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# Minimum Quality Standard Under Cournot Competition and Pollution

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**Abstract**

We extend the analysis carried out by Valletti (2000) by considering an environmental externality in a vertically differentiated duopoly where firms compete *à la* Cournot with fixed costs of quality improvement. We show that, if the weight of the external effect is high enough, the resulting minimum quality standard is indeed binding.

**JEL codes:** L13, L51, Q50.

**Keywords:** MQS, environmental externality, product quality.

# 1 Introduction

In this note we extend the analysis carried out by Valletti (2000) by considering an environmental externality in a vertically differentiated duopoly where firms compete *à la* Cournot and incur in fixed costs of quality development. Valletti (2000) shows that quantity-setting behaviour implies a non-binding minimum quality standard (MQS), as Cournot competition is milder than Bertrand's. This conclusion, however, is based on a setting where undesirable environmental implications are not modelled and therefore any quality distortion is solely driven by the firms' profit incentives. Instead, our results show that the presence of a negative external effect increasing in industry output, if large enough, implies that the MQS will indeed bite at the regulated equilibrium, bringing about an increase in qualities and welfare and a decrease in the externality.

# 2 The model

We consider a duopoly market for vertically differentiated products supplied by single-product firms. The demand side is modelled *à la* Mussa and Rosen (1978). There is a continuum of consumers whose types are identified by $\theta$, uniformly distributed with density equal to one in the interval $[0, \Theta]$ (so that total demand is equal to $\Theta$). Parameter $\theta$ represents the consumers' marginal willingness to pay for quality. Each consumer is assumed to buy at most one unit of the vertically differentiated good in order to maximise the following surplus function:

$$U = \theta q_i - p_i, \tag{1}$$

where $q_i \in [0, Q]$ indicates the quality of the product and $p_i$ is the market price at which that variety is supplied by firm $i = H, L$, with $q_H \geq q_L$. Therefore, the consumer who is indifferent between $q_H$ and $q_L$ is identified by the level of marginal willingness to pay $\widehat{\theta}$ that solves

$$\widehat{\theta} q_H - p_H = \widehat{\theta} q_L - p_L, \tag{2}$$

and therefore $\widehat{\theta} = (p_H - p_L) / (q_H - q_L)$. Thus, market demand for the high-quality good is $x_H = \Theta - \widehat{\theta}$. We assume partial market coverage, so that there is another consumer, identified by $\widetilde{\theta}$, who is indifferent between buying $q_L$ or not buying at all:

$$\widetilde{\theta} q_L - p_L = 0, \tag{3}$$

whereby $\widetilde{\theta} = p_L/q_L$ and the demand for the inferior variety is $x_L = \widehat{\theta} - \widetilde{\theta}$. Accordingly, we can define consumer surplus as follows:

$$CS = \int_{\widetilde{\theta}}^{\widehat{\theta}} (kq_L - p_L)dk + \int_{\widehat{\theta}}^{\Theta} (zq_H - p_H)dz. \tag{4}$$

This is what one needs to use in order to model Bertrand behaviour, while inverse demands

$$p_H = (\Theta - x_H)\, q_H - q_L x_L$$
$$p_L = (\Theta - x_H - x_L)\, q_L \tag{5}$$

are to be used under Cournot competition.

On the supply side, as in Ronnen (1991) and Motta (1993), *inter alia*, firms incur in convex fixed costs of quality improvement $C_i = cq_i^2$, $i = H, L$. Variable costs are assumed away. Hence profit functions are $\pi_H = p_H x_H - cq_H^2$ and $\pi_L = p_L x_L - cq_L^2$.

Production entails a negative environmental externality $s = b(x_H + x_L)^2$, with $b > 0$, measuring the negative impact of production on the environment. Also, note that consumers are assumed to be myopic, in the sense that (1) does not account for the presence of pollution.[1] Social welfare is determined

[1] It is worth noting that this is not a crucial assumption, as admitting the possibility for consumers to be environmentally concerned, with $U = \theta q_i - p_i - s$ would not modify the expressions of $\widehat{\theta}$ and $\widetilde{\theta}$ resulting from $\widehat{\theta} q_H - p_H - s = \widehat{\theta} q_L - p_L - s$ and $\widetilde{\theta} q_L - p_L - s = -s$, respectively.

by the sum of profits and consumer surplus, minus the environmental externality:

$$W = CS + \pi_H + \pi_L - s. \tag{6}$$

Competition takes place in two stages. In the first, firms choose qualities and in the second they compete in quantities. Moves are simultaneous in both stages, and the solution concept is the subgame perfect equilibrium by backward induction.

# 3 Results

To begin with, we characterise optimal outputs for any given quality pair:

$$x_H^N = \frac{\Theta(2q_H - q_L)}{4q_H - q_L};\ x_L^N = \frac{\Theta q_H}{4q_H - q_L} \tag{7}$$

where superscript $N$ stands for Nash equilibrium. The explicit derivation of the Cournot equilibrium is omitted as it is known from Motta (1993).

We now turn to the first stage where the quality game takes place. We will prove our results by manipulating the set of the first order conditions in the two alternative cases under consideration, i.e., with or without MQS. The relevant profit functions are:

$$\begin{gathered}\pi_H = \frac{q_H\left[\Theta^2\left(2q_H - q_L\right)^2 - cq_H\left(4q_H - q_L\right)^2\right]}{\left(4q_H - q_L\right)^2} \\ \pi_L = \frac{q_L\left[\Theta^2 q_H^2 - cq_L\left(4q_H - q_L\right)^2\right]}{\left(4q_H - q_L\right)^2}\end{gathered} \tag{8}$$

With no MQS, the first order conditions for non cooperative profit maximization are:

$$\frac{\partial \pi_H}{\partial q_H} = \frac{\Theta^2\left(16q_H^3 - 12q_H^2 q_L + 4q_H q_L^2 - q_L^3\right) - 2cq_H\left(4q_H - q_L\right)^3}{\left(4q_H - q_L\right)^3} = 0, \tag{9}$$

$$\frac{\partial \pi_L}{\partial q_L} = \frac{\Theta^2 q_H^2\left(4q_H + q_L\right) - 2c\left(4q_H - q_L\right)^3}{\left(4q_H - q_L\right)^3} = 0. \tag{10}$$

In the regulated case, the government introduces an MQS aimed at affecting directly the behaviour of firm $L$. Firm $H$'s FOC remains unchanged, while the regulator solves:

$$\frac{\partial W}{\partial q_L} = \frac{\Theta^2 \left[4bq_H\left(3q_H - q_L\right) + q_H^2\left(4q_H + 3q_L\right)\right] - 4cq_L\left(4q_H - q_L\right)^3}{2\left(4q_H - q_L\right)^3} = 0. \quad (11)$$

For any pair of generic qualities $(q_H, q_L)$, MQS regulation is binding (and therefore brings about an increase in both qualities) if $\partial W/\partial q_L > \partial \pi_L/\partial q_L$. Environmental effects being absent or not taken into account, we know from Valletti (2000) that this does not apply. Here, however, the presence of a negative externality implies that

$$sign\left\{\frac{\partial W}{\partial q_L} - \frac{\partial \pi_L}{\partial q_L}\right\} = sign\left\{4b\left(3q_H - q_L\right) - q_H\left(4q_H - q_L\right)\right\} \quad (12)$$

whereby

$$\frac{\partial W}{\partial q_L} > \frac{\partial \pi_L}{\partial q_L} \text{ for all } b > \frac{q_H\left(4q_H - q_L\right)}{4\left(3q_H - q_L\right)} \quad (13)$$

and conversely. Therefore, if $b$ is sufficiently large, the regulator attains a welfare increase by introducing a binding MQS.

Correspondingly, social welfare

$$W = \frac{\Theta^2\left[q_H\left(12q_H^2 - 5q_Hq_L + q_L^2\right) - 2b\left(3q_H - q_L\right)^2\right] - 2c\left(q_H^2 + q_L^2\right)\left(4q_H - q_L\right)^2}{2\left(4q_H - q_L\right)^2} \quad (14)$$

is positive for all

$$b < \frac{\Theta^2 q_H\left(12q_H^2 - 5q_Hq_L + q_L^2\right) - 2c\left(q_H^2 + q_L^2\right)\left(4q_H - q_L\right)^2}{2\Theta^2\left(3q_H - q_L\right)^2} \quad (15)$$

with

$$\frac{\Theta^2 q_H\left(12q_H^2 - 5q_Hq_L + q_L^2\right) - 2c\left(q_H^2 + q_L^2\right)\left(4q_H - q_L\right)^2}{2\Theta^2\left(3q_H - q_L\right)^2} > \frac{q_H\left(4q_H - q_L\right)}{4\left(3q_H - q_L\right)} \quad (16)$$

for all

$$c < \frac{\Theta^2 q_H \left(12q_H^2 - 3q_H q_L + q_L^2\right)}{4\left(q_H^2 + q_L^2\right)\left(4q_H - q_L\right)^2} \tag{17}$$

Note also that both firms' profits are positive for all

$$c < \min\left\{\frac{\Theta^2 \left(2q_H - q_L\right)^2}{q_H\left(4q_H - q_L\right)^2}, \frac{\Theta^2 q_H^2}{q_L\left(4q_H - q_L\right)^2}\right\} \tag{18}$$

so that any

$$c < \min\left\{\frac{\Theta^2 \left(2q_H - q_L\right)^2}{q_H\left(4q_H - q_L\right)^2}, \frac{\Theta^2 q_H^2}{q_L\left(4q_H - q_L\right)^2}, \frac{\Theta^2 q_H \left(12q_H^2 - 3q_H q_L + q_L^2\right)}{4\left(q_H^2 + q_L^2\right)\left(4q_H - q_L\right)^2}\right\} \tag{19}$$

ensures the positivity of profits and welfare for a generic quality pair.

The foregoing discussion can be summarised in

**Proposition 1** *If*

$$c < \min\left\{\frac{\Theta^2 \left(2q_H - q_L\right)^2}{q_H\left(4q_H - q_L\right)^2}, \frac{\Theta^2 q_H^2}{q_L\left(4q_H - q_L\right)^2}, \frac{\Theta^2 q_H \left(12q_H^2 - 3q_H q_L + q_L^2\right)}{4\left(q_H^2 + q_L^2\right)\left(4q_H - q_L\right)^2}\right\}$$

*then profits and welfare are positive for all* $q_H > q_L$, *and*

$$\frac{\Theta^2 q_H \left(12q_H^2 - 5q_H q_L + q_L^2\right) - 2c\left(q_H^2 + q_L^2\right)\left(4q_H - q_L\right)^2}{2\Theta^2\left(3q_H - q_L\right)^2} > \frac{q_H\left(4q_H - q_L\right)}{4\left(3q_H - q_L\right)}.$$

*Therefore, any*

$$b \in \left(\frac{q_H\left(4q_H - q_L\right)}{4\left(3q_H - q_L\right)}, \frac{\Theta^2 q_H \left(12q_H^2 - 5q_H q_L + q_L^2\right) - 2c\left(q_H^2 + q_L^2\right)\left(4q_H - q_L\right)^2}{2\Theta^2\left(3q_H - q_L\right)^2}\right)$$

*brings about the adoption of a binding MQS regulation.*

The intuition behind this result is the following. In a partially covered market with no environmental externality the only problem is the distortion of hedonic qualities driven by profit incentives, and Cournot competition is soft enough to imply that the MQS will not bite (as we know from Valletti, 2000). If, conversely, a negative externality increasing in industry output hinders welfare, there appears a tradeoff between increasing market coverage and decreasing pollution. The balance between the two determines whether the MQS is binding, which happens to be the case whenever the marginal social cost associated to the external effect is high enough.

### 3.1 An example

Here we show a numerical example using appropriate parameter values. Setting $\Theta = 1$, $c = 1/2$ and $b = 1/10$, and solving numerically the relevant system of FOCs (i.e., (9-10) in the unregulated case and (9-11) in the regulated case), we obtain:

**Table 1.**

| | no MQS | MQS |
|---|---|---|
| $q_H$ | 0.25194 | 0.25232 |
| $q_L$ | 0.09022 | 0.09924 |
| $x_H$ | 0.45083 | 0.44547 |
| $x_L$ | 0.27458 | 0.27726 |
| $CS$ | 0.04017 | 0.04111 |
| $s$ | 0.05262 | 0.05223 |
| $W$ | 0.00975 | 0.00981 |

The results with no MQS replicate those in Motta (1993) and Valletti (2000). In the regulated case, (i) both qualities increase, and the same happens to consumer surplus and social welfare; while (ii) industry output, the degree of vertical differentiation and the external effect decrease as compared to the unregulated setting. In this respect, it is worth noting that total output shrinks to contain the environmental consequences of production, and the increase in consumer surplus is driven by the fact that a lower degree of differentiation entails lower prices, more than offsetting the negative effect caused by the output reduction.

## References


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