“Should I stay or should I go?”: Weather forecasts and the economics of “short breaks”

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Quaderni - Working Paper DSE N°1034
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Weather forecasts and the economics of “short breaks”

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October 12, 2015

Abstract The aim of this paper is to model the decisions of tourists and a monopolist firm when weather forecasts are available. Before deciding whether to go on holiday or not, but after the firm has decided and posted its price, tourists can look at weather forecasts. Our results show that the price chosen by the firm and the corresponding equilibrium profit are decreasing as a function of the accuracy of weather forecasts. Consumers, instead, are better off the more accurate weather forecasts become. Managerial and policy implications are also derived.

Keywords weather forecast, firm pricing, tourist behaviour.

JEL Classification D83, L12, L83.
1. Introduction

For the 2009 spring Bank Holiday Monday, the Met Office and the BBC forecasted constant thundery showers in Bournemouth, a large coastal resort town in the south-west of England. Indeed, temperatures of 22 °C made it one of the hottest day of the year. According to Mark Smith, head of Bournemouth council’s tourism department, inaccurate weather reports led to the loss of 25,000 visitors, and an estimated cost of over a million pounds.1 Similar concerns are expressed by Italian tourism entrepreneurs. For the 2009 Easter weekend, hotel owners in the Province of Rimini, one of the leading seaside resorts in the north of Italy, publicly blamed overly pessimistic weather forecasts by a leading national TV channel.2 Following wrong forecasts on two spring weekends in 2010, which caused, according to hotel owners, a 3 million euro loss, the Veneto region, one of the leading Italian tourist regions, decided to establish a public-private partnership to produce specific weather forecasts for Jesolo, an important tourism town in the province of Venice.3

The above examples, and there are many others, suggest the importance that tourism firms attribute to weather forecasts. This judgment seems correct: according to Confesercenti, an Italian business association, 76% of Italian tourists usually consult weather forecasts when organizing their holiday.4 Weather forecasts are a crucial factor affecting tourism demand, especially for short holidays, or excursions, in which tourists do not book or, if they do, they do so only a few days in advance. For many tourist destinations, this form of tourism constitutes an increasingly important source of activity and income.

The aim of this paper is to model the decisions of tourists and firms when weather forecasts are available. For this reason, we develop a model to help in understanding the effects of weather forecasts.

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1 Bournemouth Echo, “Poor weather forecast cost Bournemouth millions, says council”, 28th May, 2009, retrieved at www.bournemouthecho.co.uk.
2 Il Resto del Carlino, “Meteo terroristi smentiti per pasqua”, 14th April, 2009.
forecasts, and in particular their accuracy, on tourists and firm decisions, and, in turn, on their welfare. The model considers a single tourist firm facing a population of tourists who have heterogeneous tastes with respect to the utility they attach to holiday consumption. Moreover, their utility is also affected by a state of the world, which corresponds to “good weather” or “bad weather”. Before deciding whether to go on holiday or not, but after the firm has decided and posted its price, tourists can look at a signal about the state of the world, i.e. the weather forecast. Therefore, tourists’ demand is conditional on the signal they receive, while the tourist firm chooses its price in order to maximize its expected profit.

Our results show that the price chosen by the firm and the corresponding equilibrium profit are decreasing as a function of the accuracy of weather forecasts. Clearly, low levels of accuracy in weather forecasts harm the firm (and possibly the consumers) when forecasters erroneously predict bad weather, as the examples previously reported suggest, but benefit the firm when forecasters erroneously predict good weather. A priori, then, the effect is ambiguous. It turns out that, by making tourists more informed, forecast accuracy reduces demand elasticity following a good signal, and this pushes the firm to raise its price. The opposite happens following a bad signal, and the latter effect prevails. Therefore, the model shows that a lack of accuracy in weather forecasts is in fact beneficial for firms. As for both managerial and policy implications, we suggest that the definition of what is a good or a bad state is crucial. Indeed, a good weather state can be interpreted as those weather (exogenous) conditions that are necessary for tourists to express a positive evaluation of their holiday. Such an evaluation, in turn, will depend on the level of utility that the firm and the destination manager can (endogenously) provide to tourists. This implies that appropriate investments by the firm and the destination authority can mitigate the negative impact of more accurate weather forecasting.

In tourism research, a significant number of papers have considered the effect of weather and climate conditions on tourism demand and destination attractiveness (Harrison et al., 1999;
Lohmann and Kaim, 1999; Hamilton et al., 2005; Gomez-Martin, 2005; Berrittella et al., 2006; Agnew and Palutikof, 2006; Alvarez-Diaz and Rossello-Nadal, 2010). These works, which adopt different methodologies, confirm the importance of weather and climate for tourism in several dimensions.

In addition, there is an interdisciplinary literature studying the economic impact of weather forecasts (Katz and Murphy, 1997a). Several case studies have been produced, studying the impact of weather forecasts on specific sectors, including agriculture (Mjelde and Penson, 2000), energy (Considine et al., 2004), fishery (Costello et al., 1998) and transportation (Craft, 1998). However, evidence for the tourism sector is negligible. As a theoretical background, this literature usually uses a Bayesian rational choice approach, in which a single decision-maker must take action whose value depends on realized weather conditions (Katz and Murphy, 1997b; Nelson and Winter, 1964; Cerdá Tena and Quiroga Gómez, 2011). Our model is close to this approach, but it also considers explicitly the interaction of decisions between the supply side and the demand side of the market.5

In tourism, and this is shared by other sectors such as agriculture, the quality of the good supplied by a firm is also determined by factors such as weather conditions, which are not under the control of either the firm or the consumers (Candela and Cellini, 1998); however, what is peculiar about tourism is that weather forecasts can directly affect consumers’ behaviour.

The paper is organized as follows. Section 2 describes the model, while in Section 3 the main results are presented and discussed. In Section 4 we further elaborate on the results, by deriving implications for firm strategy and destination management. Finally, Section 5 concludes.

2. The model

The model can be summarized as follows. Two sets of agents are considered: i) tourists and ii) a tourist firm (a hotel) acting as a monopolist in the tourist destination. Tourists are heterogeneous in

5 For a model considering the effect of weather forecasting on firms in a competitive market, see Babcock (1990).
terms of their willingness to pay for the service sold by the firm (i.e. the holiday), and such a willingness to pay also depends on weather conditions. Weather forecasts are available to tourists before they make their decision (buying or not buying the holiday) but after the firm has chosen its price, which is assumed not to be contingent on realized weather conditions or weather forecasts. The details of the demand and the supply sides of the model are now described.

2.1 Demand

Formally, \( \theta_i \) denotes the willingness to pay of tourist \( i \) when the weather state is “good”. \( \theta \) is uniformly distributed over the interval \([0;1]\) across consumers, whose mass is normalized to 1. A good weather state may correspond to sunny days for a weekend in a seaside destination, or the presence of natural snow for a ski resort. When the weather state is “bad”, however, the willingness to pay is reduced to \( \alpha \theta \) with \( 0 < \alpha \leq 1 \). In a more sophisticated interpretation, to which we will return in Section 4, a good state corresponds to a positive holiday evaluation, and a bad state to a negative one. Evaluation depends on weather, together with other factors. Formally, denote with \( w \in \mathbb{R} \) a continuous evaluation of weather and with \( a \in \mathbb{R} \) a continuous evaluation of all the other factors influencing the holiday experience (e.g. natural and cultural attractions). We shall assume that the state is good if \( w + a \) is above a reference level, which without loss of generality we can normalize to 0, and negative otherwise.

The demand function is perfectly inelastic: the tourist either buys a holiday or does not buy. Calling \( p \) the price fixed by the firm, the tourist utility function is:

\[
u = \theta - p \tag{1}\]

if the tourist buys the holiday and the weather state is good, while it is:

\[
u = \alpha \theta - p \tag{2}\]
if the tourist buys the holiday and the weather state is bad. The reservation utility when the tourist does not buy is normalized to 0. The tourist buys the holiday, paying the price, before the weather state is realized. However, the tourist knows the probability distribution of weather conditions. In particular, she knows that the \textit{ex ante} probability of a good state is $0 < r < 1$ (the state is bad with the complementary probability). Such a probability is common knowledge, and can be interpreted as the historical frequency of “good” and “bad” weather in that particular season. Moreover, before buying a holiday, tourists receive a signal of the weather conditions, i.e. the weather forecast. The signal is correct with a probability of $\frac{1}{2} \leq q < 1$, so it conveys useful information to tourists.\footnote{The assumption that $q \geq \frac{1}{2}$ is at the same time natural and not restrictive. With $q < \frac{1}{2}$, signals would be more likely to be correct than incorrect, and that case can be brought back to the case under consideration just by switching the signals’ labels.} Also $q$ is common knowledge.\footnote{Notice that we are assuming that tourists have access to a single source of information, and no bias exists in the forecast of good and bad weather states. Incidentally, Nelson and Winter (1964) show that such biases can be socially optimal. We will briefly consider the role of biases in Section 4.} As a matter of notation, we will use $g$ and $b$ to denote the realized good and bad states, and "g" and "b" to denote the corresponding signals. Therefore, applying the Bayes theorem, we get:\footnote{For Bayes’ theorem, for instance, $\Pr(b|"b") = \frac{\Pr(b)\Pr("b"|b)}{\Pr("b")}$. The probability of bad weather is $1 - r$; the probability that a bad signal is observed when the true state is bad (i.e. the signal is correct) is $q$; a bad signal is observed if the weather is bad and the signal is correct (probability $(1 - r)q$) or the weather is good but the signal is incorrect (probability $r(1 - q)$).}

\begin{align*}
\Pr(b|"b") &= \frac{(1-r)q}{(1-r)q + r(1-q)} \\
\Pr(g|"b") &= \frac{r(1-q)}{(1-r)q + r(1-q)} \\
\Pr(g|"g") &= \frac{rq}{rq + (1-r)(1-q)} \\
\Pr(b|"g") &= \frac{(1-r)(1-q)}{rq + (1-r)(1-q)}
\end{align*}

Given such \textit{ex post} probabilities on weather states, we can write the tourist’s expected utility when buying the holiday, conditional on the signals received:
\[ E(u|g^\prime) = \frac{rq}{rq+(1-r)(1-q)} \theta + \frac{(1-r)(1-q)}{rq+(1-r)(1-q)} \alpha \theta - p \] (7)

\[ E(u|b^\prime) = \frac{(1-r)q}{(1-r)q+r(1-q)} \alpha \theta + \frac{r(1-q)}{(1-r)q+r(1-q)} \theta - p \] (8)

Conditional on "g", the tourist buys the holiday when the expected utility is larger than or equal to 0, i.e. consumers are assumed to be risk-neutral. This is the case if:

\[ \frac{rq}{rq+(1-r)q} \theta + \frac{(1-r)(1-q)}{rq+(1-r)(1-q)} \alpha \theta - p \geq 0 \] (9)

Condition (9) determines a threshold \( \hat{\theta}_{g^\prime} \equiv p[rq+(1-r)(1-q)]/r + a(1-r)(1-q) \) such that the tourist buys the holiday if and only if her willingness to pay is larger than or equal to \( \hat{\theta}_{g^\prime} \). Similarly, conditional on "b", the tourist buys the holiday when the expected utility is larger than or equal to 0. This is the case if:

\[ \frac{(1-r)q}{(1-r)q+r(1-q)} \alpha \theta + \frac{r(1-q)}{(1-r)q+r(1-q)} \theta - p \geq 0 \] (10)

Condition (10) determines a threshold \( \hat{\theta}_{b^\prime} \equiv \frac{p[(1-r)q+r(1-q)]}{r(1-q)+a(1-r)q} \) such that the tourist buys the holiday if and only if their willingness to pay is larger than or equal to \( \hat{\theta}_{b^\prime} \). It can be shown that \( \hat{\theta}_{g^\prime} \leq \hat{\theta}_{b^\prime} \), where the inequality is strict if \( \alpha < 1 \). As it is intuitive, a good signal induces to buy even consumers with a relatively low willingness to pay.

Given \( \theta \) distribution, the demand functions, conditional on the signals, are derived:

\[ D_{g^\prime}(p) = 1 - \hat{\theta}_{g^\prime} = 1 - \frac{p[rq+(1-r)(1-q)]}{rq+a(1-r)(1-q)} = 1 - \beta_{g^\prime} p \] (11)

\[ D_{b^\prime}(p) = 1 - \hat{\theta}_{b^\prime} = 1 - \frac{p[(1-r)q+r(1-q)]}{r(1-q)+a(1-r)q} = 1 - \beta_{b^\prime} p \] (12)
where $\beta^{-g}, \beta^{-b} \in [1, +\infty)$ are the slope of the linear demand functions. Clearly, $\beta^{-g} \leq \beta^{-b}$ implies $D^{-g}(p) \geq D^{-b}(p)$ for any $p$ (with the equality holding only if $\alpha = 1$). Demand functions are represented in Figure 1.

**Figure 1: Demand functions**

![Diagram of demand functions](image)

2.2 The supply side and the equilibrium

The tourist firm operates at zero marginal and fixed costs, and chooses its price, in order to maximize profits (or, equivalently, revenues), before the signal is observed. Also the firm is risk-neutral.

When the firm fixes its price, the probability that tourists will receive a good signal is $[rq + (1 - r)(1 - q)]$. In that case, the firm’s profit will be $pD^{-g}(p)$. With a probability of $[(1 - r)q + r(1 - q)]$, however, consumers will receive a bad signal. In that case, the firm’s profit will be $pD^{-b}(p)$. Therefore, firms’ expected profits are:

$$E\Pi = [rq + (1 - r)(1 - q)]pD^{-g}(p) + [(1 - r)q + r(1 - q)]pD^{-b}(p) = pED(p)$$

(13)
where $ED(p)$ corresponds to (ex ante) expected level of demand. The first-order condition for profit maximization $\frac{\partial E\Pi}{\partial p} = 0$ is:

$$\left[rq + (1 - r)(1 - q)\right] \left[1 - 2\beta_{p}\cdot p\right] + \left[r(1 - q) + (1 - r)q\right] \left[1 - 2\beta_{p}\cdot p\right]$$

(14)

from which we get the equilibrium price:

$$p^* = \frac{1}{2\beta_{p}[rq+(1-r)(1-q)]}$$

(15)

Since $Pr(\"g\") = [rq + (1 - r)(1 - q)]$ and $Pr(\"b\") = [r(1 - q) + (1 - r)q]$, we will define:

$$E\beta = \beta_{p}\cdot[rq + (1 - r)(1 - q)] + \beta_{b}\cdot[r(1 - q) + (1 - r)q]$$

(16)

i.e. $E\beta$ corresponds to the expected slope of the demand function. Therefore, we can rewrite (15) as $p^* = \frac{1}{2E\beta}$. This shows that equation (15) identifies an “elasticity rule” (Tirole, 1988), which is typical of monopoly pricing, posing an inverse relationship between market power and demand elasticity.

Plugging (15) into (13) we obtain equilibrium expected profit:

$$E\Pi^* = \frac{1}{4E\beta}$$

(17)

Notice that the expected demand level in equilibrium is constant and equal to $\frac{1}{2}$. A graphical representation of the equilibrium is shown in Figure 2. The dark grey area corresponds to equilibrium profit.

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9 Second-order conditions are satisfied since the objective function is strictly concave in $p$. 
3. Results: weather forecast accuracy, price and social welfare

In this section we investigate how weather forecast accuracy ($q$) affects a firm’s price and profit and consumer surplus. There are two ways to motivate our analysis. On the one hand, scientific literature in meteorology has witnessed a significant increase in the reliability of weather forecasts in recent decades, although an exact estimate of weather forecast accuracy is not an easy task (Thornes, 1996). Therefore, it is interesting to study what reactions we should expect from firms and tourists. On the other hand, the tourist firms’ complaints we mentioned in the Introduction can be read, in light of our model, as pointing at the negative effect of a lack of forecast accuracy on their performance. So, we can try to verify their claim, irrespectively of its scientific support. Furthermore, the analysis allows also to compare a situation where weather forecasts are available ($q > \frac{1}{2}$) and a situation in which, in fact, they are not ($q = \frac{1}{2}$).

3.1 Weather forecast accuracy and equilibrium price and profits

The results obtained in Section 2.2 show that the effect of $q$ on price and profit is uniquely determined once the effect on $E\beta$ is known. Therefore, our first proposition determines such an effect. The proof is in the Appendix.
**Proposition 1** The expected slope of demand is increasing in relation to weather forecast accuracy \( q \). Therefore, equilibrium price and profit are decreasing in relation to \( q \).

The intuition for this result is as follows. In the Appendix we show that \( \frac{d\beta \cdot q}{dq} < 0 \) and \( \frac{d\beta \cdot \nu}{dq} > 0 \): more accurate weather forecasts decrease price sensitiveness when the signal is good (i.e. they make demand less elastic, because they increase the expected utility that tourists can obtain from their holiday) and increase price sensitiveness when the signal received is bad (i.e. they make demand more elastic, because they reduce the expected utility that tourists can obtain from their holiday). It turns out that the second effect prevails, so that more accurate weather forecasts reduce the equilibrium price.

3.2 Weather forecast accuracy and consumer surplus

In order to determine the effect of weather forecast accuracy on tourists, we will look at the impact of \( q \) on net expected utility. It turns out that all tourists are (weakly) better off following an increase in forecast accuracy, from which Proposition 2 follows.

**Proposition 2** Consumer surplus is increasing in relation to weather forecast accuracy.

Tourists benefit from accurate weather forecasting for two reasons. First of all, the price is lower; second, weather forecasts allow tourists to make more informed decisions. As a matter of fact, it is this superior information that weather forecasts provide to tourists that puts pressure on firms, leading to lower prices.

4. Discussion: managerial and policy implications

To summarize, the results obtained in the previous section suggest that accurate weather forecasts, rather than inaccurate ones, may be detrimental to firm profitability. Therefore, tourist firms should in fact be concerned about the increasingly accurate weather forecasts that scientific and technical knowledge are able to provide.
In order to mitigate the negative effect of weather forecasts, the firm itself and the destination authority should act in order to decrease the relative importance of weather in determining holiday evaluation. In particular, in the more sophisticated interpretation of what states are in the model (Section 2.1), a good state corresponds to a positive holiday evaluation, and a bad state to a negative one, with evaluation depending on weather conditions \((w)\), together with all the other factors \((a)\), such as cultural and natural attractions, that contribute to the holiday experience. While weather conditions are clearly exogenous, the firm and the destination authority may invest to increase \(a\), i.e. the quality of the service offered or the attractiveness of the destination. The higher \(a\) is, the lower the value of \(w\) that is needed to produce a positive holiday experience, i.e. for a given distribution of weather conditions, private and public investments may in fact increase \(r\), minimizing demand elasticity and reducing the marginal impact of \(q\) on firm profitability.\(^{10}\)

At the same time, the technical feasibility of more accurate forecasting does not necessarily translate into forecasts that are actually more accurate. A full analysis endogenizing accuracy is outside the scope of this paper (see Anbarci et al. 2011 for a recent attempt in this direction), but a brief, and relatively informal, discussion of the role of symmetry in forecasting accuracy (i.e. that signals are equally precise for good and bad states) is in order. In fact, allegations of tourist firms against forecast providers often consist in blaming them for their “pessimism” or “alarmism”, which would make their bad weather forecasts only less reliable. In the Appendix we show that the general way to express the impact of \(q\) on \(E\beta\) can be written as follows:

\[
\frac{dE\beta}{dq} = \frac{dPR("g")}{dq} \beta \cdot g^r + \frac{d\beta("g")}{dq} Pr("g") + \frac{dPR("b")}{dq} \beta^r \cdot b^r + \frac{d\beta("b")}{dq} Pr("b") \tag{18}
\]

In fact, \(q\) should be interpreted as the subjective tourists’ perception of accuracy, as long as it is common knowledge with the firm. As a consequence, pessimism and alarmism would imply a value of \(q\) higher than its objective value only for bad signals, implying in (18) an increase both in

\(^{10}\) Notice that in Equation (A8) in the proof of Proposition 1, \(\frac{dE\beta}{dq} \to 0\) for \(r \to 1\).
\( Pr\left( b^* \right) \) and \( \frac{d\beta \left( b^* \right)}{dq} \). In turn, this would increase further the value of \( \frac{dE\beta}{dq} \) with a negative impact on
the firm’s price and profit. It follows that if i) bad weather forecasts are unreliable, but ii) tourists are not aware of this, then initiatives by destination authorities to produce unbiased forecasts may be profit enhancing for the firms in the destination.

5. Conclusions

In this paper we developed a model to assist in understanding the effect of weather forecasts on tourists’ and firms’ decisions. Our results show that the price chosen by the firm and the corresponding equilibrium profit are decreasing in relation to the accuracy of weather forecasts: contrary to the common claims of tourist firms, accurate weather forecasts, rather than inaccurate ones, may be detrimental to their profitability. Accurate weather forecasts allow tourists to make more informed decisions, and therefore put pressure on prices. Firms (and destination authorities) can mitigate the negative effect of weather forecasts by actively investing in improving the tourist experience irrespectively of weather (for instance, through culture-based attractions), while the public provision of weather forecasts by destination authorities may be justified only in the presence of biased, pessimistic forecasts.

In future work, we would like to study the effects of various forms of insurance on firms’ and tourists’ behaviour, by relaxing the assumption of risk neutrality on the tourists’ and firms’ side. For tourist firms, weather derivatives (Bank and Wiesner, 2011) may be an important tool, to a large extent unexplored. As an alternative, tourist firms (and destination authorities) could themselves insure tourists against the risk of bad weather conditions. A recent attempt in this direction has been made by Jesolo,\(^{11}\) in the Veneto region of Italy. Such investigations could be performed both theoretically and through field experiments, where real tourists’ reactions to different insurance schemes can be analysed.

\(^{11}\) See http://www.jesolospiagge.it/en/suntanned-or-your-money-back.html.
Appendix

Proof of Proposition 1. The impact of $q$ on $E\beta$ is given by

\[
\frac{dE\beta}{dq} = \frac{dPr(g^*)}{dq} \beta_{g^*} + \frac{d\beta(g^*)}{dq} Pr(g) + \frac{dPr(b^*)}{dq} \beta_{b^*} + \frac{d\beta(b^*)}{dq} Pr(b^*) \tag{A1}
\]

We first compute, through simple derivations, $\frac{d\beta(g^*)}{dq}$, $\frac{d\beta(b^*)}{dq}$, $\frac{dPr(g^*)}{dq}$ and $\frac{dPr(b^*)}{dq}$:

\[
\frac{d\beta(g^*)}{dq} = -\frac{r(1-r)(1-\alpha)}{[r(1-q)+\alpha(1-r)q]^2} < 0 \tag{A2}
\]

\[
\frac{d\beta(b^*)}{dq} = \frac{r(1-r)(1-\alpha)}{[rq+\alpha(1-r)(1-q)]^2} > 0 \tag{A3}
\]

\[
\frac{dPr(g^*)}{dq} = 2r - 1 \tag{A4}
\]

\[
\frac{dPr(b^*)}{dq} = 1 - 2r \tag{A5}
\]

Substituting (A2)-(A5) into (A1), we get:

\[
\frac{dE\beta}{dq} = (2r - 1) \frac{rq+(1-r)(1-q)}{rq+\alpha(1-r)(1-q)} - \frac{[r(1-r)(1-\alpha)][rq+(1-r)(1-q)]}{[r(1-q)+\alpha(1-r)q]^2}
\]

\[
+(1 - 2r) \frac{r(1-q)+\alpha(1-r)q}{r(1-q)+\alpha(1-r)q} - \frac{[r(1-r)(1-\alpha)][(1-r)q+r(1-q)]}{[rq+\alpha(1-r)(1-q)]^2} \tag{A6}
\]

which has the same sign as:

\[
[rq+(1-r)(1-q)][r(1-q)+\alpha(1-r)q]^2[(2r-1)(rq+\alpha(1-r)(1-q)) - r(1-r)(1-\alpha)]
\]

\[
-\frac{[r(1-q)+\alpha(1-r)(1-q)]^2[(2r-1)(r(1-q)+\alpha(1-r)q) - r(1-r)(1-\alpha)]}{(2r-1)(r(1-q)+\alpha(1-r)q) - r(1-r)(1-\alpha)} \tag{A7}
\]

Simplifying (A7) one obtains:

\[
r^2(1-r)^2(2q-1)(r(1-\alpha)+\alpha) \tag{A8}
\]

which is a positive quantity.
Proof of Proposition 2. In order to show that consumer surplus is increasing in $q$ we prove that the expected net utility for each consumer is (weakly) greater after an increase in $q$.

First, we note the following. Since $\tilde{\theta}_{g^*} \leq \tilde{\theta}_{b^*}$ there are three types of consumers: i) consumers who buy no matter what the signal is; ii) consumers who buy only when the signal is “good”; iii) consumers who do not buy, irrespective of the signal.

Consider consumers initially buying when receiving both signals, and suppose first that they do not change their decisions after an increase in $q$. Their expected net utility in equilibrium is given by:

$$r\theta + (1 - r)\alpha\theta - p^*$$

(A9)

which is increasing in $q$ since the equilibrium price is decreasing in $q$. If consumers change their decision after the variation in $q$, it follows that they can get a higher expected utility.

Consider consumers initially not buying when receiving both signals, and suppose first they do not change their decision after the variation in $q$. Their expected net utility is 0. These consumers are clearly unaffected by the increase in $q$. If consumers change their decision after the variation in $q$, it follows that they can get a positive expected utility by doing so.

Finally, consider consumers who buy only when receiving a good signal. The expected net utility for those agents is:

$$rq\theta + (1 - r)(1 - q)\alpha\theta - p(rq + (1 - r)(1 - q))$$

(A10)

The derivative of this quantity with respect to $q$ is:

$$\theta[r - \alpha(1 - r)] - \frac{dp^*}{dq}(rq + (1 - r)(1 - q)) - p(2r - 1) > 0$$

(A11)

Since $\frac{dp^*}{dq} < 0$, the derivative is surely positive if $\theta[r - \alpha(1 - r)] - p(2r - 1) > 0$. Let us assume first that $(2r - 1) > 0$. Since the tourist goes on holiday when the signal is good, it must be
\[ \theta \geq \frac{p[rq+(1-r)(1-q)]}{rq+a(1-r)(1-q)}. \] Then, if we prove that \[ \frac{p[rq+(1-r)(1-q)]}{rq+a(1-r)(1-q)} [r - \alpha(1 - r)] - p(2r - 1) \geq 0, \] this implies that \[ \theta [r - \alpha(1 - r)] - p(2r - 1) > 0 \] (since \(2r - 1 > 0\) implies \(r - \alpha(1 - r) > 0\)). This is proven in a few steps:

\[
\frac{p[rq+(1-r)(1-q)]}{rq+a(1-r)(1-q)} [r - \alpha(1 - r)] - p(2r - 1) \geq 0 \quad (A12)
\]

\[ r(1 - r)(1 - \alpha) > 0 \quad (A13) \]

Suppose now that \((2r - 1) < 0\), but \(r - \alpha(1 - r) > 0\). Then \(\theta [r - \alpha(1 - r)] - p(2r - 1) > 0\).

Finally, suppose that \(r - \alpha(1 - r) < 0\). Since the tourist does not go on holiday when the signal is bad, it must be \(\theta < \frac{p[rq+(1-r)(1-q)]}{rq+a(1-r)(1-q)}\). Then, if \[ \frac{p[rq+(1-r)(1-q)]}{rq+a(1-r)(1-q)} [r - \alpha(1 - r)] - p(2r - 1) \geq 0, \] then \(\theta [r - \alpha(1 - r)] - p(2r - 1) > 0\). Again, this is proved in a few steps:

\[
[rq + (1 - r)(1 - q)][r - \alpha(1 - r)] \geq rq + \alpha(1 - r)(1 - q)(2r - 1) \quad (A14)
\]

\[ r(1 - r)(1 - \alpha) > 0 \quad (A15) \]

Also in this case, if tourists change their choice after the variation in \(q\), this means that an even higher utility is obtained by acting in this way.

References


