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Patent Races with Dynamic Complementarity

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Patent Races with Dynamic Complementarity*

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Abstract

Recent models of multi-stage R&D have shown that a system of weak intellectual property rights may lead to faster innovation by inducing firms to share intermediate technological knowledge. In this article I introduce a distinction between *plain* and *sophisticated* technological knowledge, which has not been noticed so far but plays a crucial role in determining how different appropriability rules affect the incentives to innovate. I argue that the positive effect of weak intellectual property regimes on the sharing of intermediate technological knowledge vanishes when technological knowledge is sophisticated, as is likely to be the case in many high tech industries.

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1 Introduction

In the presence of technological spillovers, a profit-maximizing firm may decide to voluntarily reveal its intermediate innovative knowledge to rivals so as to benefit from their "cooperation" in the subsequent stages of the research (De Fraja (1993)). Building on this insight, Bessen and Maskin (2009) and Fershtman and Markovich (2009) have developed models of multi-stage R&D in which a system of weak intellectual property rights may foster innovation by inducing firms to share intermediate technological knowledge[1].

In this article I argue that this literature may have over-estimated the potential benefits from a leaky system. I introduce a distinction between two kinds of technological knowledge: *plain* and *sophisticated*, where technological knowledge is *sophisticated* if it requires a voluntary (even if cost less) act to be acquired; by contrast, knowledge is *plain* when it is apparent to everybody once it has been disclosed.[2]

In order to better understand this distinction, let us consider a few examples. The solution to a simple coded *rebus*[3] is a good example of plain knowledge. Similarly, the improved graphical user interface of a new operating system, or particular functional tools incorporated in a word processor, are plain technological knowledge. On the other hand, consider a complex mathematical proof, such as the proof of Fermat's last theorem. The mathe-

[1] The role of disclosure in ensuring cumulative progress has also been studied, among others by Scotchmer and Green (1990), Scotchmer (1991), Gallini (1992), Anton and Yao (2004), Bessen (2005) and Denicolò and Franzoni (2004) in different frameworks.

[2] That is, when knowledge is plain it is as if an inventor could simply send a message to the public: "to progress in research you shall use technology z" and this is a *sufficient* condition to let "z" become common knowledge to receivers.

[3] A *rebus* is a word-image riddle by which the only correct reading is veiled through figures (where, for example, a picture of an eye stands for "I").

matical argument is very complex and proceeds by a long sequence of steps. Thus, even if the inventor discloses the innovative knowledge, he or she cannot take it for granted that those exposed to the proof have in fact understood it. Here, innovative knowledge is "sophisticated" as agents can remain ignorant even after disclosure.

The distinction between plain and sophisticated knowledge plays a crucial role in how different appropriability systems shape the incentives to innovate in patent races with dynamic technological complementarities. To see this point, reconsider the reason why an inventor may want to disclose his superior intermediate technological knowledge to its competitors. This is to enable the rival to conduct research on an equal footing in the next stages of the race, and hence to increase his research effort. Since under a weak property regime the final innovation is not fully appropriable, the leader may then benefit from the rival's eventual success.

However, I contend that when innovative knowledge is sophisticated the rival may prefer to remain ignorant and free-ride on the leader's R&D effort in the last stages of the patent race. This behaviour is prevented only if the rival can be "forced" to acquire the innovative knowledge, as in the plain knowledge case. In the sophisticated case, however, free-riding is a feasible strategy. This means that knowledge acquisition will only take place if it is *incentive compatible.*

To study this insight in greater depth, I develop a simple model of a multi-stage patent race among two firms. The innovation is commercializable only after all stages have been completed. I compare two different patent regimes: the *strong* protection regime, where the first inventor alone can utilise the invention, and the *weak* protection, in which both firms can utilise the new technology irrespective of who achieved it.

I show that in the plain technology case, weak protection can be socially desirable in terms of both the pace of innovation and expected consumer surplus. However, in the case of sophisticated knowledge, this result is reversed and a strong patent protection is typically socially desirable.

The rest of the article is organized as follows. In the next section I introduce the model. In sections 3 and 4 I characterize firms' equilibrium R&D investment under both policy regimes; two subsections in Section 4 consider separately the effects of technological knowledge being plain or sophisticated in a weak protection regime. I then analyze the overall probability of innovation in Section 5 and the expected consumer surplus in Section 6. In Section 7 I consider the effects due of the introduction of licensing contracts between firms in the strong regime. The last section summarizes the main results and concludes the paper.

2 The Model

Consider a multi-stage patent race where two risk-neutral firms, $i = 1, 2$, conduct the research.[4] The acquisition of innovative knowledge is sequential. Each stage of research, when completed, produces an intermediate technological knowledge that is necessary to proceed to the the next stage.

At each period t firms select simultaneously and non-cooperatively a level

[4]Limiting the number of firms to two is for convenience only. When three or more firms are considered, similar results are obtained numerically.

of effort $x_t^i \in [0, \bar{x}_t]$. The upper-limit $\bar{x}_t$ is uncertain.[5] It is drawn from a probability distribution $\mathcal{F}$ on the interval $(0, 1)$, independently and identically at the beginning of each period. For instance, it follows a uniform distribution.

For simplicity, suppose that there are two stages of research, $s = 1, 2$, and three periods $t = 0, 1, 2$. In the first periods, $t = 0, 1$, firms produce research outcomes that are uncertain and depend on the effort exerted in R&D by each contender. Specifically, each innovation requires one unit of time to be developed, and any level of R&D investment is mapped into a probability to successfully complete the current stage at the end of each period. In the last period, $t = 2$, the innovation process is over and, if all steps have been developed, profits are realized otherwise firms earn zero payoffs.
Note that, as a consequence of the described staggered research process, firms obtain zero payoffs when there is even a single period without improvements in research. Hence, they have incentives to speed up the process even without the spur of competition, i.e. waiting one period is very costly.

Let us denote denote firm i's current level of technological knowledge by s_t^i. All firms share the same initial level of knowledge which is standardized to zero, i.e., $s_0^i = 0$, $\forall i$. However, at the end of each period, knowledge may

[5] Uncertainty on the upper limit is not necessary for the results, but it captures the idea that difficulties in research are unknown at the beginning of the race, and it is analytically convenient.

increase by one unit as the outcome of firm's R&D effort. Specifically,

$$s_{t+1}^i = \begin{cases} s_t^i + 1 & \text{with probability} \quad x_t^i \\ s_t^i & \text{with probability} \quad 1 - x_t^i \ . \end{cases} \tag{1}$$

Hence, even if both firms are initially symmetric, over time their level of knowledge can differ. The firm that leads the race is called "leader" while the laggard is the "follower". More specifically, a firm i is defined a leader if $s_t^i \geq t$ and we denote by L_t the number of leaders at period t.

In this model, firms' knowledge may increase also because of imitation which is assumed to be cost less and instantaneous. The only way to protect intermediate inventions from imitation during the race is to keep them secret from rivals. Thus, the leader must decide whether to *disclose* its knowledge or to *keep* it *secret*.

The effects of disclosure depends on the nature of technological knowledge. When technology is plain, the follower automatically reaches the leader's technological knowledge. By contrast, in the sophisticated technology case, the follower choose either to *ignore* or *utilise* the disclosed intermediate knowledge.[6] This is represented in Figure 1 as one further decision node in the game tree.

However, I assume that secrecy is no longer feasible when the good is commercialized. One can imagine that when the innovation is eventually brought

[6] As a social phenomenon, the reluctance of the recipient to accept knowledge from the outside (the "not invented here" syndrome) is well documented, see Katz and Allen (1982) and Szulanski (2000). For example, in a software development context, this is the tendency of both individual developers and organizations to reject suitable external solutions to software development problems only because those were not internally developed, - in other words, there are no other factors that dictate an internally developed solution would be superior (http://www.developer.com).

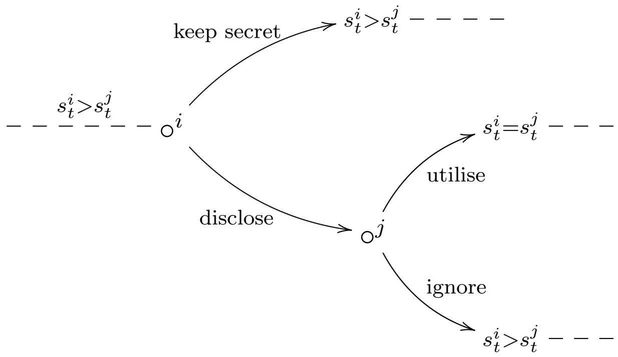

Figure 1: Cooperation and knowledge sharing at any period t when technological knowledge is sophisticated.

to the market, a cost less process of reverse-engineering may take place.

At the last period, if all stages of research have been completed, the innovation is commercializable. For simplicity, the demand function for the new product is assumed to be linear

$$P = \alpha - Q \tag{2}$$

where $\alpha \in \mathbb{R}^+$ denotes the size of the market.

Firms' payoffs depend on patent policy, which determines whether imitation is lawful or not. As in Fershtman and Markovich (2009), I assume that only the final innovation is patentable. Thus, intermediate discoveries are left with no legal protection during the race, but they may be kept secret.[7] As for the final innovation, I study two alternative regimes. The first, called "strong", prevents imitation, thereby creating a barrier to enter the final

[7]In other words, only the new good which encompasses all previous inventions, meets the patentability requirements. Specifically, the non-obviousness requirement, which specifies the size of the innovative step needed to qualify for patent protection.

market. This guarantees monopoly profits to the first inventor

$$\Pi^m = \Pi^m(\alpha) \ , \tag{3}$$

whereas the laggard obtains nothing.

The alternative regime, called “weak”, allows perfect, cost less imitation. In this case, as soon as the new good is developed by one firm, it can be produced and commercialized by both. Thus the market is always a duopoly, and firms equally share profits

$$\Pi^d = \Pi^d(\alpha, \delta) \leq \frac{\Pi^m}{2} \tag{4}$$

where the parameter $\delta \in [0, 1]$ captures the intensity of product market competition. This allows me to study in a reduced form all possible competitive configurations ranging from Bertrand ($\Pi^d = 0$) to perfect collusion ($\Pi^d = \Pi^m/2$).

To summarize, the timing of the game is as follows:

- At $t = 0$, firms are symmetric, a value $\bar{x}_0$ is drawn, and firms set their R&D efforts simultaneously. Nature then determines which firm succeeds. Firms observe the progress in research of each contender.

- At $t = 1$, a value $\bar{x}_1$ is drawn, and the leader may decide to disclose or keep secret its superior knowledge. If technology is sophisticated, the follower decides whether to ignore or acquire the new technology when it is available. Finally, R&D efforts are again chosen simultaneously. Nature determines which firm succeeds.

- At $t = 2$, if at $t = 1$ at least one firm succeeded, the good is commercialized and firms earn profits.

3 Strong Patent Regime

To study the sub-game perfect equilibria of the model, I proceed backwards starting from the product market equilibrium.
When imitation is prohibited by law, inventors can patent the new product and exclude competitors in the final market. Hence, at $t = 2$, there are three possible alternative scenarios. First, the innovation has not been attained, both firms obtain zero profits. In the second scenario, only one firm has developed and patented the good so monopoly prevails in the product market. In the third case, both firms have completed all stages and both can therefore apply for a patent. However, only one patent can be granted and so, one inventor is chosen randomly. Thus, the product market is again a monopoly.

Proceeding backwards, I consider period $t = 1$. Again, three cases can arise. However, differently from before, inventors can not apply for exclusive patent protection on the intermediate technology. Thus, the only effective protection tool for the leader is secrecy.
To understand the leader's incentives to keep the intermediate technological knowledge secret or to disclose it, suppose that at least one firm grabbed intermediate technological knowledge, so that $L_1 \geq 1$. Let use denote a leader by l. After having observed the realized value $\overline{x}_1$, each firm l selects

a level of R&D expenditure so as to maximize the following expected payoff function

$$U_1 = x_1^l \cdot \left[\Pi^m + \left(1 - (1+y)^{L_1 - 1}\right) \cdot \frac{\Pi^m}{2}\right] - c \cdot x_1^l \tag{5}$$

where y denotes the opponent's expected intensity of research.

Note that (5) is decreasing in the number of firms active in research, i.e., L_1 when $y > 0$. Every intermediate leader, therefore, will keep its superior technological knowledge secret and thereby force the rival to quit competition, setting R&D investment to zero.

In this case, due to the linearity of payoffs, leader's equilibrium R&D investment is either $x_1^{l*} = \bar{x}_1$ or zero. To avoid a proliferation of cases, however, it is convenient to focus on the set of parameter values where research is always pursued when there is one leader in the market. To be precise, I assume that $\Pi^m \geq c$, which, given the linearity of the demand function (2), can be better expressed in terms of market size,[8]

$$\alpha \geq 2\sqrt{c}. \tag{6}$$

Hence, in this model, broad patents imply that competition in research can occur only when both firms have innovated at the first stage. The following

[8] By looking at the market size I am making the ground ready for comparison between different IPRs regimes. In fact, although it might seem natural to focus on profits, in a strong regime firms may reach monopoly profits, whereas in a weak regime firms may earn only half duopoly profits. Hence, market size can be easily adopted as a measure that is common across regimes.

lemma characterizes the equilibria when there is competition in research.

Lemma 1. *Suppose that both firms have succeeded in the first period. Then, there exists a threshold $\hat{x}_1$ such that*

(i) if $\bar{x}_1 < \hat{x}_1$, the unique equilibrium investment in R&D is $x_1^{l^*} = \bar{x}_1$, $\forall l$, maximum effort,

(ii) if $\bar{x}_1 \geq \hat{x}_1$, the unique (symmetric) equilibrium investment in R&D is $x_1^{l^*} = \hat{x}_1$, $\forall l$, limited effort. $\square$

The intuition behind this result is simple. If there is no competition in research, each firm always finds it profitable to invest in R&D to achieve the latest innovation and, due to the linearity of the payoff function, the investment level is set at the upper limit. With competition, on the other hand, expected profits are reduced proportionally to the rival's effort in R&D, which again depends on the realized $\bar{x}_1$. If this value is "low", i.e., below the threshold $\hat{x}_1$, both firms invest at the maximum level. This is because, conditional on being successful, each firm has high probability of excluding the rival through patents.[9] By contrast, when the upper limit is "high", i.e., above the threshold $\hat{x}_1$, expected profits are not large enough to reward both contenders. Here, firms must reduce their individual R&D investment so as to break-even.

It is important to note that $\hat{x}_1$ is increasing in α. Thus, the probability that the realized upper limit will be above or below the threshold depends directly on the market size. Specifically, the larger the market, the higher

[9]The probability that both firms will be jointly developing the next stage is small and it gradually ceases to exist when the realized upper limit is small, i.e., $(\bar{x}_1)^2$.

the probability that both firms will exert the maximum R&D effort.

Proceeding backwards to the first period, each firm i solves the following problem,

$$\max_{x_0 \in [0, \bar{x}_0]} U_0 = x_0^i \cdot \sum_{k=1}^{2} y^k (1-y)^{1-k} \cdot E_0[U_1^* | L_1 = k] - c \cdot x_0^i \ , \tag{7}$$

where $E_0[U_1^{i*} | L_1 = k] \geq 0$ denotes the expected profits from being leader. Note that this payoff function is again linear in effort and decreasing in y. However the slope can now be negative even without competition from the other firm, i.e., $y = 0$. In other words, there are innovations whose size does not allow investments at the first stage even when research is granted to a monopolist firm.
The next lemma characterizes all equilibria.

Lemma 2. *In the first period, there exists a threshold $\hat{x}_0$ such that*

(i) if $\hat{x}_0 < 0$ both firms do not invest in R&D,

(ii) if $\bar{x}_0 < \hat{x}_0$, the unique equilibrium investment in R&D is $x_0^{i^*} = \bar{x}_0 \ \forall i$, maximum effort,

(iii) if $\bar{x}_0 \geq \hat{x}_0$, the unique (symmetric) equilibrium investment in R&D is $x_0^{i^*} = \hat{x}_0 \ \forall i$, limited-effort. $\square$

Of course, given the recursive structure of the race, the equilibria described above are analogous to the previous lemma. Again, the risk of facing tough competition in the future can reduce investment at the current stage.

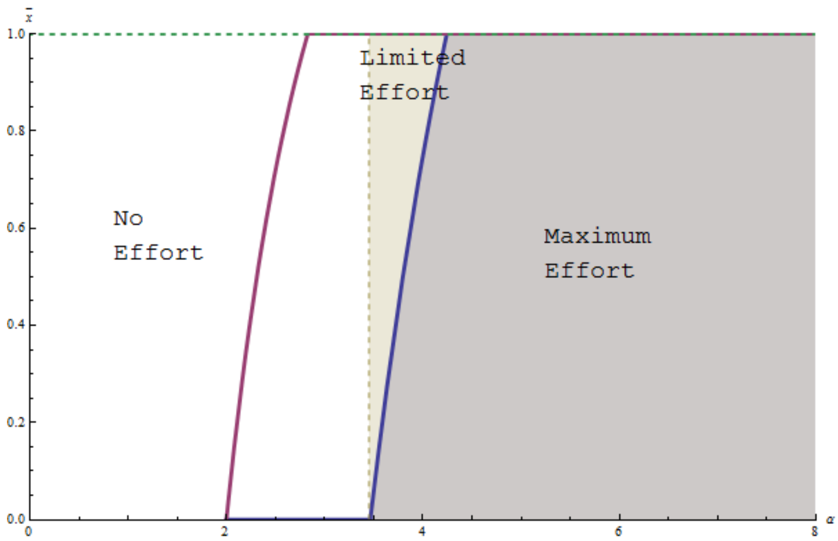

Figure 2: Strong patent protection regime: equilibrium cut-off functions $\hat{x}_1$ red curve and $\hat{x}_0$ blue curve, for $c = 1$.

Furthermore, condition i) of the lemma tells us how large the demand parameter should be for firms to start investing in research. To be precise, firms will invest in the first period if

$$\alpha \geq 2 \cdot \sqrt{3c}\ . \tag{8}$$

Taking stock of all the results obtained so far, Figure 2 depicts the two thresholds $\hat{x}_0$ and $\hat{x}_1$ as functions of the market size. We observe various patterns of R&D corresponding to three different parameter regions: *no effort, limited effort,* and *maximum effort* region.

Specifically, the white area covers the no effort region where all firms do not invest in R&D. In the shaded areas, all firms invest at the first stage and at the second stage. However, for a small interval of values, the level of R&D investment at the first stage depends on the realized upper limit. Therefore, individual R&D intensity can be lower than the realized upper limit, i.e., limited effort, or exactly the upper limit, i.e., maximum effort.

As a concluding remark for this section, it is perhaps worth noting that the notion of technological knowledge does not play any role in a strong protection regime. This is simply because leaders do not have any incentive to share their technological knowledge with followers.[10]

4 Weak Patent regime

In a regime of "weak" protection, there is no legal shield available for first inventors, imitation is cost less and trade secret protection is not feasible at the final step of the race. Hence, irrespective of who invented first, both rivals will be competing on an equal footing in the product market.

In this setting, firms are effectively playing a game of private provision of a public (from the viewpoint of the firms) good, i.e., innovative technological knowledge. In this game, the leader always has an incentive to share its innovative knowledge to the laggard so as to let him contribute to the provision of the public good. However, there naturally arises an incentive for the laggard to parasitize the leader's effort. Because R&D investment is chosen independently and simultaneously, the scope for free-riding depends on whether the laggard can commit to apprehend innovative knowledge, as in the sophisticated technological knowledge case, or not, as in the plain technological knowledge.

[10]In a more general case, however, leaders could ask for money in exchange for their superior knowledge. That is, firms can sign licensing contracts. Here I assume that any form of licensing is forbidden for anti-trust reasons or, simply, impossible. This assumption will be relaxed in Section 7.

4.1 Plain Technological Knowledge

Let us first consider the case of plain technological knowledge by which a disclosed intermediate technology becomes common knowledge to all players. As before, I solve the game proceeding backwards.

Recall that at the end of the race, i.e., $t = 2$, neither secrecy nor intellectual property law permit the leader to exclude its rivals, therefore firms equally split duopoly profits

$$\Pi^d(\alpha, \delta) = (1 - \delta) \cdot \frac{\Pi^m}{2} \quad . \tag{9}$$

Going back to $t = 1$, suppose that at least one firm improved its position in the race, i.e., $L_1 \geq 1$. Now every technological leader sets a level of R&D expenditure so as to maximize the following expected payoff function

$$U_1^w = \left[1 - (1 - x_1^l)(1 - y)^{L_1 - 1}\right] \cdot \Pi^d - c \cdot x_1^l \tag{10}$$

where y is the opponent's equilibrium R&D expenditure.

In this case, and differently with respect to the strong regime, the payoff function (10) is increasing in L_1 for $y > 0$. Therefore if only one firm goes ahead in the race, the leader will always prefer to make its superior technological knowledge freely accessible rather than to keep it secret. Thus, a weak regime of patent protection allows a shift from a strategy of secrecy to a spontaneous emergence of cooperation.

At this point, it is important to examine the role of plain technological knowledge. First recall that under this assumption a simple public message is a sufficient condition to pool contenders' abilities. Therefore, whereas with broad patents competition in research occurs only when firms improve jointly at the first stage, here, whichever firm progresses in the race, the other advances as well. Thus, firms are always symmetric at the second stage. Building on this observation, the following lemma characterizes all equilibrium outcomes at this stage.

Lemma 3. *In the second period, if at least one firm has succeeded in the first period, there exists a threshold $\hat{x}_1^w$ such that*

(i) if $\hat{x}_1^w < 0$ neither firm invests in R&D,

(ii) if $\bar{x}_1 < \hat{x}_1^w$, the unique equilibrium investment in R&D is $x_1^{l^*} = \bar{x}_1$ $\forall l$, maximum effort,

(iii) if $\bar{x}_1 \geq \hat{x}_1^w$ the unique (symmetric) equilibrium investment in R&D is $x_1^{l^*} = \hat{x}_1^w$ $\forall l$, limited effort. $\square$

As for the strong patent case, the equilibrium R&D investment depends on a cut-off value $\hat{x}_1^w$. If the realized upper limit is above this value, firms jointly moderate their R&D efforts, or else both exert the maximum effort in research. Although similar in many ways, this equilibrium differs from earlier behaviour because it is driven by the underlying public good game played by firms. Indeed, as a result of the lack of patent protection, each firm wants to free ride on the rival's effort but both end up sharing the costs of research as the only symmetric non-cooperative equilibrium in this game. Again the threshold is a function of market size, and it allows us to pin down the region of parameter values necessary to let firms do R&D at this stage of the race. That is,

$$\alpha \geq 2 \cdot \sqrt{\frac{2c}{(1-\delta)}} \ . \tag{11}$$

By contrasting this condition with the one corresponding to the strong patent protection regime (6), we observe an important drawback of a system with narrow patents in a multi-stage innovation race.[11] That is, because the lack of legal barriers in the product market reduces revenues to first inventors, inventions need a higher final demand to be pursued at this stage. More specifically, let us denote by ρ the ratio between the above condition and the one for the strong protection regime (6), then we have that

$$\rho = \sqrt{\frac{2}{(1-\delta)}} \ . \tag{12}$$

It is straightforward to see that ρ is always larger than one and that, the more intense is competition in the product market, the higher is the ratio.

Going back to $t = 0$, each firm i will solve the following problem

$$\max_{x_0^i \in [0, \overline{x}_0]} \ U_0^w = \left[x_0^i + y \cdot (1 - x_0^i)\right] \cdot E_0[U_1^{w^*} | L_1 = 2] - c \cdot x_0^i \ , \tag{13}$$

where $E_0[U_1^{w^*} | L_1 = 2]$ is the expected payoff when all firms are active in research at the next stage. The next lemma characterizes equilibria.

Lemma 4. *At the first period there exists a threshold $\hat{x}_0^w$ such that*

(i) if $\hat{x}_0^w < 0$ neither firm invests in R&D,

[11] As emphasized, among others, by Green and Scotchmer (1995) and Denicolò (2000).

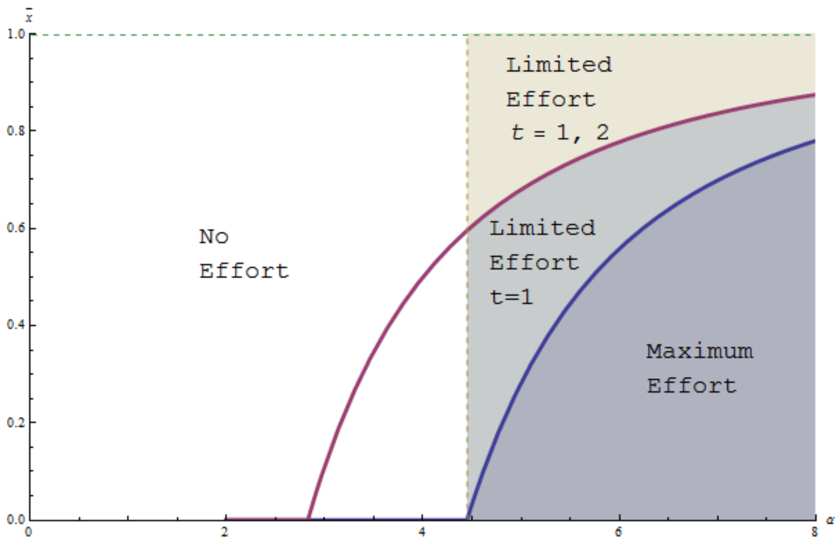

Figure 3: Weak patent protection regime: equilibrium cut-off functions, $\hat{x}_1^w$ red curve and $\hat{x}_0^w$ blue curve, for $\delta = 0$ and $c = 1$.

(ii) if $\bar{x}_0 < \hat{x}_0^w$, the unique equilibrium investment in R&D is $x_0^{i^*} = \bar{x}_0$ $\forall i$, maximum effort,

(iii) if $\bar{x}_0 \geq \hat{x}_0^w$, the unique (symmetric) equilibrium investment in R&D is $x_0^{i^*} = \hat{x}_0^w$ $\forall i$, limited effort. □

Again it is immediate to compute the condition under which the research process can start. That is,

$$E_0[U_1^{w^*} | L_1 = 2] \geq c \ . \tag{14}$$

To summarize the results, Figure 3 shows the two thresholds $\hat{x}_1^w$ and $\hat{x}_0^w$ for the case of perfect collusion in the product market. Because collusion ensures the highest possible reward to inventors under a weak regime, fixing $\delta = 0$ constitutes the "upper bound" case.

As we may conclude from the comparison with Figure 2 of the previous section, firms are now more likely to exert lower individual levels of effort than in the strong protection regime. Broad patents, however, do not necessarily produce higher aggregate levels of R&D. The reason is that, in a strong patent protection regime, a leader does not share technological knowledge with rivals. By contrast, in a weak regime, firms eagerly reveal information and cooperate in research. Thus, narrow patents will ensure a higher overall number of firms active in research at the intermediate stage.

4.2 Sophisticated Technological Knowledge

I next consider the case of sophisticated technological knowledge. Now, the follower can pretend it has not acquired the intermediate technology even when it is publicly available and imitation is cost less. In this model, I assume that the follower can decide either to *ignore* or *utilise* the intermediate knowledge after it has been disclosed by the leader. This gives the follower the opportunity to commit not to conduct any second-stage research if this commitment is profitable.

To better understand the motives behind this decision, suppose that at $t = 1$ only one firm innovated, i.e., $L_1 = 1$. Suppose that it is an equilibrium for the follower, denoted by f, to choose to ignore the new technology. At this point, the leader updates its beliefs about the rival's R&D effort. And thus, leader's payoff function becomes

$$U_1^{w^l} \quad = \quad x_1^l \cdot (\Pi^d - c) \tag{15}$$

which equals equation (10) but with $y = 0$. For the same reason, we have that the follower's expected payoff is

$$U_1^{wf} \quad = \quad y^l \cdot \Pi^d \tag{16}$$

where y^l now denotes the leader's R&D effort.

Recall that, by assumption (6), research on the latest invention is always pursued by a monopolist firm at this stage. Therefore the leader will select $x_1^{l^*} \equiv y^{l^*} = \bar{x}_1$ at equilibrium.

Turning attention to the follower's decision to ignore the available knowledge, this is a sub-game perfect equilibrium as long as the payoff the follower expects when it quits research, but the leader act as a monopolist, is larger than that from participating actively in research. This condition is the following

$$U_1^{wf^*} = \bar{x}_1 \cdot \Pi^d \geq U_1^{w^*} \ . \tag{17}$$

In other words, we may say that the decision to utilise the new technology is not "incentive compatible" when the above condition is met. This leads to the following result.

Lemma 5. *Suppose only one firm succeeded in the first period, then the follower will* ignore *the disclosed knowledge whenever the stand-alone R&D investment of leader is greater than the corresponding investment when both firms are active. Otherwise, the follower will* utilise *the disclosed knowledge and invest in research.* □

Note that this lemma hinges on the linearity of payoffs but the insight it delivers is far more general. The reason is simple. If technological knowledge is sophisticated, firms are no longer making an effort simultaneously at the second stage. In contrast to the plain case, the follower now has the possibility to move first, ignoring the available technological knowledge, and thus setting its effort to zero. Indeed, this possibility turns the strategic interaction among firms into a *sequential public good game*, e.g., Varian (1994). That is, the follower will decide how much contributing to the public good prior than the leader and the chosen contribution will be common knowledge. In this case, as long as the follower's decision to quit research has the power to adequately raise the stand-alone contribution of the leader, utilising the freely available technological knowledge may not be incentive compatible.

Going back to the first period, each firm i solves the following problem

$$\begin{aligned} \max_{x_0^i \in [0,\bar{x}_0]} U_0 &= x_0^i \cdot \Big[(1-y) \cdot E_0[U_1^{w^{l^*}} | L_1 = 1] + y \cdot E_0[U_1^{w^*} | L_1 = 2]\Big] + \\ &+(1-x_0^i) \cdot y \cdot E_0[U_1^{w^{f^*}} | L_1 = 1] - c \cdot x_0^i \end{aligned} \quad (18)$$

The next lemma characterizes the equilibria at this stage.

Lemma 6. *In the first period, when technological knowledge is sophisticated and there is a weak regime of patent protection, R&D efforts depend on a threshold $\hat{x}_0^{ws}$ that is non-greater than the corresponding cut-off value $\hat{x}_0^{w}$ in the plain knowledge case. This is true for every α and δ.*

Clearly, the expected gains from being a follower at the second stage will translate into lower incentives to innovate in the first period. Given the linear structure of the model, this reduction is captured by a cut-off value function $\hat{x}_0^{ws}$ that is smaller with respect to the case of plain technological knowledge. Moreover, it is a matter of algebra to show that the difference does not increase with market size α and in the intensity of competition δ. Again, as the demand for innovation is larger and competition is less intense, the free riding behaviour reduces and eventually ceases to be a problem for innovation.

Finally, the condition on market size above which the research process can start is the following

$$E_0[U_1^{w^{l^*}}|L_1 = 1] \geq c \ , \tag{19}$$

which leads to the following result:

Corollary 1. *In a regime of weak patent protection when technological knowledge is sophisticated, the minimum level of market size required to stimulate research is smaller than the one needed when technological knowledge is plain.*

In summary, when firms are facing innovations with weak patent protection, the nature of technological knowledge involved can strongly alter the equilibrium behaviour at every step of the race. The direction of such change is unambiguous, innovations with sophisticated technological knowledge reduce the overall effectiveness of a weak system in developing cumulative inventions. This is because it may not only result in the number of firms active in research being reduced but also because the enhanced opportunity to free ride at the second stage provides stronger incentives to reduce investment in the first period as well.

5 Expected Probability to Innovate

In this section, I examine some of the implications of different patent protection regimes. Specifically, I look at the expected probability to innovate or *pace of innovation*. That is, I follow the preceding equilibrium analysis for each regime to pin down the probability that all steps of innovation are accomplished by at least one firm. This is with the purpose of building a benchmark solution which can provide a basis for a comparison between strong and weak patent protection.

From this perspective, studying first the probability to innovate also permits the effects of a legal system on the innovation patterns to be studied separately from those on consumers' well-being. Then, in the next section I examine how consumer surplus is affected by both regimes.

5.1 Strong Patent Regime

In a system ensuring strong patent protection, cooperation in research is never spontaneous. Indeed, if only one firm succeeded at the first stage, it will not disclose for free the new intermediate knowledge to the follower. Therefore, only successful innovators will move forward in the race and invest according to the equilibrium strategy of Lemma 1 and Lemma 2.

To compute the expected probability I proceed backwards. At the second period, the probability that the second stage is developed by at least one firm depends on the number of leaders and on the realized value $\bar{x}_1$. Because the upper limit is uniformly drawn, we have:

$$\mu_1(L_1) = \begin{cases} 0 & \text{if } L_1 = 0 \\ \int_0^1 \bar{x}_1 d\bar{x}_1 & \text{if } L_1 = 1 \\ \int_0^{\hat{x}_1} \bar{x}_1(2-\bar{x}_1)d\bar{x}_1 + \int_{\hat{x}_1}^1 \hat{x}_1(2-\hat{x}_1)d\bar{x}_1 & \text{if } L_1 = 2 \ . \end{cases} \tag{20}$$

Going back to the first period, it is a matter of algebra to compute the probability that all steps of innovation are accomplished. This can be expressed in a reduced form as

$$\mu = \int_0^1 \sum_{k=1}^{2} p_k \cdot \mu_1(k) \ d\bar{x}_0 \ , \tag{21}$$

$$p_k > 0 \quad \forall k \ \text{ iff } \ \alpha > 2\sqrt{3c}$$

where p_k denotes the probability that after the first period there are k firms still active in research.

The pace of innovation (21) is illustrated in Figure 4 as a function of market size, and for a fixed cost parameter. As expected, this probability exhibits an s-shaped curve. This is because, as discussed before, there are three different parameter regions in which R&D can occur with various intensities. More specifically, there is zero R&D investment for low-demand inventions, positive investment but of limited intensity for inventions of intermediate size and finally, investment of maximum intensity in the remaining cases. At this point, the curve touches an upper-bound $\bar{\mu} \approx 0.4$ and is constant.

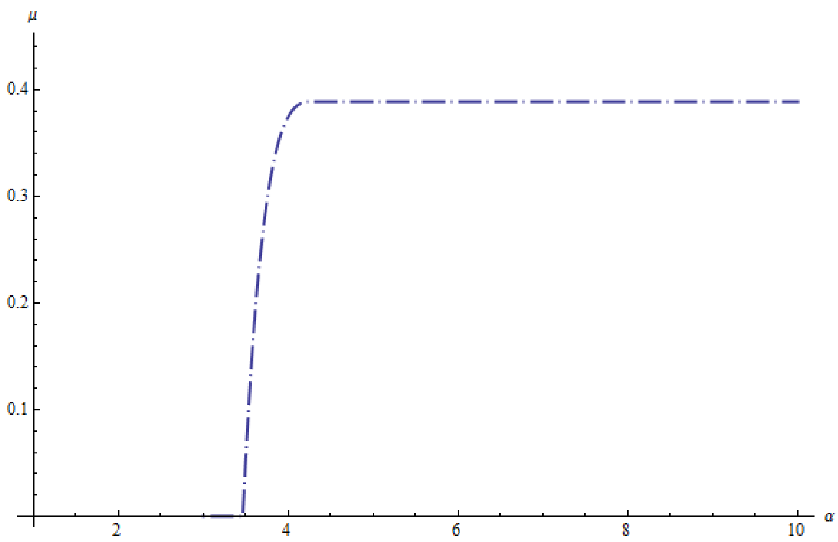

Figure 4: Pace of innovation under a strong patent protection regime ($c = 1$).

5.2 Weak Patent Regime

Now, consider a regime of weak patent protection, beginning with the case of plain technological knowledge.

5.2.1 Plain Technological Knowledge

When the legal regime is such that patents are too narrow to protect revenues in the product market, then leaders are incentivized to disclose voluntarily intermediate technological knowledge. In addition, if technological knowledge is plain, followers cannot commit to preventing this technological sharing. Therefore, if at least one firm moves forward in the race, the other firm advances as well and both will invest in research as described in Lemma 3.

Again, I proceed backwards to compute the pace of innovation. In the

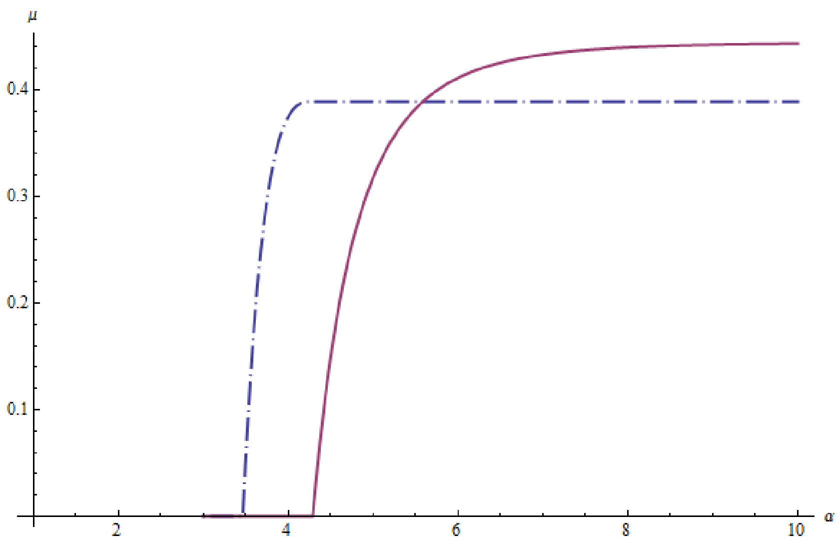

Figure 5: Comparison between the pace of innovation under a strong regime (dot-dashed line) and a weak regime (solid line) with plain technological knowledge. Fixed $c = 1$ and $\delta = 0$.

second period, the probability to develop the last stage is

$$\mu_1^w(L_1) = \begin{cases} 0 & \text{if } L_1 = 0 \\ \int_0^{\hat{x}_1^w} \bar{x}_1(2-\bar{x}_1)d\bar{x}_1 + \int_{\hat{x}_1^w}^1 \hat{x}_1^w(2-\hat{x}_1^w)d\bar{x}_1 & \text{if } L_1 > 0 \end{cases} \tag{22}$$

Thus, we have that the overall pace of innovation is

$$\mu^w = \int_0^1 \sum_{k=1}^{2} p_k^w \cdot \mu_1^w(k)\; d\bar{x}_0 \;, \tag{23}$$

$$p_k^w > 0 \quad \forall k \;\; \text{iff} \;\; E_0[U_1^{w^*} | L_1 = 2] \geq c$$

where p_k^w denotes the probability that after the first period there are k firms active in research.

To better understand by how much the strong and the weak regimes are different, Figure 5 plots both probability curves as a function of market size. As the figure illustrates, the (solid) curve representing the probability of success under a weak regime is positive for higher levels of market size and increases at a slower rate with respect to the (dot-dashed) curve representing the same probability under a strong regime.

Note that the two curves are monotonic and cross at $\alpha \approx 5.5$. Hence, although broad patents deliver a higher pace of innovation for intermediate values of market size, there exists a level of α above which the pace of innovation is enhanced by a weak system. And note that this must be true as long as the probability of success in a weak regime reaches an upper bound, here $\bar{\mu}^w \approx 0.44$, which is higher than the corresponding value for a strong patent regime.

The following proposition states the general form of this result:

Proposition 1. *For every $\delta < 1$, there exists a threshold value $\hat{\alpha}$ such that*

- *if $\alpha < \hat{\alpha}$ the pace of innovation under weak protection is smaller than under strong protection,*
- *if $\alpha \geq \hat{\alpha}$, instead, the pace of innovation under weak protection is higher than under strong protection.*

Moreover the threshold is non-decreasing in δ. $\square$

The above result can be easily understood when taking into account, as emphasized among others by Bessen and Maskin (2009) and Fershtman and Markovich (2009), the role of technological complementarity in multi-stage innovations. For instance, when complementarity is better exploited by technological transfers, relaxing competition through weak patent protection might increase the pace of innovation. Of course, a weak patent protection regime reduces the expected rewards for inventors (that are strictly positive for $\delta < 1$). In a strong regime, on the other hand, the possibility to exclude rivals via patents provides high-powered incentives to exert effort in research but kills cooperation. Indeed, knowledge is never voluntarily transfered by leaders.[12] Thus, there exists a tension between inducing cooperation and incentivizing research with higher rewards. As the market size increases, however, such a trade-off finds a solution whereby a weak patent protection regime should prevail in a market for innovations with larger demand.

5.2.2 Sophisticated Technological Knowledge

Under the assumption of sophisticated technological knowledge, the pace of innovation in a weak system may change. Now when one single firm improves one step in the race and discloses its superior knowledge, the follower can decide whether to utilise the new technology or not at its own advantage.

Starting from the second period, by Lemma 5, we have that the probability that the second stage is developed is

$$\mu_1^{ws}(L_1) = \begin{cases} \int_0^{\hat{x}_1^w} \bar{x}(2-\bar{x})d\bar{x} + \frac{(\hat{x}_1^w)^2}{2} & \text{if } L_1 = 1 \\ \mu_1^w(L_1) & \text{otherwise .} \end{cases} \tag{24}$$

[12] Although it may be transferred by means of licensing contracts.

Proceeding backwards, the overall pace of innovation is

$$\mu^{ws} = \int_0^1 \sum_{k=0}^{2} p_k^{ws} \cdot \mu_1^{ws}(k)\ d\bar{x}_0 \tag{25}$$

$$p_k^{ws} > 0 \quad \forall k \ \text{ iff } \ E_0[U_1^{w^{l^*}} | L_1 = 1] \geq c \ \ ,$$

where p_k^{ws} is the probability that at least k firms will be active in research at the second stage. Contrasting (23) with (25) ensues the following result:

Proposition 2. *The pace of innovation in a regime of weak patent protection with sophisticated technological knowledge is slower than the corresponding one in the plain technological knowledge case.* □

Corollary 2. *When $\mu^w > 0$ the difference $(\mu^w - \mu^{ws})$ is non-negative and tends to zero if and only if $\alpha \to \infty$.* □

In summary, the probability of innovating under a weak system is higher when the technology is plain. Such a difference, however, reduces as long as the prize from innovation increases and yet, it vanishes only in the limit case where market size is infinite.

At this point our simple model demonstrates that inventions with sophisticated technological knowledge might represent a limiting factor in overall system performance with weak protection. The main logic behind this result is that, although a regime encouraging cooperation may generate faster pace of innovation, the free riding problem introduced by a weak patent protection regime could weaken this positive outcome if firms are able to commit to and to quit research whenever it is convenient.

6 Consumer Surplus

The analysis of the previous section has shown how different patent systems affect the pace of innovation. To better assess all the potential benefits from either systems, it is convenient to explore now the overall effects on consumers well-being.

A regime of strong patent protection always results in a monopoly in the product market, and it carries relevant deadweight-losses and poor consumer surplus.[13] By contrast, a weak system can count on competition to provide a larger range of alternatives that are more desirable from the viewpoint of consumers. However, consumers seek a higher surplus combined with a satisfactory probability that the innovation process is achieved. Hence, I shall consider a measure that puts together the pace of innovation and the surplus that is realized once all steps of innovation are accomplished.

Given the demand function (2), the consumer surplus can easily be computed (see the appendix) and represented in a reduced form as a function of

[13] I am implicitly considering that a patent's life is infinite. As we will discuss next, an "optimized" patent regime may allow for finite patent life and this may reduce the expected deadweight-loss under a strong regime. However, restricting attention to this sub-optimal case will bolster the argument against a weak patent system.

δ and α,

$$CS(\alpha, \delta) = \frac{\alpha^2}{8} \left(1 + \delta^{\frac{1}{2}}\right)^2 \quad . \tag{26}$$

This function encompasses both cases of duopoly and monopoly in the product market. For instance, in the presence of a duopoly with perfect colluding firms, i.e., $\delta = 0$, equation (26) gives a consumer surplus that is the same as that of a monopolistic market. Hence, let this value be denoted by $CS^m(\alpha) \equiv CS(\alpha, 0)$.

By combining the consumer surplus in the product market with the pace of innovation, I obtain a function representing the *expected consumer surplus*, here denoted by ECS. Clearly, this function depends on the legal regime in place and therefore it can be described by two distinct curves. If patent protection is strong, it is

$$ECS = \mu \cdot CS^m(\alpha) \ , \tag{27}$$

whereas, if patent protection is weak, it is

$$ECS^w = \begin{cases} \mu^w \cdot CS(\alpha, \delta) & \text{if TK is plain} \\ \mu^{ws} \cdot CS(\alpha, \delta) & \text{if TK is sophisticated} \quad . \end{cases}$$

Fixing perfect collusion in the retail market, i.e., $\delta = 0$, Figure 6 shows both curves when technological knowledge is plain. Recall that, for δ close to zero, both regimes offer roughly the same consumer surplus and the main

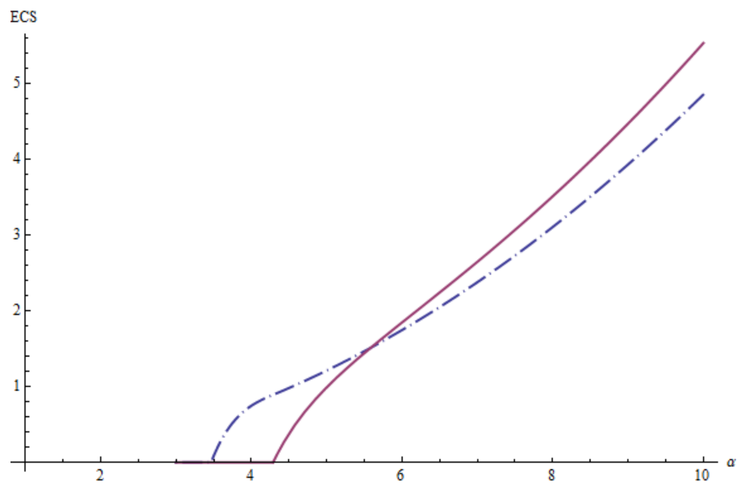

Figure 6: Comparison between the expected consumer surplus under a strong regime (dot-dashed line) and under a weak regime (solid line) with plain technological knowledge. Fixed $c = 1$ and $\delta = 0$.

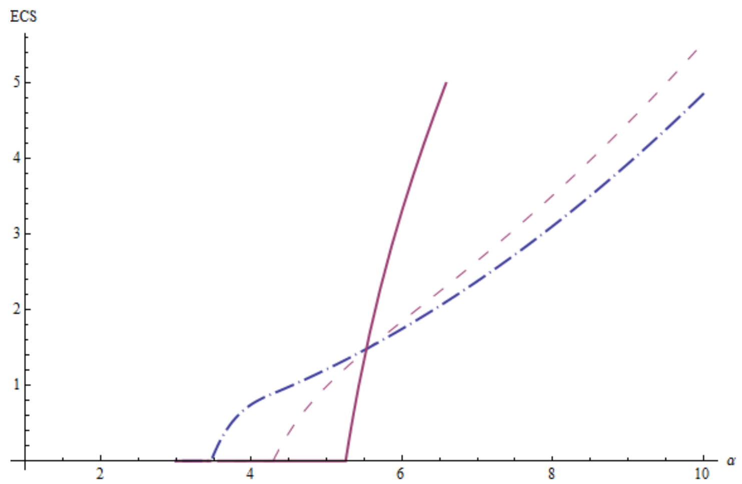

Figure 7: Comparison between the expected consumer surplus under a strong regime (dot-dashed line) and under a weak regime for $\delta = 1/3$ (solid line) and $\delta = 0$ (dashed line) with plain technological knowledge. Fixed $c = 1$.

difference is the pace of innovation. Again, hence, a weak regime performs better with inventions of larger market size.

Suppose that there is a more intense competition in the product market, i.e., $\delta = 1/3$. By looking now at Figure 7, the curve for a weak regime rotates when δ switches to positive values. This movement can easily be explained. On the one hand, firms' incentive to participate in the race are weakened, i.e., the initial value of market size must be larger. On the other hand, the difference between the expected consumer surplus in the two regimes grows exponentially larger as the market size increases.[14]

This comparatively static result adds to the growing literature and the common view that a weak patent system yields substantially larger social benefits than a strong regime. Next I am going to show an example by which, under the assumption of sophisticated technological knowledge, the positive effects of a weak system could vanish.

Suppose there is a policy-maker that is supposed to maximize the consumer surplus. The policy-maker knows the true demand parameter α or he can estimate its generating distribution.[15]

Figure 8 depicts the ECS curves as a function of the intensity of competition in the product market.

The expected consumer surplus in a strong regime is obviously a constant function of the intensity of competition. Whereas the ECS from a weak system varies in a non-monotonic way with respect to δ. This is simply because

[14] Note that the expected consumer surplus (26) increases exponentially in α and it grows faster for $\delta > 0$.

[15] As is considered in Bessen and Maskin (2009).

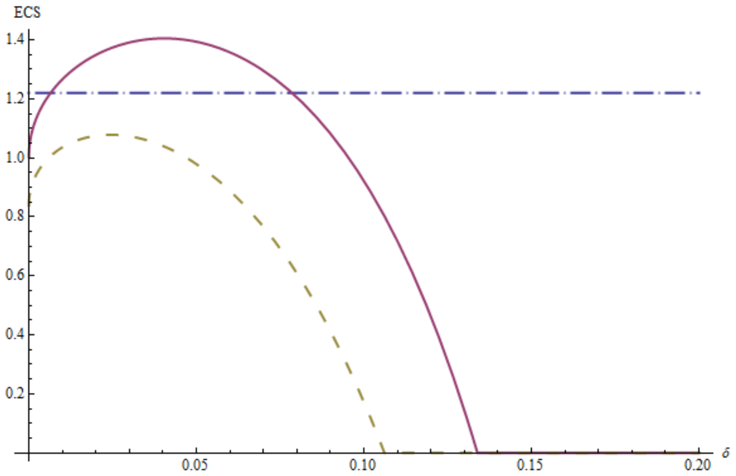

Figure 8: Comparison among the expected consumer surplus under a strong regime (dot-dashed line) and under a weak regime with plain technological knowledge (solid line) and under a weak regime with sophisticated technological knowledge (dashed line). Fixed $c = 1$ and $\alpha = 5$.

the pace of innovation is monotonically decreasing in the intensity of competition from a positive value towards zero, whereas the consumer surplus is monotonically increasing.

More importantly, we observe that as far as the case with plain technological knowledge is considered, there exists a set of parameter values of δ such that ECS is enhanced by a weak system. This result, however, is strikingly overturned in the sophisticated case.

In summary, as this simple example shows, a policy-maker should carefully take into account the appropriate definition of technological knowledge before adopting a weak system of patent protection.

7 Licensing

In general firms are able to enter contracts to share technology and seek positive licensing fees. To capture the role of licensing contracts in a simple way, suppose that the follower could offer a contingent contract to induce the leader's disclosure. As long as offers leave the leader's expected profits untouched after disclosure, technological sharing can be achieved even in a strong patent protection regime.[16] By restricting attention to the upper bound case, i.e., $\delta = 0$, the following result ensues from licensing:

Proposition 3. *The overall performance of a strong patent protection system is enhanced by the presence of licensing contracts. Moreover, if $\delta = 0$, the expected consumer surplus of a strong regime is greater than or equal to that of a weak regime with plain technological knowledge.*

As illustrated in Figure 9, licensing substantially improves social welfare in a strong regime, thus compensating some of the advantages highlighted for a weak regime. And yet, for $\delta > 0$, a weak regime could perform better. However, a full comparative statics on such other cases is beyond the scope of this article. This is simply because a full comparison between the two regimes should also consider a strong patent system that is "optimized". In that case, a strong system may compensate the improved performance of a weak patent regime by setting, for instance, adequate length and breadth of patents.

[16] For example, the contract could state that if the licensee develops the new invention, it keeps all profits. By contrast, if both firms develop the last stage invention at the same time, profits accrue to the earlier innovator only.

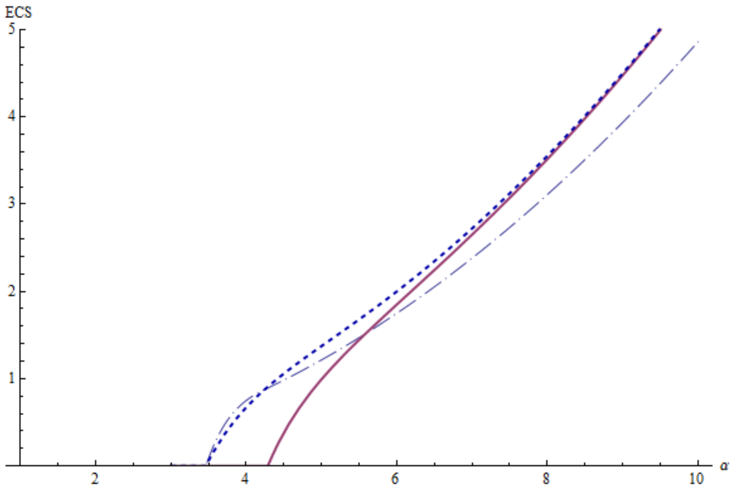

Figure 9: Comparison between the expected consumer surplus under a strong regime without licensing (dot-dashed line), with licensing (dashed line) and under a weak regime with plain technological knowledge (solid line). Fixed $c = 1$ and $\delta = 0$.

8 Conclusions

In their influential article, Bessen and Maskin (2009) claim that the reason why stronger intellectual property rights might not promote innovation has to do with sequential complementary innovations. The concern of this article is that this literature may have over-estimated the potential benefits of a weak system.

To address this issue, I developed a simple model of multi-stage patent race among two firms. At each step, firms will produce intermediate technological knowledge that they may decide to share with the rival. I study the incentives to cooperate and share new competences under two alternative definitions of technological knowledge which the standard literature has not explored so far. Hence, I studied the patent race when technological knowledge is either *plain*, whereby it becomes common knowledge once disclosed, or *sophisticated*, by which even if freely accessible, it transmits only when firms decide to utilise it. I compare two different patent regimes: the *strong* protection regime, where the first inventor alone can utilise the invention, and the *weak* protection, in which both firms can utilise the new technology irrespective of who achieved it.

I show that in the plain technology case, weak protection can be socially desirable in terms of both the pace of innovation and expected consumer surplus. However, in the case of sophisticated knowledge, this result is reversed and a strong patent protection is typically socially desirable. The reason is simple. Because R&D investment in a weak regime can be seen as a public good from the viewpoint of firms, lagging behind firms may want to commit to fully free-ride on leader's effort. However, this strategy is feasible only when technological knowledge is sophisticated, whereas it is prevented in the case of plain technology.

The model demonstrates, further, that if firms are allowed to contract licensing fees for technology sharing, broad patents provide a faster pace of innovation than a weak system. Therefore the potential benefits of a weak system are limited to the smaller dead-weight losses eventually generated in the product market. Nevertheless, it is pointed out that this outcome should be better examined and contrasted with an optimized strong patent regime, by which length and breadth are set to maximize social welfare.

Finally, a straightforward policy implication comes out of this work. That is, the policy-maker should carefully look at the technological nature of inventions when deciding to ease protection rules intended to foster cooperation.

Appendices

A Proofs

Proof of Lemma 1.

At the first period, if both firms succeed, i.e., $L_1 = 2$, the payoff function (5) is linear in x_1^l, and its slope depends on y. Therefore I can define the level of y such that the slope of (5) vanishes. That is,

$$\hat{x}_1 \equiv 2 \cdot \left(1 - \frac{c}{\Pi^m}\right) \tag{28}$$

By the linearity of the demand function, this is equivalent to

$$\hat{x}_1 = 2 \cdot \left(1 - \frac{4c}{\alpha^2}\right) \tag{29}$$

Notice that $\hat{x}_1$ is also the level of rival's effort such that leader's expected reward is zero, irrespective of its own effort. Hence, if this threshold exhibits negative values, the optimal action of both firms is to choose zero investments. However, if restricting attention to $\alpha > 2\sqrt{c}$, this case is ruled out.

At the beginning of $t = 1$ an upper limit $\bar{x}_1$ is drawn. It can be either one of two alternative scenarios. First, consider that $\bar{x}_1 < \hat{x}_1$ whereby expected profits are positive for any rival's level of R&D. Hence, a unique equilibrium exists in which both agents exert the highest level of effort, i.e., $x_1^{l^*} = \bar{x}_1 \; \forall l$.

Consider, next, the opposite case in which $\bar{x}_1 \geq \hat{x}_1$. Here, multiple equilibria arise. There are two possible asymmetric equilibria: either $x_1^{l*} = \bar{x}_1$ and $y = 0$ or vice-versa. However there is also a unique symmetrical equilibrium in which effort is $x_1^{l*} = y = \hat{x}_1$. In this equilibrium, both firms earn zero expected profits. **Q.E.D.**

Proof of Lemma 2.

At period $t = 0$, the upper limit $\bar{x}_1$ has been not drawn yet. However, firms face uniform priors about its realization. Thus, by using Lemma 1, expected payoffs can be computed for every level of $\bar{x}_1$ and then averaged over all possible upper-limit values. If $L_1 = 1$, we have that

$$E_0[U_1^*|L_1 = 1] = \int_0^1 \bar{x}(\Pi^m - c)d\bar{x} = (\Pi^m - c)/2 \quad , \tag{30}$$

whereas if $L_1 = 2$, we have that

$$E_0[U_1^*|L_1 = 2] \quad = \quad \int_0^{\min\{\hat{x}_1,1\}} \left(\bar{x} \cdot (2-\bar{x})\frac{\Pi^m}{2} - c \cdot \bar{x}\right) d\bar{x} \; . \tag{31}$$

Again, I can define a level of rival's R&D such that the slope of (7) vanishes. That is,

$$\hat{x}_0 \equiv \frac{E_0[U_1^*|L_1 = 1] - c}{E_0[U_1^*|L_1 = 1] - E_0[U_1^*|L_1 = 2]} \quad . \tag{32}$$

Notice that (32) is a function of market size and, after some algebra, it can be shown that it is non-negative when $\alpha \geq 2 \cdot \sqrt{3c}$. When $\alpha < 2 \cdot \sqrt{3c}$ the slope is negative and all firms minimize the function selecting zero R&D effort. Finally note that $\bar{x}_1 \geq 1$ is equivalent to $2 \cdot \sqrt{2c}$ which is smaller than $2 \cdot \sqrt{3c}$.

This implies that, if firms exert positive effort at the first stage, it must be true that they will exert the maximum effort at the second stage. This observation simplifies a lot (32) that now reduces to

$$\hat{x}_0 \equiv 3 \cdot \left(1 - \frac{12c}{\alpha^2}\right) \tag{33}$$

The rest of the proof is analogous to the previous lemma. **Q.E.D.**

Proof of Lemma 3.

In a weak patent regime with plain technological knowledge, intermediate inventors disclose for free their technological knowledge and followers cannot prevent sharing. Therefore, by the linearity of (10), the equilibrium R&D investments depend again on a cut-off function

$$\hat{x}_1^w \equiv 1 - \frac{8c}{(1-\delta) \cdot \alpha^2} \; . \tag{34}$$

And in an analogous manner to the analysis conducted before, we have the reported solutions. **Q.E.D.**

Proof of Lemma 4.

At the second period, firms are always symmetric, either $L_1 = 2$ or $L_1 = 0$. Because the upper limit is random, expected payoff must be averaged for all possible realizations. That is,

$$E_0[U_1^{w^*} | L_1 = 2] = \int_0^{\hat{x}_1} \left[1 - (1-\bar{x})^2\right] \Pi^d - c \cdot \bar{x} \; d\bar{x} + \int_{\hat{x}_1}^1 (\Pi^d - c) \; d\bar{x} \tag{35}$$

Therefore it can be defined a new threshold $\hat{x}_0^w$. Whereas, the remaining parts of the proof are analogous to earlier lemmas. **Q.E.D.**.

Proof of Lemma 5.

Suppose initially that the firm chooses to "uilise" the available technology. Again the decision is taken after $\bar{x}_1$ is drawn and the realization known. Thus, two cases are possible. Suppose first that $\bar{x}_1 \leq \hat{x}_1^w$, and recall that the decision to utilise the intermediate knowledge is an equilibrium if

$$\bar{x}_1 \Pi^d \leq \bar{x}_1(2 - \bar{x}_1)\Pi^d - c\bar{x}_1$$

But this expression is equivalent to $\bar{x}_1 \leq \hat{x}_1^w$, as it has been assumed at the beginning. Hence, it is a sub-game perfect equilibrium in this case.

Suppose next that $\bar{x}_1 < \hat{x}_1^w$, now can prove by contradiction that the decision of utilising the new technology is not an equilibrium. Specifically, the above condition can be rewritten as $\bar{x}_1 \leq \hat{x}_1^w$ that is against the initial assumption. Therefore it is not an equilibrium in this case. **Q.E.D.**

Proof of Lemma 6.

If innovation is of the sophisticated type. It does not alter competition when both firms succeed at the second period. It changes instead incentives when lagging behind firms can decide to quit the race depending on the realized $\bar{x}_1$.

Given the recursive linear structure of payoffs, this R&D pattern reflects into a cut-off that is necessarily lower than the corresponding function for the plain case. Hence, averaging payoffs over all possible realization of $\overline{x}_1$, we have that

$$E_0[\hat{U}_1^{f^*}] = \int_0^{\hat{x}_1^w} \bar{x}[(2\Pi^d - c) + \bar{x})]d\bar{x} + \int_{\hat{x}_1^w}^1 \bar{x}\Pi^d d\bar{x} \tag{36}$$

$$E_0[\hat{U}_1^{l^*}] = \int_0^{\hat{x}_1^w} \bar{x}[(2\Pi^d - c) + \bar{x}]d\bar{x} + \int_{\hat{x}_1^w}^1 \bar{x}[\Pi^d - c]d\bar{x} \tag{37}$$

Thus, in the same manner as before, I define a new threshold $\hat{x}_0^{ws}$ and by some simple algebra it can be shown that this is non-greater than $\hat{x}_0^w$. The remaining part of the proof is the same as that in previous cases. **Q.E.D.**

Proof of proposition 1. If $\delta < 1$, notice that when $\alpha \to \infty$ then the pace of innovation in a weak regime reaches an upper-bound that is 0.44 which is above the corresponding value in a strong regime, i.e., 0.40.
Recall further that the pace of innovation in a weak regime is continuous and takes positive values for α that are higher than in the case of strong patent protection. Therefore there must be a finite value of α such that both curves crosses.

Proof of proposition 2.

It comes straightforward from Lemma 5 and 6.

Proof of Proposition 3.

As a first step, notice that in a weak patent protection regime the introduction of licensing does never alter equilibria. This is because if firms know that the leader will disclose for free its intermediate knowledge then, the latter can not credibly seek for positive fees.

In a strong regime, instead, I suppose that followers are able to offer the type of contract described in the text. Therefore their payoff function becomes

$$U_1^f = x_1^f(1 - y^l)\Pi^m - c \cdot x_1^f \ . \tag{38}$$

However, given the structure of the contract, leaders are not going to reduce their equilibrium R&D effort and so, $x_1^{l^*} = \bar{x}_1$. This implies that a follower will exert positive effort if $(1-\bar{x}^l)\Pi^m \geq c$, therefore if $\bar{x} < 1-c/\Pi^m$, otherwise it sets R&D effort to zero. This possibility turns into a the following cut-off at the first period,

$$\tilde{x}_0 = \frac{E_0[U_1^{l^*}] - c}{E_0[U_1^{l^*}] + E_0[U_1^{f^*}] - E_0[U_1^*]} \tag{39}$$

and so the expected probability to innovate μ^{sl} is computed and can be contrasted with the previous result. **Q.E.D.**

B Consumer Surplus

As a first step consider the product market as a monopoly. If all steps of innovation are accomplished, and assuming null marginal cost in production, the equilibrium quantity produced is $q^m = \frac{\alpha}{2} = p^m$. Thus, $\Pi^m = \frac{\alpha^2}{4}$. And so it is a matter of simple algebra to compute the social surplus in this case $S^m = \frac{\alpha^2}{8}$.

Then, suppose that the product market is a duopoly. Now, the corresponding

equilibrium quantity is arguably greater than that under monopoly: $q^d = (1+\gamma)q^m$ with $\gamma \geq 0$. Therefore, $p^d = \alpha - \frac{\alpha}{2}(1+\gamma)$.
Hence, by the definition of duopoly profits, i.e., $\Pi^d \equiv (1-\delta)\Pi^m/2$, we obtain $\gamma = (1-\delta)^{1/2}$. Finally, by substituting the resulting γ into the function of social surplus under duopoly, this turns out to be equation (26).

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