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Abstract

We introduce two bonds in a standard New-Keynesian model to study the role of segmentation in bond markets for the determinacy of rational expectations equilibria. We use a strongly-separable utility function to model short-term bonds providing transaction services for the purchase of consumption goods. Long-term bonds, instead, provide the standard services of store of value. We obtain a fully analytical solution for the bond pricing kernel, allowing to endogenize the term spread within the model. In this way, we study equilibrium determinacy properties within a context embedding the full information derived from term structure of interest rates. Our results show that, when utility is weakly separable between consumption and bonds, the Taylor principle holds only conditional to a non-linear relation between output and inflation targeting coefficients of monetary policy rule. Achieving solution determinacy requires to constraint policy coefficients to lie within bounds depending on structural parameters of the model. This paper provides an analytical setting useful for several generalizations to address the stability properties in dynamic models including the term structure of interest rates, induced by policy rules.

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1 Introduction

The Taylor principle has become one of the pillars of modern normative analysis of monetary policy. In a nutshell, it prescribes that the central bank should adjust the nominal rate of interest more than one-for-one as a response to changes in the inflation rate. In the standard New Keynesian models, the Taylor principle alone pins down the equilibrium inflation rate. The recent experience of 'quantitative easing' measures implemented by several central banks has stressed once more the strict link existing between the maturity structure of government bonds and the implementation of monetary policy.

In what follows, we focus on the role of term structure of interest rates and of endogenous term spreads in defining the conditions for determinacy of a Rational Expectation Equilibrium (REE, henceforth). Our goal is to study the role of interest rates associated to bonds of different characteristics to understand if the standard 'Taylor principle' of monetary policy is still valid. The Taylor principle, as discussed by Taylor (1993) and states that policy rate should react more than proportionally with respect to changes in inflation rate.

We consider a cashless economy where bonds provide transaction or 'liquidity' services in place of money. The model includes two types of bonds, namely liquid and illiquid. Liquid bonds are providers of services that facilitate transactions in the market for consumption goods. They play the same role of real money balances in the New Keynesian monetary models. Like for real money balances, we include the real quantity of liquid bonds directly into the utility function. Illiquid bonds are mere forms of financial investments that allow to carry income across time periods and, as such, enter the intertemporal budget constraint. An alternative to this formulation consists in including transaction costs in the consumer's budget constraint, like in Sims (1994).

After modelling bonds that provide transaction services and those that do not, we go one step forward. We suggest that there is a relation between the provision of transaction services and the maturity structure of government bonds. We interpret short-term securities as imperfect substitutes for consumption, and long-term bonds as perfect substitutes. Thus, short-term public debt performs the function traditionally attributed to 'money' as a medium of exchange. This approach can be motivated from several points of view. This way, we model two bonds with different intrinsic characteristics in general equilibrium. The function of each bond refers to a different maturity.

In our framework, purchasing long-dated bonds and holding them until maturity implies that a households receives a stream of interest-rate income and 'stores the face value of a bond' until the maturity period. This way, a household 'locks in' the resources for the face value of the bond. Hence, the longer the maturity of a bond, the more limited its capability of providing opportunities for consumption smoothing until expiry, should negative shocks occur.

The spirit of our framework is similar to the proposition of Tobin (1958) about the idea of liquidity preferences as a proxy for risk attitude. In a world with money, the demand for transactions in the consumption-good market generates a demand for liquidity services in the asset market. This is the so-called transaction demand for money. The key point is that the demand for liquidity services is related to the maturity profile of a portfolio of financial assets. In particular, the longer the maturity, the larger the propensity to adjust portfolio holdings between the assets and money balances.[1]

This paper aims to present the most simplified model which allows the simultaneous presence of two interest rates in a general equilibrium model with otherwise standard features of real and nominal rigidities. We abstract from an explicit role for money because we intend to focus on the role of the short-term interest rate in the monetary transmission mechanism. In this sense, modelling money would allow us to account only for an additional buffer in the transmission of shocks. This choice allows us to deal with the most tractable and transparent model available. Moreover, as argued by Woodford (2003), the absence of money is not at odds with neutrality proposition.

In the present framework, the term structure emerges as an affine representation where the expectation hypothesis holds in log-linear approximation. If bonds providing transaction services are part of the utility function in a strongly separable way, as in the standard neo-keynesian model, the parameters of the monetary policy rule should lie in the same region described in the literature for models built without the term structure for determinacy to hold. Hence, regardless of the vehicle providing liquidity services, what really matters for determinacy is whether the demand for transaction services is linear with respect to consumption. Instead, when the liquidity services enter in a weakly separable way, the standard Taylor principle does not hold any longer. In the region of determinate solutions, there is a nonlinear relation between the inflation targeting coefficient and the output targeting coefficient. Thus, modelling several government bonds with strongly separable utility does not determine an important change in determinacy conditions. The dramatic change occurs only with a weakly separable utility, for which the determinacy conditions are no longer standard.

Rational expectations equilibria are affected by the interplay between fiscal and monetary policy. The role of fiscal policy is essential since this allows to widen the range of parameters for which determinacy exists. In the language of Leeper (1991) and Sims (1994), we find that determinacy is obtained either by considering active-monetary with passive-fiscal or, alternatively, a passive-monetary with active-fiscal regime. A passive

[1] The discussion of Tobin (1958) also introduces the idea imperfect substitutability between different types of bonds and money. This is indeed at the heart of a variety of recent contributions, including Andres, Lopez-Salido, and Nelson (2004), Canzoneri and Diba (2005) and Canzoneri, Cumby, Diba, and López-Salido (2011).

fiscal policy arises from setting the tax revenues to react to the outstanding real level of total public debt.

The explanation for our results has to do with our modelling assumption that make two interest rates coexist in a general equilibrium. Including multiple bonds is irrelevant for the determinacy conditions of the model with strongly separable utility, since it does not change how the parameters of monetary policy rules enter the model. This changes for the case with weakly separable arguments, where the traditional arguments for determinacy are strongly dependent on the core model parameters.

This paper fills a gap in the current literature on determinacy and monetary policy initiated by Bullard and Mitra (2002). In particular, we consider an explicit role for the term structure of interest rates and solving for an endogenous term spread. A similar question is addressed by McCough, Rudebusch, and Williams (2005). However, their model does not allow to solve analytically for an endogenous term spread in the standard New Keynesian setting. It should be stressed that we obtain a full analytical solution for bond prices without imposing any *ad-hoc* assumption on the pricing kernel, or on the role of the expectations hypothesis of interest rates. Part of the results contained in this paper challenge what has been considered by Canzoneri, Cumby, Diba, and López-Salido (2011), where Taylor principle does not hold when bonds together with money provide transaction services.

The remainder of the paper is organized as follows. The following section introduces the reader to the main modelling framework. Section 3 describes the calibration of the benchmark model. We also comment on selected impulse responses to compare the coherence of the model with the patterns documented in the current literature. Section 4 discusses the model reduction, derivation of the pricing kernel, and the pricing of government bonds. Section 5 presents the main results of the model about determinacy. Section 6 extends to the case of a household utility that is weakly-separable between consumption and liquid bonds. Section 7 concludes. An additional appendix material contains the set of proofs of analytical results.

2 The Model

The general feature of our model is to consider the explicit role of bonds as providers of transactional services. We assume the existence of two types of bonds: liquid and illiquid. For their intrinsic nature, liquid bonds perform the function of 'money'. They are held by the representative agent in order to facilitate liquidity services. Illiquid bonds are held for investment purposes. Both types of bonds pay an interest rate which in equilibrium differs because of two elements, namely the explicit role of transaction services and the endogenously determined term premia.

2.1 Households

We assume the existence of an infinite number of heterogeneous agents indexed on the real line between 0 and 1. Each i-th agent maximizes the following utility function:

$$U_t = \sum_{t=0}^{\infty} \beta^t u\left(C_{it}, \frac{B_{i1t}}{P_t}, L_{it}\right) \tag{1}$$

where the instantaneous utility function $u(\ldots)$ is defined as:

$$u\left(C_{it}, \frac{B_{i1t}}{P_t}, L_{it}\right) = \frac{C_{it}^{\left(1-\frac{1}{\sigma}\right)}}{\left(1-\frac{1}{\sigma}\right)} + \chi \frac{\left(\frac{B_{i1t}}{P_t}\right)^{\left(1-\frac{1}{\sigma}\right)}}{\left(1-\frac{1}{\sigma}\right)} - \frac{L_{it}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \tag{2}$$

where C_{it} indicates the amount of consumption expressed by each i-th agent, B_{i1t} indicates the amount of short-term bond holdings (here indexed with 1); the general price level is given by P_t. Instantaneous utility depends positively from C_{it} and B_{i1t}, while negatively from labor supply L_{it}. In (2), σ indicates the intertemporal elasticity of substitution, χ is a scale parameter and η is the Frisch labor elasticity. The structure of financial assets of the present model economy is similar to Ljungqvist and Sargent (2007). The Government issues two types of bonds: one is a one-period bond, B_{i1t}, the short term bond. A second type of bond, B_{i2t}, lives for two periods. Both bonds pay a return each period R_{1t} for the one-period bond, and R_{2t} for the two-period bond, in gross terms. The price of both bonds is the reciprocal of their respective gross returns.

The representative agent solves an intertemporal portfolio allocation problem by maximizing the utility function subjected to the budget constraint:

$$\frac{B_{i1t}}{R_{1t}P_t} + \frac{B_{i2t}}{R_{2t}P_t} = \frac{B_{i1t-1}}{P_t} + \frac{B_{i2t-1}}{R_{1t}P_t} + \frac{W_t L_{it}}{P_t} - C_{it} - T_{it} \tag{3}$$

where W_t is the nominal wage, T_{it} is the tax collected, assumed to be lump sum. Given the structure of financial assets, from (3), the two-period bond issued at time $t-1$, when it comes at t it has one period left before maturity: this makes it like a one-period bond. Therefore, the price of a two period bond with one period left until maturity is given by $1/R_{1t}$. Clearly, the structure of financial markets assumed in this paper allows the presence of a secondary market. This setting generalizes what has been introduced by Andres, Lopez-Salido, and Nelson (2004), Marzo and Zagaglia (2008) and Harrison (2012) by presenting a model with mild assumptions underlying the introduction of term structure of interest rates. In particular, we do not consider any explicit role for transaction costs: short-term bond B_{i1t} is inserted in the utility function, playing a similar role of money in traditional monetary models. This allows the definition of a bond pricing kernel depending on preference parameters. The first order conditions with respect to C_{it}, L_{it}, B_{i1t} and B_{i2t}, are given, respectively, by:

$$\begin{aligned} C_{it}^{-\frac{1}{\sigma}} &= \lambda_t && (4) \\ L_{it}^{\frac{1}{\eta}} &= \lambda_t W_t && (5) \\ \chi b_{i1t}^{\chi(1-\frac{1}{\sigma})-1} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} &= \frac{\lambda_t}{R_{1t}} && (6) \\ \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1} R_{1t+1}} &= \frac{\lambda_t}{R_{2t}} && (7) \end{aligned}$$

Equation (4) indicates the first order with respect to consumption; equation (5) defines the optimal labor supply choice and equates the disutility from work effort to the real wage weighted by the marginal utility of consumption; equation (6) is the optimal intertemporal allocation of short-term (one period) bonds B_{i1t} and (7) is the result of the optimal choice of long-term (two periods) bonds B_{i2t}; λ_t is the Lagrange multiplier. In particular, we expressed the bond demand in real terms, after having defined $b_{i1t} = B_{i1t}/P_t$, $b_{i2t} = B_{i2t}/P_t$.

By reshuffling (6) - (7) we get the following expression for the demand of liquid bonds:

$$\chi b_{i1t}^{\chi(1-\frac{1}{\sigma})-1} = \left[\frac{R_{2t} - R_{1t} E_t R_{1t+1}}{R_{1t} R_{2t}}\right] C_t^{-\frac{1}{\sigma}} \tag{8}$$

It is immediate to verify that demand for liquid bonds (after dropping subscript index i) satisfies the following properties, provided that $\chi\left(1 - \frac{1}{\sigma}\right) < 1$:

$$\begin{aligned} &\frac{\partial b_{1t}}{\partial C_t} > 0 \qquad &&\frac{\partial b_{1t}}{\partial R_{1t}} > 0 \\ &\frac{\partial b_{1t}}{\partial R_{1t+1}} > 0 \qquad &&\frac{\partial b_{1t}}{\partial R_{2t}} < 0 \end{aligned}$$

The intuition goes as follows. The increase in consumption increases the demand for short bonds, since they are employed for transaction. On the other hand, the increase in current, R_{1t}, and expected rate of return on short-term bonds, R_{1t+1}, increases the demand for short bonds, since investors tend to favour investment with higher return, given the same level of risk. In the same guise, the increase in the return of long-term bonds, R_{2t}, depresses the demand for short-term bonds.

To complete the demand side, we assume the existence of a large number of differentiated goods indexed over the real line between 0 and 1. This allows each firm to have a control of the price of her final good to be sold, since output becomes demand determined. Following the approach by Dixit and Stiglitz (1977), we assume that the consumption bundle C_{it} demanded by each agent $i \in [0, 1]$ is a CES type aggregate of all the $j \in [0,1]$ varieties of final goods produced in this economy, as described by:

$$C_{it} = \left[\int_0^1 c_t^i (j)^{\frac{\theta-1}{\theta}} \, dj \right]^{\frac{\theta}{\theta-1}} \tag{9}$$

where θ is the elasticity of substitution between different varieties of goods produced by each firm j. To guarantee the existence of an equilibrium, the elasticity θ is restricted to be bigger than one. Standard optimization problem for the choice of the optimal composition of bundle (9) lead to the following constant-elasticity inverse demand function:

$$\frac{c_t^i (j)}{C_{it}} = \left[\frac{p_t (j)}{P_t} \right]^{-\theta} \tag{10}$$

where $p_t (j)$ is the price of variety j and P_t is the general price index defined as:

$$P_t = \left[\int_0^1 p_t (j)^{1-\theta} \, dj \right]^{\frac{1}{1-\theta}} \tag{11}$$

As $\theta \to \infty$, the demand function becomes perfectly elastic, and the differentiated goods are perfect substitutes. The aggregate price level P_t is beyond the control of each individual firm. Similar steps can be applied to public expenditure G_t, so that aggregate demand is defined as the sum of private and public consumption for each variety goods: $C_t (j) + G_t (j) = Y_t (j)$, which after aggregating over all varieties $j \in [0,1]$ becomes: $C_t + G_t = Y_t$.

In order to simplify, we assume the existence of a symmetric equilibrium where all agents make the same choice *ex-post*. Therefore, we can drop index i from all equations in the model.

2.2 The pricing kernel

Including bonds into the utility function makes the intertemporal pricing scheme of bonds peculiar. To shed further light on this, let us consider a model with no short-term bonds in the utility function, obtained by setting $\chi = 0$. In this case, following Ljungqvist and Sargent (2007), we obtain:

$$\beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \frac{\lambda_t}{R_{1t}} \tag{12}$$

$$\beta E_t \frac{\lambda_{t+1}}{\pi_{t+1} R_{1t+1}} = \frac{\lambda_t}{R_{2t}} \tag{13}$$

The pricing kernel M_{t+1} is defined as:

$$M_{t+1} = \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} \tag{14}$$

As it is well known, the pricing kernel is the tool to price recursively the entire term structure, starting from the shortest maturity bond. In our case, it is obvious to see that the pricing of the two-periods bond is, after a recursive application of the kernel, given by:

$$\beta^2 E_t \frac{\lambda_{t+2}}{\pi_{t+1}\pi_{t+2}} = \frac{\lambda_t}{R_{2t}} \tag{15}$$

By the same sort of argument, if we generalize to j-th period bond, we obtain:

$$\beta^j E_t \frac{\lambda_{t+j}}{\lambda_{t+j-1}\left(\pi_{t+1}\dots\pi_{t+j-1}\pi_{t+j}\right)} = R_{jt}^{-1} \tag{16}$$

The classical approach to the term structure implies that expected future short term interest rates determine long-term interest rates. This defines the well known expectations hypothesis (EH, henceforth), which in our case can be simply stated as: $R_{2t} = R_t E_t R_{1t+1}$. From equation (13), we obtain:

$$R_{2t}^{-1} = \beta E_t \left[\frac{\lambda_{t+1}}{\pi_{t+1}\lambda_t}\right] E_t R_{1t+1}^{-1} + cov_t \left[\beta \frac{\lambda_{t+1}}{\pi_{t+1}\lambda_t}, R_{1t+1}^{-1}\right] \tag{17}$$

which, after using (12) becomes:

$$R_{2t}^{-1} = R_t^{-1} E_t R_{1t+1}^{-1} + cov_t \left[\beta \frac{\lambda_{t+1}}{\pi_{t+1}\lambda_t}, R_{1t+1}^{-1}\right] \tag{18}$$

From (18), we observe that the EH holds if and only if utility is linear in consumption, such that $\frac{\lambda_{t+1}}{\pi_{t+1}\lambda_t} = 1$, and when the stochastic process of π_t, so that the covariance term becomes zero. In the case under exam, instead, with the inclusion of bonds into the utility function, we observe that the pricing kernel is affected by utility terms. In fact, by taking advantage of the first order conditions (6)-(7), we can rewrite (18) as follows:

$$R_{2t}^{-1} = \left[R_t^{-1} - \frac{\chi b_{1t}^{\chi\left(1-\frac{1}{\sigma}\right)-1}}{\lambda_t}\right] E_t R_{1t+1}^{-1} + cov_t \left[\beta \frac{\lambda_{t+1}}{\pi_{t+1}\lambda_t}, R_{1t+1}^{-1}\right] \tag{19}$$

Thus, by setting $\chi = 0$ in (19) we obtain exactly the setting outlined in (18). From (19) we immediately obtain the kernel expression such that:

$$\begin{aligned} M_{t+1} &= \beta E_t \frac{\lambda_{t+1}}{\lambda_t \pi_{t+1}} = \\ &= \left[R_t^{-1} - \frac{\chi b_{1t}^{\chi\left(1-\frac{1}{\sigma}\right)-1}}{\lambda_t}\right] \end{aligned} \tag{20}$$

Thus, by plugging (21) into (19), we observe that the interaction between the preference structure and the pricing structure is also reflected in the second order terms. This shows that including bonds with a sort of 'liquidity preference' motivation in the model delivers a non-standard representation of the stochastic pricing equations for all financial assets. Finally, taking advantage of (4) into (14), we have:

$$M_{t+1} = \beta E_t \left(\frac{C_t}{C_{t+1}} \right)^{\frac{1}{\sigma}} \frac{1}{\pi_{t+1}} \tag{21}$$

To obtain a version of the pricing kernel (14) that can be used for the analysis, we need a solution for consumption and inflation as a function of the shocks of the system.

2.3 Firms

We assume the presence of a continuum of monopolistically competitive firms distributed on the unit line $[0, 1]$, indexed by $j \in (0, 1)$. Each individual firm faces a downward sloped demand curve for her differentiated product $Y_t(j)$:

$$P_t(j) = \left[\frac{Y_t(j)}{Y_t} \right]^{-\frac{1}{\theta}} P_t \tag{22}$$

It is well known that demand function (22) can be directly derived by following the details from Dixit and Stiglitz (1977).
The production function of each variety j employs only labor as input and it is given by:

$$Y_t(j) = A_t L_t^{\alpha}(j) \tag{23}$$

Note that all firms producing j varieties are subjected to an homogenous technological shock A_t, for which we assume the following structure (in log-linear terms):

$$a_t = (1 - \rho_a) a + \rho_a a_{t-1} + a_t^{1/2} \sigma_a \epsilon_t^a \tag{24}$$

where ϵ_t^a is an innovation term distributed according to a standardized Normal distribution. The structure of the equation (24) includes an heteroskedastic innovation. As it will be clear later, the term structure will be dependent on the shocks of the system: technically speaking the shock of the model represent the principal component of the yield curve, interpreted according to the specificity modelled. One factor is here modeled as business cycle component modeled via (24). Nominal rigidities are modeled via price rigidities modeled through Calvo (1983) method of price adjustment. Each seller sets each period a new price with probability $1 - \alpha$, with $\alpha \in (0, 1)$, independent of time since last change. Parameter α indicates the degree of price stickiness.

Let us define the evolution of the price level. Let P_t be the general price level index, and be $\wp_t$ the new price chosen at date t, by all sellers . Thus the price level is given by:

$$P_t^{1-\theta} = \left[\int_0^1 p_t(i)^{1-\theta} di\right] = (1-\alpha)\wp_t^{1-\theta} + \alpha \int_0^1 p_{t-1}(i)^{1-\theta} di \tag{25}$$

which is equivalent to write, given the definition of the general price level:

$$P_t^{1-\theta} = (1-\alpha)\wp_t^{1-\theta} + \alpha P_{t-1}^{1-\theta} \tag{26}$$

To determine the price level we need the choice of $\wp_t$. It is interesting to note that the optimal choice of $\wp_t$ depends only upon the current and the expected future evolution of the entire sequence of $\{P_t\}_{t=0}^{\infty}$, so there is no need to know other aspects of the price distribution.

Firms set their own price by maximizing the following profit function:

$$\Omega_t(j) = E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[\lambda_{t+k}(j) p_t^{1-\theta}(j) P_{t+k}^{\theta} Y_{t+k} - \omega\left(p_t(j)^{-\theta} P_{t+k}^{\theta} Y_{t+k}\right)\right] \tag{27}$$

By taking the First Order Condition with respect to $p_t(j)$, we obtain:

$$E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left\{(1-\theta)\lambda_{t+k}(j)\left(\frac{p_t(j)}{P_{t+k}}\right)^{-\theta} Y_{t+k} + \omega'(\cdot)\theta p_t^{-1-\theta}(j) P_{t+k}^{\theta} Y_{t+k}\right\} = 0 \tag{28}$$

After simplifying we have:

$$E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left\{\lambda_{t+k}(j) p_t(j) - \omega'(\cdot)\frac{\theta}{\theta-1}\right\} = 0 \tag{29}$$

After imposing the ex-post homogeneity condition (assuming that the pricing problem solved by each firm is equal for all firms producing the i-th varieties), and the resource constraint, we obtain:

$$E_t \sum_{k=0}^{\infty} (\alpha\beta)^k \left[u'(Y_{t+k})\frac{p_t(j)}{P_{t+k}} - \left(\frac{\theta}{\theta-1}\right)\omega'\left(Y_{t+k}\left(\frac{p_t(j)}{P_{t+k}}\right)^{-\theta}\right)\right] = 0 \tag{30}$$

where $\omega(\cdot)$ is the utility function representing the preferences towards work vs. leisure, which in our case is given by:

$$\omega(\cdot) = \frac{L_{it}^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \tag{31}$$

In this paper we abstract from the explicit definition of price distortions induced by monopolistic competition and nominal price rigidities. For a more general treatment, we address the reader to Woodford (2003) and Schmitt-Grohé and Uribe (2005), Schmitt-Grohé and Uribe (2007). Taking advantage of the production function, the first order condition on consumption given by, the first order condition of the pricing problem is given by:

$$\left[\frac{p_t(i)}{P_t}\right]^{1-\theta+\frac{\theta}{\alpha}\left(1+\frac{1}{\eta}\right)} = \frac{\theta}{(\theta-1)\alpha}\left(1+\frac{1}{\eta}\right)\frac{1}{\lambda_t}A_t^{-\frac{1}{\alpha}\left(1+\frac{1}{\eta}\right)}Y_t^{\frac{1}{\alpha}\left(1+\frac{1}{\eta}\right)-1} \tag{32}$$

After log-linearization, we obtain the following expression for the aggregate supply function:

$$\beta\pi_{t+1} = \pi_t - ky_t + \mu_a a_t + \mu_g g_t \tag{33}$$

where:

$$k = \frac{(1-\delta)(1-\delta\beta)}{\delta}\left\{\frac{\sigma S_c\left[1+\eta(1-\eta)\right]+\alpha\eta}{\sigma S_c\left[\alpha\eta(1-\theta)+\theta(1+\eta)\right]}\right\} \tag{34}$$

$$\eta_a = \frac{(1-\delta)(1-\delta\beta)}{\delta}\left[\frac{1+\eta}{\alpha\eta(1-\theta)+\theta(1+\eta)}\right] \tag{35}$$

$$\eta_g = \frac{(1-\delta)(1-\delta\beta)}{\delta}\left[\frac{\alpha\eta g}{S_c\sigma\left[\alpha\eta(1-\theta)+\theta(1+\eta)\right]}\right] \tag{36}$$

Interestingly, the aggregate supply function given in (33) depends on the exogenous shocks of the system, a_t, g_t. The traditional formulation of the model is often expressed in terms of the output gap: this would allow to get rid of the explicit formulation of the shock from the AS curve, since the definition of the potential output is a linear combination of the shock. We keep this formulation in terms of actual output, since it allows a neater derivation of the kernel, given the solution of the full model in terms of the shocks of the system.

2.4 Fiscal policy

The Government Budget Constraint in nominal terms is given by:

$$\frac{B_{1t}}{R_{1t}P_t} + \frac{B_{2t}}{R_{2t}P_t} = \frac{B_{1t-1}}{P_t} + \frac{B_{2t-1}}{R_{1t}P_t} + G_t - T_t \tag{37}$$

where G_t indicates the government expenditure, net of interest expenses. We assume that bond demand expressed by each i-the agent matches the supply supplied by the government according to the following equilibrium conditions:

$$B_{1t} = \int_0^1 B_{i1t}di; \qquad B_{2t} = \int_0^1 B_{i2t}di;$$

In the same fashion, total fiscal revenues are equal to the sum of taxes paid by each i-the agent:

$$T_t = \int_0^1 T_{it} di$$

Government spending follows a stochastic process given by:

$$g_t = (1 - \rho_g) g + \rho_g g_{t-1} + g_t^{1/2} \sigma_g \epsilon_t^g \quad (38)$$

Equation (38) represents a policy shock included in the model, ϵ_t^g is an i.i.d. standard normal shock with zero mean and constant volatility σ_g^2. The solution of the pricing kernel and the term structure is a function of two shocks: a technological shock, representing business cycle fluctuations, and a policy shock, represented by g_t in (38). This opens interesting questions about the response of both short and long rates with respect to policy changes and their feedback into the entire model economy, as we will see later.

According to the fiscal theory of price level determination (FTPL, henceforth), the comparative evaluation of alternative monetary policy rules should not be thought in isolation from an explicit design of the fiscal policy stance. As suggested by Leeper (1991) and Sims (1994), we introduce a fiscal rule whereby taxes react to the outstanding level of real public debt:

$$T_t = \psi_0 + \psi \frac{B_{1t-1}}{P_t} + \psi \frac{B_{2t-1}}{P_t} \quad (39)$$

The parameter capturing the strength of the tax response to debt fluctuations is given by ψ, which is set equal for both short and long-term debt. We follow Leeper (1991) and define fiscal policy as 'passive' for:

$$\left|\beta^{-1} - \psi\right| < 1 \quad (40)$$

and active otherwise.

Given the model structure, for short-term bonds we have an explicit demand function that depends on the preference structure of the investor. The demand for long-term bonds is derived endogenously, thus representing a residual adjustment in the government budget constraint. This amounts to saying that long-term bonds absorb residual demand that is not satisfied by the supply conditions for short-term bonds. Implicitly, our model designs the behavior of a secondary market for government bonds by allowing for trading of long-term bonds (with a two-period maturity, as it is assumed here) at each point in time.

2.5 Monetary policy

We consider the role of monetary policy rules in tying down the determinacy properties of a REE. We study the role of variants of the standard interest rate rule proposed by Taylor (1993):

$$R_{1t} = R_1 \left(\frac{\pi_{t+n_\pi}}{\pi}\right)^{\phi_\pi} \left(\frac{Y_{t+n_y}}{Y}\right)^{\phi_y} \left(\frac{R_{1t+n_R-1}}{i}\right)^{\phi_R} \tag{41}$$

where ϕ_π, ϕ_y and ϕ_R indicate the response of the policy rate R_{1t} to inflation, output and lagged R_{1t} over different time horizons (n_π, n_y , n_i). All the parameters of the policy rule are restricted to be positive. This general setting can be simplified as:

$$R_{1t} = (1-\rho)\left(\phi_\pi \pi_t + \phi_y y_t\right) + \rho R_{1t-1} \tag{42}$$

Given (41), the demand for short term bonds is fully determined.

3 Calibration and impulse responses

In this section we present the procedure adopted to calibrate the model together with a short impulse-response analysis, in order to discover the main dynamic properties of model economy under study. The scope of this section is to show the basic properties of the model conditional to the evolution of exogenous shocks.

The model is calibrated on quarterly U.S. data from for the period 1960:1-2010:3. The parameter values are reported in Table 1. The annual inflation rate for the sample period is 4.09%. The steady-state short-term interest rate R_1 is 5.58%, obtained as the mean of 3-month Treasury Bill Rate. To capture the steady state value of long-term bonds, we take the mean of the 10-year return of government bond, given by 6.5 %. The steady state level of output has been obtained as the mean of quarterly GDP, constant prices, seasonally adjusted over the sample period 1960:1-2010:3. This number has been normalized by considering the civilian population considered over the same sample period, according to the methodology described by Kim (2000). The intertemporal discount rate implied by above informations is set equal to 0.99, as it is standard in the current literature. We also assume that the inverse of risk aversion coefficient in the utility function σ has been set equal to 0.5, together with Frisch labor supply elasticity η equal to 1, and the scale parameter χ has been set equal to 0.3, as in Galí (2008). The labor share in the production function α is set equal to 0.67, as it is customary in the current real business cycle literature. The share of consumption over GDP is set to be 0.57, implying a public expenditure to GDP ratio equal to 0.43, an high value if compared to the true data, given the absence of investments from the model. Price rigidity parameter δ is set equal to

2/3, implying an average price duration of three quarters, consistent with the empirical evidence. In the same way, the elasticity of substitution between differentiated goods is equal to 6, as commonly assumed in the traditional New Keynesian models. The parameter representing the response of fiscal revenue to outstanding short and long term debt is set to be 0.05. The steady state level of labor supply is given by $L = 0.33$, implying a 1/3 ratio of working activities to non-working activities. The steady state level of total public debt is set to 33% over GDP, equal to the average of US Federal public debt to GDP ratio for the sample period considered. The short term debt has been left to be free: for the simulation reported, we set as a benchmark value 40% of the total level of debt, implying a 60% of the long term debt. Finally, the monetary policy assumed for impulse-response function is the standard Taylor rule with both contemporaneous inflation and output targeting, with $\phi_\pi = 1.5$ and $\phi_y = 0.5$ and $\rho = 0$.

The autoregressive coefficient for the shocks are: $\rho_A = 0.9$, $\rho_A = 0.5$, for technological and public expenditure shock, respectively. Standard deviation are, instead: $\sigma_A = 0.007$, and $\sigma_G = 0.01$, as considered in the current literature (for σ_A) and according to the standard deviation of the public consumption for the US economy for the sample period considered.

The model is log-linearized around the deterministic steady state. We report the impulse response functions from a one-standard deviation shock to technology and public expenditure in Figures 1 and 2, respectively. From Figure 1 we observe that technological shock expands output and consumption but reduces labor effort. The increase in aggregate demand raises inflation rate, with a consequent increase of both short and long nominal interest rates. The level of both short and long term bonds in real terms decreases because of the increase of the inflation rate. This determines a reduction of tax revenue. Labor supply decreases, because the productivity shock is perceived as a windfall gain.

The results for an expansionary government expenditure shock are reported in Figure 2: output increases, consumption increase, inflation and both short term and long term rates increase. Obviously, the increase of public debt (both short and long term in real terms) make taxes to increase, too. Interestingly, the reaction of short term rate is stronger than long term one, highlighting a smoothed out effect, as it is customary for term structure models.

Overall, the impulse response functions provide evidence of patterns similar to those of the standard New Keynesian model, with the additional feature of the interplay between short and long term interest rates. Under this perspective, the inclusion of two bonds does not generate any counter-intuitive or bizarre behavior of the response of the system to the shocks considered.

4 An analytic solution for the pricing kernel

4.1 Model solution and the kernel

The first step consists in log-linearizing the model around the deterministic steady state. This is now a standard procedure and we are not going to describe it in full detail. A technical appendix with the full log-linearized version is available upon request. The results from our dimension-reduction solution can be collected in the following proposition:

Proposition 1 *The reduced form model can be represented as follows:*

$$\pi_{t+1} = \frac{1}{\beta}\pi_t - \frac{k}{\beta}\eta_{ya}a_t - \frac{k}{\beta}\eta_{yg}g_t \tag{43}$$

$$\begin{aligned} b_{2t+1} + \eta_{\pi 1}\pi_{t+1} + \eta_{ba2}a_{t+2} + \eta_{ba1}a_{t+1} + \eta_{bg2}g_{t+2} + \eta_{bg1}g_{t+1} = \\ = \eta_{b2}b_{2t} - \eta_\pi \pi_t - \eta_{ba}a_t - \eta_{bg}g_t \end{aligned} \tag{44}$$

where coefficients η_{ya}, η_{yg}, $\eta_{\pi 1}$, η_{ba2}, η_{ba1}, η_{bg2}, η_{bg1}, η_{b2}, η_π, η_{ba}, η_{bg} *are given in Appendix 1.*

Proof 1 *See Appendix 2.*

The functional form of the model described in (43) and (44) can be directly employed in the analytical solution of the kernel, which is explicitly discussed in the following proposition.

Proposition 2 *The analytical solution to the pricing kernel is:*

$$m_{t+1} = \lambda_0 + \lambda_1 a_t - \lambda_2 g_t - \eta_1 a_t^{1/2}\sigma_a \epsilon_{t+1}^a - \eta_2 g_t^{1/2}\sigma_g \epsilon_{t+1}^g \tag{45}$$

where coefficients are:

$$\begin{aligned} \lambda_0 &= \delta - f_\pi - (1-\rho_a)\, a\left(\frac{\eta_{ca}}{\sigma} + \alpha_a\right) + (1-\rho_g)\, g\left(\frac{\eta_{cg}}{\sigma} - \alpha_g\right) \\ \lambda_1 &= \frac{\eta_{ca}(1-\rho_a)}{\sigma} - \alpha_a\rho_a \\ \lambda_2 &= \frac{\eta_{cg}(1-\rho_g)}{\sigma} + \alpha_g\rho_g \\ \eta_1 &= \frac{\eta_{ca}}{\sigma} + \alpha_a \\ \eta_2 &= \frac{\eta_{cg}}{\sigma} + \alpha_g \end{aligned}$$

Proof 2 *See Appendix 2.*

The solution presented in the previous proposition depends on the assumptions of the microfounded model. We should stress that we do not need a numerical solution of the model written in state-space form to price government bonds in our framework. Differently from most contributions in macro-finance, we obtain an analytical solution for the pricing kernel that is fully consistent with the model structure.

4.2 Bond pricing

The structural model has two state variables governing the dynamics of the pricing kernel, namely a_t and g_t. Since the pricing kernel is conditionally lognormal, the short term rate R_{1t} is given by:

$$R_{1t} = -\log E_t \exp\left(m_{t+1}\right) \tag{46}$$

which, given lognormality, becomes:

$$R_{1t} = -E_t m_{t+1} - \frac{1}{2}\text{var}_t\left(m_{t+1}\right) \tag{47}$$

Equation (47) shows that fluctuations in the short rate depend on a combination of changes in the conditional mean and variance of the pricing kernel. The conditional mean of the log of the pricing kernel is given by:

$$E_t m_{t+1} = \lambda_0 + \lambda_1 a_t - \lambda_2 g_t \tag{48}$$

while conditional variance is:

$$\text{var}_t\left(m_{t+1}\right) = \eta_1^2 a_t \sigma_a^2 + \eta_2^2 g_t \sigma_g^2 \tag{49}$$

Interestingly, this model allows for time variation in the conditional variance of the kernel, owing to the time-varying volatility of the shocks. The types of shocks included here capture business cycle patterns, in the case of a technology shock, and a policy-related shock, in the case of the fiscal policy shock. To get a constant conditional variance, we can set $\eta_1 = \eta_2 = 0$. However, this condition is fairly restrictive, since we have seen that coefficients η_1, η_2 are function of the core parameters of the model.

By combininig (47) with (48) and (49), the solution of the short-rate interest rate can be written as:

$$R_{1t} = -\lambda_0 - \left(\lambda_1 + \frac{\eta_1^2}{2}\sigma_a^2\right) a_t + \left(\lambda_2 - \frac{\eta_2^2}{2}\sigma_g^2\right) g_t \tag{50}$$

We can now further generalize the previous argument by extending the pricing scheme to longer-term government bond. In what follows, we present a general formulation to price a k-maturity bond and to extend the analytics to illiquid bonds.

Following Atkeson and Kehoe (2008), let us consider the price of a k-th period maturity bond p_t^k:

$$p_t^k = \log E_t \exp\left(m_{t+1} + p_{t+1}^{k-1}\right) \tag{51}$$

Our goal is to derive the affine recursive pricing formula. We set the price of a k-th period maturity bond as a function of the state variables a_t and g_t, as follows:

$$p_t^k = -A_k - B_k a_t - C_k g_t \tag{52}$$

The solution is collected in the following Proposition.

Proposition 3 *The affine recursive coefficients of k-th maturity bond prices are given by:*

$$A_k = -\lambda_0 + A_{k-1} + B_{k-1}\left(1-\rho_a\right)a + C_{k-1}\left(1-\rho_g\right)g \tag{53}$$

$$B_k = \rho_a B_{k-1} - \frac{\sigma_a^2}{2}\left(\eta_1^2 + B_{k-1}^2\right) - \lambda_1 \tag{54}$$

$$C_k = \lambda_2 + \rho_g C_{k-1} - \frac{\sigma_g^2}{2}\left(\eta_2^2 - C_{k-1}^2\right) \tag{55}$$

with $A_1 = \lambda_0$, $B_1 = \lambda_1 + \frac{\eta_1^2\sigma_a^2}{2}$, $C_1 = \frac{\eta_2^2\sigma_g^2}{2} - \lambda_2$.

Proof 3 *See Appendix 2.*

The yield R_{kt} on a k maturity bond can be expressed as:

$$R_{kt} = -\frac{p_t^k}{k} \tag{56}$$

which, by using (52), becomes:

$$R_{kt} = \frac{1}{k}\left(A_k + B_k a_t + C_k g_t\right) \tag{57}$$

We can now compute the term spread, i.e. the difference between long-term R_{kt} and short-term yield R_{1t}, which by using (50) and (57), becomes:

$$R_{kt} - R_{1t} = \left(\frac{A_k}{k} + \lambda_0\right) + \left(\frac{B_k}{k} + \lambda_1 + \frac{\eta_1^2\sigma_a^2}{2}\right)a_t + \left(\frac{C_k}{k} - \lambda_2 + \frac{\eta_1^2\sigma_g^2}{2}\right)g_t \tag{58}$$

Differently from Atkeson and Kehoe (2008), our setting does not allow for a parallel shift in the yield curve, since all yield change differently after a shock to a_t or g_t. This is due to the assumptions made in (24) (38), which are not random walk. Moreover, this implies a general set of formula for recursive terms of the affine coefficients A_k and B_k which are non-linear, as proved in Proposition 3. On the other hand, equation (58) shows that the difference between a k-maturity bond and the short-rate bond is mainly due to exogenous shock fluctuations. Other than this, the two yields differ for a constant term captured by $\left(\frac{A_k}{k}+\lambda_0\right)$.

We can now pin down the equations governing the long-term rate for $k = 2$.. We can collect the results in the following Corollary:

Corollary 1 *For a two-period bond, the yield and the term spread are respectively given by:*

$$R_{2t} = \frac{1}{2}\left(A_2 + B_2 a_t + C_2 g_t\right) \tag{59}$$

$$R_{2t} - R_t = \left(\frac{A_2}{2}+\lambda_0\right) + \left(\frac{B_2}{2}+\lambda_1+\frac{\eta_1^2\sigma_a^2}{2}\right)a_t + \left(\frac{C_2}{2}-\lambda_2+\frac{\eta_2^2\sigma_g^2}{2}\right)g_t \tag{60}$$

where the coefficients are:

$$A_2 = B_1\left(1-\rho_a\right)a + C_1\left(1-\rho_g\right)g \tag{61}$$

$$B_2 = \rho_a B_1 - \frac{\sigma_a^2}{2}\left(\eta_1^2 + B_1^2\right) - \lambda_1 \tag{62}$$

$$C_2 = \lambda_2 + \rho_g C_1 - \frac{\sigma_g^2}{2}\left(\eta_2^2 - C_1^2\right) \tag{63}$$

with B_1 and C_1 defined in Proposition 1

Proof 4 *By setting $k = 2$ in (52), (53)-(55), (57) and (58), rearrange and simplify, it is immediate to get the results stated in the text.*

We can rewrite equation (60) in a more suitable fashion, so that the link between return on long term bond and short term bond can be represented by:

$$R_{2t} = R_t + \eta_0 + \eta_a a_t + \eta_g g_t \tag{64}$$

where:

$$\eta_0 = \left(\frac{A_2}{2}+\lambda_0\right) \tag{65}$$

$$\eta_a = \left(\frac{B_2}{2}+\lambda_1+\frac{\eta_1^2\sigma_a^2}{2}\right) \tag{66}$$

$$\eta_g = \left(\frac{C_2}{2}-\lambda_2+\frac{\eta_2^2\sigma_g^2}{2}\right) \tag{67}$$

From equation (64), if the stochastic processes for a_t and g_t are removed, then the two returns R_t and R_{2t} differ only for a constant term. Therefore, in general equilibrium, two interest rates may coexist with a constant wedge if there is no uncertainty. Second, the wedge between the two fluctuates with a drift if uncertainty is added. We would get a similar result if we imposed exogenously a relation between liquid and illiquid bonds, like the one in equation (64). For example, this can be obtained by assuming that the relation between the two rates evolves according to:

$$R_{2t} = HR_t Z_t^{\nu} \tag{68}$$

with H is a constant, Z_t is stochastic term, for which we can assume an autoregressive structure. The log-linearized version of (68) is:

$$\widetilde{R}_{2t} = \eta_0 + \widetilde{R}_t + \nu\zeta_t \tag{69}$$

where $\eta_0 = \log H$, $\zeta_t = \log Z_t - \log Z$, where the letter without time subscript indicates the steady state for the same variable. If we express ζ_t as a linear combination of a_t, g_t we can immediately get a representation very similar to that reported in (64):

$$\widetilde{R}_{2t} = \eta_0 + \widetilde{R}_t + \nu\left(\xi a_t + (1-\xi)\, g_t\right) \tag{70}$$

The representation under (70) is qualitatively similar to (68). In fact, if the focus of the analysis is on determinacy conditions of a REE induced by term structure, the two representations under (68)-(70) do not imply any differences in the outcome for determinacy. In other words, it does not matter for determinacy whether the link between the rates of return of short and long term bonds is imposed in an exogenous way - like in the case of equation (70) -, or whether it is explicitly derived by following the procedure leading to (68). The advantage from the endogenous derivation of the link between R_{2t} and R_t consists in the fact that coefficients are functions of all the core parameters of the model.

The approach resulting in equations (68)-(70) is also considered in Marzo and Zagaglia (2008). The different results obtained in Marzo and Zagaglia (2008) arise mainly from the modelling strategy for the nature of the bonds. In fact, in Marzo and Zagaglia (2008), illiquid bond are subject to transaction costs in the representative agent's budget constraint. This delivers a different functional form for the aggregate supply function which explictly depends on the short term interest rate, thereby creating a different transmission channel of the short term rate to term structure.

A final remark is due about the Expectations Hypothesis (EH). Taking advantage of the log-linearized reduced form of the model, we can derive the following relation (expressed in log-linear terms) between long and short rate:

$$R_{2t} = R_{1t} + R_{1t+1} \tag{71}$$

Equation (71) is the log-linear representation of the expectations hypothesis. In other words, the inclusion of bonds in the utility in a weakly separable way, does not induce a violation of the EH. To get a model setting where the EH appears to be violated, it would be necessary to include stronger frictions among different types of bonds or money, as outlined in a preliminary work done by Marzo and Zagaglia (2008).

5 Determinacy of rational expectations equilibria

In this section we study the determinacy properties of the model. Each determinacy condition is derived conditional to a specific monetary policy rule. We consider seven variants of the rules proposed in (41)-(42): a contemporaneous absolute inflation targeting rule, a backward-looking and a forward-looking rule for absolute inflation targeting. In addition, we study a set of flexible inflation-targeting rule that include output targeting. In this case, the rules studied are based on the combination of current inflation and output targeting, forward inflation and current output, as well as current inflation and output coupled with interest rate smoothing.

Using the log-linearized version of the model, we can reduce the system to the aggregate supply function (43), the Taylor rule (42), the government budget constraint (44) and the following version of the intertemporal IS equation:

$$E_t y_{t+1} - S_g E_t g_{t+1} + \sigma S_c E_t \pi_{t+1} = y_t - S_g g_t + \sigma S_c R_{1t} \tag{72}$$

where S_g and S_c indicate, respectively, the share of public expenditure and consumption over GDP. Another equation of the system is given by the aggregate supply curve given by (33). For what concerns determinacy analysis we can drop from the aforementioned equations all terms involving exogenous stochastic processes a_t and g_t , since they do not impact on the dynamic properties of the model. As a general remark, after plugging the Taylor rule for R_{1t} into equation (72) and in the reduced form government budget constraint (37), we can reduce further the model down to a three-equation system in the variables π_t,y_t and b_{2t}. This system can then be represented in matrix form as follows:

$$AZ_{t+1} = BZ_t \tag{73}$$

where vector Z_t is given by $Z_t = [\pi_t, y_t, b_{2t}]'$, and matrices A and B are properly defined according the specific setting adopted. We can rewrite the system as follows:

$$Z_{t+1} = \Gamma Z_t \tag{74}$$

with $\Gamma = A^{-1}B$. Matrix Γ includes the driving dynamical properties of the system and the determinacy analysis is entirely focused on it.

5.1 Pure inflation targeting

The monetary policy rule here studied are given by:

$$R_{1t} = \phi_\pi \pi_t \tag{75}$$

$$R_{1t} = \phi_\pi \pi_{t+1} \tag{76}$$

$$R_{1t} = \phi_\pi \pi_{t-1} \tag{77}$$

Rule (75) is a simple representation of the pure inflation target regime, while (76) indicates a pure expected inflation targeting, and (77) represents a lagged inflation targeting. These policy rules have been analysed also by Bullard and Mitra (2002) and Lubik and Marzo (2007) in a model with one type of bond and money in the utility function with strongly-separable preferences. After including rules (75)-(77) into (72) and (37) and re-arranging, we obtain a three-equation system which can be represented as (74).

The determinacy conditions for a REE induced by the two rules (75) and (76) are stated in Proposition 4. The backward inflation targeting rule implies an upper bound for the coefficient ϕ_π that is discussed in Proposition 5.

Proposition 4 *Given $\phi_\pi > 0$, conditions for determinacy of a REE to be unique under a Taylor Rule of types (75)-(76) are given by:*

$$\phi_\pi > 1 \tag{78}$$

$$1 - \beta < \psi < 1 + \beta \tag{79}$$

Or, alternatively:

$$\phi_\pi < 1 \tag{80}$$

$$1 - \beta > \psi \qquad \psi > 1 + \beta \tag{81}$$

Proof 5 *See Appendix 2.*

According to the results outlined in Proposition 4, the Taylor principle for a model with the term structure does hold with a pure inflation-targeting rule. This is not the case for a model where the monetary policy rule involves a backward-looking inflation targeting, such as (77):

Proposition 5 *Given $\phi_\pi > 0$, conditions for determinacy of a REE to be unique under a Taylor Rule (77) are given by:*

$$1 < \phi_\pi < 1 + \frac{2(1+\beta)}{\sigma S_c k} \tag{82}$$

$$1 - \beta < \psi < 1 + \beta \tag{83}$$

Or, alternatively:

$$\begin{aligned} \phi_\pi &< 1 \qquad (84) \\ 1-\beta &> \psi \qquad \psi > 1+\beta \qquad (85) \end{aligned}$$

Proof 6 *See Appendix 2.*

From these results, we observe that the rule for backward inflation targeting generates an upper bound for the inflation targeting coefficient. In this sense, the model partially confirms the findings from the existing literature based on models with the term structure. Our framework replicates the interaction active-monetary, passive-fiscal regime outlined in Leeper (1991) and shows the perfect dualism of the inflation targeting procedure. A perfectly determinate equilibrium can be reached also with an inflation targeting coefficient lower than one (ϕ_π), provided that fiscal policy is set to be active, or non-responding to the outstanding path of debt. The novelty consists in the presence of an upper bound for the backward-looking inflation targeting rule.

5.2 Flexible inflation targeting

We also consider a set of monetary policy rules with a flavor for output stabilization. These rule fall under the headline of flexible inflation targeting according to Svensson (2003). In what follows we focus on two variants, including a standard Taylor rule with contemporaneous targeting of inflation and output, and an alternative including expected inflation targeting together with current output targeting.

The results for the classical Taylor rule are collected in the following Proposition:

Proposition 6 *Under simple Taylor Rule with contemporaneous inflation and output targeting given by:*

$$R_{1t} = \phi_\pi \pi_t + \phi_y y_t \qquad (86)$$

Provided that $\phi_\pi, \phi_y > 0$ *conditions for determinacy of a REE to be unique are:*

$$k\phi_\pi + \phi_y > \frac{1-\beta}{\sigma S_c \beta} \qquad (87)$$

and:

$$k(\phi_\pi - 1) + \phi_y (1-\beta) > 0 \qquad (88)$$

$$1-\beta < \psi < 1+\beta \qquad (89)$$

Alternatively, the REE is determinate if either (87) or (88) or both have the reverted inequality and:

$$\phi_\pi < 1 \tag{90}$$
$$1-\beta > \psi \qquad \psi > 1+\beta \tag{91}$$

Proof 7 *See Appendix 2.*

Conditions (87)-(89) highlight a tension between ϕ_π and ϕ_y, provided that $\phi_\pi > 1$. These results confirm the findings by Bullard and Mitra (2002) and Lubik and Marzo (2007).
A second type of Taylor rule is represented by an expected inflation targeting coupled with an objective for current output targeting. This is obtained by setting $\rho = 0$ in (42), and by replacing current inflation with the expected inflation rate:

$$R_{1t} = \phi_\pi E_{t+1}\pi_{t+1} + \phi_y y_t \tag{92}$$

The results relative to the rule (92) are summarized in the following Proposition.

Proposition 7 *Undet expected inflation and current output targeting rule given by (92), provided that $\phi_\pi > 0, \phi_y > 0$, conditions for a REE to be unique are:*

$$1 < \phi_\pi < \frac{2(1+\beta)}{k\sigma S_c\sigma} + \phi_y\frac{2(1+\beta)}{k} + 1 \tag{93}$$

and:

$$1-\beta < \psi < 1+\beta \tag{94}$$

Alternatively, the REE is determinate if either (93) is not satisfied to get determinacy, condition (94) must be replaced by:

$$1-\beta > \psi \qquad \psi > 1+\beta \tag{95}$$

Proof 8 *See Appendix 2.*

Even in this case, condition (93) identifies an upper bound for the inflation targeting coefficient, conditional to the size of the output targeting. Fiscal policy inserts another degree of freedom, letting policy maker to choose between the combination *active*-monetary and *passive*-fiscal or vice-versa.

5.3 Interest-rate smoothing

By setting $\rho \neq 0$, we obtain a Taylor rule with interest rate smoothing, as represented in equation (42). In this case the results are collected in Proposition 8:

Proposition 8 *With rule (42), provided that $\phi_\pi, \phi_y, \rho > 0$, conditions for a REE to be unique are:*

$$k(\phi_\pi + \rho - 1) + (1 - \beta)\phi_y > 0 \tag{96}$$

$$\rho < \beta \tag{97}$$

$$1 - \beta < \psi < 1 + \beta \tag{98}$$

Alternatively, if either (96) or (97) are not satisfied, REE determinacy is obtained by replacing (98) by:

$$1 - \beta > \psi \qquad \psi > 1 + \beta \tag{99}$$

Proof 9 *See Appendix 2.*

The results about determinacy here stated confirm what has been obtained in a simple New Keynesian model without the term structure. Therefore, under this perspective, the insertion of the term structure appears to be irrelevant in terms of determinacy analysis, since Taylor principle is fully satisfied, as discussed in Bullard and Mitra (2002). A possible interpretation of these findings involves the role of the EH, which holds perfectly in the log-linear version of the model. The following step consists in testing if these results are still obtained when a different assumption is made for the transaction services provided by the government bonds.

6 Robustness analysis

To what extent are the results presented in the previous sections are model-dependent? Would alternative assumptions on bond transaction services confirm the determinacy results obtained earlier? To provide a proper answer to this question, we change the functional form of instantaneous utility, and consider a weakly separable utility with bonds and consumption:[2]

$$u_t = \frac{\left[C_t^\gamma b_{1t}^{1-\gamma}\right]^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} - \frac{L_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \tag{100}$$

with $b_{1t} = B_{1t}/P_t$. From (100), we observe that short-term bond b_{1t} are treated as if they were money, since they directly provide utility to the representative agent with a direct interaction with consumption. The First Order Condition with respect to consumption

[2]In the following notation, we drop the household index i, for simplicity.

(4) is now replaced by:

$$\gamma C_t^{\gamma\left(1-\frac{1}{\sigma}\right)-1} b_{1t}^{(1-\gamma)\left(1-\frac{1}{\sigma}\right)} = \lambda_t \tag{101}$$

Moreover, the first-order condition with respect to b_{1t} is now given by:

$$(1-\gamma) C_t^{\gamma\left(1-\frac{1}{\sigma}\right)} b_{1t}^{(1-\gamma)\left(1-\frac{1}{\sigma}\right)-1} + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}} = \frac{\lambda_t}{R_{1t}} \tag{102}$$

Thus, rearranging (101) and (102), the demand for b_{1t} is now given by:

$$b_{1t} = \frac{(1-\gamma)}{\gamma} C_t \frac{R_{1t} R_{2t}}{(R_{2t} - R_{1t+1} R_{1t})} \tag{103}$$

It is not difficult to check that short-term bond demand (103) still respects the usual properties: it is increasing in consumption C_t and R_{1t}, R_{1t+1} and decreasing with respect to R_{2t}.

In order to reduce the model, we can write the log-linearized version of the First-Order Condition with respect to C_t given in (101) as follows:

$$\left[\gamma\left(1-\frac{1}{\sigma}\right)-1\right] c_t + (1-\gamma)\left(1-\frac{1}{\sigma}\right) b_{1t} = \lambda_t \tag{104}$$

The log-linearized version of the demand for liquid bonds (103) is instead:

$$b_{1t} = c_t + \frac{1}{(1-\beta R_1)} R_{1t} - \frac{R_1 \beta}{(\pi - \beta R_1)} R_{2t} + \frac{R_1 \beta}{(\pi - \beta R_1)} R_{1t+1} \tag{105}$$

Equations (104)-(105) represent the key ingredients to figure out the key aspect of the model. In equation (104), the Lagrange multiplier depends on nominal rates through equation (105), differently from the standard case, where λ_t depends on C_t only. This feature produces a strong impact on the functional form of Intertemporal IS and aggregate supply equation. In fact, the intertemporal IS curve depends on both expected and current short term rate in log-linear terms:

$$y_{t+1} - \sigma S_c \alpha_{R1} R_{1t+1} + \sigma S_c \pi_{1t+1} + g\sigma g_{t+1} = y_t - \sigma S_c \alpha_R R_{1t} + g\sigma g_t \tag{106}$$

while aggregate supply now becomes:

$$\beta \pi_{t+1} = \pi_t - k y_t + \eta_{as} a_t + \eta_{gs} g_t + \eta_{RS} R_{1t} \tag{107}$$

where all coefficients are reported in Appendix 1.

From equation (107) we note that the introduction of weak separability in the utility function modifies the functional form of the AS equation in a substantial way, since now the short term interest rate directly affects the expected inflation rate together with exogenous shocks. A similar result would have been obtained after the introduction of transaction costs in the representative agent's budget constraint. Intuitively, this means that monetary policy, by controlling the short term rate R_{1t}, directly affects the firm's costs and her ability to borrow from banks. An increase in short term rate R_{1t} has the effects of increasing the cost structure of firms, implying an increase of expected inflation as direct consequence. On the other hand, it is not difficult to check that the EH holds in log-linear terms, and equation (71) applies to this context too.

The model can be reduced by following exactly the same steps adopted in the benchmark case previously examined and the system can be set in the form highlighted by (73), with vector of variables still given by: $Z_t = [\pi_t, y_t, b_{2t}]'$. Matrices A and B from (73) are now given by:

$$A = \begin{bmatrix} \beta & 0 & 0 \\ [\sigma S_c (1 - \alpha_R \phi_\pi)] & (1 - \sigma S_c \alpha_{R1} \phi_y) & 0 \\ \mu_{\pi 1} & \mu_{y1} & 1 \end{bmatrix} \tag{108}$$

$$B = \begin{bmatrix} (1 + \eta_{RS} \phi_\pi) & -(k - \eta_{RS} \phi_y) & 0 \\ \alpha_R \sigma S_c \phi_\pi & (1 - \sigma S_c \alpha_R \phi_y) & 0 \\ \mu_\pi & \mu_y & \frac{(1-\psi)}{\beta} \end{bmatrix} \tag{109}$$

while matrix Γ becomes:

$$\Gamma = \begin{bmatrix} \frac{(1+\eta_{RS}\phi_\pi)}{\beta} & -\frac{(k-\eta_{RS}\phi_y)}{\beta} & 0 \\ \varphi_1 & \varphi_2 & 0 \\ \varphi_3 & \varphi_4 & \frac{(1-\psi)}{\beta} \end{bmatrix} \tag{110}$$

where all coefficients are reported in Appendix 1.

With the present setting at hands, the following result holds:

Proposition 9 *With rule (42), provided that $\phi_\pi, \phi_y, \rho > 0$, conditions for a REE to be unique are:*

$$argmax \left\{ \bar{\phi}_{\pi 2}, \bar{\phi}_{\pi 3} \right\} \;<\; \phi_\pi < \frac{k - \eta_{RS} \phi_y}{k \alpha_R (1 + \eta_{RS}) + \eta_{RS} \phi_y} \tag{111}$$

$$\frac{1}{\sigma S_c \alpha_{R1}} \;<\; \phi_y \tag{112}$$

with

$$\bar{\phi}_{\pi 2} = \frac{\sigma S_c \phi_y (\alpha_R + \beta \alpha_{R1}) - (1 + \beta)}{\eta_{RS} (1 - k S_c \alpha_R)}$$

$$\bar{\phi}_{\pi 3} = \frac{(1 - \beta) \sigma S_c \phi_y + \eta_{RS} \sigma S_c \phi_y - \sigma S_c k}{\eta_{RS} \sigma S_c \phi_y + k \alpha_R (\sigma S_c - \eta_{RS})}$$

$$1-\beta<\psi<1+\beta \tag{113}$$

Alternatively, if either (111) or (112) are not satisfied, REE determinacy is obtained by replacing (113) by:

$$1-\beta>\psi \qquad \psi>1+\beta \tag{114}$$

Proof 10 *See Appendix 2.*

These results highlight the presence of a set of non-linear bounds for monetary policy parameters ϕ_π and ϕ_y. This property depends entirely on the setting adopted for bond modelling approach. In fact, under the assumption of weakly separable utility function between consumption and short-term bonds modifies both aggregate supply and intertemporal IS curve. This makes the Taylor principle no longer a sufficient condition for determinacy, since inflation targeting coefficient is now dependent on output targeting coefficient.

To provide a visual representation of the implications of the results from Proposition 9, we simulate the evolution pattern of ϕ_π and ϕ_y conditional to two different values for the parameter representing the intertemporal elasticity of substitution in consumption σ. Figure 3 reports the evolutions of bounds: ϕ_y has a range between 0 and 10. The parameters of the model are exactly the same as those described in Table 1, apart from γ, which has been set equal to 0.8, in order to assign a larger weight to consumption in the instantaneous utility. The top panel in Figure 3 is plotted for a value of σ equal to 2, while in the bottom panel we set $\sigma=0.5$. In both pictures, the determinacy region is denoted by a text label. Outside the bounds, we obtain indeterminacy unless we change the fiscal policy stance, as described by condition (114). Therefore, Figure 3 has been drawn by considering a fiscal policy setting based on equation (113), which identifies a passive fiscal policy. From the top panel of Figure 3, we observe that determinacy when $\sigma=2$ obtains only when there is an almost one-to-one increase in both ϕ_π and ϕ_y.

From an intuitive point of view, with $\sigma>1$, the marginal utility of consumption is decreasing with respect to b_{1t}. In this case, a negative shock to inflation implies an increase in demand for b_{1t}, a lower marginal utility of consumption, and a lower labor supply. On the other hand, if $\sigma<1$, the marginal utility of consumption is decreasing with respect to b_{1t}. Hence, a negative shock to inflation implies a decrease in demand for short-term bonds b_{1t} together with an increase in marginal utility of consumption and higher labor supply. Therefore, elasticity of intertemporal substitution is crucial to determine the determinacy region induced by Taylor rule. Differently from a model with a strongly separable utility function, in the present context, the core parameters of the model play a crucial role in the definition of the determinacy region.

7 Conclusion

This paper explores the role of the term structure of interest rates in a simple New Keynesian mode. We focus on the ability of simple Taylor rules to generate determinate equilibria. The results show that what really matters for determinacy is not just the inclusion of both long and short term bonds in a full-fledged dynamic macro model. Rather, the key issue lies in the way the transaction services of alternative bonds are modelled.

We show that, with short-term bonds entering the utility function in a weakly separable way with consumption, the requirement for monetary policy parameters to generate determinate equilibria are very similar to what is obtained in the literature for models without the term structure. However, if transaction services enter the utility function in a weakly separable way, the bounds for the inflation targeting coefficient of the Taylor rule becomes non-linear. The results are then no longer clear-cut, since the bounds depend strongly on other core parameters of the model, such as the elasticity of intertemporal substitution of consumption.

In our framework, the expectations hypothesis holds and bond pricing scheme follows an affine structure. Under this perspective, the difference between long and short-term interest rates in a log-linear approximation is due to exogenous shocks hitting both the level and the slope of the term structure. Our results shed light on the role of modelling liquidity services in the definition of determinacy conditions and internal dynamics of the model. The role of fiscal policy becomes also evident in affecting bond pricing through a government spending shock. Assigning a proper role to the fiscal stance, as in the Fiscal Theory of the Price Level, allows for a wider characterization of the equilibrium conditions compared with what has been proposed in the literature.

Several extensions can be envisaged. At first sight, the introduction of money would provide an additional buffer, whose role for the determinacy of equilibria should be assessed. With the modelling of money balances, the role of money supply rules should be contrasted with that of standard interest rate rules.

A Coefficients

A.1 Model equations (sections 2-6)

$$\eta_{ca} \equiv \frac{\sigma\left(1+\frac{1}{\eta}\right)}{S_c\sigma\left(\frac{1}{\eta}+1-\alpha\right)+\alpha}$$

$$\eta_{cg} \equiv \frac{S_g\sigma\left(1+\frac{1}{\eta}\right)-\alpha}{S_c\sigma\left(\frac{1}{\eta}+1-\alpha\right)+\alpha}$$

$$\eta_{ya} \equiv \frac{\sigma\left(1+\frac{1}{\eta}\right)-\alpha\eta_{ca}}{\sigma\left(1+\frac{1}{\eta}\right)}$$

$$\eta_{yg} \equiv \frac{\alpha\eta_{cg}}{\sigma\left(1+\frac{1}{\eta}\right)}$$

$$\eta_b = \left\{\left[\chi\left(1-\frac{1}{\sigma}\right)-1\right]\left(1-\frac{\beta R_1}{\pi}\right)\right\}^{-1}$$

$$\eta_{\pi 1} = \frac{\phi_\pi}{\beta}+\frac{1}{\beta}\left(\frac{b\eta_b\beta}{\pi}-1\right)-\frac{bR_2\left(1+\eta_b\right)\phi_\pi}{R_1}+\frac{(1-\psi)}{\pi}\left[bR_2+\frac{R_2}{R_1}-b\eta_b\beta R_1R_2\right]$$

$$\eta_{ba2} \equiv \phi_y\eta_{ya}+\frac{b\eta_b\beta\eta_{ca}}{R_2\sigma\pi}-\frac{\eta_{ca}}{\sigma}$$

$$\eta_{ba1} \equiv \frac{\eta_{ca}}{\sigma}-\frac{b\eta_b\eta_{ca}\pi}{\beta\sigma}+\frac{b\left(1-\psi\right)\eta_bR_1^2\eta_{ca}}{\sigma\pi}-\left(\frac{b\eta_b\beta}{\pi}-1\right)k\eta_{ya}-\phi_\pi k\eta_{ya}+ \\ -\frac{b\left(1+\eta_b\right)R_2\phi_y\eta_{ya}}{R_1}$$

$$\eta_{bg2} \equiv \phi_y\eta_{yg}-\frac{b\eta_b\beta\eta_{cg}}{\sigma\pi}+\frac{\eta_{cg}}{\sigma}$$

$$\eta_{bg1} \equiv \frac{b\eta_b\eta_{cg}R_2}{\sigma R_1}-\frac{\eta_{cg}}{\sigma}+\frac{b\left(1-\psi\right)\eta_bR_1^2\eta_{cg}}{\sigma\pi}-\frac{gR_2}{b_2}-k\phi_\pi\eta_{yg}+ \\ -\left(\frac{b\eta_b\beta}{\pi}-1\right)k\eta_{yg}-\frac{b\left(1+\eta_b\right)R_2\phi_y\eta_{yg}}{R_1}$$

$$\eta_{b2} \equiv \frac{R_2}{\pi}\left(\frac{1}{R_1}-\psi\right)$$

$$\eta_\pi \equiv \phi_\pi\left[\frac{b\left(1-\psi\right)}{\pi}\eta_bR_2+\frac{1}{\beta}\right]$$

$$\eta_{ba} \equiv \phi_y\eta_{ya}\left[\frac{b\left(1-\psi\right)}{\pi}\eta_bR_2+\frac{1}{\beta}\right]+\frac{b\left(1-\psi\right)R_1\eta_b\eta_{ca}}{\beta\sigma}$$

$$\eta_{bg} \equiv \frac{b\left(1-\psi\right)R_1\eta_b\eta_{cg}}{\beta\sigma} - \phi_y\eta_{yg}\left[\frac{b\left(1-\psi\right)}{\pi}\eta_b R_2 + \frac{1}{\beta}\right]$$

A.2 Variant discussed in Section 7

$$\begin{aligned}
\alpha_{R1} &= (1-\gamma)\left(1-\frac{1}{\sigma}\right)\eta_{bR} \\
\alpha_{R} &= (1-\gamma)\left(1-\frac{1}{\sigma}\right)\eta_{bR} - 1 \\
\eta_{bR} &= \frac{(\pi-\beta R_1)-\beta R_1\left(1-\beta R_1\right)}{(\pi-\beta R_1)\left(1-\beta R_1\right)} \\
k &= \frac{(1-\delta)\left(1-\delta\beta\right)}{\delta}\left\{\frac{\sigma S_c\left(1+\eta\left(1-\alpha\right)\right)+\alpha\eta}{\sigma S_c h_s}\right\} \\
\eta_{as} &= \frac{(1+\eta)}{h_s}\left[\frac{(1-\delta)\left(1-\delta\beta\right)}{\delta}\right] \\
\eta_{gs} &= \frac{g\alpha\eta}{\sigma S_c h_s}\left[\frac{(1-\delta)\left(1-\delta\beta\right)}{\delta}\right] \\
\eta_{Rs} &= \frac{g\alpha\eta\alpha_{RS}}{h_s}\left[\frac{(1-\delta)\left(1-\delta\beta\right)}{\delta}\right] \\
h_s &= \alpha\eta\left(1-\theta\right)+\theta\left(1+\eta\right)
\end{aligned}$$

$$\begin{aligned}
\mu_{\pi 1} &= \frac{R_2}{b_2}\left\{\frac{b_1\eta_{bR}\phi_\pi}{R_1} - \frac{b_1\phi_\pi}{R_1} - \frac{b_2\phi_\pi}{R_2} - \frac{b_2\phi_\pi}{R_2\beta}\left(1+\eta_{RS}\phi_\pi\right) - \frac{b_2\phi_y f_p}{R_2}\right\} \\
f_p &= \frac{1}{1-\sigma S_c\alpha_{R1}}\left[\frac{\sigma S_c}{\beta}\left(\alpha_R\phi_\pi-1\right)\left(1+\eta_{RS}\phi_\pi\right)-\alpha_R\sigma S_c\phi_\pi\right] \\
f_y &= \frac{1}{1-\sigma S_c\alpha_{R1}}\left[1-\frac{\sigma S_c}{\beta}\left(\alpha_{R1}\phi_\pi-1\right)\left(k-\eta_{RS}\phi_y\right)-\alpha_R\sigma S_c\phi_y\right] \\
\mu_{y1} &= \frac{R_2}{b_2}\left\{\frac{b_1}{R_1 S_c} + \frac{b_1\eta_{bR}\phi_y}{R_1} - \frac{b_1\phi_y}{R_1} - \frac{b_2\phi_y}{R_2} - \frac{b_2\phi_\pi}{R_2\beta}\left(k-\eta_{RS}\phi_y\right) - \frac{b_2\phi_y f_y}{R_2}\right\} \\
\mu_\pi &= \frac{R_2}{b_2}\left\{\frac{b_1\left(1-\psi\right)\eta_{bR}\phi_\pi}{\pi} - (1-\psi)\left(\frac{b_1}{\pi}+\frac{b_2}{\pi R_1}\right) - \frac{(1-\psi)\,b_2\phi_\pi}{\pi R_1}\right\} \\
\mu_y &= \frac{R_2}{b_2}\left\{\frac{b_1\left(1-\psi\right)}{\pi S_c} + \frac{(1-\psi)\,b_1\eta_{bR}\phi_y}{\pi} - \frac{(1-\psi)\,b_2\phi_y}{\pi R_1}\right\}
\end{aligned}$$

$$
\begin{aligned}
\varphi_1 &= \frac{\sigma S_c \phi_\pi \eta_{RS}\left(\alpha_R \phi_\pi - 1\right) + \alpha_R \phi_\pi \sigma S_c\left(1-\beta\right) - \sigma S_c}{\beta\left(1 - \sigma S_c \alpha_{R1} \phi_\pi\right)} \\
\varphi_2 &= \frac{\left(k - \eta_{RS}\phi_y\right)\sigma S_c\left(1 - \alpha_R \phi_\pi\right) + \beta\left(1 - \alpha_R \sigma S_c \phi_y\right)}{\beta\left(1 - \sigma S_c \alpha_{R1}\phi_\pi\right)} \\
\varphi_3 &= \alpha_{31}\left(1 + \phi_\pi \eta_{RS}\right) - \alpha_{32}\alpha_R \sigma S_c \phi_\pi + \mu_\pi \\
\varphi_4 &= \left(\eta_{RS}\phi_y - k\right)\alpha_{31} + \alpha_{32}\left(1 - \alpha_R \sigma S_c \phi_y\right) + \mu_y \\
\alpha_{31} &= \frac{\sigma S_c\left(1 - \alpha_R \phi_\pi\right)\mu_{y1} - \left(1 - \sigma S_c \alpha_{R1}\phi_y\right)\mu_{\pi 1}}{\beta\left(1 - \sigma S_c \alpha_{R1}\phi_y\right)} \\
\alpha_{32} &= -\frac{\mu_{y1}}{1 - \sigma S_c \alpha_{R1}\phi_y}
\end{aligned}
$$

B Schur-Cohn criterion

B.1 2×2 matrix

The characteristic polynomial for a generic 2×2 matrix A is $x^2 - \mathrm{tr}\left(A\right)x + \det\left(A\right) = 0$. From La Salle (1986), conditions for the two roots to lie outside the unitary circle are given by:

$$
\begin{aligned}
\left|\det\left(A\right)\right| &> 1 \qquad (115) \\
\left|\mathrm{tr}\left(A\right)\right| &< 1 + \det\left(A\right) \qquad (116)
\end{aligned}
$$

In particular, condition (116) can be split in the following two inequalities:

$$
\begin{aligned}
1 + \det\left(A\right) + \mathrm{tr}\left(A\right) &> 0 \qquad (117) \\
1 + \det\left(A\right) - \mathrm{tr}\left(A\right) &> 0 \qquad (118)
\end{aligned}
$$

3×3 Matrix

We collect in what follows the full set of conditions to be satisfied by a generic 3×3 matrix B to obtain one root inside and two roots outside the unit circle. The characteristic polynomial for a 3×3 matrix is:

$$
P\left(\lambda\right) = \lambda^3 + A_2\lambda^2 + A_1\lambda + A_0 \qquad (119)
$$

where $A_0 = -\det\left(B\right)$; $A_2 = -\mathrm{tr}\left(B\right)$; $A_1 = -\mathrm{tr}\left(B\right)$; $A_0 = \left(b_{11}b_{12} - b_{21}b_{12}\right) + \left(b_{22}b_{33} - b_{32}b_{23}\right) + \left(b_{11}b_{33} - b_{31}b_{13}\right)$. Therefore, necessary and sufficient conditions ar egiven by the following restrictions on the coefficients of the characteristic polynomial (119). Thus, either:

1. CASE 1

$$
\begin{aligned}
1 + A_2 + A_1 + A_0 &< 0 \qquad (120) \\
-1 + A_2 - A_1 + A_0 &> 0 \qquad (121)
\end{aligned}
$$

or:

2. CASE 2

$$\begin{aligned} 1+A_2+A_1+A_0 &> 0 && (122)\\ -1+A_2-A_1+A_0 &< 0 && (123)\\ A_0^2-A_0A_2+A_1-1 &> 0 && (124) \end{aligned}$$

or:

3. CASE 3

$$\begin{aligned} 1+A_2+A_1+A_0 &> 0 && (125)\\ -1+A_2-A_1+A_0 &< 0 && (126)\\ A_0^2-A_0A_2+A_1-1 &< 0 && (127)\\ |A_2| &> 3 && (128) \end{aligned}$$

C Proofs

C.1 Proof of proposition 1

From the log-linearization of the First Order Condition on labor (5) and the production function, we find:

$$y_t = \left(\frac{1+\frac{1}{\eta}}{1+\frac{1}{\eta}-\alpha}\right) a_t - \frac{\alpha}{\sigma\left(\frac{1}{\eta}+1-\alpha\right)} c_t \tag{129}$$

Given (129) and the resource constraint log-linearized (see the technical appendix for details), we obtain the following equations linking consumption to the core shock hitting the economy:

$$c_t = \eta_{ca} a_t - \eta_{cg} g_t \tag{130}$$

where coefficients were reported in Appendix 1. Taking advantage of (130) we can also define the output equation, as follows:

$$y_t = \eta_{ya} a_t - \eta_{yg} g_t \tag{131}$$

with the coefficients η_{ya}, η_{yg} in Appendix 1. The log-linearized equation for liquid bond is:

$$b_{1t} = \eta_b \lambda_t - \eta_b R_{1t} + \eta_b \frac{\beta R_1}{\pi} \pi_{t+1} - \eta_b \frac{\beta R_1}{\pi} \lambda_{t+1} \tag{132}$$

Moreover, the log-linearized version of the Taylor rule with contemporaneous inflation and output targeting is:

$$R_{1t} = \phi_\pi \pi_t + \phi_y \eta_{ya} a_t - \phi_y \eta_{yg} g_t \tag{133}$$

after having substituted out for (131). Moreover, after further substitutions, equation (132) can be rewritten as follows:

$$b_{1t} = -\frac{\eta_b \eta_{ca}}{\sigma} a_t + \frac{\eta_b \eta_{cg}}{\sigma} g_t - \eta_b R_{1t} + \frac{\eta_b \beta R_1}{\pi} \pi_{t+1} + \eta_b \frac{\beta R_1 \eta_{ca}}{\sigma \pi} a_{t+1} - \eta_b \frac{\beta R_1 \eta_{cg}}{\sigma \pi} g_{t+1} \tag{134}$$

From FOC with respect to b_{2t}, after rearrangement, we obtain the following expression for R_{2t}:

$$R_{2t} = -\frac{1}{\sigma} \eta_{ca} a_t + \frac{1}{\sigma} \eta_{cg} g_t + \frac{1}{\sigma} \eta_{ca} a_{t+1} - \frac{1}{\sigma} \eta_{cg} g_{t+1} + \pi_{t+1} - R_{1t+1} \tag{135}$$

From the aggregate supply function, we have:

$$\pi_{t+1} = \frac{1}{\beta} \pi_t - \frac{k}{\beta} \eta_{ya} a_t - \frac{k}{\beta} \eta_{yg} g_t \tag{136}$$

Let the ratio between liquid and illiquid bonds to be: $b = b_1/b_2$. Taking advantage of both (43) and (133) together with their respective forward versions we obtain the following expression for the government budget constraint:

$$\begin{aligned} b_{2t+1} + \eta_{\pi 1} \pi_{t+1} + \eta_{ba2} a_{t+2} + \eta_{ba1} a_{t+1} + \eta_{bg2} g_{t+2} + \eta_{bg1} g_{t+1} = \\ = \eta_{b2} b_{2t} - \eta_\pi \pi_t - \eta_{ba} a_t - \eta_{bg} g_t \end{aligned} \tag{137}$$

Q.E.D.

C.2 Proof of proposition 2

In order to achieve a solution for the full model, we start by noting that equation (43) does not depend on b_{2t}. Therefore, we can solve for π_t from (43) and then substitute out into (44) to solve for b_{2t}. From (43), we note immediately that $\beta^{-1} > 1$. This implies an explosive root. Therefore, following Sargent (1979), we can solve (43) as follows:

$$\pi_{t+1} = \frac{-\left(\frac{1}{\mu_2}\right) L^{-1}}{\left(1 - \frac{1}{\mu_2} L^{-1}\right)} \left[\eta_{ya} a_t + \eta_{yg} g_t\right] \tag{138}$$

Where μ_2 is the explosive root of (43). By applying lag polynomial L to (138) we find:

$$\pi_{t+1} = \frac{-\left(\frac{1}{\mu_2}\right)}{\left(1 - \frac{1}{\mu_2} L^{-1}\right)} \left[\eta_{ya} a_{t+1} + \eta_{yg} g_{t+1}\right] \tag{139}$$

By using the definition of lag polynomial as in Sargent (1979):

$$\begin{aligned}\pi_{t+1} &= -\frac{k}{\mu_2}\sum_{i=0}^{\infty}\left(\frac{1}{\mu_2}\right)^i\left[\eta_{ya}a_{t+1}+\eta_{yg}g_{t+1}\right]= \\ &= \frac{k\eta_{ya}}{\mu_2}\sum_{i=0}^{\infty}\left(\frac{1}{\mu_2}\right)^i a_{t+i+1}+\frac{k\eta_{yg}}{\mu_2}\sum_{i=0}^{\infty}\left(\frac{1}{\mu_2}\right)^i g_{t+i+1}\end{aligned} \tag{140}$$

Applying the definition of stochastic processes for a_t and g_t given in (24) and (38) and developing the series in (140).

$$\begin{aligned}\pi_{t+1} &= \frac{k\eta_{ya}}{\mu_2}\left[\frac{(1-\rho_a)\,a}{1-\frac{1}{\mu_2}}+\sum_{i=0}^{\infty}\left(\frac{\rho_a}{\mu_2}\right)^i a_{t+1}\right]+ \\ &\quad + \frac{k\eta_{yg}}{\mu_2}\left[\frac{(1-\rho_g)\,g}{1-\frac{1}{\mu_2}}+\sum_{i=0}^{\infty}\left(\frac{\rho_g}{\mu_2}\right)^i g_{t+1}\right]\end{aligned} \tag{141}$$

Thus, applying the formula to compute the sum of infinite terms:

$$\begin{aligned}\pi_{t+1} &= \frac{k\eta_{ya}}{\mu_2}\frac{(1-\rho_a)\,a\mu_2}{(\mu_2-1)}+\frac{k\eta_{ya}}{\mu_2}\left(\frac{\mu_2}{\mu_2-\rho_a}\right)a_{t+1}+ \\ &\quad + \frac{k\eta_{yg}}{\mu_2}\frac{(1-\rho_g)\,g\mu_2}{(\mu_2-1)}+\frac{k\eta_{yg}}{\mu_2}\left(\frac{\mu_2}{\mu_2-\rho_g}\right)g_{t+1}\end{aligned} \tag{142}$$

Thus, simplifying and considering the definition of the root $\mu_2=\beta^{-1}$, after rearranging, we obtain the following solution for π_t:

$$\pi_t = f_\pi + \alpha_a a_t + \alpha_g g_t \tag{143}$$

where:

$$\begin{aligned} f_\pi &= \frac{k\eta_{ya}(1-\rho_a)\,a\beta}{(1-\beta)}+\frac{k\eta_{yg}(1-\rho_g)\,g\beta}{(1-\beta)} \\ \alpha_a &= \frac{k\eta_{ya}\beta}{(1-\beta\rho_g)} \\ \alpha_g &= \frac{k\eta_{yg}\beta}{(1-\beta\rho_g)}\end{aligned}$$

To solve for b_{2t}, insert the solution for π_t from (143) together with the expression for (24) and (38) into (44). After rearranging, we obtain the following expression for b_{2t} ready to be solved:

$$b_{2t+1} = h + \eta_{b2}b_{2t} - \delta_a a_{t+1} - \delta_g g_{t+1} \tag{144}$$

where:

$$\begin{aligned}
\gamma_0 &\equiv \eta_{\pi 1}\alpha_a + \eta_{ba2}\left(1-\rho_a\right)a + \eta_{bg2}\left(1-\rho_g\right)g + \eta_\pi f_\pi \\
h &\equiv \frac{\gamma_a\left(1-\rho_a\right)a}{\rho_a} + \frac{\gamma_g\left(1-\rho_g\right)g}{\rho_g} - \gamma_0 \\
\delta_a &\equiv \eta_{\pi 1}\alpha_a + \eta_{ba2}\rho_a + \eta_{ba1} + \frac{\gamma_a}{\rho_a} \\
\delta_g &\equiv \eta_{\pi 1}\alpha_g + \eta_{ba2}\rho_g + \eta_{bg1} + \frac{\gamma_g}{\rho_g}
\end{aligned}$$

By applying standard methods, we can rewrite and solve (144) as follows (see Sargent (1979)):

$$\begin{aligned}
b_{2t+1} &= \frac{h}{1-\eta_{b2}L} - \frac{1}{1-\eta_{b2}L}\left(\delta_a a_{t+1} + \delta_g g_{t+1}\right) = \\
&= \frac{h}{1-\eta_{b2}} - \delta_a \sum_{i=0}^{\infty} \eta_{b2}^i a_{t-i+1} - \delta_g \sum_{i=0}^{\infty} \eta_{b2}^i g_{t-i+1} = \\
&= \frac{h}{1-\eta_{b2}} - \delta_a \sum_{i=0}^{\infty} \left(\rho_a\eta_{b2}\right)^i a_{t+1} - \delta_g \sum_{i=0}^{\infty} \left(\rho_g\eta_{b2}\right)^i g_{t+1}
\end{aligned} \tag{145}$$

Where in the last line we took advantage of (24) and (38). Finally, by applying the infinite sum of series, the solution to equation (145) can be rewritten as:

$$b_{2t+1} = \frac{h}{1-\eta_{b2}} - \frac{\delta_a}{1-\rho_a\eta_{b2}}a_t - \frac{\delta_g}{1-\rho_g\eta_{b2}}g_t \tag{146}$$

Therefore, the solution of system is fully captured by equations (143) and (146). We can now solve explicitly for the pricing kernel. Thus, taking advantage of the definitions of shocks, the first order conditions on consumption log-linearized, the resource constraint log-linearized and the definition of the stochastic processes, we obtain:

$$\begin{aligned}
c_{t+1} - c_t &= \eta_{ca}\left(a_{t+1} - a_t\right) - \eta cg\left(g_{t+1} - g_t\right) = \\
&= \eta_{ca}\left(1-\rho_a\right)a + \eta_{ca}\left(\rho_a - 1\right)a + \eta_{ca}a_t^{1/2}\sigma_a\epsilon_{t+1}^a - \eta_{cg}\left(1-\rho_g\right)g + \\
&- \eta_{cg}\left(\rho_g - 1\right)g - \eta_{cg}g_t^{1/2}\sigma_g
\end{aligned} \tag{147}$$

Using (147) and (143) lagged forward for π_{t+1}, after using again the shocks definition, we obtain the solution for the kernel equation (21) as a function of the exogenous forces of the system a_t, g_t reported in the text.
Q.E.D.

C.3 Proof of proposition 3

Let us start with $k = 1$. To find the price of liquid, short term bond, let us set $p^0_{t+1} = 0$ in (52) to get:

$$\begin{aligned} p^1_t &= \log E_t \exp\left(m_{t+1} + p^0_{t+1}\right) = \log E_t \exp\left(m_{t+1}\right) = \\ &= E_t m_{t+1} + \frac{1}{2}\mathrm{var}_t\left(m_{t+1}\right) = \\ &= -A_1 - B_1 a_t - C_1 g_t \end{aligned} \tag{148}$$

Now, by using (50) together with (148):

$$-A_1 - B_1 a_t - C_1 g_t = -\lambda_0 - \left(\lambda_1 + \frac{\eta_1^2}{2}\sigma_a^2\right) a_t + \left(\lambda_2 - \frac{\eta_2^2}{2}\sigma_g^2\right) g_t \tag{149}$$

After equating coefficients it is immediate to get the definitions of A_1, B_1, C_1, as stated in the text.
For $k > 1$, we need to write coefficients of k as functions of coefficients at $k-1$. We can rewrite (52) as follows:

$$p^{k-1}_{t+1} = -A_{k-1} - B_{k-1}a_{t+1} - C_{k-1}g_{t+1} \tag{150}$$

Thus, by applying the definition of (24) and (38), we obtain:

$$\begin{aligned} p^{k-1}_{t+1} = -A_{k-1} - B_{k-1}&\left[\left(1-\rho_a\right)a + \rho_a a_t + a_t^{1/2}\sigma_a \epsilon^a_{t+1}\right] - \\ &+ C_{k-1}\left[\left(1-\rho_g\right)g + \rho_g g_t + g_t^{1/2}\sigma_g \epsilon^g_{t+1}\right] \end{aligned} \tag{151}$$

Therefore, by using (48)-(49) together with (151)

$$\begin{aligned} E_t\left(m_{t+1} + p^{k-1}_{t+1}\right) = \lambda_0 + \lambda_1 a_t - \lambda_2 g_t - A_{k-1} - B_{k-1}\left(1-\rho_a\right)a - B_{k-1}\rho_a a_t - \\ + C_{k-1}\left(1-\rho_g\right)g - C_{k-1}\rho_g g_t \end{aligned} \tag{152}$$

$$Var_t\left(m_{t+1} + p^{k-1}_{t+1}\right) = \frac{1}{2}\left[\sigma_a^2\left(\eta_1^2 + B^2_{k-1}\right)a_t + \sigma_g^2\left(\eta_2^2 - C^2_{k-1}\right)g_t\right] \tag{153}$$

Therefore, given lognormality, we can combine bond pricing (51)-(52) with (152)-(153):

$$\begin{aligned} -A_k - B_k a_t - C_k g_t = \lambda_0 + \lambda_1 a_t - \lambda_2 g_t - A_{k-1} - B_{k-1}\left(1-\rho_a\right)a - \\ + B_{k-1}\rho_a a_t - C_{k-1}\left(1-\rho_g\right)g - C_{k-1}\rho_g g_t + \\ + \frac{\sigma_a^2}{2}\left(\eta_1^2 + B^2_{k-1}\right)a_t + \frac{\sigma_g^2}{2}\left(\eta_2^2 - C^2_{k-1}\right)g_t \end{aligned} \tag{154}$$

By equating coefficients, we obtain the claim of the proposition.
Q.E.D.

C.4 Proof of proposition 4

We divide the proof in three parts, each relative to a specific monetary rule (75)- (77).

C.4.1 Current inflation rule (75)

With the insertion of (75) in (43),(44) and (72) we can set the system in the form (74), where matrix Γ is defined as:

$$\Gamma = \begin{bmatrix} \beta^{-1} & -k\beta^{-1} & 0 \\ -\left(\sigma S_c \beta^{-1} + \sigma S_c \phi_\pi\right) & \left(1 + k\sigma S_c \beta^{-1}\right) & 0 \\ \alpha_{\gamma 1} & -\alpha_{\gamma 2} & \beta^{-1}\left(1-\psi\right) \end{bmatrix} \tag{155}$$

Coefficients are given by:

$$\begin{aligned}
\alpha_{\pi 1} &= \frac{1}{\beta}\left(\frac{bR_1\eta_b}{\beta} - 1\right) + \frac{\phi_\pi}{\beta}\left(1 - b\pi - bR_1\eta_b\right) + \\
&\quad + R_1\left(1-\psi\right)\left[\frac{1}{\beta}\left(b + \frac{1}{R_1}\right) - \frac{bR_1\eta_b\eta_{cg}}{\pi}\right] \\
\alpha_y &= \frac{k}{\beta}\left[1 - \frac{bR_1\eta_b}{\beta} - \phi_\pi\right] \\
\alpha_\pi &= -\frac{\left(1-\psi\right)\phi_\pi}{\beta} \\
\alpha_{\gamma 1} &= \frac{1}{\beta}\left(\sigma S_c \alpha_y - \alpha_{\pi 1}\right) - \alpha_y \sigma S_c \phi_\pi - \alpha_\pi \\
\alpha_{\gamma 2} &= \frac{k}{\beta}\left(\sigma S_c \alpha_y - \alpha_{\pi 1}\right) + \alpha_y
\end{aligned}$$

To get determinacy, we need two roots of matrix (155) to be outside the unit circle, and one inside since public debt b_{2t} is a predetermined variable. Since (155) is upper triangular, we can concentrate on the submatrix A_{11} given by:

$$A_{11} = \begin{bmatrix} \beta^{-1} & -k\beta^{-1} \\ -\left(\sigma S_c \beta^{-1} + \sigma S_c \phi_\pi\right) & \left(1 + k\sigma S_c \beta^{-1}\right) \end{bmatrix} \tag{156}$$

Trace and determinant of submatrix A_{11} are, respectively:

$$\operatorname{tr}\left(A_{11}\right) = 1 + \frac{1}{\beta} - \frac{\sigma k S_c}{\beta} \tag{157}$$

$$\det\left(A_{11}\right) = \frac{1 + k\sigma S_c \phi_\pi}{\beta} \tag{158}$$

It is immediate to verify that condition (115) is automatically verified if $\phi_\pi > 0$. In the same way, condition (117) is verified if $\phi_\pi > 1$, while by setting $\phi_\pi > 0$ it is sufficient to verify condition (118). This ensures that submatrix A_{11} has one root inside and one outside the unit circle. To get another root inside the unit circle we need the following condition to be satisfied:

$$\frac{|1-\psi|}{\beta}<1 \tag{159}$$

which delivers the result stated in the text.
When both conditions $\phi_\pi>1$ and (159) are satisfied, two roots are inside and one is outside the unit circle. If, on the other hand, $\phi_\pi<1$, then two roots will already be inside the unit circle. Therefore, to get one root inside the unit circle, we require the following additional condition on fiscal policy:

$$\frac{|1-\psi|}{\beta}>1 \tag{160}$$

which implies:

$$\psi<1-\beta \qquad \psi>1+\beta \tag{161}$$

This proves the result for the current absolute inflation targeting rule (75).
Q.E.D.

C.4.2 Expected inflation rule (76)

Given the monetary rule under (76), matrix Γ and B in the system (74) is given by:

$$\Gamma=\begin{bmatrix}\beta^{-1} & -k\beta^{-1} & 0\\ -\sigma S_c\beta^{-1}(1-\phi_\pi) & 1-k\sigma S_c\beta^{-1}(1-\phi_\pi) & 0\\ \alpha_{31} & -k\alpha_{31}+\alpha_y & \beta^{-1}(1-\psi)\end{bmatrix} \tag{162}$$

where:

$$\begin{aligned}\alpha_{\pi 3}=\;& bR_1\eta_b-1-\frac{2b\pi\phi_\pi}{\beta}+\frac{\phi_\pi}{\beta}\left[1-\frac{k}{\beta}\sigma S_c(\phi_\pi-1)\right]+\frac{R_1(1-\psi)}{\beta}\left(b+\frac{1}{R_1}\right)+\\ &+\frac{(1-\psi)}{\beta}\phi_\pi+\frac{b(1-\psi)R_1\eta_b}{\beta}\phi_\pi-\frac{bR_1^2}{\beta\pi}(1-\psi)\eta_b\beta\eta_{cg}\end{aligned}$$

$$\alpha_{y2}=\frac{k\phi_\pi}{\beta}\left[1+\frac{1}{\beta}-\frac{\sigma S_c k(\phi_\pi-1)}{\beta}\right]+\frac{k}{\beta}\left(bR_1\eta_b-1-\frac{2b\pi\phi_\pi}{\beta}\right)$$

$$\alpha_{31}=-\frac{\sigma S_c(1-\phi_\pi)\alpha_{y2}+\alpha_{\pi 3}}{\beta}$$

Again, we observe that the structure of matrix Γ in (162) is upper-left triangular. We can then concentrate on the submatrix A_{11} given by:

$$A_{11}=\begin{bmatrix}\beta^{-1} & -k\beta^{-1}\\ -\sigma S_c\beta^{-1}(1-\phi_\pi) & 1-k\sigma S_c\beta^{-1}(1-\phi_\pi)\end{bmatrix} \tag{163}$$

To get determinacy for the full system we require that two eigenvalues of the system to be outside the unitary circle and one inside, since public debt b_{2t} is a predetermined variable. Trace and determinant of submatrix (163) are given, respectively, by:

$$\begin{aligned} \operatorname{tr}(A_{11}) &= 1+\frac{1}{\beta}-\frac{\sigma k S_c\,(1-\phi_\pi)}{\beta} && (164)\\ \det(A_{11}) &= \frac{1}{\beta} && (165) \end{aligned}$$

From the Schur-Cohn criterion, it is immediate to check that condition (115) is fully satisfied. Condition (117) implies:

$$\sigma k S_c\,(\phi_\pi-1)+2(1+\beta)>0 \tag{166}$$

which is satisfied if and only if $\phi_\pi > 1$. From (118), we obtain:

$$\sigma k S_c\,(1-\phi_\pi)>0 \tag{167}$$

which can be rewritten as:

$$-\sigma k S_c\,(\phi_\pi-1)<0 \tag{168}$$

which is satisfied if and only if $\phi_\pi > 1$, as well.
The remaining part of the proof follows exactly the same steps described for the current inflation targeting rule outlined earlier. *Q.E.D.*

C.5 Proof of proposition 5

Rule (77) requires a different setting for the analysis, given the time-indexing of the system. Therefore, let us define the vector $Z_t = [R_{1t}, \pi_t, y_t, b_{2t}]'$. Matrix Γ in this case is:

$$\Gamma = \begin{bmatrix} 0 & \phi_\pi & 0 & 0 \\ 0 & \frac{1}{\beta} & -\frac{k}{\beta} & 0 \\ \sigma S_c & -\frac{\sigma S_c}{\beta} & \left(1+\frac{k\sigma S_c}{\beta}\right) & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & \frac{1-\psi}{\beta} \end{bmatrix} \tag{169}$$

where:

$$\alpha_{\pi 4} = \frac{bR_1\eta_b}{\pi\beta}-\frac{1}{\beta}+\phi_\pi+\frac{(1-\psi)}{\beta}\left(b+\frac{1}{R_1}\right)-\frac{bR_1^2\,(1-\psi)}{\pi}\eta_b\eta_{cg}$$

$$\alpha_{y3} = \frac{k}{\beta}\left[\frac{(1-\psi)}{\beta}\left(b+\frac{1}{R_1}\right)-1+\frac{b\eta_b R_1}{\pi}\right]$$

$$\alpha_{R1} = \frac{\pi b\eta_b}{\beta}+\frac{bR_1}{\beta}$$

$$\alpha_R = \frac{(1-\psi)}{\beta}\left(1+\frac{bR_1\eta_b}{\pi}\right)$$

$$\alpha_{41} = \alpha_{y3}\sigma S_c - \alpha_R$$

$$\alpha_{42} = \phi_\pi \alpha_{R1} - \frac{1}{\beta}\left(\sigma S_c \alpha_{y3} + \alpha_{\pi 4}\right)$$

$$\alpha_{43} = \alpha_{y3} + \frac{k}{\beta}\left(\sigma S_c \alpha_{y3} + \alpha_{\pi 4}\right)$$

To get determinacy, we need two roots of the matrix (169) to be outside the unit circle and two inside. Given the upper block-triangular structure of the matrix Γ in (169), we can concentrate on the submatrix Γ_{11}, given by:

$$\Gamma_{11} = \begin{bmatrix} 0 & \phi_\pi & 0 \\ 0 & \frac{1}{\beta} & -\frac{k}{\beta} \\ \sigma S_c & -\frac{\sigma S_c}{\beta} & \left(1 + \frac{k\sigma S_c}{\beta}\right) \end{bmatrix} \tag{170}$$

We apply the Schur-Cohn criterion in order to detect the presence of two roots inside and one outside the unit circle for submatrix Γ_{11} in (170). In this case, $A_0 = \beta^{-1}$, $A_1 = \beta^{-1}$, $A_2 = -\left(1 + \beta^{-1} + \beta^{-1}\sigma k S_c\right)$. It it then immediate to check that by applying conditions (120)-(121) for Case 1 we obtain a contradiction. Therefore, from condition (122) of Case 2, we obtain (after simplifying):

$$\sigma S_c k\left(\phi_\pi - 1\right) > 0 \tag{171}$$

which is satisfied if and only if $\phi_\pi > 1$. On the other hand, from condition (123) we find:

$$-2\left(1+\beta\right) + \sigma S_c k\left(\phi_\pi - 1\right) < 0 \tag{172}$$

which is certainly satisfied if and only if:

$$\phi_\pi < 1 + \frac{2\left(1+\beta\right)}{\sigma S_c k} \tag{173}$$

Finally, in order to verify condition (124), after substituting out the definitions for A_0, A_1, A_2 given previously and simplifying, we need to check if the following inequality is satisfied:

$$\frac{\phi_\pi^2 k^2 \sigma^2 S_c^2}{\beta} + \phi_\pi k \sigma S_c\left(1 + \frac{1}{\beta} + \frac{\sigma S_c k}{\beta}\right) + (1-\beta) > 0 \tag{174}$$

which is certainly satisfied since $\beta < 1$. These conditions ensures we have two roots inside and one outside the unit circle. To get another root inside the unit circle, condition (159) needs to be satisfied. As we have seen before, this implies confition (79), stated in the text.

On the other hand, if condition $\phi_\pi > 1$ is not satisfied, then the same reasoning applied in the proof of Proposition 2 can be reapeted here without any further change.

Q.E.D.

C.6 Proof of proposition 6

With rule (86) matrix Γ of the system (74) can be written as:

$$\Gamma = \begin{bmatrix} \frac{1}{\beta} & -\frac{k}{\beta} & 0 \\ \sigma S_c\left(\phi_\pi - \frac{1}{\beta}\right) & \frac{k\sigma S_c}{\beta} + 1 + \sigma S_c \phi_y & 0 \\ \gamma_{31} & \gamma_{32} & \frac{(1-\psi)}{\beta} \end{bmatrix} \tag{175}$$

where:

$$\gamma_{y1} = \phi_y\left(1 + \frac{k\sigma S_c}{\beta} + \sigma S_c \phi_y\right) - \frac{b\pi\phi_y}{\beta}(1+\eta_b) + \frac{k}{\beta}(1 - bR_1\eta_b) - \frac{k\phi_\pi}{\beta}$$

$$\gamma_{y2} = \frac{(1-\psi)}{\beta}(1 + bR_1\eta_b)\phi_y$$

$$\begin{aligned}\gamma_{\pi 1} =& \frac{b\eta_b}{\beta}(R_1 - \pi\phi_\pi) + \frac{\phi_\pi}{\beta}(1 - b\pi) + \phi_y\sigma S_c\left(\phi_\pi - \frac{\sigma S_c}{\beta}\right)\frac{1}{\beta} + \\ &+ (1-\psi)R_1\left(\frac{1}{\beta}\left(b + \frac{1}{R_1}\right) - \frac{b}{\pi}\eta_b R_1 \eta_{cg}\right)\end{aligned}$$

$$\gamma_{\pi 2} = \frac{(1-\psi)}{\beta}(1 + bR_1\eta_b)\phi_\pi$$

$$\gamma_{31} = \frac{1}{\beta}(\sigma S_c \gamma_{y1} - \gamma_{\pi 1}) - \gamma_{y1}\sigma S_c \phi_\pi - \gamma_{\pi 2}$$

$$\gamma_{32} = -\frac{k}{\beta}(\sigma S_c \gamma_{y1} - \gamma_{\pi 1}) - \gamma_{y1}(1 + \sigma S_c \phi_y) - \gamma_{y2}$$

Even in this case, to get determinacy we need two roots of matrix (175) outside and one inside the unit circle. Given the upper-left triangular structure of the matrix Γ in (175), we can concentrate on the 2×2 submatrix G_{11}, here given by:

$$G_{11} = \begin{bmatrix} \frac{1}{\beta} & -\frac{k}{\beta} \\ \sigma S_c\left(\phi_\pi - \frac{1}{\beta}\right) & \left(\frac{k\sigma S_c}{\beta} + 1 + \sigma S_c\phi_y\right) \end{bmatrix} \tag{176}$$

Therefore, trace and determinant of matrix (176) are, respectively, given by:

$$\operatorname{tr}(G_{11}) = 1 + \frac{1}{\beta} + \frac{\sigma k S_c}{\beta} + \sigma k S_c \phi_y \tag{177}$$

$$\det(G_{11}) = \frac{1 + \sigma S_c(\phi_y - k\phi_\pi)}{\beta} \tag{178}$$

From Schur-Cohn criterion, condition (115) can be split in two parts: i) $det > 1$ is immediately satisfied, given the assumption $\phi_\pi, \phi_y > 0$; ii) $det > -1$, identifies the following bound:

$$k\phi_\pi + \phi_y > \frac{1-\beta}{\sigma S_c \beta} \tag{179}$$

Condition (117) is immediately satisfied, given that ϕ_π, ϕ_y are assumed to be positive. On the other hand, condition (118) directly implies:

$$k\left(\phi_\pi - 1\right) + \phi_y\left(1-\beta\right) > 0 \tag{180}$$

With conditions (179)) and (180) we have that one root of submatrix G_{11} in (176) will be inside and another root will be outside the unit circle. To get determinacy for the full system subsumed by matrix (175) we need another root to be inside the unit circle. This is obtained by considering condition (159): this implies condition (89), stated in the text. However, if conditions (179)-(180) are violated, then the same reasoning applied in the proof of Proposition 2 applies here.
Q.E.D.

C.7 Proof of proposition 7

With rule (92) matrix Γ of system (74) is now given by:

$$\Gamma = \begin{bmatrix} \frac{1}{\beta} & -\frac{k}{\beta} & 0 \\ \frac{\sigma S_c(1-\phi_\pi)}{\beta} & \frac{k\sigma S_c}{\beta} + 1 + \sigma S_c \phi_y & 0 \\ \lambda_{31} & \lambda_{32} & \frac{(1-\psi)}{\beta} \end{bmatrix} \tag{181}$$

where:

$$\lambda_\pi = \frac{\phi_\pi}{\beta^2}\left[1 + k\sigma S_c\left(1-\phi_\pi\right)\right] + + \frac{bR_1\left(1-\psi\right)}{\beta\pi}\eta_b\phi_\pi - \frac{\phi_y\sigma S_c\left(1-\phi_\pi\right)}{\beta} - \frac{bR_1^2\left(1-\psi\right)\eta_b\eta_{cg}}{\pi} + \\ + \frac{b\eta_b R_1}{\beta\pi} - \frac{b\pi\eta_b}{\beta^2}\phi_\pi - \frac{bR_1}{\beta^2}\phi_\pi - \frac{1}{\beta} + \frac{(1-\psi)}{\beta^2}\left(b + \frac{1}{R_1}\right) + \frac{(1-\psi)}{\beta^2}$$

$$\lambda_{y1} = \frac{\pi b\eta_b\phi_y}{\beta} + \frac{bR_1\phi_y}{\beta} + \frac{(1-\psi)}{\beta}\left(\phi_y + k\phi_\pi\right) - \frac{k\pi b\eta_b\phi_\pi}{\beta^2} + \frac{k\pi b\eta_b\phi_\pi}{\beta\pi} - \frac{kbR_1\phi_\pi}{\beta^2} - \\ + \frac{k\left(1-\pi\right)}{\beta}\left(b + \frac{1}{R_1}\right)\frac{k\phi_\pi}{\beta^2} + \left(\frac{k\phi_\pi}{\beta} - \phi_y\right)\left[1 + \sigma S_c\phi_y + \frac{k\sigma S_c\left(1-\phi_\pi\right)}{\beta}\right]$$

$$\lambda_y = \frac{bR_1\left(1-\psi\right)\eta_b\phi_y}{\beta\pi}$$

$$\lambda_{31} = -\frac{\sigma S_c\left(1-\phi_\pi\right)\lambda_{y1} + \lambda_\pi}{\beta}$$

To get determinacy, we need two roots of matrix (181) to be outside and one inside the unit circle. Once again, given the upper-left triangular structure of (181) we can concentrate on the 2×2 submatrix H_{11}, here given by:

$$H_{11} = \begin{bmatrix} \frac{1}{\beta} & -\frac{k}{\beta} \\ \frac{\sigma S_c(1-\phi_\pi)}{\beta} & \left(\frac{k\sigma S_c}{\beta} + 1 + \sigma S_c\phi_y\right) \end{bmatrix} \tag{182}$$

Trace and determinant of H_{11} in (182) are, respectively, given by:

$$\begin{aligned} \operatorname{tr}(H_{11}) &= 1+\frac{1}{\beta}+\frac{\sigma k S_c(1-\phi_\pi)}{\beta}+\sigma S_c\phi_y && (183) \\ \det(H_{11}) &= \frac{1+\sigma S_c\phi_y}{\beta} && (184) \end{aligned}$$

Condition (115) is immediately satisfied, given that $\phi_\pi, \phi_y > 0$. Condition (117) implies the following upper bound:

$$\phi_\pi = 1+\frac{2(1+\beta)}{k\sigma S_c}+\phi_y\frac{(1+\beta)}{k} \tag{185}$$

On the other hand, condition (118) implies:

$$k(\phi_\pi-1)+\phi_y(1-\beta)>0 \tag{186}$$

which is certainly satisfied if $\phi_\pi > 1$. With conditions (185)) and (186), one root of submatrix H_{11} in (186) will be inside and another root outside the unit circle. To get determinacy for the full system we need the additional conditions on fiscal policy, which will capture the position of the third root. Implementing condition (159) will imply (94), stated in the text.

When one of (185) or (186), or both, are violated, then the same reasoning applied in the proof of Propositions 2-4 applies here, originating bounds (95).

Q.E.D.

C.8 Proof of proposition 8

With rule (42) matrix Γ of system (XX) is now given by:

$$\Delta = \begin{bmatrix} \delta_{11} & \delta_{12} & -\delta_{13} & 0 \\ 0 & \frac{1}{\beta} & -\frac{k}{\beta} & 0 \\ \sigma S_c & \frac{\sigma S_c}{\beta} & \left(1+\frac{k\sigma S_c}{\beta}\right) & 0 \\ -\delta_{41} & \delta_{42} & \delta_{43} & \frac{(1-\psi)}{\beta} \end{bmatrix} \tag{187}$$

where vector Z_t is given by as $Z_t = [R_{1t}, \pi_t, y_t, b_{2t}]'$. where:

$$\begin{aligned}
\delta_{11} &= \rho + \phi_y \sigma S_c \\
\delta_{12} &= \frac{\phi_\pi - \phi_y \sigma S_c}{\beta} \\
\delta_{13} &= \frac{k}{\beta}\left(\phi_\pi - \phi_y \sigma S_c\right) + \phi_y \\
\delta_{41} &= \left[\phi_y \sigma S_c + \rho - \frac{b}{\beta}\left(\pi \eta_b + R_1\right)\right]\left(\rho + \phi_y \sigma S_c\right) + \\
&\quad + \delta_y \sigma S_c + \frac{(1-\psi)}{\beta}\left(1 + \frac{b R_1}{\pi}\eta_b\right) \\
\delta_{42} &= \frac{1}{\beta}\left[\sigma S_c \delta_y - \left(\phi_\pi - \phi_y \sigma S_c\right)\delta_{\rho 1} - \delta_{\pi 1}\right] \\
\delta_y &= \phi_y - \frac{k}{\beta}\left[\frac{b \eta_b R_1}{\pi} - 1 + \frac{(1-\psi)}{\beta}\left(b + \frac{1}{R_1}\right) - \phi_y \sigma S_c\right] \\
\delta_{\rho 1} &= \rho + \phi_y \sigma S_c - \frac{b}{\beta}\left(\pi \eta_b + R_1\right) \\
\delta_{\pi 1} &= \frac{1}{\beta}\left[\frac{b \eta_b R_1}{\pi} - 1 + \frac{(1-\psi)}{\beta}\left(b + \frac{1}{R_1}\right) - \phi_y \sigma S_c\right] - \frac{b R_1^2 (1-\psi)\eta_b \eta_{cg}}{\pi}
\end{aligned}$$

Matrix (187) is upper-left triangular. In this case, to get determinacy we need two roots inside and two outside the unit circle, since both R_{1t} and b_{2t} are predetermined. We can start by focusing on the 3×3 submatrix D_{11} given by:

$$D_{11} = \begin{bmatrix} \delta_{11} & \delta_{12} & -\delta_{13} \\ 0 & \frac{1}{\beta} & -\frac{k}{\beta} \\ \sigma S_c & \frac{\sigma S_c}{\beta} & \left(1 + \frac{k\sigma S_c}{\beta}\right) \end{bmatrix} \tag{188}$$

By applying apply now the Schur-Cohn criterion for 3×3 matrix, from (119), we have:

$$A_0 = -\frac{\rho}{\beta} \tag{189}$$

$$A_1 = \frac{1}{\beta} + \rho + \frac{\rho}{\beta} + \rho\frac{k\sigma S_c}{\beta} + \frac{\phi_y \sigma S_c}{\beta} + \phi_\pi \frac{k\sigma S_c}{\beta} \tag{190}$$

$$A_2 = \rho + \phi_y \sigma S_c + \frac{1}{\beta} + 1 + \frac{k\sigma S_c}{\beta} \tag{191}$$

Let us start with Case I. From condition (120), we obtain: $k\left(\phi_\pi + \rho - 1\right) + (1-\beta)\phi_y < 0$. Moreover, from condition ((121), we obtain a contradiction, given our assumptions on parameters' sign. Consider now Case II. Condition (122) implies:

$$k\left(\phi_\pi + \rho - 1\right) + (1-\beta)\phi_y > 0 \tag{192}$$

which is satisfied if $\phi_\pi > 1$condition (123) is immediately satisfied. By applying condition (124), we obtain:

$$\begin{aligned}&\frac{\rho^2}{\beta^2} - \frac{\rho}{\beta}\left(\rho + \phi_y \sigma S_c + \frac{1}{\beta} + 1 + \frac{k\sigma S_c}{\beta}\right) + \frac{\rho}{\beta} + \rho + \\ &\qquad + \frac{1}{\beta} + \frac{k\sigma S_c}{\beta}(\phi_\pi + \rho) + \phi_y \frac{\sigma S_c}{\beta} - 1 > 0\end{aligned} \tag{193}$$

After adding and subtracting $\frac{k\sigma S_c}{\beta}$ to (193) and rearrange, we obtain:

$$\begin{aligned}&\phi_y \frac{\sigma S_c}{\beta}(1-\rho) + \frac{k\sigma S_c}{\beta}(\phi_\pi + \rho - 1) + \frac{k\sigma S_c}{\beta}\frac{(\beta - \rho)}{\beta} + \\ &\qquad + \left[\frac{\rho^2}{\beta^2} - \frac{\rho^2}{\beta} + \rho - 1 + \frac{1}{\beta} - \frac{\rho}{\beta^2}\right] > 0\end{aligned} \tag{194}$$

Adding and subtracting $\frac{\rho}{\beta}$ to the term in square bracket of (194) and rearrange, we obtain that the inequality in (194) can be satisfied if and only if:

$$\left(1 - \frac{\rho}{\beta}\right)(1-\rho)\frac{(1-\beta)}{\beta} > 0 \tag{195}$$

which is satisfied if and only if $\rho < \beta$, as stated in the text, since $\beta < 1$, by assumption. When these conditions are satisfied, one root will be inside and two outside the unit circle. To get determinacy for the system captured by matrix (187) we need another root inside the unit circle, which is obtained by setting:

$$\frac{|1-\psi|}{\beta} < 1 \tag{196}$$

which, after taking advantage of the absolute value properties, delivers the result stated in (98).
When condition (96) or (97), or both, are not satisfied we require the following condition on fiscal policy, such that:

$$\frac{|1-\psi|}{\beta} > 1 \tag{197}$$

which implies condition (99) stated in the text.
Q.E.D.

C.9 Proof of proposition 9

As in previous case, we can concentrate our attention on the 2×2 submatrix given by:

$$\Gamma_{11} = \begin{bmatrix} \frac{(1+\eta_{RS}\phi_\pi)}{\beta} & -\frac{(k-\eta_{RS}\phi_y)}{\beta} \\ \varphi_1 & \varphi_2 \end{bmatrix} \tag{198}$$

The determinant is given by:

$$\det\left(\Gamma_{11}\right)=\frac{1-\alpha_R\sigma S_c\phi_y+\phi_\pi\eta_{RS}\left(1-k\alpha_R S_c\right)}{\beta\left(1-\alpha_{R1}\sigma S_c\phi_y\right)} \tag{199}$$

The trace is:

$$\operatorname{tr}\left(\Gamma_{11}\right)=\frac{1+\eta_{RS}\phi_\pi}{\beta}+\frac{\sigma S_c\left(k-\eta_{RS}\phi_y\right)\left(1-\alpha_R\phi_\pi\right)+\beta-\beta\alpha_R\sigma S_c\phi_y}{\beta\left(1-\sigma S_c\alpha_{R1}\phi_y\right)} \tag{200}$$

Condition $\det>1$ from (115) implies :

$$1-\alpha_R\sigma S_c\phi_y+\phi_\pi\eta_{RS}\left(1-k\alpha_R S_c\right)>\beta\left(1-\alpha_{R1}\sigma S_c\phi_y\right) \tag{201}$$

which, after rearrangement becomes:

$$\phi_\pi>\frac{\phi_y\sigma S_c\left(\alpha_R-\beta\alpha_{R1}\right)-(1-\beta)}{\eta_{RS}\left(1-\alpha_R k S_c\right)}\equiv\bar{\phi}_{\pi 1} \tag{202}$$

On the other hand, condition $\det>-1$, is satisfied if and only if:

$$\phi_\pi>\frac{\sigma S_c\phi_y\left(\alpha_R+\beta\alpha_{R1}\right)-(1+\beta)}{\eta_{RS}\left(1-kS_c\alpha_R\right)}\equiv\bar{\phi}_{\pi 2} \tag{203}$$

Bounds $\bar{\phi}_{\pi 1}$ and $\bar{\phi}_{\pi 2}$ previously defined are both upper bounds. To establish which of the two bounds in (202)-(203) are binding, let us verify if $\bar{\phi}_{\pi 2}>\bar{\phi}_{\pi 1}$. This condition is verified if and only if:

$$\sigma S_c\phi_y\left(\alpha_R+\beta\alpha_{R1}\right)-(1+\beta)>\phi_y\sigma S_c\left(\alpha_R-\beta\alpha_{R1}\right)-(1-\beta) \tag{204}$$

which is verified if:

$$\phi_y>\frac{1}{\sigma S_c\alpha_{R1}} \tag{205}$$

Therefore, if condition (205) is verified, bound $\bar{\phi}_{\pi 2}$ given in (203) applies.
Moreover condition (118) is satisfied if and only if:

$$\begin{gathered}\sigma S_c k+2(1+\beta)-\beta\sigma S_c\phi_y\left(\alpha_R+\alpha_{R1}\right)-\sigma S_c\left(\alpha_R+\alpha_{R1}\right)\phi_y-\sigma S_c\phi_y\eta_{RS}> \\ \phi_\pi\left(\sigma S_c\alpha_R k+\eta_{RS}k\alpha_R S_c-2\eta_{RS}+\sigma S_c\phi_y\eta_{RS}\right)\end{gathered} \tag{206}$$

which, after rearrangement, becomes:

$$\phi_\pi<\frac{\sigma S_c k+2(1+\beta)-\sigma S_c\phi_y\left(\alpha_R+\alpha_{R1}\right)(1+\beta)-\sigma S_c\phi_y}{\sigma S_c\alpha_R k+\eta_{RS}k\alpha_R S_c-2\eta_{RS}+\sigma S_c\phi_y\eta_{RS}} \tag{207}$$

As ancillary result, it is not difficult to prove that $\left(\alpha_{R1}-\alpha_R\right)=1$. From condition (117), we obtain:

$$\phi_\pi>\frac{(1-\beta)\sigma S_c\phi_y+\eta_{RS}\sigma S_c\phi_y-\sigma S_c k}{\eta_{RS}\sigma S_c\phi_y+k\alpha_R\left(\sigma S_c-\eta_{RS}\right)}\equiv\bar{\phi}_{\pi 3} \tag{208}$$

Therefore, by reshuffling the above conditions, we can show that determinacy obtains if and only if:

$$argmax\left\{\bar{\phi}_{\pi 2}, \bar{\phi}_{\pi 3}\right\} \quad < \quad \phi_\pi < \frac{k - \eta_{RS}\phi_y}{k\alpha_R\left(1+\eta_{RS}\right) + \eta_{RS}\phi_y} \tag{209}$$

$$\frac{1}{\sigma S_c \alpha_{R1}} \quad < \quad \phi_y \tag{210}$$

Conditions (209)-(210) identify the presence of one root inside and another outside the unit circle for submatrix (198). To check determinacy for the whole system, we need an additional conditions on the fiscal policy side in order to have an additional roots inside the unit circle. This is obtained by setting:

$$\frac{|1-\psi|}{\beta} < 1 \tag{211}$$

which is equivalent to state:

$$1 - \beta < \psi < 1 + \beta \tag{212}$$

Alternatively, if one or all of (209)-(210) do not hold, conditions (211) should be replaced by:

$$\frac{|1-\psi|}{\beta} > 1 \tag{213}$$

Or, equivalently:

$$1 - \beta < \psi; \qquad \psi > 1 + \beta \tag{214}$$

Q.E.D.

References

Andres, J., J. D. Lopez-Salido, and E. Nelson (2004): "Tobin's Imperfect Asset Substitution in Optimizing General Equilibrium," *Journal of Money, Credit, and Banking*, 36(4), 665–690.

Atkeson, A., and P. J. Kehoe (2008): "On the Need for a New Approach to Analyzing Monetary Policy," Federal Reserve Bank of Minneapolis Research Department Staff Report No. 412.

Bullard, J., and K. Mitra (2002): "Learning about Monetary Policy Rules," *Journal of Monetary Economics*, 49(5), 1105–1129.

Calvo, G. (1983): "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics*, 12, 383–398.

Canzoneri, M. B., R. Cumby, B. T. Diba, and D. López-Salido (2011): "The Role of Liquid Government Bonds in the Great Transformation of US Monetary Policy," *Journal of Economics, Dynamics and Control*, 35(2), 282–94.

Canzoneri, M. D., and B. T. Diba (2005): "Interest Rate Rules and Price Determinacy: the Role of Transaction Services of Bonds," *Journal of Monetary Economics*, 52(2), 329–343.

Dixit, Avinash, K., and J. Stiglitz (1977): "Monopolistic Competition and Optimum Product Diversity," *American Economic Review*, 67, 297–308.

Galí, J. (2008): *Monetary Policy, Inflation and the Business Cycle*. Princeton University Press.

Harrison, R. (2012): "Asset Purchase Policy at the Effective Lower Bound for Interest Rates," Working Paper 444, Bank of England.

La Salle, J. P. (1986): *The Stability and Control of Discrete Processes*. Springer Verlag, New-York and Berlin.

Leeper, E., M. (1991): "Equilibria Under Active and Passive Fiscal and Monetary Policies," *Journal of Monetary Economics*, 27(2), 129–147.

Ljungqvist, L., and T. J. Sargent (2007): *Recursive Macroeconomic Theory*. MIT Press.

Lubik, T. A., and M. Marzo (2007): "An Inventory of Simple Monetary Policy Rules in a New Keynesian Macroeconomic Model," *International Review of Economics and Finance*, 16, 15–36.

Marzo, M., and P. Zagaglia (2008): "Determinacy of interest rate rules with bond transaction services in a cashless economy," Working Paper 24-08, Bank of Finland.

McCough, B., G. D. Rudebusch, and J. Williams (2005): "Using a Long-Term Interest Rate as the Monetary Policy Instrument," *Journal of Monetary Economics*, 52(5), 855–879.

SARGENT, T. (1979): *Macroeconomic Theory*. Academic Press.

SCHMITT-GROHÉ, S., AND M. URIBE (2005): "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model: Expanded Version," NBER Working Paper No. 11417.

——— (2007): "Optimal, Simple and Implementable Monetary and Fiscal Rules," *Journal of Monetary Economics*, 54(6), 1702–1725.

SIMS, C., A. (1994): "A Simple Model to Study the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381–399.

SVENSSON, L. (2003): "What is wrong with Taylor Rules ? Using Judgment in Monetary Policy through Targeting Rule," *Journal Economic Literature*, 41(3), 426–477.

TAYLOR, JOHN, B. (1993): "Discretion versus Policy Rules in Practice," *Carnegie-Rochester Conference Series on Public Policy*, 39, 195–214.

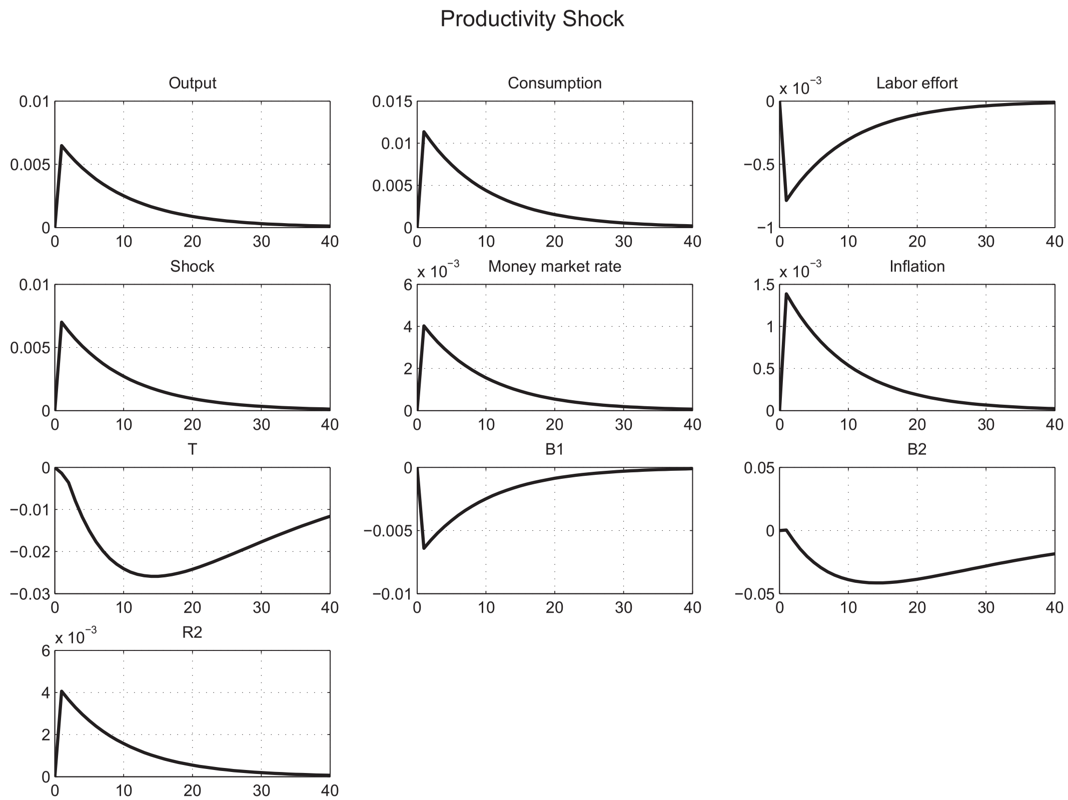
TOBIN, J. (1958): "Liquidity Preference as Behavior Towards Risk," *Review of Economic Studies*, 25(2), 65–86.

WOODFORD, M. (2003): *Interest and Prices*. Princeton University Press.

Table 1: Parameter Values

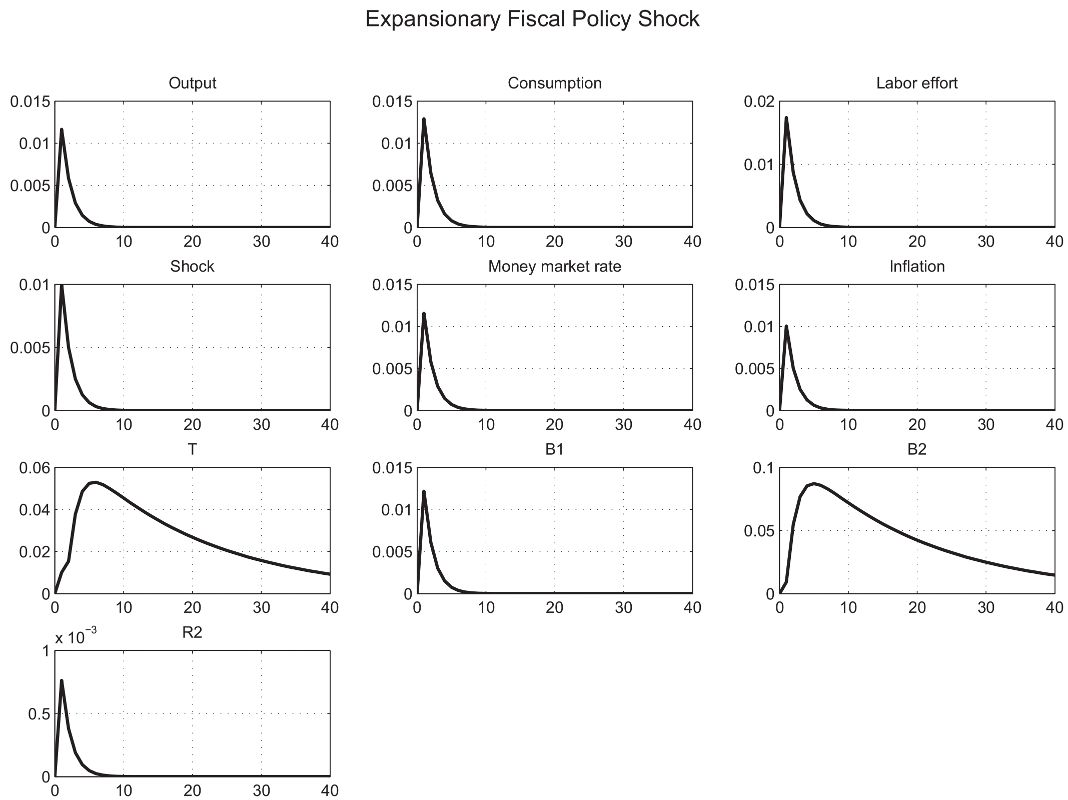
Parameter	Value
β	0.99
σ	0.5
η	1
α	0.67
δ	2/3
θ	6
ψ	0.05
ρ_A	0.9
ρ_G	0.5
σ_A	0.007
σ_G	0.01

Figure 1: Impulse responses: technology shock



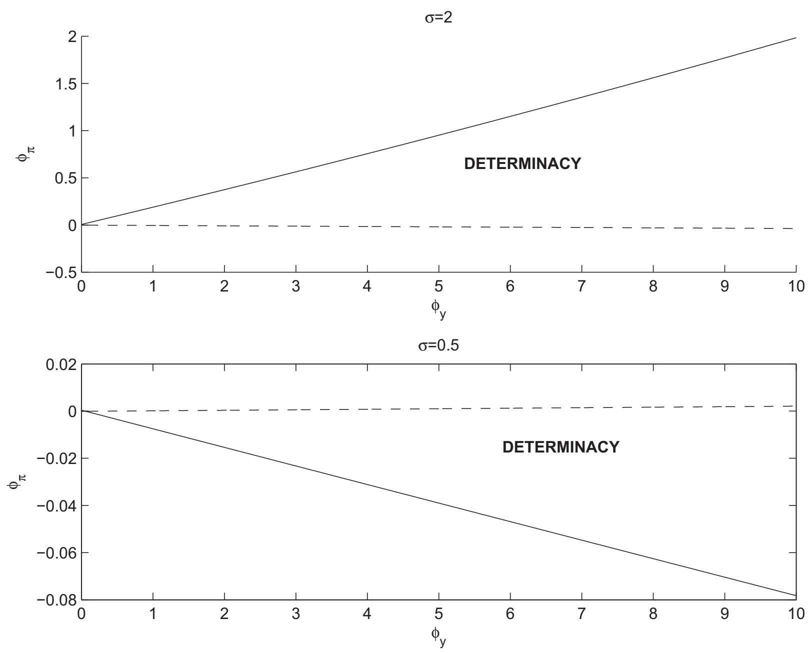
Response of each variable of the system after a one percentage standard deviation technology shock

Figure 2: Impulse responses: fiscal policy shock



Response of each variable of the system after a one percentage standard deviation fiscal policy shock

Figure 3: Determinacy regions for model with weakly separable utility



Both panels represent the simulations relative to bounds established by condition (111) by varying paramter ϕ_y. The top panel is obtained by setting $\sigma = 2$, while bottom panel is obtained with $\sigma = 0.5$.

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