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Abstract

A specific concept of structural model is used as a background for discussing the structurality of its parameterization. Conditions for a structural model to be also causal are examined. Difficulties and pitfalls arising from the parameterization are analyzed. In particular, pitfalls when considering alternative parameterizations of a same model are shown to have lead to ungrounded conclusions in the literature. Discussion of observationally equivalent models related to different economic mechanisms are used to make clear the connection between an economicall meaningful parameterization and an economically meaningful decomposition of a complex model. The design of economic policy is used for drawing some practical implications of the proposed analysis.

Keywords: structural model, recursive decomposition, exogeneity, causality, model identification, observationally equivalent models, structural invariance, misleading constraints.

JEL Classification: C10, C18, C50, C51, C54

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*The work underlying this paper started in 1984 under the heading " A Fallacious Argument in Econometric modeling" and since that period took advantage of a large number of comments from many colleagues. Incorporating these comments, the present version benefited from particularly useful comments from G. Wunsch and from the participants of the Seminar of the Economic Department of the University of Bologna (I)

Contents

1 Introduction

In this paper, we develop a concept of structural econometric model through a specific model building strategy and embed the concept of causality within the framework of a suitably constructed structural model, justifying accordingly that a structural model be also called a causal model. We show that the structurality of a model depends both on its probabilistic structure *and* on its parameterization. We also expose some difficulties, and pitfalls, in the treatment of identification and of reparameterization, particularly when treating observationally equivalent models through different parameterizations.

Following Mouchart, Wunsch and Russo (2015), we rely on a specific approach to structural modeling that combines two main econometric traditions. On the one hand, one of these traditions starts from a "theory " (*i.e.* economic theory) and develops a structural model from the statistical implications of an economic theory. This approach has been proposed by the Cowles Commission (CC), in particular Koopmans and Havelmoo. On the other hand, another tradition starts from the idea of a "Data Generating Process " (DGP), representing how the data have been generated: a structural model then looks for a structure underlying the DGP. This approach has been launched by D. Sargan at LSE and further developed by D. Hendry and others.

The order of exposition is as follows. In the next three sections, we propose a statistical approach to the concept of structural model by successively examining econometric models as a class of statistical models and the recursive decomposition of a model as a device for providing explanatory power to a model. We also show that the parameterization of a decomposed model is part of the explanatory process. The structurality of the model is related to field knowledge, in particular under some form of economic theory, and to properties of stability, or invariance, with respect to a class of interventions or of changes of the environment. Section 5 presents two major difficulties when treating the parameterization of a structural model. A first one deals with illegitimate constraints when blending two parameterizations of a same model; we show, in particular, that this error has been made repeatedly in the econometric literature. Another difficulty deals with the structural interpretation of the parameterizations of two different economic mechanisms leading nevertheless to observationally equivalent models. The last section takes an helicopter view of the achievements of this paper and points out, in particular, some implications for the design of an economic policy based on an econometric model.

2 Econometric Models as Statistical Models

In this paper, we approach an econometric model as a particular type of statistical model.

Formally, a statistical model $\mathbf{M}$ may be viewed as a set of probability distributions, explicitly:

$$\mathbf{M} = \{S,\, P^{\omega}\ \omega \in \Omega\} \tag{1}$$

where S, the sample space or observation space, is the range space of an observable random variable (or vector of variables) and for each $\omega \in \Omega$, P^{ω} is a probability distribution on the sample space, *i.e.* the sampling distribution. In other words, ω is a characteristic, or parameter, of the corresponding distribution and Ω describes the set of all sampling distributions belonging to the model. The basic idea is that the data are to be analyzed *as if* they were a realization of one of those distributions. A statistical model can accordingly be viewed as a set of plausible hypotheses regarding the Data Generating Process (for short, DGP); for more detail see Mouchart and Russo (2011), and Wunsch, Mouchart and Russo (2014).

A statistical model is based on a stochastic representation of the world. The random component of the model delineates the frontier, or the internal limitation, of the statistical explanation. More explicitly, the randomness represents what is not explained by the model, while the parameters of the distributions are the cornerstone of the statistical explanation.

A structural econometric model endeavors at unfolding the structure of an underlying economic mechanism assumedly generating the observed data. In the words of Illari and Williamson (2012): "A mechanism for a phenomenon consists of entities and activities organized in such a way that they are responsible for the phenomenon". This definition is general enough to be applicable to social contexts too. For economic phenomena, a mechanism may be viewed as a mathematical structure that models choices of economic agents or institutions through which economic activity is guided and coordinated. This mathematical structure provides a representation of a mechanism either in a deterministic form when the model is assumed to provide a complete explanation of a phenomenon or in a probabilistic form when the explanation is considered as an incomplete one. An econometric model, being a statistical one, belongs to the second alternative and takes the form of a conditional distribution where the endogenous variable is the one generated by the mechanism and the conditioning variables are those under which the mechanism is operating.

3 Recursive decomposition and sub-mechanisms

An econometric model is not always structural. Its aim may be to provide an insightful description of the observed data without the ambition of unfolding the mechanisms and sub-mechanisms underlying the DGP. Thus, many macro-econometric models as well as models of financial econometrics are based on the literature concerning the empirical issue of interest and on some stylized facts that the author aims to interpret. These models are of a descriptive nature, as they do not aim at unfolding a structural mechanism underlying a DGP.

3.1 Recursive decomposition

A structural econometric model is not only aimed at providing a stochastic *representation* of a global mechanism, but should also provide an *explanation* of that process. Once the model is dealing with a large number of variables, the usual way of explaining a complex process is given by a decomposition of the global mechanism into an ordered sequence of simpler sub-mechanisms. This is the objective of a recursive decomposition of a statistical model.

More explicitly, let us consider a partition of the data X into p components: $X = (X_1, X_2, \cdots X_p)$. Suppose that the components of X have been ordered in such a way that in the complete marginal-conditional decomposition of the joint distribution of X:

$$\begin{aligned} p_X(x \mid \omega) \quad = \quad & p_{X_p|X_1,X_2,\cdots X_{p-1}}(x_p \mid x_1, x_2, \cdots x_{p-1}, \theta_{p|1,\cdots p-1}) \cdot \\ & p_{X_{p-1}|X_1,X_2,\cdots X_{p-2}}(x_{p-1} \mid x_1, x_2, \cdots x_{p-2}, \theta_{p-1|1,\cdots p-2}) \cdot \\ & \cdots p_{X_j|X_1,X_2,\cdots X_{j-1}}(x_j \mid x_1, x_2, \cdots x_{j-1}, \theta_{j|1,\cdots j-1}) \cdot \\ & \cdots p_{X_1}(x_1 \mid \theta_1) \end{aligned} \tag{2}$$

each component of the right hand side is characterized by mutually independent parameters, *i.e.* variation-free in a sampling theory framework:

$$\omega = (\theta_{p|1,\cdots p-1}, \theta_{p-1|1,\cdots p-2} \cdots, \theta_1) \in \Theta_{p|1,\cdots p-1} \times \Theta_{p-1|1,\cdots p-2} \cdots \times \Theta_1 \tag{3}$$

or a priori independent in a Bayesian framework. When each factor in (2) stands for a univariate conditional distribution, equations (2) and (3) characterize a *completely recursive system*. When some, or all, factors represent the conditional distribution of a vector of variables, equations (2) and (3) characterize a *partially recursive, or block-recursive, system*.

The right-hand side of equation (2) may be interpreted as an ordered sequence of p sub-mechanisms, each one characterized by a distribution generating a variable conditionally on (an increasing set of) conditioning variables.

This is in line with Illari and Williamson (2012) and is compatible with an interpretation of the recursive decomposition in terms of sub-mechanisms in a structural model, as detailed in Wunsch, Mouchart, Russo (2014). Equation (3) requires that there are no restrictions binding the parameters of different factors of (2), in particular that there are no common parameters, and allows one to interpret each factors of the decomposition as independent sub-mechanisms.

A reference to the Simultaneous Equations Model (SEM) may be useful. In its standard formulation, the structural form of the SEM may be written as follows.

$$By + Cz = u \qquad u \sim \mathcal{N}(0, \Sigma) \qquad u \perp\!\!\!\perp z \tag{4}$$

The condition $u \perp\!\!\!\perp z$ implies that the equation in (4) is derived from the conditional distribution of $(y \mid z)$ and the conditional expectation $\mathbb{E}\,(y \mid z)$ is given by the reduced form:

$$y \,=\, \Pi\, z \,+\, v \quad \text{where } \Pi \,=\, -B^{-1}C \qquad v \,=\, B^{-1}u \tag{5}$$

$$\begin{aligned} By \;&+\; Cz \,=\, u && (6)\\ y \;&+\; (B - I)y \,+\, Cz = u && (7)\\ y \;&=\; (I - B)y - Cz + u && (8) \end{aligned}$$

This model is said *recursive* when the matrix B is lower triangular (along with the usual normalization rule of making the elements of the main diagonal equal to 1) and Σ is diagonal. Under a normality assumption, this recursive system of equations stands for an ordered sequence of conditional expectations and the errors are mutually independent. In this case, the SEM corresponds to a completely recursive decomposition of the joint distribution of the endogenous variables y conditionally on the exogenous variables z and the global mechanism generating $(y \mid z)$ is "explained" through an ordered sequence of sub-mechanisms represented by an ordered sequence of conditional univariate distributions.

When the SEM is not recursive, equation (4) does not refer to conditional distributions, does not stand for conditional expectations (along with errors) and may be interpreted as referring to "notional" sub-mechanisms in a spirit similar to that of counterfactuals. For instance, consider a two-equations elementary market model, generating price and quantity under a competitive equilibrium. The equation representing the quantity demanded as a function of the price and other exogenous variables, may be interpreted as the notional demand that would operate *if* the prices were exogenously fixed whereas the econometric model specifies that the actually observed prices, and quantities, have been jointly generated under an equilibrium mechanism.

From a causality, or explanation, point of view, equation (5) reflects a situation of block-recursivity: a global mechanism "explains" that vector z "causes" vector y but no explanation is given about the functioning of that mechanism. Only if model (4) were recursive would one obtain an explanation of the functioning of that global mechanism through a causal ordering in terms of sub-mechanisms.

3.2 Explanation and structurality

The recursive decomposition provides a structure for the explanation of a complex system through an ordered sequence of sub-mechanisms that compose the global mechanism. However, conditions (2) and (3) are not sufficient for ensuring that a given recursive decomposition is *the* valid one among the $p!$ possible recursive decompositions (corresponding to the number of ordered permutations of p variables). We also require conditions for ensuring that each factor of the product in (2) represents a valid sub-mechanism.

The required conditions are twofold. Firstly, each putative sub-mechanism, represented by a specific conditional distribution, should be congruent with field knowledge, *i.e.* with economic theory, as long as this one may be viewed as an organized synthesis of a large body of out-of-sample observations relative to the phenomenon of interest.

Secondly, the putative sub-mechanism should be stable, or invariant, relatively to a large class of interventions or of modifications of the environment. Indeed, a conditional distribution that would be different, say, for each observation could not be deemed to represent an underlying structure neither would it be useful, at least for accumulating empirical information. This refers to the fundamental issue of defining a "population of reference"; indeed neither an economic theory nor an econometric model may reasonably claim to be "universal" in time and in space.

The discussion of models such as (5) often concerns the distinction between the use of the model to make causal statements or simply use it to characterize a relationship between the observed data, *i.e.* a purely descriptive model. If model (5) correctly represents a (block-)causal relationship, it should be invariant under interventions that change any of the independent variables z. Conversely if (5) reveals not to be invariant under a wide range of interventions, it may be appropriate to describe a statistical relationship supported by the data, but it will not be explanatory. Moreover model (5) highlights a possible connection between the invariance on the one hand, and the relationship between explanation and counterfactual dependence on the other hand, *i.e.* dependence with respect to unobserved, or unobservable, exogenous changes. In other words invariance is a relative concept: a relationship can be invariant under a set of interventions and not under another one. Said differently, in the social and behavioral sciences the concept of invariance can be considered without appealing to a law of nature universally accepted, a notion that would be difficult to accept in the area of social sciences.

The stability requirement concerns both the structure of the model, *i.e.* the recursive decomposition as a set of sub-mechanisms (see for instance Richard (1980) for a change of exogeneity), and the characteristic of the sub-mechanisms, *i.e.* the parameters, see for instance Hendry and Mizon (1982).

When elaborating an economic policy on the basis of an econometric model, the structural validity of the model is of crucial importance, in particular because descriptive or non-structural models would provide unreliable forecasts or policy evaluations. This criticism is known in the literature as Lucas critique (Lucas, 1976), the essence of which is well summed up in this sentence: " ... given that the structure of an econometric model consists of optimal decision rules of economic agents, and that optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models " (Lucas, 1976, p. 41). It is worth noting that this criticism has strongly encouraged the structural approach in macroeconometric modeling.

More generally, the issue of invariant parametrization may be particularly complex. Indeed, the characteristics, or parameters, of a conditional distribution stand for the characteristics of an economically meaningful sub-mechanism. The tradition in economic theory is to view actual behaviors as a result of an optimization process. A substantial difficulty may be to decide what, in that optimization, is actually exogenous, *i.e.* considered as given.

To take a simple example, when optimizing a consumption plan under a budget constraint, this assumes that the level of income has been exogenously fixed, maybe under some equilibrium with respect to leisure and opportunity to increase disposable income (see *e.g.* Deaton, 1982). Endogenizing this process substantially increases the complexity of the model and should therefore be operated only when necessary for a correct specification of the model. If the endogenization of income is not operated in cases where it should have been, the price to be paid is not only a loss of consistency of the estimator but also a loss of stability of the the underlying parameters being actually estimated. As a matter of fact, building a model involves not only an empirical check on the parametric stability, by means for instance of hypothesis testing, but also a substantial amount of field knowledge that possibly could point out toward relevant changes of environment.

3.3 Parsimonious modelling

Consider now a conditional distribution, say

$$p_{X_j|X_1,X_2,\cdots X_{j-1}}(x_j \mid x_1, x_2, \cdots x_{j-1}, \theta_{j|1,\cdots j-1}), \tag{9}$$

identified as representing a plausible sub-mechanism. As a matter of fact, field knowledge, or a formal test of hypothesis, often suggests that not all conditioning variables are actually active in that sub-mechanism. More precisely, there may be a subset of the conditioning variables, or of functions of the conditioning variables, say $\mathcal{I}_{j-1}$, such that:

$$X_j \perp\!\!\!\perp X_1, X_2, \cdots X_{j-1} \mid \mathcal{I}_{j-1} \quad i.e.\ p_{X_j|X_1,X_2,\cdots X_{j-1}} = p_{X_j|\mathcal{I}_{j-1}}. \tag{10}$$

$\mathcal{I}_{j-1}$ may be called the *information set* relevant for the $j-th$ sub-mechanism, although more formally it is a σ-field rather than a set of variables. Under these conditions (10), the product, in (2), is condensed into:

$$p_{X_1,X_2,\cdots X_p} = \prod_{1\leq j\leq p} p_{X_j|\mathcal{I}_{j-1}} \tag{11}$$

where $\mathcal{I}_0$ stands for an initial condition. Equation (11) may be called a *condensed recursive form* and represents the actually relevant structure of the global mechanism, see also Mouchart and Russo (2011).

3.4 Causal and Structural Models

When all the above conditions are satisfied:

- The conditioning variables entering the information set may be viewed as exogenous and the conditioned variables may be viewed as endogenous, *for that specific sub-mechanism.* Thus, a particular variable is not endogenous or exogenous in itself, but relatively to a particular sub-mechanism.

- The conditioning variables may also be viewed as jointly *causing* the endogenous variable, viewed as an outcome (or, effect) variable; the corresponding conditional distribution provides a characterization of the *direct effect* of the cause on the outcome.

- The model may accordingly be called structural or causal.

- The global mechanism may be represented graphically by means of a Directed Acyclic Graph (for short, DAG)(for an introduction to DAG, see for instance, Pearl, 2009), although the specification of the information sets and of the condensed recursive form are often out of the scope of a DAG.

Remark. It is worth mentioning that the concept of exogeneity has a long history in econometrics. The works of the Cowles Commission in the late Forties and the early Fifties have been path-breaking and are still influential nowadays; in particular, Koopmans(1950) puts emphasis on exogeneity in dynamic models. Later, Barndorff-Nielsen(1978), in a purely statistical framework, developped general conditions for the separation of inference, introducing the concept of a *cut* in a statistical model. Florens, Mouchart and Rolin (1980) and Florens and Mouchart (1985) bridge Koopmans and Barndorff-Nielsen works and provide a coherent account of exogeneity integrating the separation of inference in dynamic and in non-dynamic models. Engle, Hendry and Richard (1983) present a classification of different concepts of exogeneity met in the econometric literature and display their connections with exogeneity through the introduction of supplementary conditions. Florens and Mouchart (1985) not only provide a basic concept of exogeneity, but also make the concept explicit in different levels of model specification, namely, global, initial, and sequential, before combining those concepts of exogeneity with non-causality. This analysis is further developed in Florens, Mouchart and Rolin (1993). ■

4 Structural model and parametrization

Let us now consider *how* to build a structural econometric model, *i.e.* a statistical model that would be an appropriate representation of a (global) economic mechanism in such a way that it would also provide an explanation of the working of that global mechanism. In Section 3, it has been mentioned that a natural strategy for enhancing the understanding of a complex multivariate mechanism would decompose that mechanism into an ordered sequence of simpler sub-mechanisms: this is the objective of the recursive decomposition. Therefore a first requirement is that each factor of the recursive decomposition should provide a suitable representation of a sub-mechanism stated in the form of a conditional distribution.

From a strictly statistical point of view, model (1) admits any arbitrary parametrization; in other words a parametrization is just a labeling system for a set of distributions. From a structural point of view, the parametrization labels an ordered sequence of conditional distributions, in the form of (2) or (11), each one representing a structurally relevant sub-mechanism. Thus a structural parametrization is endowed with an interpretation bound to the explanation of the global mechanism and of its constituting sub-mechanisms; it is also endowed with a stability property that ensures the structurality of the parametrization and allows for the accumulation of statistical information.

These ideas may be illustrated by the following simple example. Consider a model representing a market where y stands for the price of a given commodity and z for the quantity and assume that conditionally on the past up to time $t-1$, namely (Y^{t-1}, Z^{t-1}), the bivariate model is constructed as follows:

$$y_t \sim N(\mu_{y,t}\, ,\, \sigma^2_{y,t}) \qquad (z_t \mid y_t) \sim N(\alpha \,+\, \beta y_t\, , \sigma^2_{z|y}) \tag{12}$$

This model is equivalent to :

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} \sim N\left[\begin{pmatrix} \mu_{y,t} \\ \mu_{z,t} \end{pmatrix} \, , \, \begin{pmatrix} \sigma^2_{y,t} & \sigma_{yz,t} \\ \sigma_{yz,t} & \sigma^2_{z,t} \end{pmatrix}\right] \tag{13}$$

and also equivalent to:

$$z_t \sim N(\mu_{z,t}\, ,\, \sigma^2_{z,t}) \qquad (y_t \mid z_t) \sim N(\gamma_t \,+\, \delta_t z_t\, , \sigma^2_{y|z,t}) \tag{14}$$

under some obvious relationships among these different parameterizations, such as:

$$\mu_{z,t} = \alpha \,+\, \beta\mu_{y,t} \quad \sigma_{yz,t} = \beta\, \sigma^2_{y,t} \quad \sigma^2_{z,t} = \sigma^2_{z|y} + \beta^2\, \sigma^2_{y,t} \tag{15}$$

Let us now assume that the model is aimed to represent the working of a monopolistic market where the offer sets the price y under a possibly unstable process whereas the demand, generating the quantity z, is just price-taking under a stable mechanism depending only on the current price. This economic structure is captured by the model (12) the parameters of which explain the operation of the global mechanism in terms of two economically meaningful sub-mechanisms. Model (13), although statistically equivalent, does not provide an adequate structure of the global mechanism and its parametrization has no interpretation in terms of the functioning of the global mechanism. Model (14) suffers from the same weakness of explanatory power as model (13): the factors of that underlying recursive decomposition do not represent the assumed economic structure. Moreover, model (12) has extracted from the apparent instability of the global process some stable, and accordingly structural, parameters that will be therefore estimable and has succeeded in isolating what is not stable. This is a simple example of the nature of structural modeling: identify relevant sub-mechanisms and identify the stable aspects of the working of economic mechanisms.

Conversely if it is assumed that price y and quantity z are fixed simultaneously following a competitive process aiming at clearing the market, the economic structures represented by models (12) and (14) are not valid representations of the economic structure. Only the global mechanism (13) provides a correct representation of the system without, however, having the possibility to decompose it into a sequence of simpler sub-mechanisms, *i.e.* without providing an explanation of the the equilibrium mechanism. More explicitly, it might be possible to introduce two *notional* concepts of offer and demand but these concepts would not be sufficient for providing an ordered causal structure representing the functioning of the global process of competitive equilibrium.

Remark. The recursive decomposition does not require neither a specification of the coordinates coding the variables nor a specification of a particular family of distribution. In other words, the recursive decomposition is non-parametric and is σ-algebraic in nature. The next step of the modeling is to, simultaneously, specify the coordinates of the variables and a parametrization of a chosen family of distributions.

5 Pitfalls with alternative parameterizations

Previous sections handle the specification and the parametrization of structural models. In a purely statistical approach, a parameterization of a statistical model is a labeling system for the distributions the set of which specifies the model. In a structural econometric model, the parameterization is the basis for the explanatory mission of the structural model, provided the parameterization bears on a structurally valid recursive decomposition.

In this section, we treat some difficulties, and possible pitfalls, when facing alternative possible parameterizations. These issues are indeed important for a proper understanding of what is at stake with the choice of the parameterization. We shall discuss two different questions. The first one is exemplified with a case of simultaneous equations. Two parameterizations are used for estimation purposes: the structural form and the reduced form. Identification restrictions on the structural form ensure the identification of the parameters characterizing the notional concepts underlying the equations of the structural form; the proper relationship between the two parameterizations has however been the object of repeated misunderstandings in the econometric literature. A second type of problems is raised when different structural models are observationally equivalent, *i.e.* correspond to a same statistical model, namely a same set of distributions. This is a matter of model identification, as different from parametric identification.

5.1 Reparameterizations suggesting erroneous constraints

5.1.1 A simple pedagogic example

Consider the following assertion: "In a univariate normal distribution, $N(\mu, \sigma^2)$, if the variance σ^2 tends to zero, the expectation μ necessarily tends to zero". As a " proof", of this assertion, consider the inverse of the coefficient of variation $\eta = \mu\sigma^{-1}$; thus $\mu = \eta\sigma$ and therefore $\sigma \to 0$ implies $\mu \to 0$.

Such an assertion and its "proof" rest on a fallacious argument. The error may be viewed as follows: the pair $\theta = (\mu, \sigma) \in \Theta$ gives one parametrization of the univariate normal family while $\lambda = (\eta, \sigma) \in \Lambda$ gives another one; both parameterizations (of the same family of distributions) have the same parameter space $\Theta = \Lambda = R \times R_+$ without restrictions on the range of variation of the parameters. The argument "$\sigma \to 0$ implies $\mu \to 0$" is therefore invalid because it is based on a relationship ($\mu = \eta\sigma$) involving two different parameterizations: (μ, σ) and (η, σ). The fallaciousness of the assertion above may be viewed graphically in Figure 1 as follows. Let us consider a fixed value e_0 of η and the corresponding subsets of the two parametrizations $T_0 = \{(\mu, \sigma) \in \Theta | \mu = \sigma\, e_0\}$ and $L_0 = \{(\eta, \sigma) \in \Lambda | \eta = e_0\}$. Clearly L_0 and T_0 correspond to each other: they represent the same set of normal distributions. To a converging sequence $\sigma \to 0$ corresponds, *in* L_0, a sequence converging to $(e_0, 0)$ and, *in* T_0, a sequence converging to $(0, 0)$. This trivial fact does not imply any relationship between μ and σ in Θ, *i.e.* Θ has a product structure, equivalently μ and σ are variation-free.

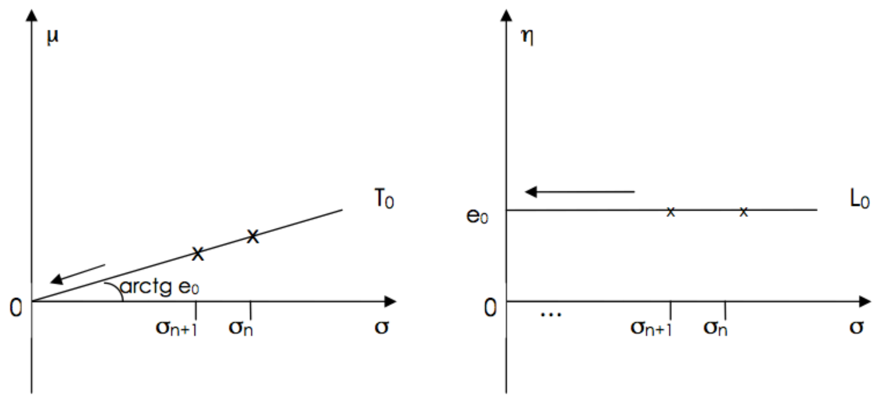

Figure 1: **A same converging sequence in two parameterizations.**

5.1.2 A case in simultaneous equations

As naive as it may seem, the previous fallacious argument has been met in less elementary situations, when considering different parameterizations of a given model. As an example, let us consider the simplest version of Haavelmo (1947)'s model:

$$\begin{aligned} c_t &= \delta_0 + \delta y_t + u_t \qquad && \text{with } \ u_t \sim IN(0, \sigma_{uu}) \\ y_t &= c_t + z_t && u_t \perp\!\!\!\perp z_t \end{aligned} \tag{16}$$

with reduced form:

$$y_t = \alpha_0 + \alpha z_t + v_t \tag{17}$$

where $\alpha_0 = \delta_0(1-\delta)^{-1}$, $\alpha = (1-\delta)^{-1}$, $v_t = (1-\delta)^{-1}u_t$. Let us also assume that $z_t \sim IN(\mu_z, \sigma_{zz})$ independently of u_t (or of v_t). The reduced form implies $\sigma_{yy} = \alpha^2\sigma_{zz} + \sigma_{vv}$ and therefore $\alpha^2 < \sigma_{yy}\sigma_{zz}^{-1}$. This (true) inequality appears to have been erroneously interpreted as a constraint on the parameter space. For instance, Genberg (1972) suggests to estimate α under the restriction $\alpha^2 < q$, where q would be a consistent estimator of the variance ratio $\sigma_{yy}\sigma_{zz}^{-1}$ and notices that if one were to use a ratio of sample moments to estimate $\sigma_{yy}\sigma_{zz}^{-1}$ the restriction $\alpha^2 < q$ would not be binding when estimating α by ordinary least squares. Remark that Gensberg's proposal could also be used to assert that Haavelmo's model implies a (wrong) restriction over σ_{zz}, namely $\sigma_{zz} < \sigma_{yy}\alpha^{-2}$.

As a matter of fact, we have two parameterizations (structural form and reduced form) of a same model, namely $(\mu_z, \sigma_{zz}, \delta_0, \delta, \sigma_{uu}) = \theta$, say and

$(\mu_z, \sigma_{zz}, \alpha_0, \alpha, \sigma_{vv}) = \lambda$, say. These parameterizations are in bijection and each is variation-free: there is no constraint among the parameters *within* a same parameterization. The variance of y, σ_{yy}, is a function of these parameters, indifferently of θ or of λ, is not a new parameter and introduces no new constraint neither on θ nor on λ. In particular, the (true) inequality $\sigma_{yy} > \alpha^2\sigma_{zz}$ does not introduce a constraint within the parametrization θ[1].

In order to ascertain how fallacious these arguments of "constraints often overlooked" (see Zellner, 1972 and Maddala, 1976) are, we consider, in rather obvious notation, two equivalent parameterizations of the set of bivariate normal distributions on (y, z):

$$\theta_* = (\mu_y, \mu_z, \sigma_{yy}, \sigma_{yz}, \sigma_{zz}) \in \Theta_* = I\!R^2 \times \mathcal{C}_{(2)} \quad (18)$$

$$\lambda_* = (\mu_z, \alpha_0, \alpha, \sigma_{vv}, \sigma_{zz}) \in \Lambda_* = I\!R^3 \times I\!R^2_+ \quad (19)$$

where $\alpha_0 = \mu_y - \alpha\mu_z$, $\alpha = \sigma_{yz}\sigma_{zz}^{-1}$, $\sigma_{vv} = \sigma_{yy} - \sigma_{yz}^2\sigma_{zz}^{-1}$ and $\mathcal{C}_{(2)}$ is the cone of the (2×2) $SPDS$ matrices. It should be pointed out that there is no restriction on the range of α, neither on that of σ_{zz} nor on that of σ_{yy}. As in the pedagogic example, inequalities like $\alpha^2 < \sigma_{yy}\sigma_{zz}^{-1}$ involve two different parametrizations and represent no restriction on any parameter space.

Before qualifying the use of such inequalities as misleading restrictions, it may be illustrative to consider a slightly more general version of the very same example. Let us consider the model:

$$\begin{aligned} (y|z) &\sim N_m(\Pi_a' z, \Sigma_{vv}) \\ z &\sim N_k(0, \Sigma_{zz}) \end{aligned} \quad (20)$$

where Π_α is a $(k \times m)$ matrix the elements of which are known functions of a vector of unknown parameters α. This model may also be written as:

$$\begin{pmatrix} y \\ z \end{pmatrix} \sim N_{m+k}\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_{yy} & \Pi_\alpha'\Sigma_{zz} \\ \Sigma_{zz}\Pi_\alpha, & \Sigma_{zz} \end{pmatrix}\right] \quad (21)$$

where $\Sigma_{yy} = \Sigma_{vv} + \Pi_\alpha'\Sigma_{zz}\Pi_\alpha$. Thus we consider two equivalent parametrizations:

$$\theta_{**} = (\Sigma_{zz}, \Sigma_{vv}, \alpha) \in \Theta_{**} = \mathcal{C}_{(k)} \times \mathcal{C}_{(m)} \times A \quad (22)$$

$$\lambda_{**} = (\Sigma_{zz}, \Sigma_{yy}, \alpha) \in \Lambda_{**} = \{(\Sigma_{zz}, \Sigma_{yy}, \alpha) \in \mathcal{C}_{(k)} \times \mathcal{C}_{(m)} \times A \ |\Sigma_{yy} - \Pi_\alpha'\Sigma_{zz}\Pi_\alpha \in \mathcal{C}_{(m)}\} \quad (23)$$

[1] But if the estimation of σ_{yy} introduces new data, then we have a case of mixed estimation, "à-la-Theil", or of mixed Bayesian estimation and the constraints may have a role in the procedure of blending two sources of data but not for introducing new constraints on the parameter space.

where A is the set of possible values for α, the "free" parameters of the (possibly overidentified) reduced form. Here it is valid to recognize that the parameter space Λ_{**} is restricted by the condition: $\Sigma_{yy} - \Pi'_\alpha \Sigma_{zz} \Pi_\alpha \in \mathcal{C}_{(m)}$, *i.e.* $\Sigma_{yy} - \Pi'_\alpha \Sigma_{zz} \Pi_\alpha (= \Sigma_{vv})$ should be $SPDS$, and thus $\Sigma_{yy}, \Sigma_{zz}, \alpha$ are not variation free . Note however that the relationship $\Sigma_{yy} - \Sigma_{vv} \in \mathcal{C}_{(m)}$, although true, does not provide any effective restriction as it grounds on two different parametrizations. The danger of overlooking this fact can be illustrated in the particular case where $m = 2, \alpha = (\beta, \gamma), \Pi_\alpha = (\beta\gamma,\ \beta) = \beta(\gamma,\ 1)$ with $\beta \in I\!R^k$ and $\gamma \in I\!R$. Thus, in this particular case:

$$\Sigma_{yy} = \Sigma_{vv} + \beta' \Sigma_{zz} \beta \begin{pmatrix} \gamma^2 & \gamma \\ \gamma & 1 \end{pmatrix}$$

Here it is valid that, in Λ_{**}, parameters α, Σ_{zz} and Σ_{yy} should be restricted by:

$$\Sigma_{yy} - \beta' \Sigma_{zz} \beta \begin{pmatrix} \gamma^2 & \gamma \\ \gamma & 1 \end{pmatrix} \in \mathcal{C}_{(2)}$$

or, more explicitly:

$$\sigma_{y_1 y_1} \geq \gamma^2 \beta' \Sigma_{zz} \beta$$
$$\sigma_{y_2 y_2} \geq \beta' \Sigma_{zz} \beta$$
$$\sigma_{y_1 y_1} \sigma_{y_2 y_2} - \sigma^2_{y_1 y_2} \geq \beta' \Sigma_{zz} \beta (\gamma^2\, \sigma_{y_2 y_2} - 2\gamma\, \sigma_{y_1 y_2} + \sigma_{y_1 y_1})$$

(Note that one of the first two inequalities is redundant). A fallacious use of the (true) relationship $\beta' \Sigma_{zz} \beta = \gamma^{-1} (\sigma_{y_1 y_2} - \sigma_{v_1 v_2})$ would be to assert that, from the above inequalities, γ is constrained to satisfy restrictions such as the following ones:

$$\frac{\sigma_{y_1 y_2} - \sigma_{v_1 v_2}}{\sigma_{y_2 y_2}} \lesseqgtr \gamma \lesseqgtr \frac{\sigma_{y_1 y_1}}{\sigma_{y_1 y_2} - \sigma_{v_1 v_2}} \ \text{ according to : } \ \sigma_{y_1 y_2} \lesseqgtr \sigma_{v_1 v_2} \tag{24}$$

$$\begin{aligned} \gamma^2\, \sigma_{y_2 y_2} (\sigma_{y_1 y_2} - \sigma_{v_1 v_2}) \quad & - \gamma [\sigma_{y_1 y_1} \sigma_{y_2 y_2} \ - \ \sigma^2_{y_1 y_2} + 2 \sigma_{y_1 y_2} (\sigma_{y_1 y_2} \ - \ \sigma_{v_1 v_2})] \\ & + \sigma_{y_1 y_1} (\sigma_{y_1 y_2} \ - \ \sigma_{v_1 v_2}) \ \leq \ 0 \end{aligned} \tag{25}$$

In other words, we disagree with the contention that (24) or (25) would mean that "certain parameter values are subject to bounds flowing from usual specifying assumptions". Indeed inequalities (24) and (25) do not represent restrictions neither on Θ_{**} nor on Λ_{**}: they only express some properties of the correspondence between Θ_{**} and Λ_{**}. As a consequence, estimating γ, for instance, under the restriction implied by consistent estimates of the bounds of inequalities (24) would either be ineffective, if a correct parameterization is employed, or would consist of using twice the same sample information if one were to step from one parameterization to another one. This later would be the case when estimating *e.g.* bounds in one parameterization before estimating the other parameterization without taking due account of the double use of the same sample information.

When pooling two sources of information as is done with bayesian methods or in pooling time series and cross-section data or in mixed estimation, the use of two parameterizations may be justifiable. In the above example, while θ_{**} may be the most natural parameterization for the final inference, it may be that there is available some prior information on, say, Σ_{yy} leading to consider also the λ_{**}-parameterization as a natural recipient for that prior information. If inference only concerns parameters common to θ_{**} and λ_{**}, *i.e.* α and/or Σ_{zz}, one may also start by first incorporating the prior information in the λ_{**}-parameterization and thereafter reparameterize in θ_{**} before incorporating the second information. Such a stepwise procedure seems to be the only available one in the case of inference on Σ_{vv}. It should nevertheless be noticed that even in such a case inequalities (24) or (25) will never appear as restriction on any parameter space.

The reader may like to compare this analysis with the debate between Maddala (1976, a and 1976,b) and Zellner (1972, 1976, a and 1976,b) that has been inconclusive for missing the issue of the relationships between alternative parameterizations.

5.2 Reparameterizations involving different but observationally equivalent sub-mechanisms

The previous section illustrates possible abuse of restrictions due to a fallacious argument involving alternative parameterizations. In this section, we consider two different structural models that are nevertheless observationally equivalent and, therefore, correspond to two different parameterizations of a same model. In such cases the two parameterizations correspond to two different (sub-)mechanisms possibly suggesting different contextually relevant parametric restrictions. As a matter of fact the price equations of the two models below represent two structurally different mechanisms, and the corresponding parameters capture quite a different economic meaning.

"Model 1" (from Bowden, 1978 b)

Consider a simple price adjustment model:

$$D_t = \alpha_1' x_t^D - \alpha_2 P_t + u_{1t} \tag{26}$$
$$S_t = \beta_1' x_t^S + \beta_2 P_t + u_{2t} \tag{27}$$
$$P_t = \mu P_{t-1} + (1-\mu) P_t^* + u_{3t} \tag{28}$$

where D_t (demand), S_t (supply) and P_t (price) are endogenous variables, x_t^D and x_t^S are exogenous variables and P_t^* is a "clearing" price defined as:

$$P_t^* = (\beta_2 + \alpha_2)^{-1}(\alpha_1' x_t^D - \beta_1' x_t^S + u_{1t} - u_{2t}) \tag{29}$$

Therefore, the reduced form of the price equation is:

$$\begin{aligned} P_t &= \mu P_{t-1} + (1-\mu)[(\beta_2+\alpha_2)^{-1}(\alpha_1' x_t^D - \beta_1' x_t^S + u_{1t} - u_{2t})] + u_{3t} \\ &= \mu P_{t-1} + (1-\mu)(\beta_2+\alpha_2)^{-1}\alpha_1' x_t^D - (1-\mu)(\beta_2+\alpha_2)^{-1}\beta_1' x_t^S \\ &\quad +(1-\mu)(\beta_2+\alpha_2)^{-1}[u_{1t} - u_{2t})] + u_{3t} \end{aligned} \tag{30}$$

"Model 2" (from Fair and Jaffee, 1972)

Consider another model given by equations (26), (27) and the alternative specification of the price adjustment mechanism

$$P_t - P_{t-1} = \eta(D_t - S_t) + u_{3t}^{**} \tag{31}$$

equivalently:

$$\begin{aligned} P_t &= P_{t-1} + \eta(D_t - S_t) + u_{3t}^{**} &(32) \\ &= P_{t-1} + \eta[\alpha_1' x_t^D - \alpha_2 P_t + u_{1t} - (\beta_1' x_t^S + \beta_2 P_t + u_{2t})] + u_{3t}^{**} \\ &= [1+\eta(\beta_2+\alpha_2)]^{-1}[P_{t-1} + \eta\alpha_1' x_t^D - \eta\beta_1' x_t^S + \eta(u_{1t} - u_{2t}) + u_{3t}^{**}] \end{aligned}$$

Therefore, the reduced form of the price equation is:

$$\begin{aligned} P_t &= [1+\eta(\beta_2+\alpha_2)]^{-1} P_{t-1} + [1+\eta(\beta_2+\alpha_2)]^{-1}\eta\alpha_1' x_t^D \\ &\quad -[1+\eta(\beta_2+\alpha_2)]^{-1}\eta\beta_1' x_t^S \\ &\quad +[1+\eta(\beta_2+\alpha_2)]^{-1}[\eta(u_{1t} - u_{2t}) + u_{3t}^{**}] \end{aligned} \tag{33}$$

Notice that (28) has sense only if $0 \leq \mu \leq 1$ whereas in (31) one should require $\eta \in I\!R_+$. Therefore the parameter spaces corresponding to Models 1 and 2 are:

$$\theta = (\alpha_1, \beta_1, \alpha_2, \beta_2, \mu, \Sigma) \in \Theta = I\!R^{k_D+k_S} \times I\!R_+^2 \times [0,1] \times \mathcal{C}_{(3)} \tag{34}$$
$$\lambda = (\alpha_1, \beta_1, \alpha_2, \beta_2, \eta, \Sigma^{**}) \in \Lambda = I\!R^{k_D+k_S} \times I\!R_+^3 \times \mathcal{C}_{(3)} \tag{35}$$

where k_D is the number of variables in x^D, k_S is the number of variables in x^S (for the sake of presentation, we assume no common exogenous variable in x^S and x^D), $\Sigma = Var(u_1, u_2, u_3)$ and $\Sigma^{**} = Var(u_1, u_2, u_3^{**})$.

The two price equations (28), along with (30) and (31) with (33), represent two different sub-mechanisms. Model 1 and Model 2 are nevertheless observationally equivalent because they provide two different parameterizations of a same set of distributions. Indeed, from the coefficients of P_{t-1} the mapping between θ and λ is given by:

$$\mu = [1 + \eta(\beta_2 + \alpha_2)]^{-1} \qquad \eta = \frac{1 - \mu}{\mu(\beta_2 + \alpha_2)} \tag{36}$$

implying that in equations (34) and (35) we indeed have:

$$\mu \in [0,\, 1] \quad \text{corresponding to} \quad \eta \in I\!R_+$$

Moreover, for the coefficients of $(u_{1t} - u_{2t})$, we may check that:

$$(1 - \mu)(\beta_2 + \alpha_2)^{-1} = [1 + \eta(\beta_2 + \alpha_2)]^{-1}\eta$$

and conclude that the two models are observationally equivalent under the identification relationship:

$$u_{3t} = [1 + \eta(\beta_2 + \alpha_2)]^{-1} u_{3t}^{**} \qquad u_{3t}^{**} = \mu^{-1} u_{3t} \tag{37}$$

As a matter of fact, equation (37) may be viewed as a short-hand notation for:

$$\sigma_{j3} = [1 + \eta(\beta_2 + \alpha_2)]^{-1} \sigma_{j3}^{**} \qquad \sigma_{j3}^{**} = \mu^{-1}\sigma_{j3} \quad j = 1, 2 \tag{38}$$

$$\sigma_{33} = [1 + \eta(\beta_2 + \alpha_2)]^{-2} \sigma_{33}^{**} \qquad \sigma_{33}^{**} = \mu^{-2}\sigma_{33} \tag{39}$$

Note that μ and Σ are variation free in Θ; so are also η and Σ^{**} in Λ. It may be nevertheless tempting to erroneously conclude from (37) , as in the previous section, that " $\mu \to 0$ implies $\sigma_{33} \to 0$ because $u_{3t} = \mu u_{3t}^{**}$ ".

Once Model 1 and Model 2 are recognized as observationally equivalent, does it imply that the two price sub-mechanisms are also equivalent? The answer is: no! Indeed, equations (28) and (31) explain the disequilibrium of the market by two different price sub-mechanisms : in equation (28) the price sub-mechanism partially adjusts the past price to the current equilibrium price whereas equation (31) partially adjusts the past price in function of the disequilibrium in quantity. Thus the parameters μ and η have a clearly different economic meaning. In other words, the choice between the specifications of Models 1 and 2 is a choice between two different parameterizations of a same (conditional) distribution and should be based on economic and contextual plausibility and on structural stability.

This example illustrates several issues. Firstly, the two models are observationally equivalent: no data may decide in favor of one against the other one. Nevertheless, the two price equations represent two structurally different sub-mechanisms, the corresponding parameters being endowed with quite a different economic meaning: only field knowledge, structural stability (or invariance) or new information may decide which one is actually structural. Secondly, again, one should operate a clear distinction between relationships among alternative parameterizations, bringing no restrictions on the parameter space, and genuine restrictions that might be used either to improve inference, if accepted, or to be subjected to testing, if put in doubt. Thirdly, alternative parameterizations may possibly bring interesting information in an encompassing spirit. For instance, if one is willing to interpret the parameters of Model 1 at the light of Model 2, the relationship (36) would tell that a value of μ close to 0 (resp. 1) in Model 1 would correspond to a great (resp. small) value of η in model 2. But this relationship implies no restriction neither on μ nor on η.

More recently, An and Schorfheide (2007) produced another example of two observationally equivalent models corresponding to two different parametrizations of a same family of conditional distributions. These two models describe different sub-mechanisms characterized by parameters with a different economic interpretation. Through an implied identification problem, they propose to work out a bayesian solution. This example shows that when specifying a structural model it is not sufficient to specify a family of distributions: a particular parametrization, with a specific economic meaning, should also be specified.

6 Concluding remarks

6.1 Summarizing: The basic framework

In this paper, we make explicit the link between causal model and structural model. In particular, the structural explanation of a model is based on a recursive decomposition of the model itself, together with the interpretation of each term of the structural decomposition as an economic sub-mechanism. When we can reach this structuring of the model, the explanatory variables can be interpreted as causal variables.

We also propose an approach for building a structural econometric model, *i.e.* a statistical model that provides (i) an appropriate representation of a global economic mechanism and (ii) an explanation of the working of that global mechanism *i.e.* each factor of the recursive decomposition should provide a suitable representation of an economically meaningful sub-mechanism.

The necessary conditions for a recursive decomposition to be interpreted as a structural model, are: (i) there is congruence with the underlying economic theory (ii) there is invariance or stability of the parameters characterizing the economic sub-mechanisms as well as of the recursive decomposition itself.

One limitation of this approach is that the explanatory power of the model relies on its recursive decomposition. A completely recursive decomposition provides a complete causal ordering of the variables. When a completely recursive decomposition is not possible, *i.e.* the case of block-recursivity where the recursive decomposition is only partial, there is a simultaneity of the action of several sub-mechanisms within the generation of a block of endogenous variables. In this case, the model cannot claim for causal effects within the block of endogenous variables because outcomes cannot cause each other simultaneously.

From a narrow statistical point of view, the parametrization of a family of distributions is arbitrary. In a structural modeling approach the issue is more subtle. Indeed, a basic requirement of structurality is the stability of the recursive decomposition, involving both the stability of the decomposition itself and the stability of the parameters of the different distributions. As a matter of fact, the stability of these characteristics is, in general, not complete. As mentioned in an example of Section 4, some factors of the decomposition may be more stable than others. Therefore the specification of the parametrization should be based on the search of the parameters that are likely to be more stable; this ensures that these parameters are endowed with a reliable economic meaning.

This paper also makes explicit some difficulties and pitfalls when handling alternative parameterizations, namely the danger of introducing illegitimate constraints in case of alternative parameterizations of a same model, or selecting among two different sub-mechanisms leading to observationally equivalent models.

6.2 On the use of models for the design of economic policy

An intervention, such as an economic policy, should be based on a structural, or causal, model rather than on a descriptive model. More specifically, econometric models used for the design of economic policy should represent actual behavior, be it macro- or micro-, rather than provide a representation based on theoretically grounded behavior. This is precisely the meaning of an econometric structural, or causal, model.

Thus the model builder should carefully check the structural stability of the model, in particular its resilience to a suitable class of transformations of the environment. Indeed the parameters and the recursive structure of the model should not be thought in a universal sense, in space and in time, and it is crucial to evaluate how the "universe" should be circumscribed. Lucas' critique may indeed be interpreted as referring to the fact that an intervention may modify the structure of the causal model, because a model is developed within a given environment and the difficulty may be to evaluate to what extent a modification of the environment might modify some properties of the causal model. Thus the strategy for building an econometric model should identify from the apparent instability of the global process the relevant sub-mechanisms and identify the stable aspects of the working of economic mechanisms.

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