - f\}$$
$$s.t.\pi_1 (r_1, r_2; s\psi, s\psi) - F_1 \geq \pi_1 (0, r_2; s, s\psi)$$
$$\pi_2 (r_2, r_1; s\psi, s\psi) - F_2 \geq \pi_2 (0, r_1; s, s\psi)$$
$$r_1 \geq 0, r_2 \geq 0, F_1 \geq 0, F_2 \geq 0$$

where $\pi_1 (0, r_2; s, s\psi)$ is defined in (7). Here the outside option for each firm is not buying the new input given that the rival firm does. $\pi_i (r_i, r_j; s\psi, s\psi)$ and $q_i (r_i, r_j; s\psi, s\psi)$ denote the third stage equilibrium D firm i profit and quantity when both firms produce the high quality good, namely:

$$\pi_i (r_i, r_j; s\psi, s\psi) = \frac{(s\psi - 2r_i + r_j)^2}{9\psi s}, \tag{8}$$

$$q_i (r_i, r_j; s\psi, s\psi) = \frac{1}{3s\psi} (s\psi - 2r_i + r_j). \tag{9}$$

As the two constraints are binding at equilibrium, we have

$$F_1 (r_1, r_2) = \pi_1 (r_1, r_2; s\psi, s\psi) - \pi_1 (0, r_2; s, s\psi),$$
$$F_2 (r_1, r_2) = \pi_2 (r_2, r_1; s\psi, s\psi) - \pi_2 (0, r_1; s, s\psi),$$

with $\pi_i (0, r_j; s, s\psi) = \frac{(s\psi+r_j)^2}{(4\psi-1)^2 s}$. The maximization problem, thus becomes:

$$\max_{r_1,r_2} \{r_1 q_1 (r_1, r_2; s\psi, s\psi) + r_2 q_2 (r_2, r_1; s\psi, s\psi) + F_1 (r_1, r_2) + F_2 (r_1, r_2) - f\}.$$

The optimal contract is then:

$$\begin{cases} r_1 = r_2 = \frac{\left(s\psi-26s\psi^2+16s\psi^3\right)}{64\psi^2-14\psi+4},\ F(r_1, r_2) = \frac{\left(248\psi^2-17\psi-272\psi^3+256\psi^4+1\right)s\psi}{4(32\psi^2-7\psi+2)^2} \text{ if } \psi > 1.585\,6, \\ r_1 = r_2 = 0,\ F(0,0) = \frac{(\psi-1)(16\psi-1)s\psi}{9(4\psi-1)^2} \text{ if } \psi < 1.585\,6. \end{cases} \tag{10}$$

This means that when the innovation is small the inventor's incentive is to set a per-unit price as low as possible, that is the optimal contract is a fixed fee. In contrast for large innovations we have a positive per-unit royalty.[6]

[6] This result is in line wih Sen and Tauman (2007).

Equilibrium magnitudes are for $\psi > 1.5856$ (superscript T stands for complete technology diffusion):[7]

$$q^T(s\psi) = \frac{\left(4\psi+16\psi^2+1\right)}{2(32\psi^2-7\psi+2)}, Q^T = \frac{\left(4\psi+16\psi^2+1\right)}{(32\psi^2-7\psi+2)}$$
$$p^T = \frac{\left(16\psi^2-11\psi+1\right)\psi s}{(32\psi^2-7\psi+2)}$$
$$\pi^T(s\psi) = \frac{\left(4\psi+16\psi^2+1\right)^2 s\psi}{4(32\psi^2-7\psi+2)^2}$$
$$\pi^T(s\psi) - F^*(r^*, r^*) = \frac{25(4\psi-1)^2 s\psi^2}{4(32\psi^2-7\psi+2)^2}$$
$$\Pi_U^T(s\psi) = 2r^* q^T(s\psi) + 2F^*(r^*, r^*) - f = \frac{\left(16\psi^2-16\psi+1\right)s\psi}{2(32\psi^2-7\psi+2)} - f$$
$$CS^T = \frac{\left(4\psi+16\psi^2+1\right)^2 s\psi}{2(32\psi^2-7\psi+2)^2},$$
$$SW^T = s\psi\frac{\left(4\psi+16\psi^2+1\right)^2}{2(32\psi^2-7\psi+2)^2} + \left(2\frac{25(4\psi-1)^2 s\psi^2}{4(32\psi^2-7\psi+2)^2} + \frac{\left(16\psi^2-16\psi+1\right)s\psi}{2(32\psi^2-7\psi+2)} - f\right)$$

Whereas for $\psi < 1.5856$, equilibrium magnitudes are:

$$q^T(s\psi) = \frac{1}{3}, Q^T = \frac{2}{3}$$
$$p^T(s\psi) = \frac{s\psi}{3}$$
$$\pi^T(s\psi) = \frac{s\psi}{9}$$
$$\pi^T(s\psi) - F^*(r^*, r^*) = \frac{\psi^2 s}{(4\psi-1)^2}$$
$$\Pi_U^T(s\psi) = 2F^*(0,0) - f = 2\frac{(\psi-1)(16\psi-1)s\psi}{9(4\psi-1)^2} - f,$$
$$CS^T = \frac{2}{9}\psi s,$$
$$SW^T = \frac{2}{9}\psi s + \left(2\frac{\psi^2 s}{(4\psi-1)^2} + 2\frac{(\psi-1)(16\psi-1)s\psi}{9(4\psi-1)^2} - f\right)$$

Comparing equilibrium variables under exclusive licensing and complete technology diffusion, we find the following results.

Proposition 1 *Under Cournot competition, the external patent holder always prefers complete technology diffusion, namely,* $\Pi_U^T(s\psi) - \Pi_U^{EL}(s\psi) > 0$. *The optimal contract is specified in (10).*

A consequence of the innovator's preferences towards technology diffusion is that the downstream market is not vertically differentiated, as only the high quality input is sold to both firms. In other words, the downstream market is characterized by a homogeneous good.

[7] The condition $\psi > 1.5856$ from the non-negativity of the optimal royalty.

Comparing total quantities, prices, profits and consumer surpluses, we find the following rankings.

for $\psi > 1.5856$	$Q_{EL} > Q > Q^T$, $\pi_2(s\psi, s) > \pi^T(s\psi) - F^*(r^*, r^*) > \pi_1(s, s\psi)$,
for $\psi < 1.5856$	$Q_{EL} > Q = Q^T$, $\pi_2(s\psi, s) > \pi^T(s\psi) - F^*(r^*, r^*) = \pi_1(s, s\psi)$
$\forall\psi$	$p_1^{EL} < p < p^T < p_2^{EL}$, $CS^T > CS^{EL} > CS$, $SW^T > SW^{EL}$

4 Vertical integration

We have considered so far, the case of an *external innovator*, i.e. the U firm does not sell the final good in the D market. Suppose now that the U producer and one of the two D firms, say firm 2, merge, in this case the vertically integrated (VI) firm is an internal patent holder and its profit consists of two parts: the profit from selling the new input to the rival D firm 1 (if it decides to license) and the profit from selling the high quality final good 2.

We consider the following three-stage game: first, the patent holder offers a contract to D firm 1, D firm 1 decides whether to accept it and finally market competition takes place.

Proceeding backwards, consider the quantity competition between the VI firm and firm 1 producing the same high quality final good. The VI firm has zero variable production costs as the new input is trasferred at the marginal cost $c_2 = 0$, whereas firm 1 incurs marginal cost r_1:

$$q_{VI}(0, r_1; s\psi, s\psi) = \frac{1}{3s\psi}(s\psi + r_1)$$

$$q_1(r_1, 0; s\psi, s\psi) = \frac{1}{3s\psi}(s\psi - 2r_1) \geq 0 \iff r_1 \leq \frac{s\psi}{2}$$

$$p_{VI} = s\psi(1 - q_{VI}(0, r_1; s\psi, s\psi) - q_1(r_1, 0; s\psi, s\psi)) = \frac{1}{3}(s\psi + r_1)$$

$$\pi_{VI}(0, r_1; s\psi, s\psi) = \frac{(s\psi + r_1)^2}{9s\psi} - f$$

$$\pi_1(r_1, 0; s\psi, s\psi) = \frac{(s\psi - 2r_1)^2}{9\psi s}$$

where $q_{VI}(0, r_1; s\psi, s\psi)$ and $q_1(r_1, 0; s\psi, s\psi)$ are obtained from expression (9) substituting properly r_i and r_j; $\pi_{VI}(0, r_1; s\psi, s\psi)$ and $\pi_1(r_1, 0; s\psi, s\psi)$ are obtained from expression (8) substituting properly r_i and r_j. The VI firm offers firm 1 the two-part tariff contract (r_1, F_1) such that:[8]

$$\max_{r_1,F_1} \{\pi_{VI}(0, r_1; s\psi, s\psi) + r_1 q_1(r_1, 0; s\psi, s\psi) + F_1\}$$
$$s.t. \pi_1(r_1, 0; s\psi, s\psi) - F_1 \geq \pi_1(0, 0; s, s\psi)$$
$$r_1 \in \left[0, \frac{s\psi}{2}\right],$$

where $\pi_1(0, 0; s, s\psi) = \frac{\psi^2 s}{(4\psi-1)^2}$, obtained from (7) is firm 1 outside option, that is firm 1 profit if it does not buy the new input thus selling the low quality final good and incurring per-unit cost equal to zero. As the first constraint is binding at equilibrium, we have:

$$\max_{r_1} \{\pi_{VI}(0, r_1; s\psi, s\psi) + r_1 q_1(r_1, 0; s\psi, s\psi) + \pi_1(r_1, 0; s\psi, s\psi) - \pi_1(0, 0; s, s\psi)\}$$

The optimal contract is then:

$$r_1^* = \frac{s\psi}{2}, F_1^* = -\frac{\psi^2 s}{(4\psi - 1)^2}$$

If we let the VI firm to set negative fees, the vertical merger implements the monopoly outcome by inducing the nonintegrated firm to produce a nil quantity (foreclosure) and compensating it for the outside option. Equilibrium magnitudes are:

$$q_{VI}(0, r_1^*; s\psi, s\psi) = \frac{1}{2} = q^m$$
$$q_1(r_1^*, 0; s\psi, s\psi) = 0$$
$$p_{VI} = \frac{1}{2}\psi s = p^m$$
$$\pi_{VI}(0, r_1^*; s\psi, s\psi) = \frac{1}{4}\psi s$$
$$\pi_1(r_1^*, 0; s\psi, s\psi) - F_1^* = \frac{\psi^2 s}{(4\psi - 1)^2}$$
$$\Pi_{VI} = \pi_{VI}(0, r_1^*; s\psi, s\psi) - f + r_1^* q_1(r_1^*, 0; s\psi, s\psi) + F_1^* = \frac{1}{4}\psi s - \frac{\psi^2 s}{(4\psi - 1)^2} - f$$

However negative fees would be clearly held to be illegal by antitrust authorities. It is clear from the analysis above that the VI firm wants to restrict as much as possible the quantity produced by the non affiliate firm so as to (at least) partially internalize the vertical externality. If the VI is constrained to nonnegative fees, it will optimally let the nonaffiliate firm to produce a positive

[8] In the following maximization problem we do not restrict the VI firm to set nonnegative fees. This allows us to make clear its incentives. We next solve the maximization problem constrained to nonnegative fees.

quantity $q_1(r_1) : \pi_1(r_1) = \frac{\psi^2 s}{(4\psi-1)^2}$. Given the Cournot equilibrium quantities, we have

$$\left(\frac{1}{3}(s\psi + r_1) - r_1\right)\frac{1}{3s\psi}(s\psi - 2r_1) = \frac{\psi^2 s}{(4\psi-1)^2}$$
$$\Longleftrightarrow r_1 = \frac{\left(\psi s(4\psi-1) - 3s\psi\sqrt{\psi}\right)}{2(4\psi-1)}$$

The optimal contract is then:

$$r_{VI}^C = \frac{\left(\psi s(4\psi-1) - 3s\psi\sqrt{\psi}\right)}{2(4\psi-1)}, F_{VI}^C = 0. \tag{11}$$

Equilibrium magnitudes are:

$$q_{VI}^C(0, r_{VI}^*; s\psi, s\psi) = \frac{\left(4\psi^2 - \psi\sqrt{\psi} - \psi\right)}{2(4\psi-1)\psi}$$
$$q_1^C(r_{VI}^*, 0; s\psi, s\psi) = \frac{\sqrt{\psi}}{(4\psi-1)}$$
$$Q_{VI}^C = q_{VI}(0, r_{VI}^*; s\psi, s\psi) + q_1(r_{VI}^*, 0; s\psi, s\psi) = \frac{\left(4\psi^2 - \psi + \psi^{\frac{3}{2}}\right)}{2(4\psi-1)\psi}$$
$$p_{VI}^C = \frac{\psi\left(4\psi - \sqrt{\psi} - 1\right)s}{2(4\psi-1)}$$
$$\pi_{VI}^C(0, r_{VI}^*; s\psi, s\psi) = \frac{\left(4\psi^2 - \psi\sqrt{\psi} - \psi\right)^2 s}{4(4\psi-1)^2\psi}$$
$$\pi_1^C(r_{VI}^*, 0; s\psi, s\psi) - F_{VI}^* = \frac{s\psi^2}{(4\psi-1)^2}$$
$$\Pi_{VI}^C = \pi_{VI}(0, r_{VI}^*; s\psi, s\psi) - f + r_{VI}^* q_1(r_{VI}^*, 0; s\psi, s\psi) + F_{VI}^* = \frac{\left(16\psi^2 - 13\psi + 1\right)s\psi}{4(4\psi-1)^2}$$
$$CS_{VI}^C = \frac{\left(4\psi + \sqrt{\psi} - 1\right)^2 s\psi}{8(4\psi-1)^2},$$
$$SW_{VI}^C = \frac{\left(4\psi + \sqrt{\psi} - 1\right)^2 s\psi}{8(4\psi-1)^2} + \frac{s\psi^2}{(4\psi-1)^2} + \frac{\left(16\psi^2 - 13\psi + 1\right)s\psi}{4(4\psi-1)^2}$$

We gather the above results as follows.

Proposition 2 *Under Cournot competition, the internal patent holder always sells the innovation to the rival firm. The optimal contract is specified in (11).*

San Martin and Saracho (2010) consider a non drastic process innovation and show that in a Cournot duopoly with homogeneous goods the optimal licensing mode is an ad valorem royalty, that is a profit sharing agreement. This result does not hold in our model: as we prove in the Appendix the ad valorem licensing mode is dominated by the two-part tariff contract defined in (11). The intuition behind this result is as follows. In San Martin and Saracho (2010) the ad valorem royalty is optimal as the internal patentee introduces the process innovation that increases total quantity (it let the rival firm to produce with a more efficient technology) and then appropriates the rival's profit up to its outside option. In our model if the innovation is diffused via an ad valorem royalty total output does not increase (as under homogeneous goods it is independent of the quality, see the status quo equilibrium quantity, expression 3). This implies that industry profits correspond to the duopoly profits that are shared according to α. Whereas under two-part tariff the internal patentee can at least partially internalize the negative externality coming from competition and approach the monopoly outcome.

5 Private and social profitability of vertical integration

We next consider the merger profitability as well as the social welfare comparing the vertical integration scenario with the vertical separation scenario (i.e., external patent holder) where complete technology diffusion takes place.

As far as the private profitability is concerned, the merger is always profitable, namely:

$$\Pi_{VI}^{C} - \left(\pi^{T}\left(s\psi\right) - F^{*}\left(r^{*}, r^{*}\right) + \Pi_{U}^{T}\left(s\psi\right)\right) = \begin{cases} \frac{5\left(256\psi^{4}-57\psi^{2}-112\psi^{3}-7\psi+1\right)s\psi^{2}}{4(32\psi^{2}-7\psi+2)^{2}(4\psi-1)^{2}} > 0 \text{ for } \psi > 1.585\,6, \\ \frac{(\psi-1)(16\psi-1)s\psi}{36(4\psi-1)^{2}} > 0 \text{ for } \psi < 1.585\,6. \end{cases}.$$

This result is in line with the new market foreclosure theory according to which VI allows the U producer monopolist to protect its monopoly power.[9]

As for the social profitability of VI, we make the following comparisons. For $\psi < 1.585\,6$, $CS_{VI}^{C} - CS^{T} = \frac{\left(4\psi+\sqrt{\psi}-1\right)^{2}s\psi}{8(4\psi-1)^{2}} - \frac{2}{9}\psi s = \psi s\left(\frac{\left(4\psi+\sqrt{\psi}-1\right)^{2}}{8(4\psi-1)^{2}} - \frac{2}{9}\right) < 0$. For $\psi > 1.585\,6$, $CS_{VI}^{C} - CS^{T} = s\psi\left(\frac{\left(4\psi+\sqrt{\psi}-1\right)^{2}}{8(4\psi-1)^{2}} - \frac{\left(4\psi+16\psi^{2}+1\right)^{2}}{2(32\psi^{2}-7\psi+2)^{2}}\right) > 0 \iff \psi > 3.407\,8$. We can conclude that for $\psi > 3.407\,8$ both the industry profit and consumer surplus are higher under VI, that is vertical integration is welfare improving. In contrast for $\psi < 3.407\,8$, the result is ambiguous as consumer surplus is lower but producer surplus is higher under VI rather than the non-merger case. However direct computations of social welfare ($SW = CS + PS$) reveal the following.

$$SW_{VI}^{C} - SW^{T} < 0 \iff \psi < 3.407\,8.$$

We gather our results in the following proposition.

Proposition 3 *Vertical integration is always privately profitable. However it is welfare improving if and only if the innovation is sufficiently large, namely* $\psi > 3.407\,8$.

[9] See Rey and Tirole 2007.

Sandonis and Faulì-Oller (2006) consider non-drastic process innovations in a horizontally differentiated Cournot duopoly and study the patentee incentives to merge with either firm in the market. They show that the merger is privately profitable for small innovations and it is welfare improving for large innovations. They argue that all profitable mergers are welfare detrimental, this also holds for homogeneous goods. More precisely, Sandonis and Faulì-Oller (2006) individuate the following trade-off: an internal patentee (VI case) better internalizes market profit with respect to an external one, however the patentee can only use one instrument (a contract for the other firm in the market) rather than two (a contract for each firm in the market). An external patentee has two instruments but it has to care about firms' outside option which depends on the royalty, in particular it increases with the royalty and decreases with the innovation size. They find that the balance of these two effects depend on the innovation size: the merger is privately profitable for small innovations, in fact for large innovations the outside option faced by the external patentee is low and so it has little relevance with respect to the availability of two instruments. In our model the merger is profitable for any innovation size ψ. In particular, in contrast with Sandonis and Faulì-Oller (2006) the internal patentee approaches the monopoly outcome as the innovation size ψ increases. In fact, the VI firm has incentive to reduce as much as possible the quantity produced by the non-affiliate, in order to internalize as much as possible the vertical externality (in the limit, if allowed, the VI firm would completely foreclose the rival firm compensating it via a negative fixed fee). However the VI firm is constrained by the nonaffiliate outside option: the higher ψ the lower is this outside option and so the lower the quantity that the nonaffiliate produces and in turn the more the VI firm approaches the monopoly outcome. The outside option for the external patentee is $\pi_i\left(0, r_j; s, s\psi\right)$ decreasing in ψ and increasing in r_j. The outside option for the internal patentee is $\pi_1\left(0, 0; s, s\psi\right) = \frac{\psi^2 s}{(4\psi-1)^2}$ independent of r and decreasing in ψ. For low ψ, both outside options are large. There is the same negative effect of Sandonis and Faulì-Oller (2006) related to the external patentee and so vertical integration dominates vertical separation. For high ψ, both outside options are small, however also in this case vertical integration dominates as the internal patentee approches the monopoly outcome.

As for social welfare, in contrast with Sandonis and Faulì-Oller (2006) we find that, under homogeneous goods, vertical integration is privately and socially profitable for high quality improvements. Vertical integration has two opposite effects on prices: on one hand, VI pushes prices down as it implies the (partial) internalization of the vertical externality; on the other hand, VI has an anticompetitive effect because the VI firm is able to (partially) foreclose the rival firm via a positive per-unit royalty. The first effect prevails for ψ high (and we have $p_{VI} - p^T < 0$), the opposite effect prevails for ψ low (and we have $p_{VI} - p^T > 0$).

6 Bertrand competition

We next extend our analysis to Bertrand competition. This extension allows us to analyse a post-innovation scenario with product differentiation. Indeed the innovator has no incentive to sell its product innovation to both firms, as under Bertrand competition with homogeneous goods, market profits are equal to zero and therefore he could not extract any surplus from the licensees.

Ex-ante, as goods are homogeneous, equilibrium price is equal to marginal cost, set to zero. Therefore, the market is covered, i.e., the demand is equal to one; firms make zero profit and social welfare coincide with consumer surplus that is:

$$SW^B(s) = CS^B(s) = \int_0^1 (\theta s)\, d\theta = \frac{s}{2}.$$

Under Bertrand competition, the patent holder will sell the new input to only one firm (otherwise the Bertrand paradox applies). We thus analyse the optimal contract under exclusive licensing, considering in turn the case of an *external patentee* and the case of an *internal patentee*.

Suppose as before that firm 1 is the non-innovating firm that produces a final good of quality $s_1 = s$ at price p_1; firm 2 is the innovating firm that produces a final good of quality $s_2 = \psi s > s$, at price p_2. The demands for the goods are the same as in (5) and (6), in particular:

$$q_1 = \frac{p_2 - p_1}{s(\psi - 1)} - \frac{p_1}{s}, q_2 = 1 - \frac{p_2 - p_1}{s(\psi - 1)}.$$

D firms profits are:

$$\pi_1 = p_1 q_1, \pi_2 = (p_2 - r_2) q_2 - F_2.$$

Bertrand competition leads to the following third stage prices and quantity equilibrium:

$$\begin{aligned} p_1(0, r_2; s, s\psi) &= \frac{s\psi - s + r_2}{4\psi - 1}, \\ p_2(r_2, 0; s\psi, s) &= \frac{2\psi(s\psi - s + r_2)}{4\psi - 1} \\ q_1(0, r_2; s, s\psi) &= \frac{(s\psi - s + r_2)\psi}{(\psi - 1)(4\psi - 1)s} \\ q_2(r_2, 0; s\psi, s) &= \frac{2\psi s(\psi - 1) - r_2(2\psi - 1)}{(\psi - 1)(4\psi - 1)s} \end{aligned}$$

with $q_2(r_2, 0; s\psi, s) > 0 \iff r_2 < \frac{2\psi s(\psi-1)}{(2\psi-1)}$.[10] Firm 1 profit is then:

$$\pi_1(0, r_2; s, s\psi) = \frac{(s\psi - s + r_2)^2 \psi}{(4\psi - 1)^2 (\psi - 1)s}$$

[10] This inequality is more stringent than the corresponding one under Cournot competition. Namely, $\frac{s(2\psi-1)}{2} > \frac{2\psi s(\psi-1)}{(2\psi-1)}$.

As for firm 2:

$$\pi_2(r_2, 0; s\psi, s) = \frac{(2\psi s(\psi-1) - r_2(2\psi-1))^2}{(4\psi-1)^2(\psi-1)s} - F_2$$

The U firm chooses the two-part tariff contract for firm 2 (r_2, F_2) such that:

$$\begin{aligned} & \max_{r_2, F_2} \Pi_U \\ s.t. r_2 &\in \left[0, \frac{2\psi s(\psi-1)}{(2\psi-1)}\right] \\ F_2 &\leq \tfrac{\left(2s\psi - r_2 + 2\psi r_2 - 2s\psi^2\right)^2}{(4\psi-1)^2(\psi-1)s} - \tfrac{(s\psi - s + r_2)^2\psi}{(4\psi-1)^2(\psi-1)s} \end{aligned}$$

with $\Pi_U = \frac{2\psi s(\psi-1) - r_2(2\psi-1)}{(\psi-1)(4\psi-1)s} r_2 + F_2 - f$. The first constraint comes from the non-negativity of q_2 and the second constraint (binding at equilibrium) ensures that firm 2 has the incentive to get the license rather than the outside option, that we assume to be equal to $\pi_1(0, r_2; s, s\psi) > 0$.[11] The optimal contract is:

$$r_2^B = 0, F_2^B = \frac{(\psi-1)s\psi}{(4\psi-1)}. \tag{12}$$

A product innovation is sold via a fixed fee, this is due to the fact that the rival firm has zero marginal cost.[12] This result is in contrast with a process innovation (Muto, 1993). Remaining equilibrium variables are:

$$q_1^B = \frac{\psi}{(4\psi-1)}, q_2^B = \frac{2\psi}{(4\psi-1)}, Q_B = \frac{3\psi}{(4\psi-1)} \tag{13}$$

$$p_1^B = \frac{(\psi-1)s}{4\psi-1}, p_2^B = \frac{2\psi s(\psi-1)}{4\psi-1}, \tag{14}$$

$$\pi_1(s, s\psi) = \tfrac{(\psi-1)s\psi}{(4\psi-1)^2} = \pi_2(s\psi, s) = \tfrac{(\psi-1)s\psi}{(4\psi-1)^2}$$

$$\Pi_U^B(s, s\psi) = \tfrac{(\psi-1)s\psi}{(4\psi-1)} - f.$$

Π_U^B is the U patent holder profit under exclusive licensing, when selling via a two-part tariff, which reduces to a fixed fee, the new input to only one D firm. For completeness we provide equilibrium consumer surplus under Bertrand competition:

$$CS^B = \tfrac{(4\psi+5)s\psi^2}{2(4\psi-1)^2}. \tag{15}$$

Consider next the case of an internal patentee, in particular assume that the U producer and firm 2 merge. The VI firm does not sell the innovation to the

[11] In fact firm 2 can always refuse the offer of the patent holder knowing that he will make the offer to the rival firm. Therefore the outside option is not zero (the status quo profit), rather it is positive and equal to the low quality firm's profit.

[12] If the rival firm had a positive marginal cost of production, say $c > 0$, (so that the innovation would include the product as well as the process) then the optimal contract would be $r_2 = \frac{c}{2} > 0$, $F_2 = \frac{4\psi s^2(\psi-1) + 4cs\psi + c^2(1-4\psi)}{4(4\psi-1)s}$.

rival so that we have an equilibrium such that the VI firm produces the high quality good at zero costs and compete with the rival firm 1 that produces the low quality good at zero cost. Equilibrium quantities, prices and CS are as in (13), (14) and (15), equilibrium profits are

$$\begin{aligned}\pi_1^B &= \tfrac{(\psi-1)s\psi}{(4\psi-1)^2} \\ \pi_{VI}^B &= 4\tfrac{(\psi-1)s\psi^2}{(4\psi-1)^2}\end{aligned}$$

As for the merger private and social profitability under Bertrand competition we find that:

$$\pi_{VI}^B - \left(\pi_2\left(s\psi, s\right) - F_2^B + \Pi_U^B\left(s\psi\right)\right) = 0, SW_{VI}^B - SW^B = 0.$$

The patent holder is indifferent between staying out of the market and vertically integrate. The same holds from the social welfare point of view. The above results are gathered in the following proposition.

Proposition 4 *Under Bertrand competition the patent holder always prefers exclusive licensing. (i) An external patent holder prefers to sell its product innovation via a fixed fee rather than a royalty (the optimal contract is specified in (12)). (ii) An internal patent holder does not license its innovation. (iii) The equilibrium prices, quantities and social welfare are independent of whether the patent holder stays out of the market or vertically integrate with either firm.*

We conclude our analysis by comparing the private and social profitability of Cournot vs Bertrand competition. Under Cournot competition, at equilibrium vertical integration takes place and technology diffusion is complete, so that we have a homogeneous high quality good in the market; in constrast under Bertrand competition, at equilibrium exclusive licensing occurs, so that the market is vertically differentiated.

We find that from the firms' point of view, the producer surplus is higher under Cournot than under Bertrand competition, namely:[13]

$$\Pi_{VI}^* - \pi_{VI}^B = \left(\tfrac{\left(16\psi^2-13\psi+1\right)s\psi}{4(4\psi-1)^2} - 4\tfrac{(\psi-1)s\psi^2}{(4\psi-1)^2}\right) = \frac{s\psi}{(4\psi-1)^2}\frac{1}{4}\left(3\psi+1\right) > 0$$

$$\tfrac{s\psi^2}{(4\psi-1)^2} + \tfrac{\left(16\psi^2-13\psi+1\right)s\psi}{4(4\psi-1)^2} - \left(4\tfrac{(\psi-1)s\psi^2}{(4\psi-1)^2} + \tfrac{(\psi-1)s\psi}{(4\psi-1)^2}\right) = \frac{1}{4}\left(4\psi-1\right)^{-2}\left(3\psi+5\right)s\psi > 0$$

As for consumer surplus and welfare, the comparison depends on the quality improvement, in particular total social welfare is higher under Cournot than under Bertrand competition for ψ sufficiently high:

$$SW^{VI} - SW^B = \tfrac{\left(11\psi-21\psi^2-2\psi^{\frac{3}{2}}+8\psi^{\frac{5}{2}}\right)s}{8(4\psi-1)^2} > 0 \iff \psi > 6.28$$

This result is clearly linked to the fact that on one hand, under Cournot, competition is milder than under Bertrand where both qualities stays on sale; on the other hand, under Cournot, complete technology diffusion arises so that the average quality is higher than under Bertrand competition.

[13] This is in line with Singh and Vives (1984).

7 Conclusion

We have analysed the optimal licensing strategy of an upstream input innovator producing a new input which improves the quality of the final goods. We have considered a duopoly downstream market and have shown that under Cournot competition complete technology diffusion takes place and the innovator always prefers to be inside the market as the vertical merger with either downstream firm is always privately profitable. It is also welfare improving for large innovations. In contrast, under Bertrand competition exclusive licensing takes place and we find an indifferent result between vertical integration and vertical separation from both the private and social welfare point of view.

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8 Appendix

8.1 Ad valorem royalty

Suppose (under Corunot competition) the internal patentee (the VI firm) decides to sell the innovation via an ad valorem royalty, that is a profit sharing agreement. Firms' profits are then: $\pi_1^{PS} = (1-\alpha)\, pq_1$, $\pi_{VI}^{PS} = pq_{VI} + \alpha pq_1$ where $\alpha \in [0,1]$ is the ad valorem royalty. The equilibrium ad valorem royalty is the α such that the licensee is indifferent between buying and not buying the innovation, i.e.:

$$\begin{aligned} \alpha \quad &: \quad (1-\alpha)\frac{s\psi}{9} = \frac{\psi^2 s}{(4\psi-1)^2} \\ \iff &\quad \alpha^* = \frac{(\psi-1)(16\psi-1)}{(4\psi-1)^2} \in (0,1)\,. \end{aligned}$$

Industry profit is $2\frac{s\psi}{9}$, $\pi_{VI}^{PS} = \frac{s\psi}{9} + \frac{(\psi-1)(16\psi-1)}{(4\psi-1)^2}\frac{s\psi}{9}$ and $\Pi_{VI}^{*} - \pi_{VI}^{PS} = \frac{(\psi-1)(16\psi-1)s\psi}{36(4\psi-1)^2} > 0$.

8.2 Social welfare comparisons

For completeness we provide equilibrium social welfare under exclusive licensing:

$$SW^{EL} = PS^{EL} + CS^{EL} = \left(\frac{\left(36\psi^2-16\psi+1\right)(\psi-1)s}{9(4\psi-1)^2} + \frac{\psi^2 s}{(4\psi-1)^2} + \frac{s}{9}\right) + \frac{\left(\psi+4\psi^2-1\right)s\psi}{2(4\psi-1)^2}$$

where $CS^{EL} = \int_{\frac{p_1}{s}}^{\frac{p_2-p_1}{s(\psi-1)}} (\theta s - p_1)\, d\theta + \int_{\frac{p_2-p_1}{s(\psi-1)}}^{1} (\theta s\psi - p_2)\, d\theta$. The remaining social welfare comparisons are: for $\psi < 1.585\,6$, $SW^T - SW^{EL} = s\left(\frac{2}{9}\psi + \left(2\frac{\psi^2}{(4\psi-1)^2} + 2\frac{(\psi-1)(16\psi-1)\psi}{9(4\psi-1)^2}\right)\right) - s\left(\left(\frac{\left(36\psi^2-16\psi+1\right)(\psi-1)}{9(4\psi-1)^2} + \frac{\psi^2}{(4\psi-1)^2} + \frac{1}{9}\right) + \frac{\left(\psi+4\psi^2-1\right)\psi}{2(4\psi-1)^2}\right) > 0$; for $\psi > 1.585\,6$, $SW^T - SW^{EL} = s\left(\psi\frac{\left(4\psi+16\psi^2+1\right)^2}{2(32\psi^2-7\psi+2)^2} + \left(2\frac{25(4\psi-1)^2\psi^2}{4(32\psi^2-7\psi+2)^2} + \frac{\left(16\psi^2-16\psi+1\right)\psi}{2(32\psi^2-7\psi+2)}\right)\right) - s\left(\left(\frac{\left(36\psi^2-16\psi+1\right)(\psi-1)}{9(4\psi-1)^2} + \frac{\psi^2}{(4\psi-1)^2} + \frac{1}{9}\right) + \frac{\left(\psi+4\psi^2-1\right)\psi}{2(4\psi-1)^2}\right) > 0$. $SW_{VI}^C - SW^{EL} = s\left(\frac{\left(4\psi+\sqrt{\psi}-1\right)^2\psi}{8(4\psi-1)^2} + \frac{\psi^2}{(4\psi-1)^2} + \frac{\left(16\psi^2-13\psi+1\right)\psi}{4(4\psi-1)^2}\right) - s\left(\left(\frac{\left(36\psi^2-16\psi+1\right)(\psi-1)}{9(4\psi-1)^2} + \frac{\psi^2}{(4\psi-1)^2} + \frac{1}{9}\right) + \frac{\left(\psi+4\psi^2-1\right)\psi}{2(4\psi-1)^2}\right)$.

For high values of ψ ($\psi > 3.407\,8$), social welfare ranking is $SW_{VI}^{C} > SW^{T} > SW^{EL}$. For $\psi < 3.407\,8$, we have $SW^{T} > SW_{VI}^{C}$ and $SW^{T} > SW^{EL}$.

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