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Abstract

We study the underground economy in a dynamic and stochastic general equilibrium framework. Our model combines limited tax enforcement with an otherwise standard two-sector neoclassical stochastic growth model. The Bayesian estimation of the model based on Italian data provides evidence in favor of an important underground sector in Italy, with a size that has steadily increased over the whole sample period. We show that this pattern is due to a persistent increase in taxation. Fiscal policy experiments suggest that a moderate tax cut, along with a stronger effort in the monitoring process, causes a sensitive reduction in the size of the underground economy and positive stimulus to the regular sector that jointly increase the total fiscal revenues.

Keywords: DSGE, Underground Economy, Tax Evasion, Bayesian Estimation, Italy
JEL: E65, O41, O52

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1. Introduction

The underground economy represents a major issue when studying an economic system because of its large impact on public finances and of its distorting effects on production through the unfair competition among firms. Furthermore, the social costs can increase because the overall tax burden becomes shared among a smaller number of citizens, thus increasing economic

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inequality, and because of the lack of labor protection for the individuals who are working in the underground market. Therefore, in studying the underground economy, the analysis of the conveniences that occur in an irregular mode of production involving the complicity of the workers themselves then takes on considerable significance. These situations allow business activities to be established in conditions that lead people to accept lower incomes and fewer guarantees in the workplace, making the birth and development of productive initiatives possible with a very low investment.

The available empirical evidence shows that the irregular economy is a relevant issue for most countries, although to different extents. In fact, the underground economy is universally widespread; it is present in developing as well as in advanced economies. In particular, there exists evidence of a growing trend for irregular economies due to the combined effects of international competition and the high fragmentation of working organizations (see e.g. Schneider et al., 2010). Furthermore, legal activities conducted underground to escape taxation appear to be the faster growing component of the irregular economy, largely because of the structure of the tax systems.

Because of its latent nature, the underground economy is difficult to measure and study empirically and, even if the law enforcement and taxation officials readily admit that the underground economy is a widespread phenomenon, it is difficult to agree on its size. Examining the literature on the underground economy, we recognize that there has been a good deal of progress in ascertaining the data and developing techniques to quantify its size and importance, even if the discussion regarding the most appropriate methodology to quantify this phenomena is still ongoing.

Taking into account the aspects mentioned above, in this paper we tackle the issue of measuring the underground economy using a structural econometric approach, which exploits equilibrium conditions from an economic model to provide estimates for unobservable variables. We follow the idea that unobserved data may be derived from a well-behaved theoretical model by using the so called theory for measurement approach, as implemented for the first time by Ingram et al. (1997). To this end, we build and estimate, using a Bayesian approach, a dynamics stochastic general equilibrium (DSGE) model that explicitly accounts for concealed transactions. The inferential procedure that we use, which is based on the Markov chain Monte Carlo methods (MCMC), allows us to estimate the dynamics of the unobservable underground economy together with the parameters of the model. This approach strongly departs from the classical methodologies proposed in literature for studying underground economy, providing, in our opinion, two main advantages. First, because it is theory-based, we believe that the DSGE methodology provides a deeper understanding of the causes of the underground economy. Second, the estimated model allows to assess from a general equilibrium perspective fiscal policy implications of the underground economy.

The model we propose combines incomplete tax enforcement à la Allingham and Sadmo (1974) with an otherwise standard two-sector neoclassical stochastic growth model. In this environment, the underground economy emerges as a result of the agents' incentives to conceal their transactions in order to escape taxation. This structure is adapted from Busato and Chiarini (2004) who are among the first to include the underground economy in a DSGE model. Our model differs from their in three main aspects. First, we allow for labor adjustments along the intensive margin. This property is important in order to relax sign restrictions on the co-movements among variables that are implied by Busato and Chiarini model. Second, we consider a richer set of exogenous shocks, which are necessary to take the model to the data. More precisely, in addition to fiscal and sectorial-specific technological shocks, we also consider preference and investment-specific shocks. These additional shocks have been proved to be important to explain the variability of data at the business cycle frequencies (see e.g. Smets and Wouters, 2007; Justiniano et al., 2010). Third, our model features a deterministic growth rate driven by labor-augmenting technological progress. Because of this property, the data do not need to be detrended before estimation.

We estimate the model using Italian quarterly data in the interval from 1982:Q1 to 2006:Q4. Italy is an interesting case study because, according to the available evidence, the relevance of the underground economy appears to be larger in this country with respect to other developed countries (see Schneider et al., 2010). Furthermore, the recent severe sovereign debt crisis in Italy requires policy makers to propose effective policies to fight against tax evasion.[1] The results of the Bayesian estimation provide evidence in favor of a sizeable underground sector in Italy, which on average accounts for 23% of GDP. This number is about 4 percentage points larger than the official estimates. According to our results, the size of the underground economy

[1]It should be stressed, however, that our method is general enough to be easily adapted to other countries.

has steadily increased over the whole sample period, and this pattern is primarily explainable by the persistent increase in taxation that occurred in Italy since the eighties. Moreover, we find that the cyclical component of the underground economy is negatively correlated with the cyclical component of the official output, thus providing evidence of a *double business cycle* in the Italian economy. The estimated model is then used to assess the implications of alternative fiscal policies. We find that at the actual tax rates, Italy is on the *slippery* side of the steady-state Laffer curve, and can improve its budgetary situation by either reducing taxes or increasing the tax enforcement. However, the analysis of transitional dynamics reveals that for the government would be optimal to undertake a mix between the above two policies as, according to our findings, this turns out to be the only fiscal adjustment that can improve welfare and, at the same time, permanently increase fiscal revenues.

This paper is organized as follows: Section 2 defines what is meant by underground economy and reviews the literature, emphasizing the methodologies that have been implemented to measure this phenomenon. The DSGE model is presented in Section 3, whereas the inferential methodologies and the description of the data are included in Section 4. The empirical results are provided in Section 5, and the policy implications are analyzed in Section 6. Finally, Section 7 concludes the paper.

2. Current approaches in estimating the underground economy

Information about underground economic activities, their magnitude and the manner in which these actions occur are difficult to obtain because they do not belong to the official economy, and the individuals involved do not want to be identified. Thus, the only possible method for quantifying the size of the underground economy is through estimation. Many attempts have been made, and several different methods are employed for this purpose, as demonstrated by a vast literature (Schneider, 2005 and Schneider and Enste, 2002, among others), even if disagreements still exist regarding the definition as well as the *best* approach for estimating the underground economy.

At this purpose, OECD addressed this question almost a decade ago (OECD, 2002), proposing definitions and harmonization procedures to integrate the underground economy into the gross national product (GNP). The objective was to provide an indirect measurement of the economic activities that are not included in the official statistics but are still relevant to the economy of a country. Furthermore, starting from the 1990s, the statistical offices of the OECD countries adopted the international definition established according to the SNA93 and SEC95 accounting systems, which represent a reference for the national accounts estimates and guarantee homogeneity in the statistical evaluation of GDP figures. To provide a definition that makes the concept of underground economy comparable and fairly uniform for the countries belonging to the European community, the European Union's statistical office (Eurostat) has provided details on how to account for the *non observed economy* and monitors compliance with these directives in their definition of the national accounts of the member countries.

In particular, *non (directly) observed economy* is composed as follows:

- The *underground economy*, which regards legal production that is not official and is not recorded because of tax and contributory evasion, tax labor regulations evasion and the non-observance of administrative rules.

- The *informal economy*, which includes all legal activities carried out by individuals, small or home enterprises (part-time secondary work, moonlighting, baby-sitting and so on) and goods and services produced and consumed within the household. For these activities, it is very difficult, or even impossible, to rely on statistical observation and measurement, even if they are not directed toward tax evasion. Thus, they are not included in the genuine underground economy, as defined above.

- The *illegal economy or criminal economy*, which includes all economic activities that violate penal norms, such as the illegal drug business, prostitution, and other criminal activities.

Throughout the paper, we refer to the underground economy as the production of legal commodities and services that are deliberately concealed from the public authorities to avoid the payment of taxes or social security contributions. This definition therefore does not include the informal and criminal activities since they do not lead to tax evasion. In other words, in our definition, the irregularities that are related to the underground economy rely on the ways that regular economic activities are conducted.

The taxonomy of the methods for estimating the underground economy most frequently encountered in the literature distinguishes between direct methods and indirect methods. Direct approaches are implemented essentially at a microeconomic level, and are mainly based on surveys of households and businesses, or on data generated by the tax supervision. Indirect methods infer the size of the underground economy by comparing macroeconomic indicators. For example, they compare the income produced annually with the income annually used for consumption, investment and savings; or alternatively they compare electric input power with industrial output, or even compare the actual currency demand with the demand for currency in the absence of taxation (Cagan, 1958; Tanzi, 1980, 1983; Gutmann, 1977).

A third methodology is model based, in which the underground economy is treated as a latent variable. This is known as *multiple indicator multiple cause* (MIMIC) which is based on a factor model (see for instance Frey and Weck-Hannemann, 1984 and Giles, 1999). This approach has an attractiveness in the context of the underground analysis, since the idea is to represent the output of the underground economy as a latent variable or index, which has causes and effects that are observable but which cannot itself be directly measured, The observed variables in the model are classified in *causal* and *indicators*, which are connected by a single unobserved index. Values of the index over time are inferred from data on causes and indicators by estimating the statistical model and predicting the index. The fitted index is then interpreted as a time series estimate of the underground economy.

The approach that we propose in this paper does not belong specifically to any of these categories, and thus we name it as *structural model approach.* In fact, we build and estimate a DSGE model with irregular transactions and we get the underground economy in the form of latent variable.

3. Model

We consider an economy that consists of a continuum of homogenous goods that are indexed by $i \in [0, 1]$, each produced by a perfectly competitive producer. Goods are sold by firms to a continuum of measure 1 of identical households for consumption and investment purposes and to the government, which collects taxes from households and firms to finance public spending. The economy is divided into a regular and an unofficial sector, and none of the transactions that occur in the latter are recorded by the government authorities. Firms therefore use factors from the underground markets to hide part of their production to evade taxation. In each period of time, however, firms face a non-negligible probability of being inspected by the fiscal authorities, convicted of tax evasion and forced to pay taxes that are augmented by a penalty surcharge. Households might also evade personal income taxation by reallocating their labor services from the regular to the underground sector. All of the interactions between firms, households and the government occur in a stochastic environment where the short-run dynamics of the economy are driven by productivity, demand, and fiscal shocks.

3.1. Firms

Each firm i uses regular labor $h^m_{i,t}$, and capital $k_{i,t}$ to produce regular output via a Cobb-Douglas production function

$$y^m_{i,t} = A_t \left(\Gamma_t h^m_{i,t}\right)^{\alpha} \left(k_{i,t}\right)^{1-\alpha} \tag{1}$$

where $\alpha \in (0,1)$, A_t is a purely transitory technological shock,[2] while Γ_t is the labor augmenting technological progress, which follows a deterministic trend of the form $\Gamma_t = \gamma\Gamma_{t-1}$ with $\gamma > 1$. Every unit of output produced is taxed at the stochastic corporate tax rate, $\tau^c_t < 1$,[3] but compliance is only partial, and firms can hide part of their production to evade taxes. To produce underground output, firms combine labor hired in the unofficial market, $h^u_{i,t}$, with capital via the following Cobb-Douglass technology:

$$y^u_{i,t} = B_t \left(\Gamma_t h^u_{i,t}\right)^{\alpha_u} \left(k_{i,t}\right)^{1-\alpha_u} \tag{2}$$

where B_t is a purely transitory technological shock, and $\alpha_u \in (0,1]$.

The assumption of sector-specific technological shocks incorporates potentially important inter-sectorial differences in labor productivity into the model.[4] This property is consistent with the available empirical evidence that documents a clear association between the level of education and participation in the irregular labor market (see e.g. Marcelli et al., 1999; Gallaway

[2]The process that governs the evolution of stochastic shocks will be introduced shortly.

[3]To keep the analysis relatively simple, our model abstracts from the endogenous fiscal policies that determine the evolution of the marginal tax rates. We thus treated the latter as aggregate stochastic disturbances.

[4]An alternative way to introduce this property into the model is to assume that the goods that are produced in the two sectors are manufactured with the same technology but that firms have to pay an additional cost for every unit of underground output produced. We prefer to rely on sector-specific production functions to take into account the labor-intensive nature of underground production activities.

and Bernasek, 2002).[5] The idiosyncratic shock B_t can also be interpreted as capturing exogenous changes in the overall labor force that primarily affect irregular workers productivity. For example, several empirical papers have documented that most workers hired under irregular work arrangements are immigrants (see e.g. Leonard, 1998). As noted by Busato and Chiarini (2004), it is reasonable to believe that these individuals have strong incentives to be very productive to increase the probability of being hired as regular workers.[6] An increase in legal immigration might therefore result in a temporary boost to the underground sector's productivity.[7]

Let $p_{i,t}^m$ and $p_{i,t}^u$ denote the price of the $i-th$ good in the regular and unofficial markets, respectively. Following Busato and Chiarini (2004), we assume that a good that is produced in the underground sector is indistinguishable from the regular goods, and therefore, at equilibrium, their prices must be the same.[8] Thus, without loss of generality, we will impose hereafter that $p_{i,t}^m = p_{i,t}^u = P_t \ \forall \ i \in [0,1]$, where P_t is the market price of each good that the perfectly competitive firms take as given. Because the goods that are produced in the two markets are homogenous, the total final output produced by a firm i at date t, namely $y_{i,t}$, can be simply defined as

$$y_{i,t} = y_{i,t}^m + y_{i,t}^u \tag{3}$$

According to equation (3), a firm is always allowed to produce a total

[5]These papers show that in urban settings, highly skilled individuals are more likely to work in the regular sector, whereas those with the lowest level of education have a higher probability of working in the irregular sector.

[6]In most of the Western European countries, for example, immigrants enter with temporary visas that are converted to permanent visas when they become regular workers.

[7]The idiosyncratic cost B_t might also capture the movements in the labor force that, by definition, are specific to the irregular sector, such as those implied by illegal immigration or by workers that are officially inactive or retired.

[8]This assumption embodies the idea that customers in the regular markets are unable to detect which products are manufactured by irregular workers. While realistic for commodities, this hypothesis is somewhat too restrictive in the case of specific services where underground transactions often result from direct agreements between customers and producers. In these circumstances, the customers' costs for regular and underground service cannot be equal at equilibrium. One way to incorporate this feature into our framework is to specify a service sector where customers face a different demand for underground and regular services and pay valued added taxes only on the latter. We, however, decided to abstract from this possibility because tax evasion of this nature is likely to be quantitatively negligible.

output of $y_{i,t}$ using only the regular technology. The unofficial productive factors are therefore not strictly necessary to produce the final output. As a result, the underground production takes place in our model primarily because the firms aim to take advantage of tax evasion.

To discourage concealed transactions, the government enforces a monitoring process. Following Allingham and Sadmo (1974), we assume that on each date t, firms face a not-zero probability $p \in (0,1)$ of being inspected and forced to pay the tax rate τ_t^c on the concealed production, augmented by a penalty surcharge factor $s > 1$. As a result, for a given market price P_t, the total expected net revenues from an amount of final output $y_{i,t}$ at time t are given by:

$$E_t \{NR(y_{i,t})\} = P_t \left[(1-\tau_t^c) y_{i,t}^m + (1 - ps\tau_t^c) y_{i,t}^u\right] \tag{4}$$

where E_t denotes the mathematical expectation operator conditional on information available at time t. This expression shows that as long as $(1-ps\tau_t^c)$ > 0, the firms have an incentive to produce the underground output because the revenues from this activity are expected to be positive.

The capital and labor markets are perfectly competitive. The cost of renting capital is equal to the nominal rental rate R_t paid per unit of capital. The total cost of labor instead depends on whether the firms hire workers in the regular or in the underground sector. More precisely, we assume that the cost of labor in the regular market is represented by the nominal wage paid for one unit of labor services W_t^m, augmented by a stochastic social security tax rate $\tau_t^s < 1$, whereas the cost of labor hired in the underground market is given by the nominal wage, W_t^u. Accordingly, the total costs for a firm i, namely TC, are defined as follows:

$$TC(h_{i,t}^m, h_{i,t}^u, k_{i,t}) = (1+\tau_t^s) W_t^m h_{i,t}^m + R_t k_{i,t} + W_t^u h_{i,t}^u \tag{5}$$

Given the equations (4) and (5), the optimal amount of final output produced by a firm i at date t is the solution of the following static problem:

$$\begin{cases} \max\limits_{h_{i,t}^m, h_{i,t}^u, k_{i,t}} E_t \{NR(y_{i,t})\} - TC(h_{i,t}^m, h_{i,t}^u, k_{i,t}) \\ s.t. \\ y_{i,t}^m = A_t \left(\Gamma_t h_{i,t}^m\right)^{\alpha} (k_{i,t})^{1-\alpha} \\ y_{i,t}^u = B_t \left(\Gamma_t h_{i,t}^u\right)^{\alpha_u} (k_{i,t})^{1-\alpha_u} \end{cases}$$

where the vector of prices $\{P_t, W_t^u, W_t^m, R_t\}$ is taken as given. The associated optimal planning satisfies the following three conditions:

$$(1-\tau_t^c)(1-\alpha)\frac{y_{i,t}^m}{k_{i,t}} + (1-ps\tau_t^c)(1-\alpha_u)\frac{y_{i,t}^u}{k_{i,t}} = \frac{r_t}{1-\tau_t^c} \tag{6}$$

$$\alpha\frac{y_{i,t}^m}{h_{i,t}^m} = \frac{w_t^m(1+\tau_t^s)}{1-\tau_t^c} \tag{7}$$

$$\begin{cases} \alpha_u \frac{y_{i,t}^u}{h_{i,t}^u} = \frac{w_t^u}{1-ps\tau_t^c} & \text{if } 1-ps\tau_t^c > 0 \\ h_{i,t}^u = 0, & \text{otherwise.} \end{cases} \tag{8}$$

where $r_t = R_t/P_t$, $w_t^m = W_t^m/P_t$ and $w_t^u = W_t^u/P_t$, respectively, denote the real rental rate, the real wage paid in the regular labor market and the real wage paid in the underground sector. Equations (6) and (7) describe the optimal demand for capital and regular labor, respectively. Equation (8) instead describes the optimal demand for underground labor services. Accordingly, as long as $1-ps\tau_t^c > 0$, a firm demands irregular labor until its marginal productivity equates to its marginal cost, where the latter is given by the real wage w_t^u discounted by the expected real revenue from an additional unit of underground output, $1-ps\tau_t^c$. Conversely, when $1-ps\tau_t^c < 0$, the firms have no incentive to hire irregular workers to produce final output because the real revenues from the underground sector are expected to be negative. In this case, total output is entirely produced with the regular technology (i.e., $h_{i,t}^u = 0$), and therefore, firms do not evade taxation.

3.2. The representative Household

The representative household has preferences in period 0 given by:

$$U_t^h = \sum_{t=0}^{\infty}\beta^t E_0\left\{\frac{(c_t/\Gamma_t)^{(1-\sigma)}-1}{1-\sigma} - \xi_t^h B_0\frac{(h_t^m+h_t^u)^{1+\xi}}{1+\xi} - B_1\frac{(h_t^u)^{1+\phi}}{1+\phi}\right\} \tag{9}$$

where $\sigma > 0$ is the inverse of the intertemporal elasticity of substitution, $\beta \in (0,1)$ is the subjective discount factor, $B_0 \geq 0$ and $B_1 \geq 0$ are preference parameters controlling for the disutility of working activities, and $\xi > 0$ and $\phi > 0$ respectively denote the inverse labor supply elasticities of aggregate and underground labor supplies. ξ_t^h represents a purely transitory demand shock that affects the marginal rate of substitution between consumption and leisure.[9]

The specification of preference implies that households take utility from consumption relative to the rate of technology Γ_t. This assumption ensures that the economy evolves along a balanced growth path. As in An and Schorfheide (2007), we interpret Γ_t as an exogenous habit component. The specification of the disutility of total hours worked $(h_t^m + h_t^u)$ is standard, and allows for perfect labor mobility across sectors. The last term in equation (9) implies that households face an idiosyncratic cost of working in the underground sector. It might be interpreted as capturing the cost associated with the lack on any social and health insurance in the underground sector.[10]

Households supply labor services per unit of time and rents to firms whatever capital they own. We assume that the capital stock, k_t, held by households evolves over time according to the following law of motion

$$k_{t+1} = \xi_t^x x_t + (1 - \delta_k)\, k_t \tag{10}$$

where x_t denotes the investment at date t, and $\delta_k \in [0, 1]$ is the capital depreciation rate. Following Justiniano et al. (2010), we assume that the efficiency with which the final good can be transformed into physical capital is random and determined by the purely transitory exogenous shock ξ_t^x. As shown in Greenwood et al. (1988), a stochastic disturbance of this type is equivalent to a sector-specific technological shock that affects the production of investment goods in a simple two-sector model. As such, this assumption is useful to capture the potentially different sources of fluctuations between consumption and investment.[11]

Households might evade income taxes by reallocating their labor services from the regular to the irregular labor markets. The underground-produced income flows, $w_t^u h_{h,t}^u$ are, therefore, not subject to the stochastic income tax rate $\tau_t^h < 1$. Under these assumptions, the household's period-by-period real

[9]This assumption has been introduced mainly because, according to the available empirical evidence, a shock to the disutility of labor of this form turns out to be particularly important to allow the actual dynamics of the worked hours in the estimated DSGE models to be captured (see e.g., Smets and Wouters, 2007).

[10]An alternative interpretation is that parameter B_1 measures the degree of households' tax morality (see e.g., Gordon, 1989).

[11]This is particularly important because both consumption and investment aggregates are treated as observable variables in the model's estimate. See section 4 for further details.

budget constraint can be written as follows:

$$c_t + x_t = (1 - \tau_t^h)(w_t^m h_t^m + r_t k_t) + w_t^u h_t^u \tag{11}$$

The utility maximization problem for the representative household can be stated as a matter of choosing the processes c_t, h_t^u and h_t^m that maximize the intertemporal utility function (9) subject to the law of motion of capital (10) and to the budget constraint (11). An optimal consumption, labor supply, and saving plan for the representative household must satisfy the following conditions:

$$\Gamma_t^{(1-\sigma)} c_t^{-\sigma} = \lambda_t$$

$$\frac{\lambda_t}{\xi_t^x} = \beta E_t \left\{ \lambda_{t+1} \left[\frac{(1-\delta_k)}{\xi_{t+1}^x} + (1 - \tau_{t+1}^h) r_{t+1} \right] \right\} \tag{12}$$

$$B_0 \left(h_t^m + h_t^u \right)^{\xi} \xi_t^h = (1 - \tau_t^h) w_t^m \lambda_t \tag{13}$$

$$B_0 \left(h_t^m + h_t^u \right)^{\xi} \xi_t^h + B_1 \left(h_t^u \right)^{\phi} = w_t^u \lambda_t \tag{14}$$

where λ_t is the Lagrange multiplier for the constraint (11). Equation (12) is the usual Euler equation that provides the intertemporal optimality condition, whereas equation (13) describes the (total) labor supply schedule. Equation (14) describes the optimal allocation of time for the working activities in the underground sector. To gain intuition regarding the determinants of the irregular labor supply, it is useful to combine (13) with (14) and solve the resulting equation with respect to h_t^u to obtain the following:

$$h_t^u = \begin{cases} \lambda_t^{\frac{1}{\phi}} \left[\frac{w_t^u - (1-\tau_t^h) w_t^m}{B_1} \right]^{\frac{1}{\phi}} & \text{if } w_t^u - (1-\tau_t^h) w_t^m \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

This equation states that households supply labor services in the underground sector as long as the wage that they earn from this activity exceeds the net real wage that they earn by working in the regular labor market. From this perspective, $1/\phi$ stands for the Frisch elasticity of irregular labor supply with respect to the net-of-taxes wage differential between the underground and the regular labor market. Additionally, for a given wage differential, the supply of irregular labor shifts to the left when parameter B_1 increases. Intuitively, to keep the same amount of irregular labor supplied, households require a higher wage gap to compensate for the increased disutility that they derive by working in the irregular sector.

3.3. Government

In each period t, the government raises taxes to finance a given amount of government consumption, g_t. For simplicity, we abstract for public debt and assume that public expenditures are selected on a balanced basis each period. The period-by-period government budget constraint can then be written as follows:

$$g_t = \tau_t^h(w_t^m h_{h,t}^m + r_t k_{h,t}) + \tau_t^c \int_0^1 (psy_{i,t}^u + y_{i,t}^m)di + \tau_t^s w_t^m \int_0^1 h_{i,t}^m di \tag{16}$$

where the first term in the right-hand side of (16) is the total fiscal revenues from personal income taxation, G_t^h; the second term is the total fiscal revenues from corporate taxation, G_t^c; and the last term is the total fiscal revenues from the social security contributions, G_t^s.

Total tax evasion at date t, namely TE_t, takes the following form:

$$TE_t = (\tau_t^s + \tau_t^h)w_t^u \int_0^1 h_{i,t}^u di + (1-p)\tau_t^c \int_0^1 y_{i,t}^u di$$

3.4. Stochastic Processes

To complete the model, we formulate productivity, demand and tax rates disturbances as a stationary VAR(1) process

$$\mathbf{z}_t = (I - \Phi)\mathbf{z} + \Phi\mathbf{z}_{t-1} + \varepsilon_t \tag{17}$$

where $\mathbf{z}_t = \left\{log(A_t), log(B_t), log(\tau_t^c), log(\tau_t^s), log(\tau_t^h), log(\xi_t^i), log(\xi_t^h)\right\}'$, $\mathbf{z}$ is a vector containing the mean values the exogenous state variables, $\Phi = diag[\rho_a, \rho_b, \rho_c, \rho_s, \rho_h, \rho_H, \rho_I]$, and $\varepsilon_t = \left\{\varepsilon_t^a, \varepsilon_t^b, \varepsilon_t^c, \varepsilon_t^s, \varepsilon_t^h, \varepsilon_t^I, \varepsilon_t^H\right\}'$ is the vector of zero-mean normal random innovations with diagonal variance-covariance matrix $\Omega = diag[\sigma_a^2, \sigma_b^2, \sigma_c^2, \sigma_s^2, \sigma_h^2, \sigma_H^2, \sigma_I^2]$.

3.5. Symmetric equilibrium

We restrict the analysis to symmetric equilibria where all firms produce the same quantity of their respective good using the same amount of official and irregular productive factors. In addition, we normalize the price P_t of goods to 1 in each period of time t. The symmetric equilibrium of the model is then formally derived by imposing the following clearing conditions for the goods and the labor markets:

$$c_t + x_t + g_t = \int_0^1 y_{i,t} di$$

$$h_t = \int_0^1 (h_{i,t}^m + h_{i,t}^u) di$$

where h_t denotes the total amount of time for working activities supplied by households at date t.

Given the assumptions made for the production functions and the preferences, the model's economy features a balanced growth path equilibrium in which the variables grow at a constant rate. It is therefore convenient to express the model in terms of detrended variables, for which there exists a deterministic steady state.[12] Thus, denoting with $\widehat{S}_t = S_t/\Lambda_t$ the original variable S_t detrended by means of its trend Λ_t, and letting $\mathbf{x}_t = (\hat{r}_t, \hat{w}_t^m, \hat{w}_t^u, \hat{w}_t^h, \hat{y}_t^m, \hat{y}_t^u, \hat{h}_t^m, \hat{h}_t^u, \hat{G}_t^c, \hat{G}_t^s, \hat{G}_t^h, \hat{c}_t, \hat{k}_t, \hat{x}_t, \hat{y}_t, \hat{h}_t)$ the vector of all endogenous variables, then a symmetric equilibrium for the economy can be formally defined as an initial condition $\widehat{k}_0 \in \mathbb{R}_+$ and a process $\{\mathbf{x}_t\}_{t=0}^{\infty}$ that, given the exogenous stochastic process $\{\mathbf{z}_t\}_{t=0}^{\infty}$, satisfies the following system of equations:

$$\hat{y}_t^m = A_t \left(\hat{h}_t^m\right)^{\alpha} \left(\hat{k}_t\right)^{1-\alpha} \tag{18}$$

$$\hat{y}_t^u = B_t \left(\hat{h}_t^u\right)^{\alpha_u} \left(\hat{k}_t\right)^{1-\alpha_u} \tag{19}$$

$$\hat{y}_t = \hat{y}_t^m + \hat{y}_t^u \tag{20}$$

$$(1-\tau_t^c)(1-\alpha)\frac{\hat{y}_t^m}{\hat{k}_t} + (1-ps\tau_t^c)(1-\alpha_u)\frac{\hat{y}_t^u}{\hat{k}_t} = \frac{\hat{r}_t}{1-\tau_t^c} \tag{21}$$

$$\alpha\frac{\hat{y^m}_t}{\hat{h}_t^m} = \frac{\hat{w}_t^m(1+\tau_t^s)}{1-\tau_t^c} \tag{22}$$

[12]The perfect foresight equilibrium (or non-stochastic steady state) of the model is derived by setting the shocks $\mathbf{z}_t$ equal to their mean values in every period .

$$\begin{cases} \alpha_u \frac{\hat{y}_t^u}{\hat{h}_t^u} = \frac{\hat{w}_t^u}{1-ps\tau_t^c}, & \text{if } 1-ps\tau_t^c \geq 0 \\ \hat{h}_t^u = 0, & \text{otherwise.} \end{cases} \tag{23}$$

$$\gamma \hat{k}_{t+1} = \xi_t^x \hat{x}_t + (1-\delta_k)\,\hat{k}_t \tag{24}$$

$$\hat{c}_t + \hat{x}_t = (1-\tau_t^h)(\hat{w}_t^m \hat{h}_t^m + \hat{r}_t \hat{k}_t) + \hat{w}_t^u \hat{h}_t^u \tag{25}$$

$$\frac{\hat{c}_t^{-\sigma}}{\xi_t^x} = \frac{\beta}{\gamma} E_t \left\{ \hat{c}_{t+1}^{-\sigma} \left[\frac{(1-\delta_k)}{\xi_{t+1}^x} + (1-\tau_{t+1}^h)\hat{r}_{t+1} \right] \right\} \tag{26}$$

$$B_0 \left(\hat{h}_t\right)^{\xi} \xi_t^h = (1-\tau_t^h)\hat{w}_t^m \hat{c}_t^{-\sigma} \tag{27}$$

$$\hat{h}_t^u = \begin{cases} \lambda_t^{\frac{1}{\phi}} \left[\frac{\hat{w}_t^u - (1-\tau_t^h)\hat{w}_t^m}{B_1} \right]^{\frac{1}{\phi}} & \text{if } \hat{w}_t^u - (1-\tau_t^h)\hat{w}_t^m \ \geq \ 0 \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

$$\hat{h}_t = \hat{h}_t^u + \hat{h}_t^m \tag{29}$$

$$\hat{G}_t^h = \tau_t^h(\hat{w}_t^m \hat{h}_t^m + r_t \hat{k}_t) \tag{30}$$

$$\hat{G}_t^c = \tau_t^c(ps\hat{y}_t^u + \hat{y}_t^m) \tag{31}$$

$$\hat{G}_t^s = \tau_t^s \hat{w}_t^m \hat{h}_t^m \tag{32}$$

4. Parameter Estimates

4.1. Method

Estimation and inference are major issues when managing DSGE models. A common solution in the empirical literature is to recur to Bayesian methods and, in particular, to MCMC algorithms (see e.g. Canova and Sala, 2009; Fernandez-Villaverde and Rubio-Ramirez, 2007). The model, defined through equations (18)-(32), is in fact a highly nonlinear system that cannot be estimated in a straightforward manner. For this reason, the system is linearized and solved to derive a more manageable reduced form. To address these nonlinearity issues, it is common practice to linearize the system through a first-order Taylor expansion around its steady state. This approximation leads to the following representation of the dynamic system:

$$\Gamma_0 \mathbf{x}_t = c_x + \Gamma_1 \mathbf{x}_{t-1} + \Gamma_z \mathbf{z}_t + \Pi \eta_t \tag{33}$$

in which $\mathbf{x}_t$ is the endogenous state vector, $\mathbf{z}_t$ is a zero mean autoregressive exogenous process with shocks ϵ_t, and η_t are the forecasting errors. Γ_0, Γ_1, Γ_z,

Π and c_x are matrices whose entries are functions of the structural parameters and of the steady states of the model. Even if (33) is an approximate version of the model, we stress that it is still a structural representation of the system that must be solved to derive its reduced form. There are many strategies available in the literature to overcome this problem, (see An and Schorfheide, 2007, for instance). In this paper, we use the algorithm implemented in Sims (2002), which leads to

$$\mathbf{x}_t = \Theta_c + \Theta_x \mathbf{x}_{t-1} + \Theta_z \mathbf{z}_t \tag{34}$$

in which the system's matrices still depend on the structural parameters $\boldsymbol{\theta}$ and on the steady states. The second relevant issue for inference is that the system cannot be estimated through standard methods because $\mathbf{x}_t$ is partially non-observable, and then, the likelihood cannot be computed. To manage this problem, the vector $\mathbf{x}_t$ is linked to a set of observable variables that are indicated by the vector $\mathbf{y}$. Using matrix notation, the observables are related to the state vector through the following relationship:

$$\mathbf{y}_t = S\tilde{\mathbf{x}}_t \tag{35}$$

in which S is a selection matrix, whereas $\tilde{\mathbf{x}}_t = (\mathbf{x}_t, \mathbf{x}_{t-1})$ is the augmented state vector that includes the eventual lagged observations. Equations (34) and (35) define a linear and a Gaussian state space system that can be handled through the Kalman filter, a recursive algorithm that allows the likelihood function $L(\mathbf{y}|\boldsymbol{\theta})$ to be precisely evaluated even in the presence of latent processes. Here, we base our inference on the Bayesian paradigm, which has proved to be successful in the empirical macroeconomic literature. In particular, Bayesian methods allow us to incorporate additional information into the parameter estimation procedure through prior distributions, which eventually reduces the risks of non-identification troubles for the parameters by adding curvature to the likelihood function. The choice of these prior distributions will be extensively described in Section 4.3. Our goal is to jointly estimate the parameter vector together with the latent process $\mathbf{x}_t$. In particular, we aim to evaluate the magnitude of the underground economy in Italy. This task can be easily handled through an MCMC algorithm. Details of the algorithm are provided in Appendix.

In this paper, all of the calculations are based on software written using the Ox©6.21 language of Doornik (2001) combined with the state space library `ssfpack` of Koopman et al. (1999) and the `LiRE` library to solve rational expectation models of Mavroeidis and Zwols (2007). Moreover, the initial value $\boldsymbol{\theta}^{(0)}$ has been set by maximizing the posterior mode $p(\boldsymbol{\theta})L(\mathbf{y}|\boldsymbol{\theta})$. Once the initial value has been set, we build a multi-chain MCMC procedure based on 4 chains of size 200,000. As stated before, the movement of the chain is characterized by random walk dynamics, i.e., $\boldsymbol{\theta}^* = \boldsymbol{\theta}^{(j-1)} + \tilde{\eta}_j$ in which $\tilde{\eta}_j \sim N(0, \Sigma)$. A rule of thumb to define a *optimal* scaling factor Σ that allows for the reasonable convergence properties of the algorithm is to guarantee an acceptance rate ranging between 25% and 35%. In our empirical application, we found a rate of approximately 28 percent.

4.2. Data

We consider the following set of *measurement* equations to link our theoretical model to the real world economy

$$\mathbf{y}_t \equiv \begin{bmatrix} \Delta c_t \\ \Delta x_t \\ \Delta G_t^c \\ \Delta G_t^s \\ \Delta G_t^h \\ \Delta w_t^h \end{bmatrix} = \begin{bmatrix} \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \\ \gamma^{(Q)} \end{bmatrix} + 100 \begin{bmatrix} \hat{c}_t - \hat{c}_{t-1} \\ \hat{x}_t - \hat{x}_{t-1} \\ \hat{G}_t^c - \hat{G}_{t-1}^c \\ \hat{G}_t^s - \hat{G}_{t-1}^s \\ \hat{G}_t^h - \hat{G}_{t-1}^h \\ \hat{w}_t^h - \hat{w}_{t-1}^h \end{bmatrix} \tag{36}$$

in which Δc_t is the consumption growth expressed in percentage terms, Δx_t is the investment growth, Δw_t^h is the change in the gross real total earnings paid in the regular market (i.e., $w_t^h = (1 + \tau_t^s) w_t^m h_t^h$), $\Delta G_t^i,\ i = c, s, h$ are the growth rates of fiscal revenues from corporate taxation, social security contributions and personal income taxation, respectively, and finally $\gamma^{(Q)} = 100 log(\gamma)$ is the common quarterly trend growth rate.

The model is estimated using the quarterly figures provided by the Italian National Institution of Statistics (ISTAT) over the full sample period 1982:1 to 2006:4.[13] All of the data are in real terms (base year 2000) and divided by the total population aged 15-64 years. The choice of the observable variables is directly guided by the theory. More precisely, given that our ultimate goal is to estimate the size and trend of the underground economy, we choose as our observables those aggregates that, according to our model, are particularly informative regarding the magnitude of underground economic activities. In this respect, the data on aggregate consumption and investment proxy the general level of economic activity in Italy; the fiscal

[13]ISTAT provides data on fiscal revenues and labor earnings annually. Quarterly figures for these series are made available by Associazione Prometeia of Bologna.

revenues data captures the incentives of firms and households to engage in underground transactions; and finally, the official labor earnings data are informative on the households' opportunity cost of supplying labor services in the underground sector.

4.3. Prior distributions and calibrated parameters

Our priors are summarized in Table 1. Overall, we considered prior densities that match the domain of the structural parameters. Starting with the underground economy-related parameters, our prior choice is mostly based on previous analysis provided by ISTAT. More specifically, the elasticity of labor in the underground production function, α_u, is assumed to be a beta random variable with a mean of 0.7026 and a standard deviation of 0.02, while the disutility of working activities in the underground economy, B_1, is assumed to follow a gamma distribution with a mean of 15.2328 and a standard deviation of 0.4. Conditional to all of the other prior parameter values, the prior means of α_u and B_1 imply a steady-state size of the underground economy (Y^u/Y) and a steady-state share of the total worked hours ascribed to the underground sector (H^u/H) of 19% and 13%, respectively. These numbers match the estimates of the underground output-to-GDP ratio and the irregular labor share provided by ISTAT over the 1982-2006 period. We firmly believe that these estimates are the most reliable available regarding the underground economy in Italy. For this reason, these values have been set as the starting point for our analysis. The inverse of the Frisch elasticity of underground labor supply ϕ is instead described by a gamma distribution with a mean of 0.06 and a standard deviation of 0.01. The prior mean was chosen to be consistent with the calibration reported in Busato et al. (2005).

For the parameters that are commonly used in the DSGE literature, our prior choice is consistent with previous studies (An and Schorfheide, 2007, Smets and Wouters, 2007, and Iacoviello and Neri, 2010 among others). More precisely, we assume that the inverse of the intertemporal elasticity of substitution σ and the inverse of the elasticity of the total labor supply ξ are distributed according to a gamma random variable, both with a mean 1 and a standard deviation respectively set to 0.05 and 0.1. The elasticity of labor in the regular production function α is assumed to follow a beta distribution with a mean of 0.65 and a standard deviation of 0.02. The capital depreciation rate δ_k is assumed to be a beta random variable centered at a quarterly rate of 2.5 percent, i.e., $\mathrm{E}[\delta_k] = 0.025$, and with a standard deviation of 0.005.

Finally, the common quarterly trend growth rate, γ^Q, is assumed to follow a Gaussian prior with a mean of 0.23 and a standard deviation of 0.5. The prior mean is chosen to match the average growth rate of actual per-capita GDP over the 1982-2006 period. This choice is consistent with the balanced growth path hypothesis.

Regarding the exogenous processes, we assume that the standard error of the innovations follow a rather dispersed inverse gamma distribution to summarize the lack of a priori information about these quantities. The persistence of the AR(1) processes (i.e., parameters ρ_a, ρ_b, ρ_c, ρ_h, ρ_s, ρ_H and ρ_I) are instead described by beta distributions with means ranging between 0.5 and 0.9 to allow for moderate to high persistence for the propagation mechanism for the exogenous shocks. In particular, the variances of these priors are relatively high to account for a wide range of possible posterior values for these parameters. This hypothesis is consistent with Smets and Wouters (2007).

Finally, the remaining parameters are fixed, either because they reflect some characteristics that are regulated ex-ante by law or because they are difficult to identify. More specifically, the steady state parameters τ_c, τ_h and τ_s, that represent the average tax rates on corporate profits and personal income and the rate of social security contributions, respectively, are fixed at 41.55%, 34.26% and 21%. These numbers are consistent with the average tax rates imposed in Italy over the 1982-2006 period.[14] Furthermore, the penalty paid by a firm once it is detected is set to 30% of the corporate tax rate. The surcharge factor is thus $s = 1.30$, consistent with the current Italian Tax Law (Busato and Chiarini, 2004). The probability p that a company is inspected is set to 3%, corresponding to the estimate reported by Busato and Chiarini (2004) using data on the number of inspected firms that was released by the Italian Ministry of Labor. The subjective discount factor β is set to 0.9840, implying a steady-state gross interest rate of 1.0186. Finally, the parameter controlling for the disutility of the total labor supply, B_0, has been set to a value that implies that households devote 19% of its time to

[14]More precisely, the values of the average tax rate on personal income and the rate of social security contributions are taken from Busato and Chiarini (2004), while the value of τ_c corresponds to the average Italian statutory corporate tax rates over the 1982-2006 period. The values for statutory tax rates are taken from the OECD Tax Database. For the 1998-2006 period, the OECD data have been augmented by 4.25% to account for the newly introduced regional corporate taxation (IRAP).

labor activities.[15] This value corresponds to the average hours worked in a quarter as a fraction of the total quarterly hours for the 1982-2006 period.

4.4. Posterior Distributions

Table 1 shows the posterior mean, mode, and the 95 percent probability interval for the structural parameters, together with the mean and the standard deviation of the prior distributions. A frequently employed and practical way to assess the identification of the parameters is to compare the prior to the posterior distributions to check if the observable variables are informative for inferential purposes. These results are displayed in Figure C.12.

A closer inspection of the parameters governing production, i.e., α and α_u, suggests that the contribution of the observed data (likelihood) is relevant. In particular, the likelihood provides a sensitive negative shift with respect to the prior information. Specifically, the elasticity of regular production to labor α has a posterior mean of .61, while the irregular labor elasticity α_u has a posterior mean of approximately .65. The observed difference between the estimates for these two parameters could be interpreted as evidence in favor of a higher output sensitivity to labor in the irregular market.

With regard to the households, the posterior estimate of the inverse of the intertemporal elasticity of substitution σ is slightly smaller than its prior counterpart even though the posterior variability is much smaller. The data therefore suggest that consumption is somewhat more sensitive to movements in the real interest rate than is implied by the prior distribution.

We also find that the posterior estimate of the labor supply elasticity in the regular market ξ is substantially larger than the a priori hypothesis, while its counterpart in the irregular market ϕ is only slightly smaller than the a priori assumption. In particular, the posterior mean of ϕ is small (0.056), suggesting that the labor supply in the underground sector is highly sensitive to movements in the net wage differential. The estimate of δ_k (0.033) implies an half-life for the capital stock of about 5 years.

The parameter controlling for the disutility of irregular labor, B_1, has a posterior mean of 15.38, which is substantially equivalent to its prior mean. This may be interpreted as a suspected lack of identification. On the other

[15] In the estimation procedure, this parameter is updated at any iteration using equation (27) evaluated at the steady-state.

Table 1: DSGE with Underground Economy - **Posterior computation (MCMC)**

	Posterior distribution			Prior information		
	Mean	Mode	95% Cred. Int.	Mean	S.E.	Type
$p(\alpha\|\mathbf{y})$	0.6104	0.6107	[0.571,0.647]	0.650	0.020	Beta
$p(\delta_k\|\mathbf{y})$	0.0339	0.0338	[0.024,0.043]	0.025	0.005	Beta
$p(\alpha_u\|\mathbf{y})$	0.6537	0.6536	[0.616,0.681]	0.7026	0.020	Beta
$p(\rho_a\|\mathbf{y})$	0.9615	0.9629	[0.935,0.979]	0.800	0.100	Beta
$p(\rho_b\|\mathbf{y})$	0.9691	0.9697	[0.952,0.981]	0.500	0.100	Beta
$p(\rho_c\|\mathbf{y})$	0.9879	0.9894	[0.963,0.999]	0.900	0.100	Beta
$p(\rho_h\|\mathbf{y})$	0.9956	0.9987	[0.979,0.999]	0.900	0.100	Beta
$p(\rho_s\|\mathbf{y})$	0.9494	0.9521	[0.906,0.981]	0.900	0.100	Beta
$p(\rho_H\|\mathbf{y})$	0.6856	0.6926	[0.483,0.842]	0.600	0.100	Beta
$p(\rho_I\|\mathbf{y})$	0.9700	0.9721	[0.937,0.989]	0.800	0.100	Beta
$p(\sigma\|\mathbf{y})$	0.9868	0.9862	[0.986,1.069]	1.000	0.050	Gamma
$p(\phi\|\mathbf{y})$	0.0559	0.0556	[0.038,0.074]	0.060	0.010	Gamma
$p(\xi\|\mathbf{y})$	1.1882	1.1848	[0.993,1.371]	1.000	0.100	Gamma
$p(B_1\|\mathbf{y})$	15.384	15.380	[14.61,16.17]	15.2328	0.400	Gamma
$p(100\sigma_a\|\mathbf{y})$	1.5310	1.5256	[1.330,1.758]	0.600	0.160	IG
$p(100\sigma_b\|\mathbf{y})$	1.0432	1.0365	[0.886,1.235]	0.600	0.160	IG
$p(100\sigma_c\|\mathbf{y})$	2.0404	2.0303	[1.782,2.356]	0.600	0.160	IG
$p(100\sigma_h\|\mathbf{y})$	1.3731	1.3676	[1.196,1.580]	0.600	0.160	IG
$p(100\sigma_s\|\mathbf{y})$	2.0884	2.0778	[1.833,2.396]	0.600	0.160	IG
$p(100\sigma_H\|\mathbf{y})$	0.6296	0.5849	[0.384,1.101]	0.600	0.160	IG
$p(100\sigma_I\|\mathbf{y})$	0.9193	0.9151	[0.708,1.162]	0.600	0.160	IG
$p(100\gamma^{(Q)}\|\mathbf{y})$	0.7043	0.6652	[0.150,1.369]	0.230	0.500	Normal

side, the estimated steady state of the underground economy is approximately 22 percent, which is about 3 percentage points larger than the official statistics provided by ISTAT, that represent our prior knowledge about the phenomenon. This empirical evidence allows to conclude that our posterior estimate of the underground economy is mainly caused by the posterior variation of α_u.

Turning to the exogenous processes, the autoregressive coefficients provide information regarding the persistence of the shocks that describe the mechanism of propagation for the exogenous shocks. Overall the coefficients are large and in some cases close to 1, thus providing strong evidence of shock persistence. The variances of the exogenous shocks are clearly identified.

Finally, some convergence diagnostics are presented in Figure C.12, where the recursive averages of the sampler have been reported. Specifically, the plot reports the evolution of $\frac{1}{i}\sum_{j=1}^{i}\boldsymbol{\theta}^{(j)}, \forall i$. Of course, it is impossible to assess the convergence properties of an MCMC algorithm through the study of only a few realizations of the chains. However a common practice is to check for the convergence of the empirical averages of the draws. (see Robert and Casella, 1999, chap. 8 for a survey on this topic.) We can say that a chain converges rapidly if the evolution of its empirical averages stabilizes after few iterations. As Figure C.12 illustrates, it is quite evident that the running averages for the algorithm stabilize quickly showing evidence of convergence for the algorithm. According to our experience, it appears that the random walk algorithm that is adopted needs about 100,000 iterations to converge to its correct expected value, and, according to these results, we discarded the first 100,000 draws from each chain to remove the dependence from the initial condition $\boldsymbol{\theta}^{(0)}$.

5. The underground economy in Italy and its sources

Having estimated the model, we now use it to address the primary questions of the paper. How large is the underground economy in Italy? How does the underground sector respond to exogenous shocks? What are the main driving forces of fluctuations in the underground output?

5.1. The size and trend of the underground economy

Figure 1 depicts the smoothed estimate of the ratio of underground production to GDP along with the 95 percent credible bands.[16] This figure summarizes how our model predicts the size and the trend of the underground economy in Italy over the 1982-2006 period. According to the results provided in Figure 1, the underground production accounts on average for 22.8% of GDP. This number is about 4 percentage points larger than the official estimates, and confirms that the underground sector is sizeable in Italy. There is also strong evidence of a steep increase in the size of the underground economy, leading to a change of about 10 percentage points over two

[16]To estimate the endogenous variables dynamics, we picked all the posterior draws from the MCMC algorithm and, for each set of these parameters, we evaluated the latent variables through the simulation smoother algorithm of de Jong and Shephard (1995). Our posterior estimate is thus the average of all of the trajectories obtained, whereas the credible bands have been computed as the 2.5 and the 97.5 percentiles of the empirical distribution. This procedure allows us to also take into account parameter uncertainty.

decades. In particular, the series started the sample around the 17.5%, and then grew slowly during the 1980s, up to the 19.5%. After that period, the series increased quickly over the nineties and then fluctuates around the 27% from 2000 until the end of the sample.[17] Additionally, the estimated model predicts two major contractions in the size of the underground economy: one from and 1992 to 1994 and the second from 2000 to 2002. In both cases, the size of the underground economy decreased by about 3 percentage points. A major expansion occurs instead in the 1997-1999 period, when the size of the underground economy increased by about 5 percentage points.

It is of obvious interest to shed light on the sources behind the predicted pattern of the underground economy. To this end, Figure 2 plots the smoothed estimates of the underground economy (in log-deviation from the steady-state) along with the historical contribution of technological, demand and fiscal factors. This picture provides an immediate visual representation of the relative contribution of each shock to the predicted dynamics of the underground economy in Italy. As the picture illustrates, the trend of the underground economy is clearly explained by the fiscal component (the sum of the three fiscal shocks), whereas its fluctuations are mostly driven by movements in the technological shocks. More specifically, it is evident that a rise in the fiscal component led to a systematic increase of the underground economy. This feature captures the effect of income and corporate taxation, which accordingly to the estimated results, have persistently increased over the 1982-2006 period, as shown in Figure C.13 which plots the smoothed estimates of each tax rate. The technological component instead, has been relevant mainly to explain variability of the underground economy around its upward trend. By contrast, the contribution of the demand shocks (the sum of preference and investment-specific shocks) is negligible over the whole sample period.

5.2. Impulse response

The response of the underground economy model to the estimated exogenous shocks can be assessed through the impulse-response functions. This assessment is shown in Figures C.5-C.11, where we graph the impulse-response functions of regular and underground production, total output (GDP), consumption, investment, and total worked hours along with the 95 percent

[17]Similar results have been found by Schneider et al. (2010).

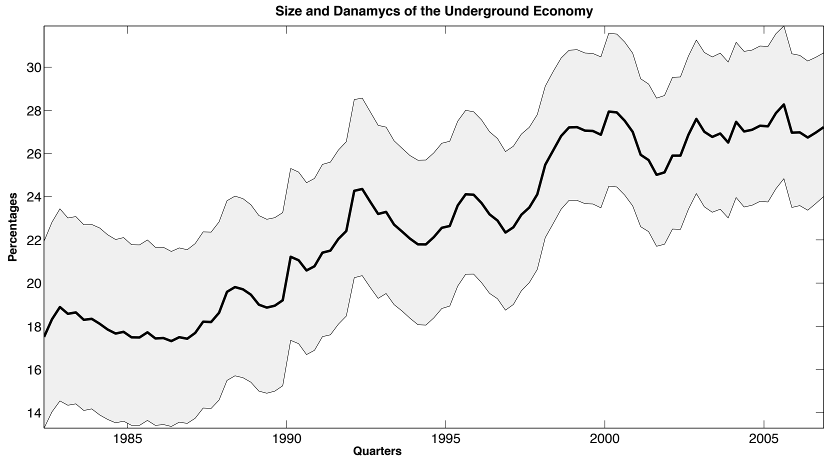

Figure 1: **The Estimated size of the underground economy.** The picture depicts the smoothed estimate of the quarterly ratio of underground production to total output. The series is depicted along with the 95 percent credible interval.

credible intervals.[18]

To begin with, we note that consumption, investment and total hours worked all increase in response to an exogenous boost in the official sector productivity, A_t (see Figure C.5). This is a well-known effect of a positive technology shock that characterizes any standard real business cycle model (see, e.g. King and Rebelo, 1999). The presence of the underground economy, however, implies an additional resource reallocation effect. Because an increase in the rate of technology A_t makes the official output relatively

[18]As for the estimated size of the underground economy, these quantities have been computed as the posterior average of the impulse response functions obtained for each draw of the MCMC algorithm. The credible intervals have been computed as the 2.5 and 97.5 percentiles of the empirical distributions obtained.

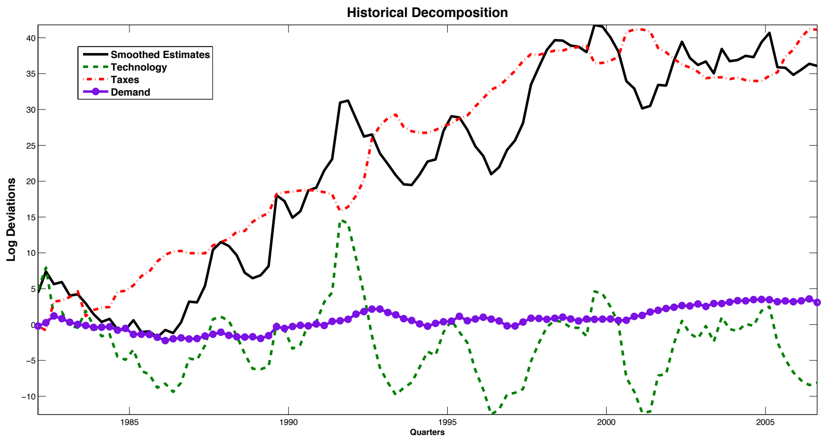

Figure 2: **Historical Decomposition of Underground Production-GDP ratio.** Note: Technology component includes sectorial-specific technology shocks. Fiscal factor includes shocks on corporate, social security and personal income tax rates. Demand component is the sum of preference and investment-specific shocks.

more productive, firms find it more convenient to produce the final output with regular workers rather than with irregular workers. Consequently, in response to a temporary boost in A_t, the total official output increases, while the underground output declines. This effect partially dampens the response of total GDP, which, in fact, increases by a lower rate relative to the official output.

The results are reverted when the economy is hit by a temporary boost in the rate of irregular sector productivity, B_t.[19] In this case, the underground level of output increases while the official level declines, as clearly appears in Figure C.6. At the posterior parameter values, the effect of this shock

[19]For instance, the effect provided by an unexpected increase in regular migration.

on the underground production is strong enough to overcompensate for the decline in official output, and thus total GDP also increases. Interestingly, this shock also affects the short-run intertemporal elasticity of substitution, making the households less willing to smooth consumption through saving. In fact, with a shock of this type, our model predicts that the response of investment is negative for several quarters after the shock, while the response of consumption is always positive and hump-shaped.

The impact of fiscal shocks are summarized in Figures C.7-C.9, where we report the effects of temporary increases in tax rates. Unsurprisingly, because taxation is distortive in our framework, the estimated model suggests that increasing taxes implies a negative response of consumption, investment, total worked hours and GDP. Furthermore, movements in taxes also imply a resource reallocation effect: the underground production increases, while the official production declines. In the case of corporate taxation (Figure C.7) and social security contributions (Figure C.8), this effect is a result of the higher (net) expected returns from underground production; however, when considering taxes on personal income (Figure C.9), the effect instead operates through a labor-supply channel. All else being equal, an increase of τ_t^h induces on impact a larger net wage-gap differential, thus pushing households to reallocate their labor services from the regular to the irregular labor market. This effect provides downward pressure on the irregular labor wage and, at the same time, upward pressure on the official labor wage. As a result, the firms find it more convenient to produce a larger portion of their outputs with irregular workers.

Finally, the effects of demand shocks are provided in Figures C.10 and C.11, where we graph the response of the economy to a temporary increase in the rate of transformation of investment in capital and a temporary boost in the disutility of total hours worked, respectively. As the pictures illustrate, these two demand shocks have a rather different impact on the economy. An unexpected increase in ξ_t^h has a depressive effect on the economy, leading to a decrease in the equilibrium level of all of the main aggregates. This effect is caused by the lower demand for both investment and consumption goods that results from a shock of this type. With larger values of ξ_t^h, in fact, households experience an increase in the disutility of labor that pushes them to substitute consumption with leisure over time. This effect results in a lower demand for both consumption and investment goods and, thus, in a decline for total production. In contrast, an unexpected increase in ξ_t^I stimulates the current investment (at the cost of a lower present consumption) and thus results in a net increase of aggregate demand to which firms respond by increasing both underground and official production. This is illustrated in Figure C.10, which shows that, with the exception of consumption, all of the main economic aggregates increase in response to this shock.

5.3. Cyclical Properties

The top panel of table 2 reports second-order moments for GDP (Y_t), official production (Y_t^m) and underground output (Y_t^u). These statistics are useful to assess the cyclical properties of the underground economy at the business cycle frequencies. As the table illustrates, the estimated model predicts that the underground production is a weakly countercyclical and highly volatile variable over the course of the business cycle. The contemporaneous correlation of this variable with GDP is, in fact, equal to -0.14, while its standard deviation is 1.4 times larger than the standard deviation of GDP. Additionally, our model predicts that the cyclical component of the underground economy is negatively correlated with those of the official output (-0.41). This result is particularly interesting as it provides evidence in favor of a *double business cycle* in the Italian economy, with the peaks of the official economy associated with the troughs of the underground economy and vice versa.[20]

The second panel of table 2 presents results from the asymptotical variance decomposition. Accordingly, the technology shock in the irregular sector (B_t) is predicted to be the primary driving force of fluctuations in the underground output. As illustrated in the table, this shock alone explains approximately 80% of the variance in underground production at business cycle frequencies. The fiscal component (the sum of the three fiscal shocks) is also quantitatively important, explaining around 8% of the overall volatility, while the contributions of the other shocks are smaller. The contribution of the fiscal component becomes quantitatively more important when the underground economy is expressed as a share of GDP, accounting in this case for 12.71% of the overall volatility. This result hinges on the property that the fiscal component also explains a large fraction of regular output volatility (10.30%). Regarding the other aggregates, we see that consumption, investment and total hours worked are all particularly sensitive to fiscal shocks.

[20]This is particularly true by taking into account that the assumptions we have made in our model do not guarantee the existence of a *double business cycle*

Table 2: Cyclical properties and variance decomposition

Business Cycle Statistics				
		$(Y_t^u; Y_t)$	$(Y_t^u; Y_t^m)$	$(Y_t^m; Y_t)$
Correlation		-0.14	-0.41	0.96
Relative standard deviation		1.4	1.01	1.38
Asymptotical Variance Decomposition (%)				
Variable	A	B	Fiscal	Demand
Y_t^u	6.22	79.92	7.82	6.04
Y_t^m	86.98	1.40	10.30	1.32
Y_t	87.62	2.09	6.99	3.30
Y_t^u/Y_t	49.20	36.73	12.71	1.35
X_t	33.65	6.57	42.78	17.00
C_t	16.32	38.50	25.01	20.17
H_t	39.48	2.91	46.87	10.74

For example, the fiscal component is the primary driving force of fluctuations in both total hours worked (46.87%) and investment (42.78). This result mirrors the intra-temporal and inter-temporal reallocation effects that occur in our model as a result of exogenous tax changes.

6. Policy Implications

In this section, we use the estimated model to assess the effects of alternative fiscal policies. The current Italian sovereign debt crisis has strengthened the urge among Italian policy makers to design suitable policies to fight tax evasion. This issue is perceived as a priority in Italy not only to increase fiscal revenues to prevent the risk of national default but also to improve fiscal equity among individuals. Given the size of the informal sector in Italy, it is clear that to achieve these targets in an efficient manner, it is of fundamental importance to know how the underground economy reacts to different fiscal and institutional stimuli. From this perspective, one of the main advantages of our approach, with respect to the more traditional methodologies, is that it provides a natural *laboratory* from which to derive general equilibrium implications of fiscal policies.

6.1. The Laffer curve

According to the OECD statistics, in the year 2009, the tax revenues to GDP ratio was 43.4% for Italy, a fiscal pressure that was higher than both the OECD and the European averages (respectively, 33.8% and 37.1%). Because of this evidence, there exists a certain agreement among Italian economists that the fiscal burden is responsible not only for tax evasion but also for discouraging foreign direct investment and weakening the competitiveness of Italian firms. In this respect, a widespread idea is that a general reduction in the tax burden would benefit the Italian economy.

A test for this claim is provided in Figure 3, which illustrates the steady-state effects to changes in the corporate tax rate τ^c. Keeping all of the other parameters fixed to their posterior mean values, the picture depicts total fiscal revenues (Laffer curve), tax evasion (in percentage terms, with respect to the overall amount of taxes due) and regular and total output as functions of the steady-state corporate tax rate. As illustrated in the left panel of the picture, the estimated steady-state Laffer curve (the continuous blue line) has the typical textbook inverted U-shape, with a maximum at τ^c equal to approximately 26%. This result conforms to previous studies (e.g. Schmitt-Grohe and Uribe, 1997; Trabandt and Uhlig, 2009), which also show the existence of a Laffer curve in standard neoclassical growth models. In our model, however, the shape of the Laffer curve is determined by two related effects. On the one hand, an increase in the corporate tax rate reduces the equilibrium level of regular output (see the continuous line in the right-bottom panel), thus lowering the tax base. This is the traditional effect of distortive taxation. On the other hand, the concealed taxes as a share of the total tax base (a measure of the strength of tax evasion) is a convex function of the corporate tax rate, which increases quickly as τ^c moves from low to high values. This is an additional effect that is due to the presence of the underground sector in the economy. For a sufficiently large τ^c, the above effects dominate the one induced by a larger tax rate, leading then to a monotonically decreasing pattern of fiscal revenues. Note that at the average corporate tax rate (the vertical line in the left panel), Italy is on the *slippery side* of the Laffer curve and therefore can improve its budgetary situation by cutting corporate taxes. Additionally, the picture illustrates that the expansion in underground production that results from the higher corporate

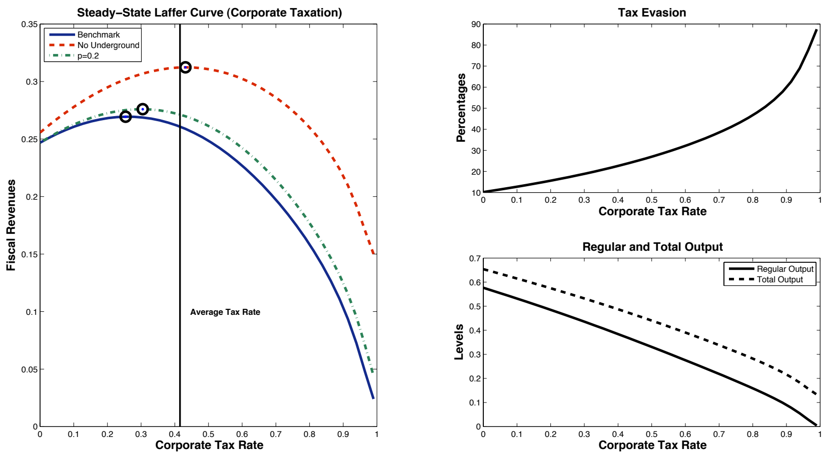

Figure 3: **Steady-state effects of corporate taxation.** This picture depicts the steady-state Laffer curve, total tax evasion and regular and total output as a function of the steady-state corporate tax rate τ^c. All the other parameters are kept fixed to their posterior mean values. Total tax evasion is expressed as a share of total taxes due.

tax rates is not strong enough to completely compensate for the decline of regular output, and thus the equilibrium level of the overall production monotonically decreases with τ^c (see the dashed line in the right-bottom panel). Hence, the estimated model predicts that in the long run, a reduction in the corporate tax rate would effectively benefit the Italian economy in terms of both higher fiscal revenues and higher total production. These results are generally confirmed in Figure 4, which illustrates the effects upon total fiscal revenues and the economic activity of changes in the income tax rate τ^h.

Figures 3 and 4 also depict the steady-state Laffer curve for an economy without the underground sector (see the dashed line).[21] The comparison

[21] The alternative Laffer curves are obtained by setting all of the parameters of the model

between the complete enforcement economy and the benchmark model highlights the importance of the underground sector for fiscal policy purposes. First of all, we note that the loss of fiscal revenues due to tax evasion is remarkable. At the average corporate tax rate, for example, the gap between revenues collected and the potential ones amounts to 11 percentage points of GDP, a results similar to that found by Busato and Chiarini (2012). Furthermore, Figure 3 illustrates that at the average corporate rate, Italy is located on the left side of the full enforcement Laffer curve and could successfully increase tax revenues by raising the tax rate. This result contrasts with the predictions of the benchmark model, thereby suggesting that lack of proper consideration for the underground sector might be dramatically misleading while undertaking fiscal policy interventions. In Figures 3 and 4, we also graph the Laffer curves by setting the probability parameter p to 0.2 instead of to 0.03 as in the benchmark case. This experiment is useful to assess the effects of a stronger enforcement effort. By comparing the continuous and dotted curves in both pictures, we see that increasing parameter p has the effect of shifting the Laffer curve to the right, and thus the gap between taxes actually collected and the potential ones decreases for any given tax rate. This means that fiscal policies which credibly raise the *perceived* probability of being detected evading, successfully increase the steady-state fiscal revenues. Alternatively, the estimated model predicts that at the actual tax rates, the Italian government might effectively raise fiscal revenues by increasing the effort in the monitoring process.

6.2. Transitional Dynamics

We now characterize transitional dynamics and welfare effects of three different fiscal policies: (i) a general tax cut; (ii) an increase in the monitoring effort; and (iii) a mix between the above two policies. Results are provided in Figures C.14 and C.15 where we plot the transition path of selected endogenous variables, and in Table 3 which summarizes the steady-state effects and welfare implications of each alternative fiscal policy. Welfare gains are evaluated at the steady-state, as percentage deviation of households utility with respect to the benchmark counterpart (ΔU), as well as over the transition path, as ratio of equivalent consumption, C_e, to the benchmark level of steady-state consumption, C_b. For each policy, C_e is calculated as the

without underground economy at their posterior mean values.

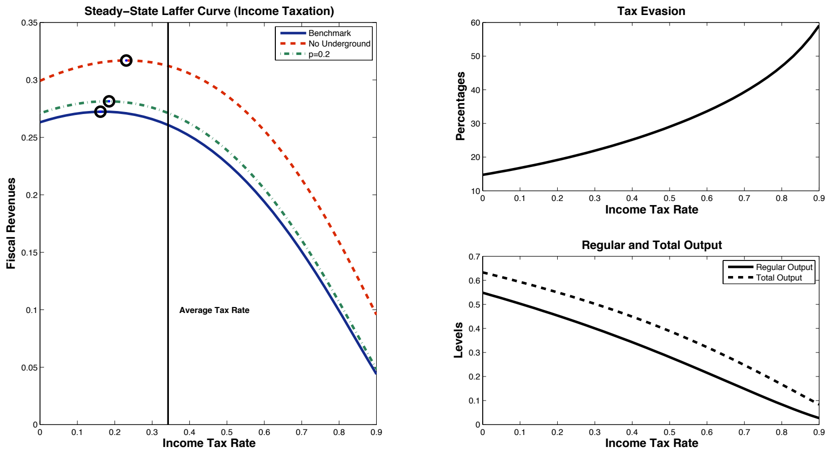

Figure 4: **Steady-state effects of personal income taxation.** This picture depicts the steady-state Laffer curve, total tax evasion and regular and total output as a function of the steady-state income tax rate τ^h. All the other parameters are kept fixed to their posterior mean values. Total tax evasion is expressed as a share of total taxes due.

permanent level of consumption that provides to households the same utility they get along the whole transition path.

6.2.1. A general tax cut

We begin our analysis by characterizing the effects of a general tax cut, assuming that each tax rate is permanently reduced by 2 percentage points. As illustrated in the first row of Table 3, a general tax cut has a positive effect on the economic activity. Relative to the benchmark economy, in the new steady-state, capital, consumption and GDP increase by 9.46%, 6.89% and 4.38%, respectively. The transition paths of these aggregates are depicted in Figure C.14. We see that consumption, GDP, and capital monotonically converge to the new steady-state, while investment overshoots.[22] The relatively slow rate of convergence of these aggregates implies that, although fiscal policy has no effect on the long-run growth, a general tax cut has a sustained impact on their growth rates for substantial periods during the transition. Hours worked also increase relative to the benchmark case, and this effect is due the excess demand that occurs in the labor market because of the lower tax rates.[23] However, despite the induced increase in hours worked, we find that a general tax cut is welfare-improving, both in the new steady-state ($\Delta U = 3.52\%$), and during the whole transition path ($C_e/C_b = 1.06$). The welfare-improvement effect of the policy is clearly driven by the consumption pattern, which more than compensates for the higher disutility arising from the increased equilibrium level of hours worked. According to our results, the policy is also effective in discouraging tax evasion, which in the long-run declines by 9.08%. In terms of transition, Figure C.14 shows that in the quarter immediately after the tax cut, tax evasion jumps below its new steady-state value, gradually increasing thereafter. This pattern mirrors the dynamic response of the concealed production, which also declines in the steady-state by 3.37%. This last effect, together with the increase in total production, implies that the tax cut reduced the steady-state size of the underground economy ($\Delta Y^u/Y$) by 7.43%. Finally, Figure C.14 shows that fiscal revenues overshoot during the transition path, staying below the benchmark level for almost 3 years after the tax cut. This result appears particularly important from the standpoint of fiscal policy, as it shows that a general tax cut, although increasing fiscal revenues in the long-run, may nevertheless worsen the budgetary situation in the short-run.

6.2.2. A stronger tax enforcement

We now assess the effects of a permanent increase in the tax enforcement effort. To this end, we set the probability parameter p to 0.10 instead of to 0.03, while holding the remaining parameters at their posterior mean values. The value assigned to parameter p has been chosen such that, in the new steady-state, total fiscal revenues are exactly the same as those resulting from the general cut tax analyzed above. This choice makes the two poli-

[22]These dynamics resemble those of a standard neoclassical growth model due to a capital stock which is below its steady-state value.

[23]This effect is apparent in Figure C.14, which shows that over the whole transition to the new steady-state, both sectorial wage rates are larger than they benchmark counterparts.

Table 3: **Steady-state effects and welfare implications**

	Steady-State Effects (%)								Welfare	
	ΔG	ΔTE	$\Delta \frac{Y^u}{Y}$	ΔY^u	ΔY^m	ΔY	ΔC	ΔK	ΔU	$\frac{Ce}{C_b}$
Tax Cut	1.71	-9.08	-7.43	-3.37	6.53	4.38	6.89	9.46	3.52	1.06
Larger p	1.71	-9.00	-3.30	-3.56	0.65	-0.26	-2.94	-1.67	-0.87	0.97
Mix	3.27	-17.2	-10.5	-6.84	7.13	4.10	4.11	7.83	2.77	1.03

Note: for each variable, steady-state effects are computed as percentage deviations from their benchmark counterparts. ΔG refers to fiscal revenues.

cies equivalent in terms of long-run impact on fiscal revenues, and therefore facilitates the comparison between them. Steady-state effects and welfare implications of the policy are reported in the second row of Table 3, while Figure C.14 depicts the transitional dynamics. Apart from the impact on tax evasion and fiscal revenues, we note that the effects of a stronger enforcement are dramatically different from those resulting from the comparable tax cut. Precisely, we observe a decline for GDP, consumption and capital in the new steady-state equilibrium, meaning that the policy has depressive effects on the economy activity. The intuition for this result is straightforward. On the one hand, since the policy leaves tax rates unchanged, a stronger enforcement effort stimulates the regular production only through the induced inter-sectorial reallocation of resources. Consequently, the underground production decreases while the regular production increases, but with a net effect on GDP that is virtually equal to zero. On the other hand, given the balanced-budget rule, the increase in fiscal revenues resulting from the policy are used by the government to finance a larger amount of wasteful public expenditures. These two effects jointly imply that the amount of resources available for the private sector decreases in equilibrium, thereby crowding-out consumption and investment both in the steady-state and during the whole transition path. Hours worked instead increase, but now the effect is driven by the excess of labor supply resulting from the negative wealth effect provided by the policy. This result is apparent in Figure C.14, which shows that both regular and unregular wages stay permanently below their benchmark counterparts. The increase in labor supply, and the declining pattern of consumption, jointly imply that the policy is welfaredeteriorating, as summarized in Table 3. As a result, although in our model the government may completely eliminate the underground production by setting the parameters p and s accordingly, this policy would nevertheless be suboptimal, as it would induce welfare looses. Finally, Figure C.14 shows that fiscal revenues increase by 4% on impact, gradually converging to the new steady-state thereafter. Unlike the tax cut policy, this finding shows that a permanently stronger enforcement increases fiscal revenues also in the very short-run.

6.2.3. Policy mix

We evaluate next the effects of a mix between the two policies analyzed above. We assume therefore that government permanently cuts each tax rate by 2 percentage points, and simultaneously increases the effort in the monitoring process, which permanently raises the probability p to 0.10. Results of this experiment are provided in the last row of Table 3, and in Figure C.15 where the transitional dynamics due to the policy mix are compared with those induced by the tax cut alone. We see that the effects of the policy resemble pretty much those resulting from a general tax cut. The policy stimulates the economic activity, and is welfare improving, although with a quantitatively smaller impact with respect to the tax cut alone. Also, we find that mixing the two policies is very effective in discouraging tax evasion, which in the quarter immediately after the policy changes falls by about 18%, and then gradually converges to the new steady-state. Again, this pattern mirrors the response of the underground production, which in the long-run decreases by 6.78%. Because of this effect, and given the expansion in GDP, the policy mix induces a significant decrease in the size of the underground economy, which in the steady-state falls by 10.5%. Most importantly, we find that the mixing the two policies raises fiscal revenues both in the steady-state (3.27%), and during the whole transition path. According to these results, for Italy would therefore be desirable to implement a general tax cut accompanied by stronger monitoring process since, as we have seen, this policy may increase welfare and, at the same time, permanently improve the budgetary situation.

7. Some concluding remarks

In this paper, we develop and estimate a two-sector DSGE model that explicitly accounts for concealed transactions in the economy. Our goal is twofold: to estimate size and trend of the underground economy and to assess the effects of alternative fiscal policies in an economic environment characterized by tax evasion.

We apply our methodology to the Italian economy, using quarterly aggregate data covering the period 1982-2006. The Bayesian estimate of the model produces a time series for the underground economy which is, on average, about 4 percentage points larger than the official estimates. Additionally, we find evidence of a steadily upward trend in the size of the underground economy, primarily driven by the persistent increase in taxation that has been observed in Italy since the eighties. As far as fiscal policy is concerned, numerical experiments based on the estimated model show that, at the actual tax rates, Italy is on the *slippery* side of the steady-state Laffer curve, and can improve its budgetary situation by either reducing taxes or increasing the tax enforcement. However, the analysis of transitional dynamics reveals that for the government would be optimal to undertake a mix between the above two policies, as this turns out to be the only fiscal adjustment that can improve welfare and, at the same time, permanently increase fiscal revenues.

To conclude, we wish to stress that the methodology presented in this paper provides a new method to estimate the underground economy. The novelty of our approach with respect to those available in the literature, is the use of a structural model where the underground economy is treated as an integral part of the economic system. This method allows to estimate the unobservable underground economy by exploiting the information provided by equilibrium conditions of the model. We believe that this methodology is quite general and can readily be applied to other countries to perform comparative analysis of the sources of the underground economy. Moreover, the theoretical model can be easily modified to assess the labor market implications. However, we left these issues for future research.

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Appendix

Appendix A. Method

The basic idea behind the MCMC is to build a Markov chain transition kernel starting from a given initial point and with a limiting invariant distribution that is equal to the posterior distribution of the quantities of interest. Under suitable conditions (see Robert and Casella, 1999, chap. 6-7), such a transition kernel converges in the distribution to the target posterior density $p(\boldsymbol{\theta}|\mathbf{y})$. These Markov chain trajectories are obtained through simulations following a two-step procedure. First, a new movement is proposed by simulating the new position from a proposal distribution, and, second, this move is accepted or rejected according to some suitable probabilities that depend on the likelihood function and on the prior distribution of the parameters $p(\boldsymbol{\theta})$. In a nutshell, given a starting value for the parameter's vector $\boldsymbol{\theta}^{(0)}$, we simulate trajectories of the Markov chain $\{\boldsymbol{\theta}^{(j)},\ j = 1,\ldots,n\}$ whose draws converge to the posterior distribution. Once convergence is achieved, the inference can be based on the generated serially dependent sample simulated from the posterior. More precisely, the estimates of the posterior means $\mathrm{E}_{p(\boldsymbol{\theta}|\mathbf{y})}[\boldsymbol{\theta}]$ are obtained by averaging over the realization of the chains, i.e., $\hat{\boldsymbol{\theta}} = n^{-1}\sum_{j=1}^{n}\boldsymbol{\theta}^{(j)}$. To account for the serial correlation induced by the Markovian nature of this procedure, we estimate the numerical standard error of the sample posterior mean using the approach implemented, for instance, in Kim et al. (1998).

In the MCMC literature, there are many different ways to propose a move for the Markov chain. Our inferential procedure is based on a Random Walk Metropolis-Hastings algorithm, as suggested, for instance, in An and Schorfheide (2007), in which the proposal distribution depends uniquely on the current state of the chain at time j, i.e., $q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(j)})$. The procedure can be summarized as follows:

`MCMC algorithm`

- Initialize the chain at $\boldsymbol{\theta}^{(0)}$
- At step $j = 1,\ldots,n$
  - Update $\boldsymbol{\theta}$ in block through a random walk Metropolis-Hastings scheme

$$\boldsymbol{\theta}^{*} \sim q(\boldsymbol{\theta}|\boldsymbol{\theta}^{(j-1)});$$

– Compute the acceptance probability $\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*)$ defined as

$$\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*) = \frac{p(\boldsymbol{\theta}^*)L(\mathbf{y}|\boldsymbol{\theta}^*)q(\boldsymbol{\theta}^{(\boldsymbol{j-1})}|\boldsymbol{\theta}^*)}{p(\boldsymbol{\theta}^{(j-1)})L(\mathbf{y}|\boldsymbol{\theta}^{(j-1)})q(\boldsymbol{\theta}^{*}|\boldsymbol{\theta}^{(j-1)})}$$

– Draw u from an $U(0,1)$ random variable. If $\alpha(\boldsymbol{\theta}^{(j-1)}, \boldsymbol{\theta}^*) \leq u$
    * Then $\boldsymbol{\theta}^{(j)} = \boldsymbol{\theta}^*$;
    * Else $\boldsymbol{\theta}^{(j)} = \boldsymbol{\theta}^{(j-1)}$;

• $j = j + 1$

Appendix B. The complete log-linearized model

In what follows, we describe the log-linearized equilibrium system of equation. Notice that coefficient evaluated at the steady-state are indicated with the sub-index ss. Taxes at the steady-state are indicated just suppressing the temporal index.

$$\hat{w}_t^m = \hat{y}_t^m - \hat{h}_t^m - \left(\frac{\tau^c}{1-\tau^c}\right)\hat{\tau}_t^c - \left(\frac{\tau^s}{1+\tau^s}\right)\hat{\tau}_t^s$$

$$\hat{w}_t^u = \hat{y}_t^u - \hat{h}_t^u - \left(\frac{ps\tau^c}{1-ps\tau^c}\right)\hat{\tau}_t^c$$

$$\hat{y}_t^m = \hat{A}_t + (1-\alpha)\hat{k}_t + \alpha\hat{h}_t^m$$

$$\hat{y}_t^u = \hat{B}_t + \alpha_u\hat{h}_t^u + (1-\alpha_u)\hat{k}_t$$

$$\hat{y}_t = \left(\frac{y_{ss}^m}{y_{ss}}\right)\hat{y}_t^m + \left(\frac{y_{ss}^u}{y_{ss}}\right)\hat{y}_t^u$$

$$\hat{h}_t = \left(\frac{h_{ss}^m}{h_{ss}}\right)\hat{h}_t^m + \left(\frac{h_{ss}^u}{h_{ss}}\right)\hat{h}_t^u$$

$$\xi\hat{h}_t + \hat{\xi}_t^h = \hat{w}_t^m - \sigma\hat{c}_t - \left(\frac{\tau^h}{1-\tau^h}\right)\hat{\tau}_t^h$$

$$\frac{\Omega_0}{\xi}\hat{\xi}_t^h + \Omega_0\hat{h}_t + \Omega_1\hat{h}_t^u = \hat{w}_t^u - \sigma\hat{c}_t$$

$$\hat{c}_t = E_t\left\{\hat{c}_{t+1}\right\} + \frac{\tau^h\left[\gamma-\beta(1-\delta_k)\right]}{\gamma\sigma\left(1-\tau^h\right)}\rho_h\hat{\tau}_t^h - \frac{\gamma-\beta(1-\delta_k)}{\gamma\sigma}E_t\left\{\hat{r}_{t+1}\right\} - \left(\frac{\gamma-\beta(1-\delta_k)\rho_I}{\sigma\gamma}\right)\hat{\xi}_t^I$$

$$\hat{k}_{t+1} = \left[\frac{(1-\delta_k)}{\gamma}\right]\hat{k}_t + \left[\frac{\gamma-(1-\delta_k)}{\gamma}\right]\hat{x}_t + \left[\frac{\gamma-(1-\delta_k)}{\gamma}\right]\hat{\xi}_t^I$$

$$c_{ss}\hat{c}_t + x_{ss}\hat{x}_t = -\Omega_2\hat{\tau}_t^h + (1-\tau^h)\left[w_{ss}^m h_{ss}^m(\hat{w}_t^m + \hat{h}_t^m) + r_{ss}k_{ss}(\hat{r}_t + \hat{k}_t)\right] + w_{ss}^u h_{ss}^u(\hat{w}_t^u + \hat{h}_t^u)$$

$$r_{ss}\hat{r}_t = (1-\tau^c)(1-\alpha)\frac{y_{ss}^m}{k_{ss}}\hat{y}_t^m + (1-ps\tau^c)(1-\alpha_u)\frac{y_{ss}^u}{k_{ss}}\hat{y}_t^u - r_{ss}\hat{k}_t - \Omega_3\hat{\tau}_t^c$$

$$\widehat{G}_t^h = \widehat{\tau}_t^h + \frac{\tau^h w_{ss}^m h_{ss}^m}{G_{ss}^h}(\widehat{w}_t^m + \widehat{h}_t^m) + \frac{\tau^h r_{ss}k_{ss}}{G_{ss}^h}(\widehat{r}_t + \widehat{k}_t)$$

$$\widehat{G}_t^c = \widehat{\tau}_t^c + \frac{\tau^c y_{ss}^m}{G_{ss}^c}\widehat{y}_t^m + \frac{\tau^c ps y_{ss}^u}{G_{ss}^c}\widehat{y}_t^u$$

$$\widehat{G}_t^s = \widehat{\tau}_t^s + \widehat{w}_t^m + \widehat{h}_t^m$$

$$\hat{w}_t^h = \frac{\tau_{ss}^s}{1+\tau_{ss}^s}\hat{\tau}_t^s + \hat{w}_t^m + \hat{h}_t^m$$

where

$$\Omega_0 = \frac{B_0(h_{ss})^\xi\xi}{B_0(h_{ss})^\xi + B_1(h_{ss}^u)^\phi}$$

$$\Omega_1 = \frac{B_1(h_{ss}^u)^\phi\phi}{B_0(h_{ss})^\xi + B_1(h_{ss}^u)^\phi}$$

$$\Omega_2 = \tau^h\left[w_{ss}^m h_{ss}^m + r_{ss}k_{ss}\right]$$

$$\Omega_3 = \tau^c\left[(1-\alpha)\frac{y_{ss}^m}{k_{ss}} + ps(1-\alpha_u)\frac{y_{ss}^u}{k_{ss}}\right]$$

Appendix C. Impulse-Response Functions

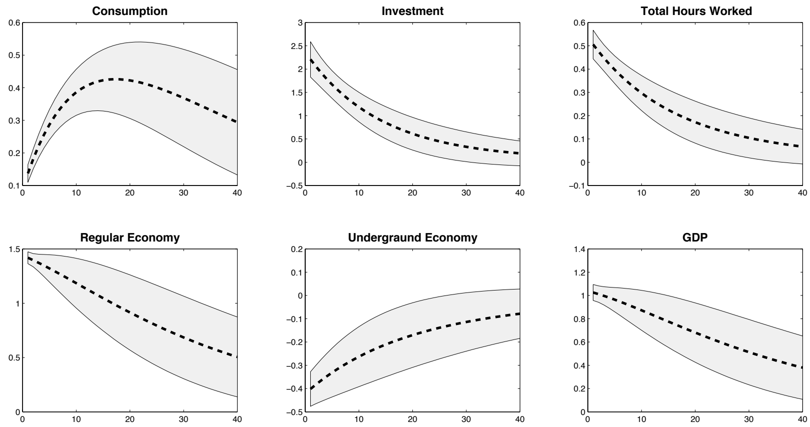

Figure C.5: **Technology Shock. Regular Economy.** A_t

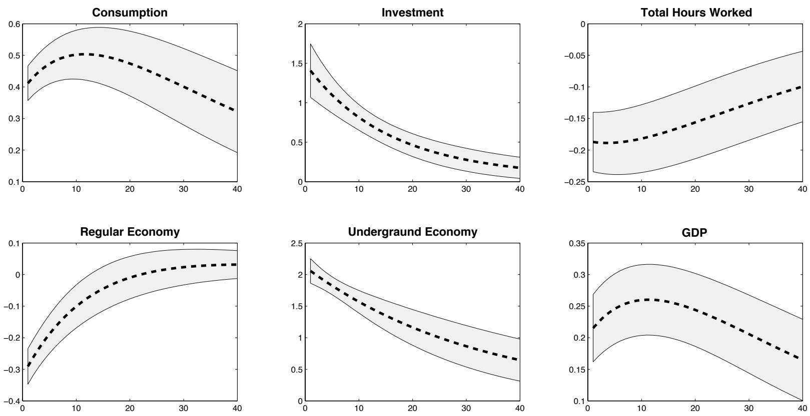

Figure C.6: **Technology Shock. Underground Economy.** B_t

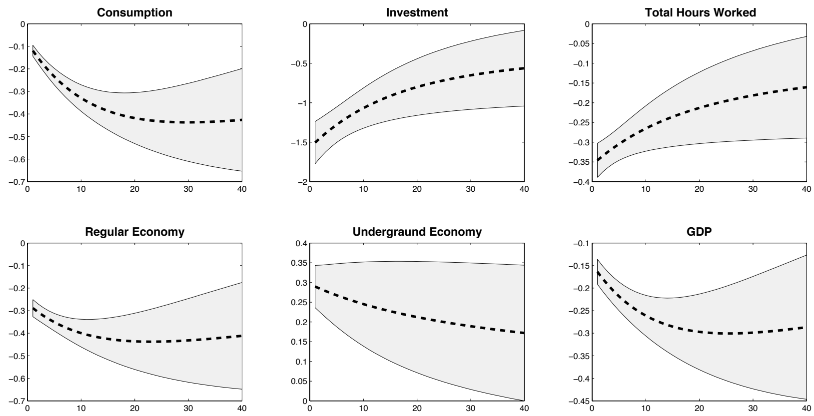

Figure C.7: **Fiscal Shock. Corporate Tax Rate.** τ_c

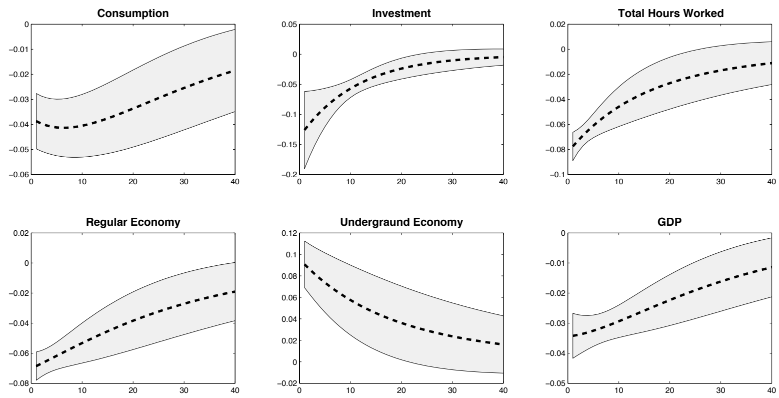

Figure C.8: **Fiscal Shock. Social Security Tax Rate.** τ_s

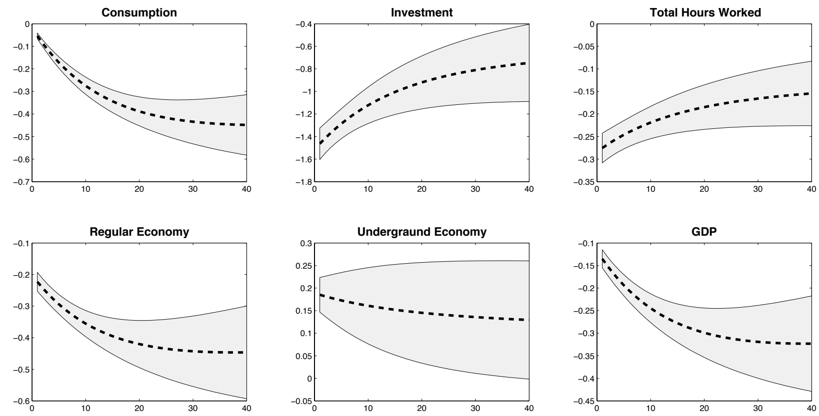

Figure C.9: **Fiscal Shock. Personal Income Tax Rate.** τ_h

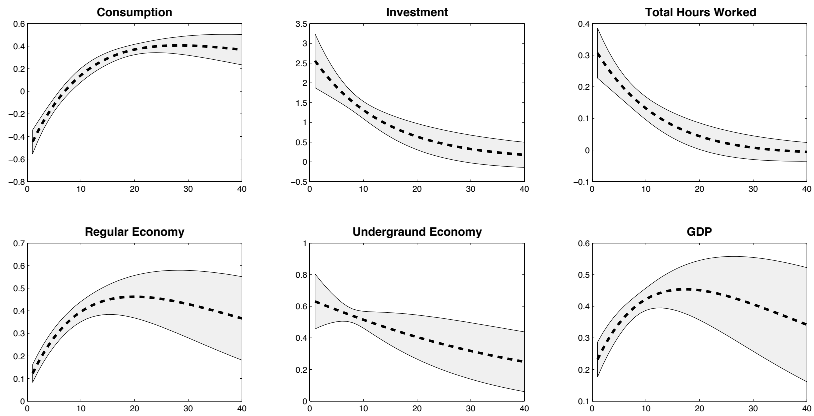

Figure C.10: **Demand Shock. Investment Shock.** ξ_I

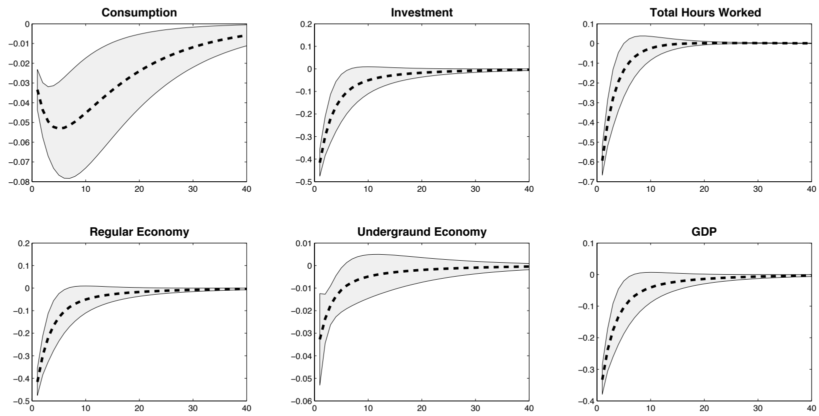

Figure C.11: **Demand Shock. Disutility of labor.** ξ_h

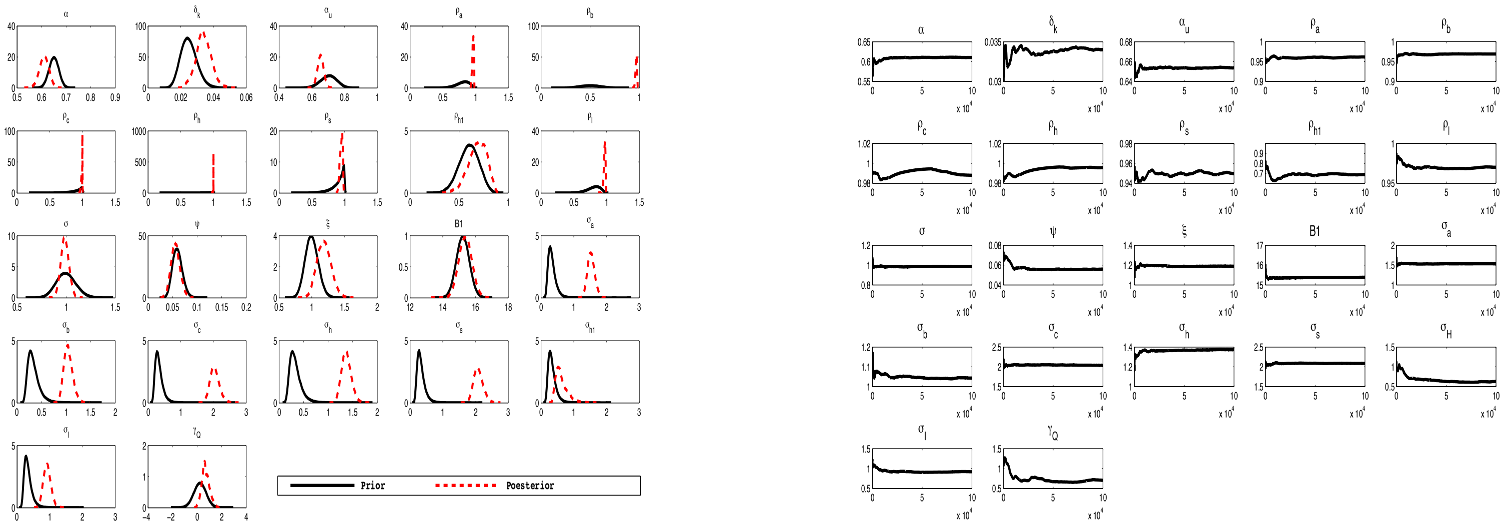

Figure C.12: Prior and posterior distribution of estimated parameters

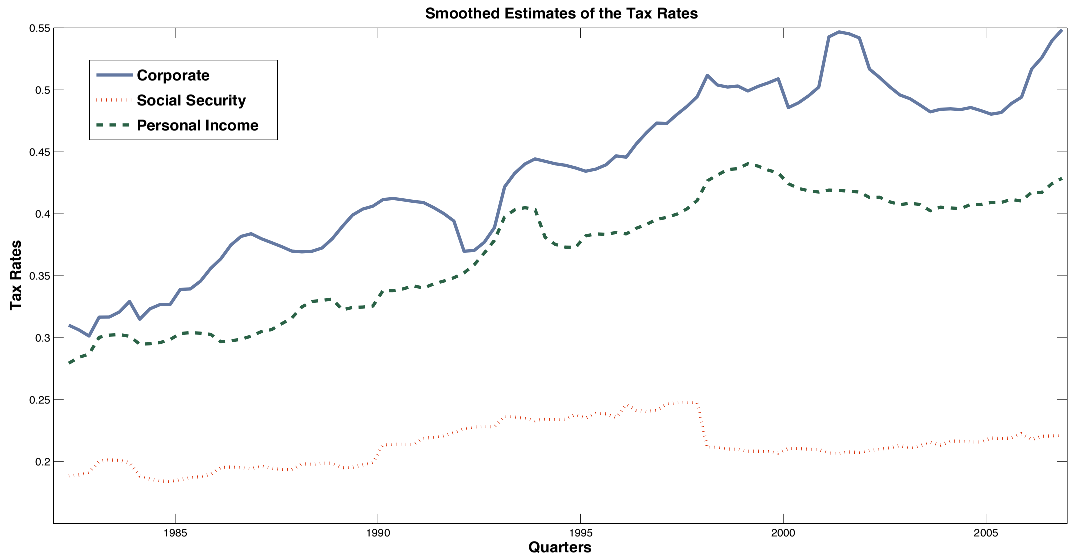

Figure C.13: Smoothed Estimate of the Tax Rates

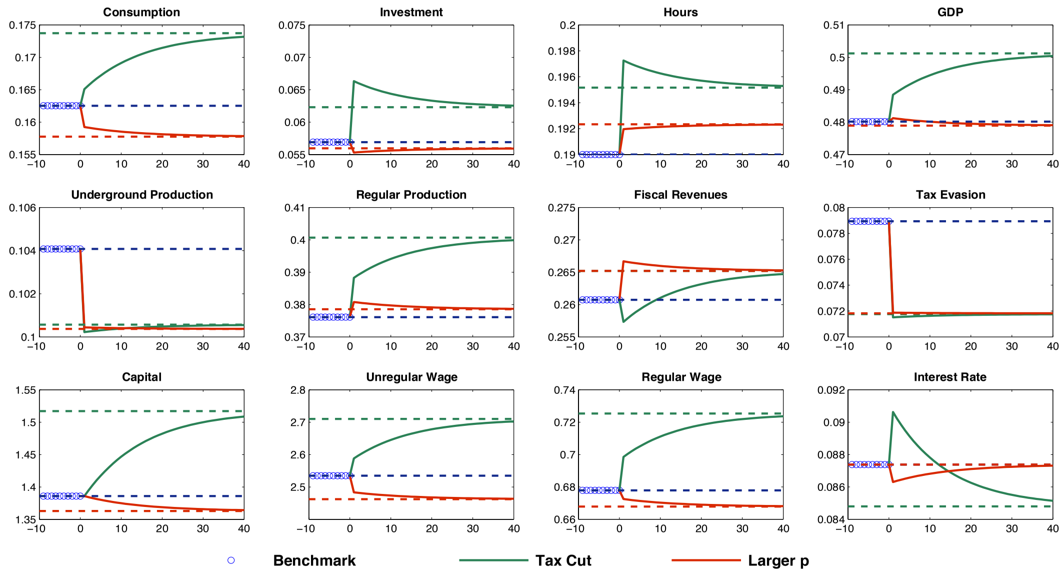

Figure C.14: Transition Dynamics

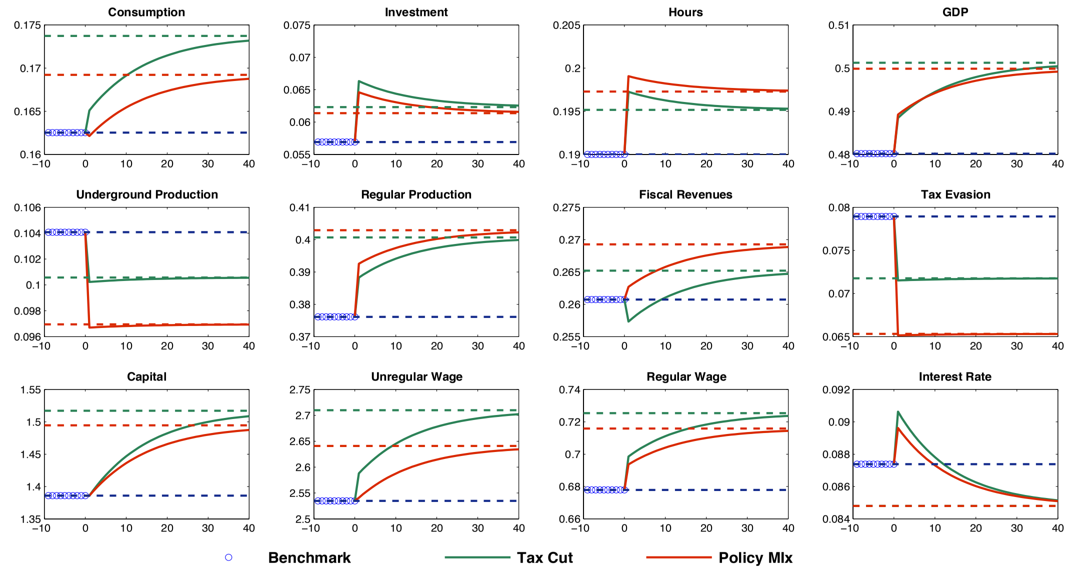

Figure C.15: Transition Dynamics

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