ISSN 2282-6483

Alma Mater Studiorum - Università di Bologna
DEPARTMENT OF ECONOMICS

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Quaderni - Working Paper DSE N°1009

Vertical flexibility, outsourcing and the financial choices of the firm*

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June 4, 2015

Abstract

We investigate the relationship between the extent of vertical flexibility and the underlying financial choices of a firm. By vertical flexibility we mean the opportunity to outsource a necessary input and to reverse the choice as input market conditions dictate. A firm simultaneously selects the portion of equity and debt and its vertical setting. Debt is provided by a lender that requires the payment of a fixed coupon over time and, as a collateral, an option to buy out the firm in certain circumstances. Debt leads to the same level of flexibility acquired by an unlevered firm. However, investment to set up a flexible technology occurs earlier. An alternative to debt is the involvement of venture capital for the production of the input. We explore this second avenue finding that the extent of outsourcing adopted is lower than for the unlevered firm, but the firm invests earlier.

Keywords: vertical integration, flexible outsourcing, debt, equity and venture capital.

JEL Classification: C61; G31; G32; L24.

*We acknowledge the financial support of the Universities of Bologna and Padova within the 2014-15 RFO scheme and Fondazione Cassa dei Risparmi di Forlì. We thank the audiences of the 2014 ASSET Conference held in Aix en Provence (France) and of a seminar held in Trieste for comments and suggestions.

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1 Introduction

The purpose of the ensuing pages is to analyse the relationship between the financial structure and the flexibility of the vertical organization of a corporate enterprise. By vertical flexibility we mean the opportunity for a firm to buy inputs from the market, i.e., to do outsourcing (OS) in a variable and reversible manner, going back to internal production if necessary. The organizational aspects of OS and flexibility are crucial for most firms which buy inputs in different and variable proportions and change quite often the extension of activity along the vertical chain of the producion process. Flexibility improves the ability to cope with uncertain scenarios and has considerable effects on competitiveness, scale of production and social efficiency[1]. OS and flexibility do not come for free since the acquisition of inputs from the market requires the setting up of a supply chain with specific logistic investment. A vertically flexible firm decides to substitute an internally produced input with an externally provided one while keeping the option of bringing back in-house (backsourcing or reshoring) the same production. In such a case it must keep alive a dedicated internal facility and the associated know-how. As a matter of fact, flexibility may turn out to be quite dear. Moreover, the costs of flexibility may be affected by technical progress, by efficiency of external markets, i.e., the opportunity of buying easily inputs from producers which may be specialized or located in low cost countries and, last but not the least, by the design of a proper capital budgeting. This final aspect is crucial: each firm should try to finance vertical flexibility in the best way in terms of the mix between equity, debt and other possible financial sources such as venture capital[2]. Unfortunately this financial aspect is often sidestepped in the analysis of both vertical relationships and flexibility since funding and organization themes are studied separately in financial[3], managerial, industrial organization and operations research literature[4]. This partly unexplored field requires to analyse jointly finance and vertical organization issues. On the real side we shall be concerned with the extent and the type of vertical flexibility, that can be secured by arms' length OS of inputs while maintaining in all cases a partial in-house prudential production. On the financial side we shall see how the mix between equity and debt or the participation of a venture capitalist may affect the extent of flexibility acquired and the time sequence of the investment in flexibility. Financial sources may be represented by new equity, debt (convertible or nonconvertible) or by a venture capitalist. We exclude from our investigation new equity raised through an IPO (initial public offering) since it tends to reduce the price of existing stock and may open the way to a loss of control[5].

Since flexible technologies reduce risk (profit volatility) they may be considered as a kind of (real) option and their price should reflect their (option) value (Amran and Kulatilaka, 1999, Ch.

[1] In a strategic environment the amount of flexibility adopted by interacting firms (Yoshida, 2012) affects uncertainty which becomes endogeneously determined. The more flexible is a firm the more uncertain becomes the scenario for the rival. In our analysis we disregard strategic market interactions. Therefore, uncertainty is exogeneous and is not dependent upon flexibility.

[2] Other financial channels may be activated by a firm. For the sake of simplicity we confine to debt, equity and the involvment of a venture capitalist.

[3] See, for a good survey of main related issues, Tirole (2006).

[4] See Van Mieghen (1999), Wang, Liu and Wang (2007), Moretto and Rossini (2012) where a good deal of literature on these latter aspects is surveyed.

[5] See Eckbo, Masulis, and Norli (2007).

16). As a result the presumption is that the value of a vertically flexible firm be weakly larger than the value of a corresponding non flexible firm. Yet, we shall see that this is not always the case whenever the cost of flexibility and the related financial aspects are properly taken into account.

Our investigation is prompted by broad casual observation and press reports[6] showing that most firms change over time their vertical production structure, expanding and/or subsequently reducing (or the other way round) the extent of OS of inputs[7]. For instance in the automotive industry most brands adopt partial OS, i.e., concomitant internal production and purchase of engines and other intermediate products from external sources. Moreover, the extent of OS is frequently changed as witnessed by the variable level of value added over revenue found in balance sheets and, indirectly, in everydays news. Since different organizational settings exhibit distinct degrees of risk it is worth seeing how the financial choices affect the degree of flexibility acquired.

Literature has recently examined vertical flexibility (Shy and Stenbacka, 2005; Alvarez and Stenbacka, 2007; Moretto and Rossini, 2012; Yoshida, 2012) scantly going into the relationship with capital structure. Contributions on the link between industrial decisions and financial structure may be found in Lederer and Singhal (1994), in Leland (1998), in Mauer and Sarkar (2005), Benaroch et al. (2012), Banerjee et al. (2014). Mainstream literature does not address the specific question about which financial decision favors vertical flexibility and OS. However, most of the few contributions, show that inefficiency arises if organizational and strategic decisions are not taken simultaneously with financial choices. Mauer and Sarkar (2005) focus on the agency cost of financing investment with debt in a dynamic stochastic framework. In a similar environment Leland (1998) digs the same topic raised in the seminal paper of Jensen and Meckling (1976). Unlike Leland (1998), Mauer and Sarkar (2005) emphasize the inefficiency of debt. In the traditional Modigliani and Miller (1958) scenario the value of a firm is given by the sum of its liabilities. Equity and debt turn out to be quite close (in certain circumstances, perfect) substitutes. However, equityholders and debtholders do not usually coincide and each group maximizes a different objective function. Shareholders maximize the equity value while debtholders maximize the debt value. The consequence is a subadditive result. Only a "social planner" would rather maximize the sum of debt and equity pursuing a first best. Mauer and Sarkar (2005) calculate the agency cost of debt as the difference between the total value of a firm where each group of stakeholders optimizes separately and the case where the whole value of the firm is jointly maximized. Equityholders, in a limited liability legal framework, tend to overinvest if they do not face the proper agency cost of debt confirming the old Jensen and Meckling (1976) wisdom. The issue of going back and forth from (complete) OS to vertical integration is studied in Benaroch and al. (2012) who analyse the particular case of service production. OS may allow a firm facing volatile demand to avoid the risk of bearing fixed costs that cannot be easily covered. By (complete) OS of services which are capital intensive the firm turns a fixed into a variable cost cutting risk. If it wants to go back to internal production it must bear each time a fixed cost. While in our model we go through the privately optimal (hence, variable) extent of OS contingent upon the capital structure adopted, in the Benaroch et al. (2012) paper the main question is about the optimal switching from (complete) OS to backsource and

[6]For instance Apple has recently increased the OS of some inputs while reducing and bringing back home other inputs. See for further examples: *The Economist* (2011, 2013), *Forbes* (2012).See also empirical assesments in Klein (2005) and Rossini and Ricciardi (2005).

[7]Examples may found in Benaroch et al. (2012).

the value of the switching option, which is bound to increase with demand volatility and the skill intensity of the production process of the input.

In Banerjee et al. (2014) the investment in a new technology, such as a flexible vertical process, financed by an external subject is seen as a joint option. Size of the investment, timing of the exercise of the option and rule concerning the sharing of returns of the investment have to be established jointly by the firm and by the financial investor. According to Banerjee et al. (2014) it is inefficient to specify a sharing mode before the venture is carried out. Bakhtiari and Breunig (2014), at firm level on longitudinal data, assess OS as a device to smooth demand uncertainty. They find an asymmetric link with demand fluctuations, i.e., OS increases substantially during slumps while does not respond much to demand increases. Some scanty data investigation on the financial counterpart of OS is attempted but it is fairly inconclusive. OS appears definitely as a shield against market contraction. In Moon and Phillips (2014) a higher level of OS makes the firm less risky in terms of cash flows. The result is a capital structure with less debt and more equity mainly in high value-added industries.

In the ensuing pages we are going to consider two alternative cases. In the first the control right over the investment decision is allocated to the firm (i.e., the shareholders), while in the second case the control belongs with an outside investor (i.e., a venture capitalist). As in Banerjee et al. (2014) both actors agree in advance over the sharing rule of the project value. While the timing of the investment is determined by one party the terms of the investment are determined by both parties. In both cases the level of OS is always set by the operating party.

As to the financial terms of the investment, in the first case we shall be concerned with debt financing. To overcome the agency problem of debt, the lender is granted an option to buy out the firm if OS becomes the main source of profits for shareholders and in-house production gets almost irrelevant. The alternative case considers a pure equity offer: ownership is shared with an outside investor (venture capitalist) without side payments (i.e., no debt service by the firm).

In our endeavor we shall couple two streams of contributions: one on vertical flexibility and the timing of adoption of a specific technology to carry out OS (Moretto and Rossini, 2012; Shy and Stenbacka, 2005; Alvarez and Stenbacka, 2007; Alipranti, Miliou and Petrakis, 2014) and another on financial choices of a firm in an uncertain dynamic framework (Leland, 1994; Lederer and Singhal, 1994; Banerjee et al., 2014; Triantis and Hodder, 1990).

With debt the firm may rush to adopt flexibility, but it may be hard to finance it unless the lender gets as a collateral an option to buy the entire firm in case flexibility turns out not to be profitable enough. The result is that (warranted) debt makes a firm invest earlier in the vertically flexible technology. When a venture capitalist is involved, again the investment occurs earlier but the extent of OS is lower than with pure equity.

The paper roadmap is the following: in section 2 we see the basic model, in section 3 we go through the value of a vertically flexible firm in the control case without debt, in section 4 we introduce debt, in section 5 we go through the case of venture capital. The epilogue is in section 6.

2 The basic set up

We consider the internal organization of a *vertically flexible* enterprise that for each unit of output to be produced needs one unit of a perfectly divisible input (*perfect vertical complementarity*). The firm has to decide whether to buy a vertically flexible technology that allows to manufacture the input in-house at the constant marginal cost d, to resort (totally or partially) to OS, in case the market price of the input, c_t, is low enough, and to reverse the choice over time (backsourcing), if c_t goes up to a sufficient extent. In the specific, the enterprise, at any time, can switch from totally making the input in-house when $\hat{c}_t \equiv \alpha c_t + (1-\alpha)d$ rises above d, to partially purchasing it if $\hat{c}_t$ falls below d and viceversa, where $\alpha \in (0, 1]$ is the outsourced share (its complement to one is the home produced portion).

Then, assuming, for the sake of tractability, that there are no fixed costs in the production of the input, the instantaneous profit function can be written as:[8]

$$\pi_t \equiv \max\{0, \ [p - d + \max(d - \hat{c}_t, 0)]\} \tag{1}$$

where p is the output market price. When $\alpha < 1$ the firm uses a linear combination of produced and procured input. It can go back to vertical integration if $\hat{c}_t$ becomes too high. Finally, to avoid default, we assume that $p - d > 0$.[9]

The sunk cost of the flexible technology is given by:[10]

$$I(\alpha) = k_1 + \frac{k_2}{2}\alpha^2 \quad \text{for } \alpha \in (0, 1] \tag{2}$$

where k_1 is the direct cost to keep internal facilities working (i.e., the cost of maintenance and updating the process for the internal production of the input) with total or partial OS. The term $\frac{k_2}{2}\alpha^2$ is the organizational cost to design and run a system devoted to obtain a cost advantage from a vertically flexible technology and to procure the input from the market (Simester and Knez, 2002). That requires setting up a supply chain of subcontractors, monitoring input quality and contract enforcement and so on.

We do not consider investment in capacity expansion, i.e., we assume that capacity is already employed to meet demand in the best way producing the input in-house. The cost to keep in operation the internal facilites is fixed whilst the organizational cost grows as the extent of OS

[8] Fixed costs would not change qualitatively our conclusions. They give rise to a hysteresis interval in the option to switch from producing the input in-house to outsourcing it. See Benaroch et al. (2012) for the consideration of fixed costs.

[9] Vertical flexibility, as stressed in the introduction, is an insurance against risk based on the maintanance of the know how and the facilities to produce the input in house. This assumption allows us to focus on differential arrangements to finance and to see how they affect the decision as to whether and when to invest in the flexible technology and as to the extent of OS.

[10] The sunk cost to build up the mixed technology is assumed quadratic only for the sake of simplicity. None of the results is altered if the investment cost is of type $I(\alpha) = k_1 + k_2\alpha^\delta$ with $\delta > 1$ as in Alvarez and Stenbacka (2007).

tends towards its maximal level[11]. Therefore, we explicitly esclude the case $\alpha = 0$ with $k_1 > 0$.[12]

Notice that, when $\alpha = 1$, the firm buys the input entirely from an independent provider, while keeping the option of returning to complete internal manufacturing.[13]

The scenario is one of dynamic uncertainty where the input market price c_t follows a geometric Brownian motion[14]:

$$dc_t = \gamma c_t dt + \sigma c dz_t \tag{3}$$

where dz_t is the increment of a Wiener process (or Brownian motion) uncorrelated over time, σ the istantaneous volatility of the market input price and γ the drift parameter. [15]

Finally, as anticipated in the introduction, we assume that the firm may finance the required investment for the flexible technology in two alterantive ways. 1) By debt, which may become convertible since it contains an option on the existing shares. 2) By venture capital to finance the production of the input. In both cases the constant discount factor is r. Shareholders, lenders and venture capitalists are all assumed to be risk neutral.[16]

3 The benchmark case: an unlevered vertically flexible or pure equity enterprise

As a benchmark, we consider the optimal OS share and the optimal investment policy of a firm entirely financed by equityholders (i.e., the unlevered firm value).

3.1 The operating value

We go through the operating firm's value in two distinct cases. In the first we consider a *vertically integrated* firm manufacturing the input in house, if $\hat{c}_t > d$, keeping the option of buying it. In

[11]The increasing cost of recurring to OS may be seen as the mirror image of a (specificity based) hold-up which grows with the share of OS as Transaction Cost Economics (TCE) that emphasizes how hold-up in OS relationships make input markets less efficient than internal production (Williamson, 1971; Joskow, 2005; Whinston, 2003).Of course generic inputs like, for instance, janitorial services do not require specific know how and cannot be modeled in this way (Anderson and Parker, 2002; Holmes and Thornton, 2008) while for other services flexibility of OS may matter a lot (Benaroch et al., 2012).

[12]The case $\alpha = 0$ with $k_1 > 0$, rapresents the standard case where the firm invests in a plant just to produce the input in house. We neglect this case.

[13]This is the case of Benaroch and al. (2012) where entry and exit occur always with $\alpha = 1$ and there is a fixed cost. In our framework, once the flexible technology has been acquired, the firm can enter and exit without further costs at any level of α.

[14]The dynamic setting adopted implies that the input market is perfectly competitive or that the forces moving the price over time do not depend on the market structure. A different approach is adopted by Billette de Villemeur, Ruble, Versaevel (2014) where an imperfect market for the input in the upstream section of production makes the firm delay entry. In such cases vertical integration regains its superiority.

[15]Input price uncertainty may be due to the exchange rate if the input is bought abroad (see Kogut and Kulatilaka, 1994; Dasu and Li, 1997; Kouvelis et al. 2001).

[16]Alternatively, under the assumption of complete capital markets, we can assume that there are some traded assets that can be used to hedge the input cost uncertainty z_t of (3). These traded assets with a riskless asset allow to construct a continuously re-balanced self-financing portfolio that replicates the value of the firm (Constantinides, 1978; Harrison and Kreps, 1979; Cox and Ross, 1976).

the second case we see an enterprise which adopts *OS*, if $\hat{c}_t < d$, acquiring a share α of the input while making in-house the remaining $1-\alpha$, keeping the option to manufacture the whole input requirement, if $\hat{c}_t$ goes further up. Since, for $\alpha > 0$, the condition $\hat{c}_t > d$ implies $c_t > d$, standard arguments lead to a general solution for the unlevered operating firm's value taking the following functional form (See Appendix A):

$$V^U(c_t;\alpha) = \begin{cases} \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} & \text{if } c_t > d \\ \\ \left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right) + \tilde{B}c_t^{\beta_1} & \text{if } c_t < d. \end{cases} \tag{4}$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive roots of the characteristic equation: $\Phi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1) + \gamma\beta - r$.

Notice that $V^U(c_t;\alpha)$ is a convex function of c_t, with $\lim_{c\to\infty} V^U(c_t;\alpha) = \frac{p-d}{r}$ and $\lim_{c\to 0} V^U(c_t;\alpha) = \frac{p-(1-\alpha)d}{r}$. Moreover, $\frac{p-d}{r}$ and $\left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right)$ are the present values of the firm associated to the two distinct vertical arrangements and, as it appears from (4), viable in-house production rules out any closure option or default. Additional terms $\tilde{A}c_t^{\beta_2}$ and $\tilde{B}c_t^{\beta_1}$ indicate respectively the value of the option to go from *vertical integration* to *OS* and the other way round. The constants $\tilde{A}$ and $\tilde{B}$ are positive and equal to (See Appendix A):

$$\begin{cases} \tilde{B}(\alpha) = \alpha B \equiv \frac{\alpha}{\beta_1-\beta_2}(r-\gamma\beta_2)d^{1-\beta_1}\frac{1}{r(r-\gamma)} \\ \tilde{A}(\alpha) = \alpha A \equiv \frac{\alpha}{\beta_1-\beta_2}(r-\gamma\beta_1)d^{1-\beta_2}\frac{1}{r(r-\gamma)}. \end{cases} \tag{5}$$

If $\alpha \to 0$, the firm is vertically integrated, both $\tilde{A}$ and $\tilde{B} \to 0$. If $\alpha \to 1$ the input is bought entirely from an independent provider. Even in this extreme case, the firm has the option to switch to internal production that (represented by $\tilde{B}c_t^{\beta_1}$) makes for a larger value of the firm than without the reversal opportunity.

3.2 The optimal OS share and investment timing

Let's now derive both the optimal investment timing and OS. The firm optimally sets the proportion of OS once the investment in the flexible technology is carried out. Therefore, by working backward, we determine the optimal α. We consider the case of a firm manufacturing in-house the input, while holding the option to switch to OS, at a future date, if c_t becomes lower than d.[17]

Then, with $c_t > d$, the problem is to select α that maximizes (4) minus the cost of setting up a dedicated production organization consistent with OS, i.e.:

$$\alpha^{*U} = \arg\max\left[\frac{p-d}{r} + \tilde{A}c_t^{\beta_2} - I(\alpha)\right] \tag{6}$$

[17] Adopting a different starting point would not make sense since the option to do OS exists only if the firm is not doing it.

where $I(\alpha)$ is given by (1). Solving (6) we get:

$$\alpha^{*U}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \leq \tilde{c}^U \\ \frac{A}{k_2} c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^U \end{cases} \tag{7}$$

where $\tilde{c}^U \equiv \left(\frac{k_2}{A}\right)^{1/\beta_2}$. Since $\frac{\partial \alpha^{*U}}{\partial c_t} < 0$, if c_t is low it is better to choose complete OS, while, as c_t increases α goes down and tends to zero for high values of c_t. In other words, as c_t rises it becomes less likely that it will fall enough to justify investment in flexibility.

Let's now turn to the optimal investment policy. Denoting by c^{*U} the input price triggering investment, the value of the option to invest (i.e. the ex-ante value of the firm), is given by:

$$F^U(c_t) = \max_{T^{*U}} E_t[e^{-r(T^{*U}-t)}] \left[V^U(c^{*U}, \alpha^{*U}(c^{*U}) - I(\alpha^{*U}(c^{*U})\right] \tag{8}$$

where $T^{*U} = \inf\{t \geq 0 \mid c_t = c^{*U}\}$ is the optimal investment timing, and $\alpha^{*U}(c^{*U})$ is the optimal OS share at entry**.** The standard method used for V^U can be applied again to find the general solution of (8) and to derive c^{*U}. In particular assuming that the current value of c_t is sufficiently high so that immediate investment is not optimal, we can prove that:

Proposition 1 *The ex-ante value can be written in compact notation as:*

$$F^U(c_t) = \begin{cases} F^U c_t^{\beta_2} & \text{for } c_t > c^{*U} \\ \frac{p-d}{r} + \tilde{A}(\alpha^{*U}(c^{*U}))c_t^{\beta_2} - I(\alpha^{*U}(c^{*U})) & \text{for } d < c_t \leq c^{*U} \end{cases}. \tag{9.1}$$

where the constant $F^U = \tilde{A}$, *while the optimal trigger is:*

$$c^{*U} = \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r} - k_1\right)}}{A}\right]^{1/\beta_2} \tag{9.2}$$

and the OS share is

$$\alpha^{*U} = \min\left[\sqrt{\frac{2\left(\frac{p-d}{r} - k_1\right)}{k_2}} \;,\; 1\right] \tag{9.3}$$

Proof See Appendix A

Notice that a solution for c^{*U} exists if $k_1/\frac{p-d}{r} < 1$.[18] As the firm maintains the ability to produce the input in-house, the ex-ante value of the option to invest is simply given by the value of the option to do OS, once the flexible technology has been adopted, i.e.:

[18] We assume that this always holds.

$$F^U(c_t) = \tilde{A}(\alpha^{*U}(c^{*U}))c_t^{\beta_2} \quad \text{for } c_t > c^{*U} \tag{10}$$

Further, as $\frac{\partial \alpha^{*U}}{\partial c_t} < 0$, it is evident from (9.2) and (9.3) that a necessary condition for having $c^{*U} > \tilde{c}^U > d$ and then $\alpha^{*U} < 1$, is $\frac{p-d}{r} - k_1 < \frac{k_2}{2}$. Otherwise it is always better to set $\alpha^{*U} = 1$. In words, the firm sets $\alpha^{*U} = 1$ if in-house input production leads to profits sufficient to cover organizational costs to buy the entire input requirement.

4 Debt funding with a take over option (warrant)

Going through the case of debt, we assume that the firm negotiates a contract with a (financial) investor to get the funds to cover part of the cost of the flexible technology paying a fixed coupon D per year. Unlike traditional (riskless) debt financing, the shareholders grant the lender a call option to buy out the firm to make the project attracting. That may occur if operative profits become very high as the market input price has gone extremely low and the flexible technology is expected to become useless. This option may be seen as a warrant on the debt, i.e., a kind of "sweetener" for the investor.[19]

The sequence of moves, in this case, is the following: first the firm and the lender decide the terms of the contract (i.e., the coupon and the buy out option in the covenant). Then, the firm optimally sets both the level of flexibility α and the investment timing while the lender chooses how much to lend and when to buy out the firm.[20]

Since the funding contract contains a specific covenant (the warrant) allowing the lender to buy out the firm it seems reasonable to assume that a rational shareholder signs the contract only if the coupon $D < p - d$. In addition, we assume that the lender, who takes over, to minimize risk continues production with the optimal share decided by the incumbent shareholders.

4.1 The operating value

As for the benchmark case, we first compute the market value of the production facility which is given by the sum of the market value of equity and of debt. In this case, the instantaneous profit is:

$$\pi_t \equiv [p - d - D + \max(d - \hat{c}_t, 0)] \tag{2bis}$$

where the technology allows the firm to manufacture the input in-house with profits $p - d - D \geq 0$.

[19] The loan may be considered as a convertible (into equity) debt. To some extent all kinds of debt may be liable to be considered as convertible into something else even if there are infinite types of conversion of debt according to the financial rules and the legal framework in which that occurs. After all each debt implies a collateral, i.e., some kind of pawn.

[20] Notice the relevance of the point concerning who sets the timing of the investment. The evaluation of debt may take place in different scenarios. We confine to a simple, realistic, framework where the lender buys out the entire equity and adopts the outsourcing setting chosen by incumbent shareholders. Of course this is not the only possible scenario. We may consider cases in which the option is not to buy the entire equity but just a chunk or cases in which the lender decides to keep flexibility without constraining to OS for ever.

4.1.1 Equity

Defining $E(c_t; D)$ as the market value of the equity, the analogous general solution for the value of levered equity is:

Lemma 1 *The value of levered equity (for incumbent shareholders) is:*

$$E(c_t;\alpha) = \begin{cases} \frac{(p-d-D)}{r} + \hat{A}c_t^{\beta_2} & \text{if} \quad c_t > d, \\ \left(\frac{p-(1-\alpha)d-D}{r} - \frac{\alpha c_t}{(r-\gamma)}\right) + \hat{B}_1 c_t^{\beta_1} + \hat{B}_2 c_t^{\beta_2} & \text{if} \quad c^l < c_t < d \\ 0 & \text{if} \quad c_t \le c^l \end{cases} \tag{11}$$

where $\beta_1 > 1$ and $\beta_2 < 0$ are the roots of the characteristic equation $\Phi(\beta)$ and c^l the level of the input price that triggers the buy out by the lender.

Proof See Appendix B

As before, the terms $\hat{A}c_t^{\beta_2}$and $\hat{B}_1 c_t^{\beta_1}$ indicate respectively the value of the option to go from *vertical integration* to *OS* and the other way round. Differently, the term $\hat{B}_2 c_t^{\beta_2}$ is the loss for the incumbent shareholders when the firm is bought out, therefore $\hat{B}_2 < 0$. This loss can be seen as a kind of *agency cost* (as in Mauer and Sarkar, 2005), that the equity has to pay to the lender. In the absence of any agency fee shareholders would excessively increase debt since they are protected by limited liability. That puts a boundary on losses which cannot exceed equity while leaving to shareholders the opportunity of getting the upside cream, i.e., profits, in bonanza times. In other words, the option of the lender to buy out the leveraged firm decreases the equity market value. $\hat{B}_2 c_t^{\beta_2}$, representing the loss due to the threat of take over by the lender, is equal to the value of the call option in the hands of the lender who has the right to buy out the firm if c_t goes below c^l.[21]

Furthermore, by the value matching and the smooth pasting conditions at $c_t = d$, we are able to show that (see Appendix B):

$$\hat{A} = \tilde{A} + \tilde{B}_2, \quad \hat{B}_1 = \tilde{B} \quad \text{and} \quad \hat{B}_2 < 0 \tag{12}$$

The constant $\hat{B}_1$ is the same regardless of whether the firm has to decide only the extent of vertical flexibility or the capital structure as well. In other words, once the investment is undertaken, the option value of flexibility to go from *OS* to *vertical integration* remains the same regardless of the way it is financed. On the contrary the option value to go OS differs with respect to the unlevered firm, since it carries the risk of being taken over. Now, the constant $\hat{A}$, may even turn out negative. Here, the novelty concerns $\hat{B}_2$ which takes into account the possible buy out by the lender if the input price goes below the threshold c^l. If the take over threat is not high (i.e., $c^l \to 0$) the option value of OS is definitely positive while, if the threat is quite high, it is not profitable to do OS and the relative option suffers. Then, the firm must consider the effect on its equity value of financing OS and flexibility with debt.

[21] c^l must be lower than the internal cost of production d for the buy out to make sense.

4.1.2 Debt

The market value of debt $D(c_t;\alpha)$, since it has no stated maturity, will be given by:

Lemma 2 *The value of debt is:*

$$D(c_t;\alpha)=\begin{cases}\frac{D}{r}+Cc_t^{\beta_2} & \text{if} \quad c_t>c^l,\\ \frac{p-(1-\alpha)d}{r}-\frac{\alpha c_t}{r-\gamma} & \text{if} \quad c_t\leq c^l.\end{cases} \tag{13.1}$$

where $C=-\frac{1}{\beta_2-1}\left[\frac{p-(1-\alpha)d-D}{r}\right](c^l)^{-\beta_2}>0$, *while the buy out trigger is:*

$$c^l=\frac{\beta_2}{\beta_2-1}\frac{(r-\gamma)}{\alpha}\left[\frac{p-(1-\alpha)d-D}{r}\right]>0 \tag{13.2}$$

Proof See Appendix B

The take over occurs when the flexible technology is expected to become useless, i.e., when the market input price has gone substantially low to suggest that it will be better to buy the input, rather than producing it, for ever. The fresh owner will behave like the former shareholders in terms of optimal strategies adopted by the firm. This assumption is a simplication. Other possible scenarios may be featured**.**

Some comparative statics shows that:

$$\frac{\partial c^l}{\partial\alpha}<0 \qquad \text{and} \qquad \frac{\partial c^l}{\partial D}<0.$$

The negative relationship between c^l and α shows the countervailing interests of the shareholdersm vis à vis the lender. If the firm sets a low level of α (i.e., it tends to be vertically integrated), the lender would find it profitable to buy the firm, i.e., $c^l\to d$. On the contrary, if the firm adopts a high α (i.e., the input is bought mainly from an independent provider), the lender prefers not to bear the risk and sticks to the coupon D. If α is high the benefit of keeping the facility to produce in-house has a low value. As for the second comparative statics inequality it appears that an increase in the coupon (the benefit for the lender) lets the trigger price decrease, i.e., the take over becomes less likely. With a larger coupon the lender gets a higher compensation that relaxes the take over threat and is less eager to buy out the firm by converting debt into equity. The assumption $p-d-D>0$ guarantees that both c^l and C are positive.

Finally, by Lemma 1 and 2, the market value of the levered firm is given by:

$$\begin{aligned}V^L(c_t;\alpha) &= E(c_t;\alpha)+D(c_t;\alpha) \\ &= \begin{cases}\frac{p-d}{r}+\hat{A}c_t^{\beta_2}+Cc_t^{\beta_2} & \text{if} \quad c_t>d,\\ \frac{p-(1-\alpha)d}{r}-\frac{\alpha c_t}{r-\gamma}+\hat{B}_1c_t^{\beta_1}+\hat{B}_2c_t^{\beta_2}+Cc_t^{\beta_2} & \text{if} \quad c^l<c_t<d\\ \frac{p-(1-\alpha)d}{r}-\frac{\alpha c_t}{r-\gamma} & \text{if} \quad c_t\leq c^l\end{cases}\end{aligned} \tag{14}$$

where, using (12) and (13.1), we are now able to isolate the constants $\hat{A}$ and $\hat{B}_2$. In particular we get (See Appendix B):

$$\hat{A} = \tilde{A} - \tilde{B}c^{l(\beta_1-\beta_2)} - C, \quad \text{and} \quad \hat{B}_2 = -\tilde{B}c^{l(\beta_1-\beta_2)} - C. \tag{15}$$

Note that, if $c^l \to 0$ (i.e., the firm is never bought by the lender), then $\hat{A} \to \tilde{A} > 0$ is always positive and $\hat{B}_2 \to 0$. We are back to the unlevered firm as in Section 4. On the contrary, if $c^l \to d$ (i.e., the firm is bought the first time it does OS), then $\hat{A} \to -\alpha d \frac{\gamma}{(r-\gamma)} \frac{1}{r} d^{-\beta_2} - C < 0$ which is always negative as well as $\hat{B}_2$. This is a crucial result in our model: even if an option value is, by definition, always non negative, it is possible that the cost of obtaining such an option exceeds its benefits making the "strategic" value of the option negative. In this case, the cost of the option handed over to the lender may rub out the value of the option to go from vertical integration to OS reducing the equity value of the firm for shareholders. This becomes evident by substituting (15) in (14), i.e.:

$$V^L(c_t;\alpha) = V^U(c_t;\alpha) - E_t[e^{-r(T^l-t)}]\tilde{B}c^{l\,\beta_1} \qquad \text{for } c_t > d, \tag{16}$$

where $T^l = \inf\{t \geq 0 \mid c_t = c^l\}$ is the buy out timing and $E_t[e^{-r(T^l-t)}] = \left(\frac{c_t}{c^l}\right)^{\beta_2}$.[22] The value of the levered firm is equal to the unlevered firm minus the discounted value of the option to go from *OS* to *vertical integration* calculated at the buy out time. As expected, the value of the firm does not depend on debt but on the covenant contained in the contract which corresponds to a shut down option (for equityholders). In words, the value of the firm depends on the (strategic) interaction between the lender and the shareholders. If such an interaction did not exist, the value of the firm would be the sum of debt and equity and the use of debt would not erode the value of equity, i.e., $V^L(c_t;\alpha) = V^U(c_t;\alpha)$. [23]

4.2 The optimal OS share and the investment timing

Since equityholders control both the decision about the OS share and the timing of the investment we proceed stating first α^{*L} and then the optimal investment trigger c^{*L}. To get the optimal α^{*L}, equityholders maximize (11) minus the cost of setting up the production organization with partial OS:

$$\alpha^{*L} = \arg\max\left[E(c_t;\alpha) - (I(\alpha) - k)\right] \tag{17}$$

where $k \leq I(\alpha)$ is the share of the investment expenditure paid by the lender who controls the amount to loan and the buy out timing. Since a rational investor will not agree to finance the firm unless k is a (financially) fair price for the debt, we set $k = D(c_t;\alpha)$ for $c_t > c^l$.[24] Then,

[22] The expected present value $E_t[e^{-r(T^l-t)}] = \left(\frac{c_t}{c^l}\right)^{\beta_2}$, can be determined by using dynamic programming (see e.g. Dixit and Pindyck, 1994, pp. 315-316).

[23] The coupon disappears when we sum debt and equity values to obtain the entire value of the firm.

[24] Note that the lender chooses the amount of the loan as a function of c_t. That is, as in Mauer and Sarkar (2005), the contract may be seen as a revolving credit line where the firm decides when to use it.

substituting in (17), we obtain:

$$\alpha^{*L} = \arg\max \left[V^L(c_t;\alpha) - k_1 - \frac{k_2}{2}\alpha^2 \right] \tag{18}$$

where $V^L(c_t;\alpha)$ is given by (14).

As before, let's consider a firm manufacturing in-house the input, while holding the option to switch to OS. Solving (18) the optimal OS share is given by:

$$[A - S(\alpha^{*L})]c_t^{\beta_2} - k_2\alpha^{*L} = 0 \tag{19}$$

where A is as in (5), $S(\alpha) = Bc^{l\beta_1-\beta_2}\left(1-(\beta_1-\beta_2)\frac{p-d-D}{p-(1-\alpha)d-D}\right) < 0$ and $S'(\alpha) > 0$. Since $\frac{\partial\alpha^{*L}}{\partial c_t} < 0$, if c_t is low it is better to choose complete OS, while, as c_t increases α goes down and tends to zero for high values of c_t. Further, if $c^l \to 0$, $S(\alpha) \to 0$, and then $\alpha^{*L} \to \alpha^{*U}$.

Defining with $F^L(c_t)$ the value of the option to invest in the vertically flexible technology, this is equal to (8) with $T^{*L} = \inf\{t \geq 0 \mid c_t = c^{*L}\}$ as the optimal investment timing. Then, going through the same steps as before, we can prove that:

Proposition 2 *The ex-ante value can be written as:*

$$F^L(c_t) = \begin{cases} F^L c_t^{\beta_2} & \text{for } c_t > c^{*L} \\ \frac{p-d}{r} + \hat{A}(\alpha^{*L}(c^{*L}))c_t^{\beta_2} + C(\alpha^{*L}(c^{*L}))c_t^{\beta_2} - I(\alpha^{*L}(c^{*L})) & \text{for } d < c_t \leq c^{*L} \end{cases}. \tag{20.1}$$

where the constant $F^L = \tilde{A}(\alpha^{*L}(c^{*L})) - \tilde{B}(\alpha^{*L}(c^{*L})c^{l(\beta_1-\beta_2)}$, *while the optimal trigger is:*

$$c^{*L} = \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r} - k_1\right)}}{A - S(\alpha^{*L})} \right]^{1/\beta_2} \tag{20.2}$$

and α^{*L}, *by (19), becomes:*

$$\alpha^{*L} = \min\left[\sqrt{\frac{2\left(\frac{p-d}{r} - k_1\right)}{k_2}}\ ,\ 1 \right] \tag{20.3}$$

Proof : See Appendix C

Substituting F^L in (20.2), we can write the value of the option to invest in the form:

$$F^L(c_t) = \tilde{A}(\alpha^{*L}(c^{*L}))c_t^{\beta_2} - E_t[e^{-r(T^l-t)}]\tilde{B}c^{l\beta_1} \ \text{ for } c_t > c^{*L} \tag{21}$$

Notice that, unlike the case of pure equity, if shareholders keep the possibility to decide both the optimal OS and the timing of the investiment, the value of investing in the new technology comes from the value of the option to do OS minus the value of the option to exit held by debt holders. By direct inspection of (9.2), (21.2) and (9.3), (21.3) the following proposition summarizes the comparison with respect to the unlevered firm.

Proposition 3 *The levered firm invests always earlier than the unlevered firm, i.e. :*

$$c^{*L} \geq c^{*U} \tag{22.1}$$

but adopts the same proportion of outsourced input, i.e.:

$$\alpha^{*L} = \alpha^{*U}. \tag{22.2}$$

Since the levered firm decides both α^{*L} and c^{*L} by maximizing only the value of equity, it does not care of the risk carried by the lender. Part of the investment is paid by the lender and the risk born by the equityholders is just the buy out option in the hands of the lender. In this case the equityholders have an incentive to invest as soon as possible to get a higher loan and reap the profits of OS as soon as possible.

Using (10), (21) and Proposition 3, we find that $F^L(c_t) < F^U(c_t)$, for $c_t > c^{*L} \geq c^{*U}$. Hence the value of the option to invest in the flexible technology is lower for the levered firm with a take over option (warrant) than for the unlevered firm, as is to be expected.

5 Venture capitalist involvement

Now let us assume that the firm offers to an outside investor, a venture capitalist (VC), a share of profits $\psi \in (0,1)$ (without side payments) to finance the flexible technology. This is just a take or leave offer. The VC may accept the offer together with the option to optimally decide when to implement the deal.[25] If the VC accepts, it has to decide the optimal trigger c^{*V} to start while the equityholders decide the outsoucing share α^{*V}.

As it appears the decision setting changes with respect to the case of debt seen before, where the equityholders retained both the decisions on the timing of the investment and the proportion of outsourced input. Now, the sequence of moves can be summarized as follows: Equityholders offer ψ, the VC decides when to invest accepting that the equityholders set α. However, as the decison on the OS share is still in the hands of the equityholders, the VC agrees to parteciapte only for the direct cost to keep internal facilities working.

Since entry takes place as usual at $c_t > d$, with the firm initially producing the input in-house, proceeding backward the equityholders first decide the OS share conditional on c_t. Then the VC knows the reaction function $\alpha^{*V}(c_t)$ and decides the optimal trigger c^{*V}. Equityholders may anticipate their offer ψ that could be announced even before entry takes place, i.e., at t.

The problem for the equityholders is to select the optimal α that maximizes (4) minus the cost of the technology after the financial cost:

$$\alpha^{*V} = \arg\max \left[(1-\psi)V^U(c_t;\alpha) - (1-\zeta)k_1 - \frac{k_2}{2}\alpha^2 \right] \tag{23}$$

[25]Notice that we can model the above setting as a sequential game where, at each time $s \geq t$, the equityholders offer ψ and the VC can accept or reject the offer. Thus, at every point of time, the VC has the action set [Accept, Reject] that can be seen as a perpetual call option. See Lukas and Welling (2014) for an application of this game to supply chains.

where $\zeta \in [0,1]$ is the share of the investiment financed by VC. Solving (23) yields:

$$\alpha^{*V}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \le \tilde{c}^V \\ \frac{(1-\psi)A}{k_2} c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^V \end{cases} \tag{24}$$

where $\tilde{c}^V \equiv \left(\frac{k_2}{(1-\psi)A}\right)^{1/\beta_2} \le \tilde{c}^U$.

Now, defining $F^V(c_t)$ as the value of the option to invest by the VC and $T^{*V} = \inf\{t \ge 0 \mid c_t = c^{*V}\}$ as the optimal investment timing, we can prove that:

Proposition 4 *The ex-ante value of the firm is:*

$$F^V(c_t) = \begin{cases} F^V c_t^{\beta_2} & \text{for } c_t > c^{*V} \\ \psi\left(\frac{p-d}{r} + \tilde{A}(\alpha^{*L}(c^{*L}))c_t^{\beta_2}\right) - \zeta k_1 & \text{for } c_t \le c^{*V} \end{cases}. \tag{26.1}$$

where the constant $F^V = 2\psi\tilde{A}(\alpha^{*L}(c^{*L}))$, *while the optimal trigger is:*

$$c^{*V} = \left[\frac{\sqrt{\frac{k_2}{\psi}\left(\psi\frac{p-d}{r} - \zeta k_1\right)}}{A}\right]^{1/\beta_2} \tag{26.2}$$

and the OS share is:

$$\alpha^{*V} = \min\left[\sqrt{\frac{\frac{1-\psi}{\psi}\left(\psi\frac{p-d}{r} - \zeta k_1\right)}{k_2}} \;,\; 1\right] \tag{26.3}$$

Proof : See Appendix D

As before substituting F^V in (26.1) we see that the value of the option to invest is equal to the option to go OS multiplied by 2ψ. So only if $\psi = \frac{1}{2}$ the shareholders and the VC evenly split the market value of the firm and the value of the option to invest is equal to the value of the option to outsource. Unlike previous cases, the condition for the existence and the finiteness of the optimal trigger is $\psi > \frac{\zeta k_1}{\frac{p-d}{r}}$, while the necessary condition for having $c^{*V} > \tilde{c}^V$ and then $\alpha^{*V} < 1$, is now $\psi\frac{p-d}{r} - \zeta k_1 < \frac{\psi}{1-\psi}\frac{k_2}{2}$.

Notice that , as it is to be expected, for values of ψ tending to the extremes of the feasible interval $(\frac{\zeta k_1}{\frac{p-d}{r}}, 1)$, it is optimal for equityholders to give up the flexible technology, i.e. $\alpha^{*V} \to 0$. If $\psi \to 1$ equityholders are selling the firm to VC, in this case it makes no sense to invest in a flexible technology. On the contrary, if equityholders announce a small profit share, i.e., $\psi \to \frac{\zeta k_1}{\frac{p-d}{r}}$, the VC invests immediately to reap the profits as soon as possible. However, investing in a flexible technology with a high c_t is too risky and the equityholders choose zero flexibility.[26]

The comparison with respect to the unlevered firm is summarized in the following proposition:

[26] This is consistend with the comparative statics of (26.2) and (26.3). That is, $\frac{\partial c^{*V}}{\partial \psi} < 0$ and $\frac{\partial \alpha^{*V}}{\partial \psi} > 0$ for $\psi \in (\frac{\zeta k_1}{\frac{p-d}{r}}, \sqrt{\frac{\zeta k_1}{\frac{p-d}{r}}})$. On the contrary $\frac{\partial c^{*V}}{\partial \psi} \ge 0$ and $\frac{\partial \alpha^{*V}}{\partial \psi} \le 0$ for $\psi \in [\sqrt{\frac{\zeta k_1}{\frac{p-d}{r}}}, 1)$.

Proposition 5 *If the flexible technology is partially financed by a venture capitalist, then:*

$$\begin{aligned} c^{*V} &\geq c^{*U} \quad \text{for } \psi \in (\frac{\zeta k_1}{\frac{p-d}{r}}, \psi_1] \\ c^{*V} &< c^{*U} \quad \text{for } \psi \in (\psi_1, 1) \end{aligned} \tag{27.1}$$

where ψ_1 *is the positive root of* $\Psi(\psi) = 2\psi^2\left(\frac{p-d}{r} - k_1\right) - 2\psi\left(\frac{p-d}{r} - k_1\right) + \psi\frac{p-d}{r} - \zeta k_1$.

While, if $\frac{\zeta k_1}{\frac{p-d}{r}} < \frac{\zeta}{2-\zeta} < 1$, *the optimal OS level is*:

$$\alpha^{*V} < \alpha^{*U} \quad \text{for all } \psi \in (\frac{\zeta k_1}{\frac{p-d}{r}}, 1) \tag{27.2}$$

Proof : See Appendix E

When ψ is low, i.e., $\psi \in (\frac{\xi k_1}{\frac{p-d}{r}}, \psi_1]$, the VC enters earlier than the unlevered firm. With ψ low, the VC is better off anticipating the time he will receive the "sure" profits from producing in-house. On the contrary, if ψ is high, i.e., $\psi \in (\psi_1, 1)$, the option value to wait for "expected" higher profits from OS prevails and the VC enters later than the unlevered firm. In addition, the equityholders choose a lower level of OS with respect to the unlevered firm if $\frac{\zeta k_1}{\frac{p-d}{r}} < \frac{\zeta}{2-\zeta}$ which, consistently with the previuos result, it is alwyas satisfied if $\psi < \xi$.

Using (10) and Proposition 4, we find that:[27]

$$\frac{F^V(c_t)}{F^U(c_t)} = 2[\psi(1-\psi)]^{1/2}\left[\frac{\left(\psi\frac{p-d}{r} - \zeta k_1\right)}{\left(\frac{p-d}{r} - k_1\right)}\right]^{1/2} \qquad \text{for } c_t > \max(c^{*V}, c^{*U})$$

Unlike the preceding case, the value of the option to invest for the VC may be higher than the option for the unlevered firm. This may occur in the odd case where ψ is much higher than ζ. The intuition: the higher the share of profits going to the VC vis à vis the capital commitment the higher is the value of the option to invest in the firm for the VC.

Finally, an open question is the determination of the share parameter ψ. The equityholders may set ψ, maximizing the portion of value they keep.[28] In this case they may announce ψ before the optimal investment timing c^{*V} by maximizing the following function:

$$\max_{\psi^*} E_t(e^{-r(T^V-t)})\left[(1-\psi)V^U(c^{*V}; \alpha^{*V}(c^{*V})) - (1-\zeta)k_1 - \frac{k_2}{2}(\alpha^{*V}(c^{*V}))^2\right].$$

[27] If in both cases $\alpha^{*V} = \alpha^{*U} = 1$ the ratio reduces to 2ψ

[28] In a different environment Banerjee et al. (2014) introduce a bargaining as to the share parameter and find that it is inefficient to set it before the investment because of the emerging time inconsistency. Only a bargaining carried out after the investment may assure temporal efficiency.

Recalling that $E_t(e^{-r(T^V-t)}) = \left(\frac{c_t}{c^{*V}}\right)^{\beta_2}$ and using Proposition 4, we able to reduce the above expression to:

$$\max_{\psi^*} \left(\frac{c_t}{c^{*V}}\right)^{\beta_2} \left[3(1-\psi)\frac{p-d}{r} - 2\frac{1-\psi}{\psi}\zeta k_1 - (1-\zeta)k_1\right] \tag{28}$$

where c^{*V} is given by (26.2). Notice that feasibility requires that, for any $\psi > 0$, the value of the firm for the equityholders at the time of the investment be positive. On the basis of the above results it is easy to show that it is never optimal to choose a value $\psi \to 1$ since it would imply a negative value for (28). Therefore, consistently with the firm value maximization by equityholders ψ^*must lie in the range $(\frac{\zeta k_1}{\frac{p-d}{r}}, 1[$.

Since c^{*V} is not monotone in ψ, the optimal share cannot be investigated analitically. Then, we resort to numerical simulations. The parameter scenario we choose is given by $\frac{p-d}{r} = 100, 70, 50$, with $\zeta = 1, 0.9, 0.6$. The cost k_1 to keep internal facilities working is normalized to one while the convex component of the organizational cost $\frac{k_2}{2}$ is set to 50.[29] The optimal share ψ^* is described in Table 1 and for α^{*V} in Table 2.[30]

$\psi^*; (\psi_1)$			$\frac{p-d}{r}$	
		100	70	50
	1	5.3%; (50.5%)	6.0%; (50.7%)	6.7%; (50.9%)
ζ	0.9	5.0%; (50.4%)	5.8%; (50.6%)	6.5%; (50.8%)
	0.6	4.2%; (50.0%)	4.9%; (50.1%)	5.6%; (50.2%)

Table 1: The Optimal share of profits ψ^* offered to the VC

α^{*V}			$\frac{p-d}{r}$	
		100	70	50
	1	0.62	0.50	0.40
ζ	0.9	0.62	0.51	0.41
	0.6	0.64	0.52	0.43

Table 2: Optimal OS share α^{*V}

The simulations reported in Table 1 and 2 confirm our results. The higher is $\frac{p-d}{r}$, i.e., the profitability of vertical integration, the lower is the risk associated with the adoption of the flexible technology and the firm can afford much OS. This is shown in Table 2. Moreover, if $\frac{p-d}{r}$ is high the shareholders will give a low share to the VC. On the contrary, if $\frac{p-d}{r}$ goes down, ψ^* must increase to induce the VC to invest. Obviously ψ^* goes down if the investment share of the VC ζ goes down, as Table 1 shows.

[29] This component has been calibrated so as to get, with $\alpha = 0.2$, a ratio 2 to 1 with the fixed cost.

[30] See Appendix F for the Simulation background.

Moreover, we observe that the level of flexibility α^{*V} increases as the share ζ decreases. The intuition can be seen reading together Table 1 and 2. In Tab 1 we see that ψ^* goes down if the involvment of the VC goes down. Then, the equityholders will increase the level of flexibility to balance the fixed cost and the organizational cost and delay the investment. Obviuosly, if $\zeta \to 0$ then $\alpha^{*V} \to \alpha^{*U}$ and $c^{*V} \to c^{*U}$.

As for the VC a further explanation may come from the comparison with Yoshida (2012) where, in a different context, the extent of flexibility chosen by one agent affects the level of uncertainty of the scenario. In a symmetric framework the more flexibility is adopted by an agent the more is chosen by the rival (flexibility is a strategic complement). The increase in (endogeneous) uncertainty associated with the extent of flexibility makes for an investment delay. In our framework this kind of simmetry is absent since the extent of flexibility is chosen (asymmetrically) only by one party, the incumbent equityholders. Their choice puts a ceiling on uncertainty making the investment occur earlier.

6 Epilogue

We have considered a firm that has to decide simultaneously the internal vertical setting and the financial structure in a dynamic stochastic framework. The firm we analyse is vertically flexible since it has an option to outsource entirely or partially a necessary input and it can reverse its choice by going back to in-house production, i.e., vertical integration. Unlike recent literature (Benaroch et al. 2012) we have not examined the choice of complete OS vis à vis vertically integrating, yet we have gone through a set of financial issues of a vertically flexible corporate organization where partial and reversible OS occur.

Flexibility comes with a cost required to set up a suitable supply chain and to keep alive the know how and the facilities to backsource the input in case market circumstances require to do so. We have investigated two possible financial avenues for the vertically flexible firm. First we have studied the case of debt financing. A lender may be willing to finance the firm that invests in flexibility if she gets a suitable "sweetener" such as an option to buy out the firm in case flexibility becomes useless. The option is required to make the lender willing to finance the corporate firm where limited liability may induce the incumbent equityholders to overinvest. With debt the shareholders rush to invest earlier with respect to a corresponding pure equity unlevered firm. The levered firm decides the level of OS and the timing of the invrestment while the lender sets only the size of the investment and the buy out time. Vertical flexibility is a cushion against risk but it is costly. If financial providers require collaterals which are too expensive it may not be worth. In such a case the value of a levered flexible firm may be lower than the value of an unleverd vertically unflexible firm and the strategic value of the option to become flexible may turn negative. We went through a second possible financial arrangement for the vertically flexible firm considering a venture capitalist financing the production of the necessary input. In this case it appears that the level of ousoucing is lower than in the case of the unlevered firm and the investment takes place earlier. As the share of the firm offered to the venture capitalist decreases ($\psi \to 0$) the behaviour of the firm converges to the unlevered case. The main results are sumed up in Table 3 below:

Levered vs Unlevered	Joint Venture vs Unlevered
$c^{*L} < c^{*U}$	$c^{*V} < c^{*U}$
$\alpha^{*L} = \alpha^{*U}$	$\alpha^{*V} < \alpha^{*U}$

Table 3: sum up of results for time of entry (c^*) and flexibility adopted (α^*)

In the end we may say that financing flexibility with a warranted debt - the only one that is consistent with an efficient allocation of debt in the presence of limited liability - induces the firm to invest earlier but not more than the unlevered firm. Then, debt makes a firm more eager to go flexible to anticipate reaping expected profits. This is consistent with common observation suggesting that debt may accelerate innovation in organizational flexibility.

The venture capitalist case provides a bunch of suggestive results. If to the venture capitalist is given a small share of the project she will invest earlier than the unlevered firm since she aims at cashing profits as soon as possible (syndrome of the poor VC) and the amount of OS adopted is lower than in the case of debt. With the VC the firm acquires less OS. It seems that the sharing of risk that the involvment of the VC implies makes the firm less eager to have a high OS as an insurance against uncertainty. As for the optimal share of profits (or simply of the firm) to be given to the VC there exists an internal solution that makes the VC solution reasonable and implementable. A conclusion out of the epilogue should be that there is no unique way to increase flexibility in the vertical organization of a firm because the way it is financed always makes a difference. Yet debt appears not only as the easiest and handiest device but seems to be able to accelerate investment and to carry it out at a level that is not lower than that financed with internal cash flow.

A Appendix: Proof of Proposition 1

The standard arbitrage and hedging arguments require that the *vertically flexible* firm value, $V^U(c_t;\alpha)$, is the solution of the following dynamic programming problems (i.e., the Hamilton-Jacobi-Bellman equations):

$$\Gamma V^U(c_t;\alpha) = -(p-d), \qquad for \;\; c_t > d \tag{A.1}$$

and

$$\Gamma V^U(c_t;\alpha) = -(p-\alpha c_t-(1-\alpha)d), \qquad for \;\; c_t < d, \tag{A.2}$$

where Γ is the differential operator: $\Gamma = -r + \gamma c\frac{\partial}{\partial c} + \frac{1}{2}\sigma^2 c^2 \frac{\partial^2}{\partial c^2}$. The solution of (A.1) and (A.2) requires the following boundary conditions:

$$\lim_{c\to\infty}\left\{V^U(c_t;\alpha) - \frac{p-d}{r}\right\} = 0 \qquad \text{if } c_t > d$$

and

$$\lim_{c\to 0}\left\{V^U(c_t;\alpha) - \left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right)\right\} = 0, \quad \text{if } c_t < d$$

where $\frac{p-d}{r}$ is the present value of the firm "making" the input, while $\left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right)$ is the present value when "buying" a share α of the input. Then, from the assumptions and the linearity of (A.1) and (A.2), using the above boundary conditions, we get:

$$V^U(c_t;\alpha) = \begin{cases} \frac{p-d}{r} + \tilde{A}c_t^{\beta_2} & \text{if } c_t > d \\ \\ \left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right) + \tilde{B}c_t^{\beta_1} & \text{if } c_t < d. \end{cases} \tag{A.3}$$

where $\beta_2 < 0$ and $\beta_1 > 1$ are, respectively, the negative and the positive roots of the characteristic equation: $\Phi(\beta) \equiv \frac{1}{2}\sigma^2\beta(\beta-1) + \gamma\beta - r$. Finally, by the value matching and the smooth pasting conditions at $c_t = d$ we obtain the two constants (Dixit and Pyndyck, 1994, p. 189):

$$\begin{cases} \tilde{B} = \alpha B \equiv \frac{\alpha}{\beta_1-\beta_2}(r-\gamma\beta_2)d^{1-\beta_1}\frac{1}{r(r-\gamma)} \\ \tilde{A} = \alpha A \equiv \frac{\alpha}{\beta_1-\beta_2}(r-\gamma\beta_1)d^{1-\beta_2}\frac{1}{r(r-\gamma)}. \end{cases} \tag{A.4}$$

which are always nonnegative and linear in α.

Since $\tilde{A} = \alpha A$, the optimal vertical arrangement is given by:

$$\begin{aligned} \alpha^{*U} &= \arg\max\left[NPV^{VI}(c_t,\alpha)\right] \\ &= \arg\max\left[\frac{p-d}{r} + \alpha A c_t^{\beta_2} - k_1 - \frac{k_2}{2}\alpha^2\right]. \end{aligned} \tag{A.5}$$

Then, the FOC is:

$$Ac_t^{\beta_2} - k_2\alpha = 0 \tag{A.6}$$

while the SOC is always satisfied. From (A.6) we obtain (7) in the text:

$$\alpha^{*U}(c_t) = \begin{cases} 1 & \text{if} \quad c_t \leq \tilde{c}^U \\ \frac{A}{k_2}c_t^{\beta_2} & \text{if} \quad c_t > \tilde{c}^U \end{cases}$$

where $\tilde{c}^U \equiv \left(\frac{k_2}{A}\right)^{1/\beta_2}$.

Let's now consider the firm's ex-ante value $F^U(c_t)$. In the range of c_t where the option to wait to invest is positive $F^U(c_t)$ is still given by the solution of the following Hamilton-Jacobi-Bellman equation:

$$\Gamma F^U(c_t) = 0, \qquad \text{for} \;\; c_t > c^{*U} \tag{A.7}$$

where c^{*U} is the threshold at which it is efficient to invest. Since when c_t approaches infinity $F^U(c_t)$ should go to zero, the solution of (A.7) requires the boundary condition, $\lim_{c\to\infty} F^U(c_t) = 0$. By the linearity of (A.7) and using the boundary condition, we obtain:

$$F^U(c_t) = F^U c_t^{\beta_2}, \tag{A.8}$$

where β_2 is the negative root of $\Phi(\beta)$. To evaluate the constant F^U and the optimal entry trigger c^{*U}, the $F^U(c_t)$ must satisfy the matching value and smooth pasting conditions:

$$F^U(c^{*U}) = NPV^U(c^{*U}, \alpha^{*U}(c^{*U})), \tag{A.9.1}$$

$$F_c^U(c^{*U}) = NPV_c^U(c^{*U}, \alpha^{*U}(c^{*U})), \tag{A.9.2}$$

where the second equality follows from $NPV_\alpha^U(c^{*U}, \alpha^{*U}(c^{*U})) = 0$ by (A.5). Conditions (A.9.1) and (A.9.2) say that the optimal share of OS α is set when the investment takes place. Substituting (A.8) into (A.9.1) and (A.9.2) we obtain:

$$F^U c^{*U\beta_2} = \frac{p-d}{r} + \tilde{A}(\alpha^{*U}(c^{*U}))c^{*U\beta_2} - k_1 - \frac{k_2}{2}(\alpha^{*U}(c^{*U}))^2$$

$$\beta_2 F^U c^{*U\beta_2 - 1} = \beta_2 \tilde{A}(\alpha^{*U}(c^{*U}))c^{*U\beta_2 - 1}$$

from which we get:

$$F^U = \tilde{A}(\alpha^{*U}(c^{*U})) \tag{A.10.1}$$

and

$$\frac{k_2}{2}(\alpha^{*U}(c^{*U}))^2 = \frac{p-d}{r} - k_1 \tag{A.10.2}$$

Simply substituting (A.6) in (A.10.2) we obtain:

$$c^{*U} = \left[\frac{\sqrt{2k_2 \left(\frac{p-d}{r} - k_1 \right)}}{A} \right]^{1/\beta_2} \qquad \text{and} \quad \alpha^{*U} = \min \left[\sqrt{\frac{\left(\frac{p-d}{r} - k_1 \right)}{k_2/2}} \ , \ 1 \right] \tag{A.11}$$

from which it is easy to show that $c^{*U} > \tilde{c}^U$ if $\frac{p-d}{r} - k_1 < \frac{k_2}{2}$.

B Appendix: Proof of Lemmas 1 and 2

The Hamilton-Jacobi-Bellman equations describing the market value of equity is the same as in (A.1) and (A.2), except that the cash flow accruing to equityholders is now $p - d - D$ for $c_t > d$ and $p - D - \alpha c_t - (1 - \alpha)d$ for the case $c^l \leq c_t < d$, where c^l is the input price triggering the lender to buy out the firm. The general solution can be expressed as:

$$p \frac{(p - d - D)}{r} + \hat{A} c_t^{\beta_2} \quad \text{if} \quad c_t > d, \tag{B.1}$$

and

$$\left(\frac{p - (1 - \alpha)d - D}{r} - \frac{\alpha c_t}{(r - \gamma)} \right) + \hat{B}_1 c_t^{\beta_1} + \hat{B}_2 c_t^{\beta_2} \quad \text{if} \quad c^l < c_t < d. \tag{B.2}$$

By the value matching and the smooth pasting conditions at $c_t = d$ and the boundary condition $E\left(c^l; \alpha\right) = 0$, we get the system:

$$\frac{(p - d - D)}{r} + \hat{A} d^{\beta_2} = \left(\frac{p - (1 - \alpha)d - D}{r} - \frac{\alpha d}{r - \gamma} \right) + \hat{B}_1 d^{\beta_1} + \hat{B}_2 d^{\beta_2} \tag{B.3.1}$$

and

$$\beta_2 \hat{A} d^{\beta_2 - 1} = -\frac{\alpha}{r - \gamma} + \beta_1 \hat{B}_1 d^{\beta_1 - 1} + \beta_2 \hat{B}_2 d^{\beta_2 - 1} \tag{B.3.2}$$

and

$$\left(\frac{p - (1 - \alpha)d - D}{r} - \frac{\alpha c^l}{r - \gamma} \right) + \hat{B}_1 c^{l\beta_1} + \hat{B}_2 c^{l\beta_2} = 0. \tag{B.3.3}$$

Solving the system made by (B.3.1) and (B.3.2), we obtain:

$$\hat{B}_1 = \tilde{B} = \frac{1}{(\beta_1 - \beta_2)} \frac{r - \gamma\beta_2}{r} \frac{\alpha d^{1-\beta_2}}{(r - \gamma)} \tag{B.4}$$

and

$$\hat{A} = \tilde{A} + \hat{B}_2 = \frac{\alpha d^{1-\beta_2}}{(r - \gamma)} \left[\frac{r - \gamma\beta_1}{r(\beta_1 - \beta_2)} \right] + \hat{B}_2. \tag{B.5}$$

Let us now consider the debt. Similarly to equity, it must satisfy the following differential equations:

$$\Gamma D(c_t;\alpha) = -D, \qquad for \;\; c_t > c^l, \tag{B.6.1}$$

and

$$\Gamma D(c_t;\alpha) = -(p - \alpha c_t - (1-\alpha)d), \qquad for \;\; c_t \leq c^l, \tag{B.6.2}$$

with the two boundary conditions:

$$\lim_{c\to\infty}\left\{D(c_t;\alpha) - \frac{D}{r}\right\} = 0$$

and

$$\lim_{c\to c^l}\left\{D(c_t;\alpha) - \left(\frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma}\right)\right\} = 0.$$

The solution is:

$$D\left(c_t;\alpha\right) = \begin{cases} \frac{D}{r} + Cc_t^{\beta_2} & \text{if} \quad c_t > c^l, \\ \frac{p-(1-\alpha)d}{r} - \frac{\alpha c_t}{r-\gamma} & \text{if} \quad c_t \leq c^l. \end{cases} \tag{B.7}$$

By imposing the value matching and the smooth pasting conditions at $c_t = c^l$ we obtain:

$$c^l = \frac{\beta_2}{\beta_2 - 1}\frac{r-\gamma}{\alpha}\left[\frac{p-(1-\alpha)d-D}{r}\right] \tag{B.8}$$

and

$$C = -\frac{1}{\beta_2 - 1}\left[\frac{p-(1-\alpha)d-D}{r}\right](c^l)^{-\beta_2} > 0. \tag{B.9}$$

Substituting (B.4) and (B.9) in (B.3.3) we get $Cc^{l\beta_2} + \tilde{B}c^{l\beta_1} + \hat{B}_2c^{l\beta_2} = 0$. Since the first and the second terms are positive, the equality is satisfied only if:

$$\hat{B}_2 < 0. \tag{B.10}$$

Finally, from (B.5) and (B.10) we are able to isolate $\hat{A}$ and $\hat{B}_2$ respectively, i.e.:

$$\hat{A} = \alpha A - \alpha Bc^{l(\beta_1-\beta_2)} - C. \tag{B.11}$$

and

$$\hat{B}_2 = -C - \alpha Bc^{l(\beta_1-\beta_2)}. \tag{B.12}$$

If $c^l \to 0$ then $\hat{A} \to \alpha A > 0$ and $\hat{B}_2 \to 0 > 0$, we are back to the unlevered firm. If $c^l \to d$ we have:

$$\hat{A} \to -\alpha d\frac{\gamma}{(r-\gamma)}\frac{1}{r}d^{-\beta_2} + \frac{p-(1-\alpha)d-D}{r}\frac{d^{-\beta_2}}{\beta_2-1} < 0$$

which is always negative.

C Appendix: Proof of Proposition 2

By (B.4), (B.5) and (B.9), we note that $\hat{A} + C = \tilde{A} + \hat{B}_2 + C = \alpha[A - Bc^{l(\beta_1-\beta_2)}]$. Then, by (14) the optimal vertical arrangement is given by:

$$\alpha^{*L} = \arg\max \left[\frac{p-d}{r} + \alpha[A - Bc^{l(\beta_1-\beta_2)}] - k_1 - \frac{k_2}{2}\alpha^2 \right] \quad \text{(C.1)}$$

and the FOC is:

$$Ac_t^{\beta_2} - k_2\alpha - Bc^{l\beta_1-\beta_2}\left(1 - (\beta_1 - \beta_2)\frac{p-d-D}{p-(1-\alpha)d-D}\right)c_t^{\beta_2} = 0. \quad \text{(C.2)}$$

Defining $S(\alpha) = Bc^{l\beta_1-\beta_2}\left(1 - (\beta_1 - \beta_2)\frac{p-d-D}{p-(1-\alpha)d-D}\right) < 0$, we are able to reduce (C.2) to:

$$[A - S(\alpha^{*L})]c_t^{\beta_2} - k_2\alpha^{*L} = 0. \quad \text{(C.3)}$$

We go through the SOC:

$$\begin{aligned} \frac{\partial FOC}{\partial \alpha} &= -k_2 - \frac{r-\gamma\beta_2}{r}\frac{1}{(\beta_1-\beta_2)}\frac{1}{(r-\gamma)}d^{1-\beta_1}(\beta_1-\beta_2)c^{*\beta_1-\beta_2}c_t^{\beta_2} \\ &\quad \left[c^{*-1}\left(1 - \left(\frac{p-d-D}{p-(1-\alpha)d-D}\right)\right) - \left(\frac{d(d+D-p)}{(p-(1-\alpha)d-D)^2}\right)\right] \end{aligned}$$

The sign depends on:

$$(1 - \left(\frac{p-d-D}{p-(1-\alpha)d-D}\right)) - \left[\frac{d(d+D-p)}{(p-(1-\alpha)d-D)^2}\right]$$

which is always positive, making for a verified SOC.

We define $F^L(c_t)$ as the value of the option to invest by the levered firm. The constant F^L and the optimal trigger c^{*L} must satisfy the matching value and smooth pasting conditions:

$$F^L c^{*L\beta_2} = \frac{p-d}{r} + \tilde{A}(\alpha^{*L}(c^{*L}))c^{*L\beta_2} - \left(\frac{c^{*L}}{c^l}\right)^{\beta_2}\tilde{B}(\alpha^{*L}(c^{*L}))c^{l\beta_1} - k_1 - \frac{k_2}{2}(\alpha^{*L}(c^{*L}))^2 \quad \text{(C.4)}$$

$$F^L\beta_2 c^{*L\beta_2-1} = \tilde{A}(\alpha^{*L}(c^{*L}))\beta_2 c^{*L\beta_2-1} - \beta_2\left(\frac{c^{*L}}{c^l}\right)^{\beta_2-1}\frac{1}{c^l}\tilde{B}(\alpha^{*L}(c^{*L}))c^{l\beta_1} \quad \text{(C.5)}$$

from which:

$$F^L = \tilde{A}(\alpha^{*L}(c^{*L})) - \tilde{B}(\alpha^{*L}(c^{*L})c^{l(\beta_1-\beta_2)} \quad \text{(C.6)}$$

$$\frac{k_2}{2}(\alpha^{*L}(c^{*L}))^2 = \frac{p-d}{r} - k_1. \quad \text{(C.7)}$$

Simply, substituting (C.3) in (C.7) we obtain:

$$c^{*L} = \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r} - k_1\right)}}{A - S(\alpha^{*L})}\right]^{1/\beta_2} \quad \text{and} \quad \alpha^{*L} = \min\left[\sqrt{\frac{\left(\frac{p-d}{r} - k_1\right)}{k_2/2}}\ ,\ 1\right] \quad \text{(C.8)}$$

D Appendix: Proof of Proposition 4

Let's define $F^V(c_t)$ as the value of the option to invest by the venture capitalist. The optimal entry trigger c^{*V} must satisfy the matching value and smooth pasting conditions:

$$F^V(c^{*V}) = \psi V^U(c^{*V}; \alpha^{*V}(c^{*V})) - \zeta k_1, \tag{D.1}$$

$$F_c^V(c^{*V}) = \psi \left[V_c^U(c^{*V}; \alpha^{*V}(c^{*V})) + V_\alpha^U(c^{*V}; \alpha^{*V}(c^{*V})) \frac{d\alpha^{*V}}{dc_t}_{|c_t = c^{*V}} \right] \tag{D.2}$$

where $\alpha^{*V}(c_t)$ is given by (24). Substituting for V^U we get:

$$F^V c^{*V\beta_2} = \psi \frac{p-d}{r} + \psi \alpha^{*V}(c^{*V}) A c^{*V\beta_2} - \zeta k_1$$

$$\begin{aligned} F^V \beta_2 c^{*V\beta_2 - 1} &= \psi \alpha^{*V}(c^{*V}) \beta_2 A c^{*V\beta_2 - 1} + \psi A c^{*V\beta_2} \frac{d\alpha^{*V}}{dc_t}_{|c_t = c^{*V}} \\ &= 2\psi \alpha^{*V}(c^{*V}) \beta_2 A c^{*V\beta_2 - 1} \end{aligned}$$

where the last equality follows from the fact that:

$$\frac{d\alpha^{*V}}{dc_t}_{|c_t = c^{*V}} = \frac{(1-\psi)\beta_2 A c_t^{\beta_2 - 1}}{k_2}_{|c_t = c^{*V}} = \beta_2 {c_t}^{-1} \alpha_{|c_t = c^{*V}} < 0.$$

By substituting it back in the matching value and smooth pasting conditions, we obtain:

$$F^V = 2\psi \alpha^{*V}(c^{*V}) A \tag{D.3}$$

and

$$\frac{\psi}{1-\psi} k_2 [\alpha^{*V}(c^{*V})]^2 = \psi \frac{p-d}{r} - \zeta k_1 \tag{D.4}$$

Simply, substituting (24) in (D.4) we obtain:

$$c^{*V} = \left[\frac{\sqrt{\frac{1-\psi}{\psi} k_2 \left(\psi \frac{p-d}{r} - \zeta k_1 \right)}}{(1-\psi)A} \right]^{1/\beta_2} \quad \text{and } \alpha^{*V} = \min \left[\sqrt{\frac{\frac{1-\psi}{\psi} \left(\psi \frac{p-d}{r} - \zeta k_1 \right)}{k_2}} \quad , \quad 1 \right] \tag{D.5}$$

from which is easy to show that $c^{*V} > 0$ if $\psi \frac{p-d}{r} - \zeta k_1 > 0$ and $c^{*V} > \tilde{c}^V$ if $\psi \frac{p-d}{r} - \zeta k_1 < \frac{\psi}{1-\psi} k_2$.

E Appendix: Proof of Proposition 5

Consider condition $c^{*V} < c^{*U}$, i.e.:

$$\left[\frac{\sqrt{\frac{1-\psi}{\psi}k_2\left(\psi\frac{p-d}{r}-\zeta k_1\right)}}{(1-\psi)A}\right]^{1/\beta_2} < \left[\frac{\sqrt{2k_2\left(\frac{p-d}{r}-k_1\right)}}{A}\right]^{1/\beta_2} \tag{E.1}$$

Since $\beta_2 < 0$ this reduces to:

$$\begin{aligned}\Psi(\psi) &= 2\psi^2\left(\frac{p-d}{r}-k_1\right)-2\psi\left(\frac{p-d}{r}-k_1\right)+\psi\frac{p-d}{r}-\zeta k_1 \\ &= \psi(\psi-1)2\left(\frac{p-d}{r}-k_1\right)+\psi\frac{p-d}{r}-\zeta k_1 > 0\end{aligned} \tag{E.2}$$

where $\Psi(\psi)$ it is a parabola convex with $\Psi(1) = \frac{p-d}{r} - \zeta k_1 > 0$, and $\Psi(0) = -\zeta k_1 < 0$. Therefore, the two roots are : $0 < \psi_1 < 1$, and $\psi_2 < 0$. In addition, since ψ must be greater than $\frac{\zeta k_1}{\frac{p-d}{r}}$ and $\Psi(\frac{\zeta k_1}{\frac{p-d}{r}}) \leq 0$, we may reduce the range to:

$$\begin{aligned} c^{*V} &< c^{*U} \quad \text{for } \psi \in (\psi_1, 1) \\ c^{*V} &\geq c^{*U} \quad \text{for } \psi \in (\frac{\zeta k_1}{\frac{p-d}{r}}, \psi_1]. \end{aligned} \tag{E.3}$$

Let us see now the the condition $\alpha^{*V} < \alpha^{*U}$, i.e.:

$$\frac{\sqrt{\frac{1-\psi}{\psi}\left(\psi\frac{p-d}{r}-\zeta k_1\right)}}{k_2} < \frac{\sqrt{2\left(\frac{p-d}{r}-k_1\right)}}{k_2}$$

from which it appears that:

$$\Sigma(\psi) = -\psi^2\frac{p-d}{r} - \psi\left(\frac{p-d}{r}-(2+\zeta)k_1\right) - \zeta k_1 < 0 \tag{E.4}$$

where $\Sigma(\psi)$ is a parabola concave, with $\Sigma(1) < 0$ and $\Sigma(\frac{\zeta k_1}{\frac{p-d}{r}}) < 0$. Further, since $\Sigma'(\psi) = -2\psi\frac{p-d}{r} - \left(\frac{p-d}{r} - (2+\zeta)k_1\right)$ it is easy to show that $\Sigma'(1) < 1$ and $\Sigma'(\frac{\zeta k_1}{\frac{p-d}{r}}) < 0$ if $-\frac{p-d}{r} + k_1 + (1-\zeta)k_1 < 0$.Therefore, if ζ is large (the VC pays for a high chunk of the investment), $\Sigma'(\frac{\zeta k_1}{\frac{p-d}{r}}) < 0$ and we may conclude that:

$$\alpha^{*V} < \alpha^{*U} \quad \text{for all } \psi \in (\frac{\zeta k_1}{\frac{p-d}{r}}, 1). \tag{E.5}$$

F Appendix: Simulation background

The shareholders' problem is to set ψ to maximize the following function:

$$\max_{\psi} \left(\frac{c_t}{c^{*V}}\right)^{\beta_2} [G(\psi)] \tag{F.1}$$

where c^{*V} is given by (D.5) and:

$$G(\psi) = 3(1-\psi)s - 2\frac{1-\psi}{\psi}k - (k_1 - k)$$

with $s = \frac{p-d}{r}$ and $k = \zeta k_1$. Since $g(\frac{k}{s}) = s - k_1 > 0$ and $g(1) = -(k - k_1) < 0$, it is never optimal to set $\psi = 1$ since that implies (F.1) negative. On the contrary, if we go towards $\psi = \frac{k}{s}$ equityholders have always a positive value of (F.1). Therefore, the acceptable range is $\psi \in (\frac{k}{s}, 1[$

To get the optimal ψ we go through the FOC:

$$f(\psi) \equiv \left[\frac{2}{\psi^2}k - 3s\right] - \left[3(1-\psi)s - 2\frac{1-\psi}{\psi}k - (k_1 - k)\right] \beta_2 \frac{1}{c^{*V}} \frac{dc^{*V}}{d\psi} = 0 \tag{F.2}$$

By totally differentiating (D.4) we get:

$$-\psi((1-\psi))\beta_2 \frac{1}{c^{*V}} \frac{dc^{*V}}{d\psi} = \frac{1}{2} \frac{1}{\psi s - k} \left[\psi^2 s - k\right]$$

which, once substituted in (F.2), reduces the FOC to:

$$\begin{aligned} f(\psi) &\equiv \left[-3s + \frac{2}{\psi^2}k\right] - \left[3(1-\psi)s - 2\frac{1-\psi}{\psi}k - (k_1 - k)\right] \left[\frac{1}{2} \frac{1}{\psi s - k}(\psi^2 s - k)(-\frac{1}{\psi(1-\psi)})\right] = 0 \\ &= (1-\psi)\left[-3s^2\psi^3 + 4s\psi^2 k + \psi s k - 2k^2\right] + (k - k_1)\psi(k - \psi^2 s) = 0 \end{aligned} \tag{F.3}$$

and the SOC is:

$$f'(\psi) \equiv \left[3s^2\psi^3 - 4s\psi^2 k - \psi s k + 2k^2\right] + (1-\psi)\left[-9s^2\psi^2 + 8s\psi k + sk\right] + (k - k_1)[k - 3\psi^2 s] \tag{F.4}$$

Calculating $f(\psi)$ in the extremes we obtain:

$$\begin{aligned} f(1) &= (k - k_1)(k - s) > 0 \\ f(\frac{k}{s}) &= -\frac{k^2(k-s)(2k - k_1 - s)}{s^2} < 0 \end{aligned}$$

To have a maximum we look, in the interval $(\frac{k}{s}, 1[$, for a root ψ^* of (F.3) with a negative value of (F.4), i.e. $f'(\psi^*) < 0$.

References

[1] Alipranti M., Milliou C. and Petrakis E. 2014. On Vertical Relations and Technology Adoption Timing. *Mimeo*

[2] Alvarez L.H.R. and Stenbacka R. 2007. Partial Outsourcing: A Real Options Perspective. *International Journal of Industrial Organization* **25:** 91-102.

[3] Amran M and Kulatilaka N. 1999. Real Options Managing strategic investment in an uncertain world. Harvard Business School press, Boston

[4] Anderson EG and Parker GG. 2002. The Effect of Learning on the Make, Buy Decision. *Production and Operations Management* **11:** 313-339.

[5] Bakhtiari S and Breunig R. 2014. Demand Uncertainty and Outsourcing, *Mimeo.*

[6] Banerjee S, Güçbilmez U and Pawlina G. 2014 Optimal exercise of jointly held real options: A Nash bargaining approach with value diversion. *European Journal of Operational Research* **239:** 565–578.

[7] Billette de Villemeur E, Ruble R and Versaevel B. 2014 Investment timing and vertical relationships. *International Journal of Industrial Organization* **33**: 110–123.

[8] Benaroch M, Webster S and Kazaz B, 2012 Impact of sourcing flexibility on the outsourcing of services under demand uncertainty. *European Journal of Operational Research* **219**: 272–283.

[9] Constantinides, G.M. 1978 Market risk adjustment in project valuation *The Journal of Finance* **33**, 603-616.

[10] Cox, J.C. and ROSs, S.A. 1976, The Valuation of Options for Alternative Stochastic Processes, *Journal of Financial Economics*, **3**, 145-166.

[11] Dasu, S. And L. Li, 1997, Optimal Operating Policies in the Presence of Exchange Rate Variability, *Management Science*, **43**, 705-722.

[12] Dixit A and Pindyck RS. 1994. *Investment under Uncertainty.* Princeton University Press: Princeton.

[13] Eaton RC and Schmitt N. 1994. Flexible Manufacturing and Market Structure. *American Economic Review* **84:** 875-888.

[14] Eckbo, B.E., R.W. Masulis, and O. Norli, 2007, Security offerings, in B.E. Eckbo, ed., Handbook of Corporate Finance: Empirical Corporate Finance, vol. 1, Chapter 6, Elsevier, North-Holland.

[15] Forbes 2012 Made in the USA: Bringing Industry Back Home, November 12th, printed edition.

[16] Harrison, J.M. and Kreps, D. (1979), “Martingales and Arbitrage in Multiperiod Securities Markets”. *Journal of Economic Theory*, **20**, 381-408.

[17] Harrison J.M., 1985, *Brownian Motions and Stochastic Flow Systems,* New York, Wiley.

[18] Holmes, T.J. and Thornton, J. 2008, Outsourcing to Limit Rent Extraction, Paper presented at the 6th Annual *International Industrial Organization Conference* (IIOC), Washington 15-17 May 2008.

[19] Jensen M. and Meckling W. 1976. Theory of the firm: Managerial behavior, agency costs, and ownership structure. *Journal of Financial Economics* **3:** 305–360.

[20] Joskow P. 2005. Vertical Integration. In *Handbook of New Institutional Economics*, Ménard C and Shirley MM (ed). Springer: Dordrecht and New York; 319-348.

[21] Klein P. 2005. The Make or Buy Decision: Lessons from Empirical Studies. In *Handbook of New Institutional Economics*, Ménard C and Shirley MM (ed). Springer: Dordrecht and New York; 435-64.

[22] Kogut, B. and N. Kulatilaka 1994 Operating flexibility, global manufacturing, and the option value of a multinational network *Management Science* **40**, 123-139

[23] Kouvelis, P., Axarloglou, K., Sinha, V. 2001. Exchange Rates and the Choice of Ownership

[24] Structure of Production Facilities. *Management Science* **47**:1063–1080.

[25] Kulatilaka N. 1988. Valuing the Flexibility of Flexible Manufacturing Systems. *IEEE Transactions on Engineering Management* **35:** 250-257.

[26] Lederer P.J. and V.R. Singhal 1994 The Effect of Financing Decisions on the Choice of Manufacturing Technologies, *International Journal of Flexible Manufacturing Systems*, **6**, 333-360

[27] Leland H.E. 1998. Agency costs, risk measurement, and capital structure. *Journal of Finance* **53:** 1213–1243.

[28] Lukas E. and Welling A. 2014. Timing and eco(nomic) efficiency of climate-friendly investments in supply chains. *European Journal of Operational Research,* **233:** 448-457.

[29] Mauer D.C. and Sarkar S. 2005. Real options, agency conflicts, and optimal capital structure. *Journal of Banking & Finance* **29:** 1405–1428.

[30] Michaels D, Lunsford JL and Stone R. 2008. Airbus Deliveries Stalled Again. A380s Could Face Delays of 3 Months; Jet Setbacks Mount. *The Wall Street Journal,* May 14; Page B6.

[31] Modigliani F. and M. Miller 1958 The Cost of Capital, Corporation Finance and the Theory of Investment *The American Economic Review*, **48**, 261-297.

[32] Moon SK and Phillips GM. 2014 Outside Purchase Contracts, Human Capital and Firm Capital Structure, *NBER Working Paper* No. 20579.

[33] Moretto M and Rossini G., 2012 Flexible OS, *Managerial and decision economics*, **33**, 47 - 59

[34] Rossini G and Ricciardi D. 2005. An Empirical Assessment of Vertical Integration on a Sample of Italian Firms. *Review of Economic Conditions in Italy* **59:** 517-32.

[35] Shy O and Stenbacka R. 2005. Partial Ooutsourcing, Monitoring Cost, and Market Structure. *Canadian Journal of Economics* **38:** 1173-1190.

[36] Simester D and Knez M. 2002. Direct and Indirect bargaining Costs and the Scope of the Firm. *Journal of Business* **75:** 283-304.

[37] The Economist. 2011 Moving back to America, May 14th, from the print edition

[38] The Economist. 2013 Reshoring manufacturing, Jan 19th, from the print edition

[39] Tirole, J. 2006 The Theory of Corporate Finance Princeton University press Princeton

[40] Triantis A.J and Hodder J.E. 1990. Valuing Flexibility as a Complex Option. *Journal of Finance* **45:** 549-565.

[41] Van Mieghen J. 1999. Coordinating Investment, Production and Subcontracting. *Management Science* **45:** 954-71.

[42] Wang LM, Liu LW, Wang YJ. 2007. Capacity Decisions and Supply Price Games under Flexibility of Backward Integration. *International Journal of Production Economics* **110:** 85-96.

[43] Whinston MD. 2003. On the Transaction Cost Determinants of Vertical Integration. *Journal of Law, Economics and Organization* **19:** 1-23.

[44] Williamson OE. 1971. The Vertical Integration of Production: Market Failure Considerations. *American Economic Review* **61:** 112-123.

[45] Yoshida J. 2012 Why Are Joint-Venture Investments Delayed? A Model of Strategic Uncertainty. Mimeo

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