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Strategic Accessibility Competition

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Abstract

We analyze the effect of competition in market-accessibility enhancement among quality-differentiated firms. Firms are located in regions with different ex-ante transport costs to reach the final market. We characterize the equilibrium of the two-stage game in which firms first invest to improve market accessibility and then compete in prices. Efforts in accessibility improvement crucially depend on the interplay between the willingness to pay for the quality premium of the median consumer and the ex-ante difference in accessibility between regions. From the social standpoint, all the accessibility investment should be carried out by the high-quality firm. Finally quality choice is endogenized.

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1 Introduction

Competing firms often differ in terms of their degree of market accessibility. This is typically the case in international or interregional trade, where firms located in different countries or regions are characterized by different transport and communication costs to reach consumers located in a specific market. This may occur for natural reasons (e.g., geographical distance) or artificial ones (e.g., for different endowments of transportation infrastructure, different access to communication technologies, or trade barriers). At the theoretical level, it is interesting to investigate firm strategic behavior when there is a trade-off for consumers between purchasing a “nearby” lower-quality variant of the good or a “far away” higher-quality one.\footnote{The opposite case, namely that of a “close” high-quality producer and a “distant” low-quality one is, in our opinion, less interesting, because it reinforces the natural demand advantage of high-versus low-quality producers.} To make our argument effective, consider the case of typical food, e.g., Italian cheese like mozzarella or Parmesan. In this case, “high quality” original producers in Italy compete with “low quality” imitators outside Italy. In the United States, for instance, a consumer in New York can choose between a low quality Parmesan produced in Wisconsin (whose producers, indeed, hold a market leader position), or a “high quality” Parmesan produced in Italy.\footnote{“Fake Parmesan wins the US’ best cheese award”, http://www.italianfoodnet.com, March 27, 2009.} Similar situations occur in services as well. In tourism, a German citizen can choose between a “low quality” destination in the Baltic or a “high quality” destination in the Mediterranean sea or outside Europe. We intend to analyze these types of situations in which quality-differentiated firms invest in market-accessibility enhancement in a strategic environment.

More specifically, two firms located in different regions compete for attracting consumers located in a third region. The transport costs for the two regions are ex-ante different, and this asymmetry is assumed to capture all the differences in market accessibility between regions. On the production side, firms’ products are vertically differentiated; on the demand side, consumers are heterogeneous with respect to their quality appreciation, while the transport cost for a specific region is identical across consumers.

First, we consider the locations and the quality levels of firms as given. Also, we assume that the ex-ante transport cost for the region where the high-quality good is produced is the highest. The strategic interaction in market accessibility is modeled as a two-stage game. At the first stage, each firm invests in
accessibility enhancement, i.e. in the reduction of transport cost (an alternative, equivalent, interpretation is that the investment is made by a regional authority). At the second stage, given the investment levels, firms set prices. We first investigate how (exogenous) differences in quality levels affect investment and thus equilibrium market shares and profitability. We show the crucial role played by willingness to pay for the quality premium of the median consumer (e.g. the increase in the gross utility due to the consumption of the high-quality good vs. the low-quality one) and the ex-ante difference in transport costs. Then we analyze the influence of quality levels and ex-ante transport costs on the accessibility investments. A welfare analysis is then conducted, showing that either in the case where total profits or overall social welfare are maximized, all the investment in accessibility enhancement should be concentrated on the high quality region. Finally, we extend the model by endogenizing quality choice.

Our model contributes different streams of literature. Firstly, it relates to the vast literature dealing with transport costs both in economic geography and industrial organization. In both cases, transport costs are generally taken as exogenous, while location is a firm variable of choice (see, for example, Helpman and Krugman (1985), Grossman (1992), Hotelling (1929), Launhardt (1993), Thisse and Wildasin (1995), Piga and Poyago-Theotoky (2005)). Very few contributions, instead, have analyzed the importance of strategic interactions among firms in reducing the transport cost burden. Lambertini et al. (2003) analyze the strategic effect of different intensity in profitability of transport cost abatement on the equilibrium features in a single region. Lambertini and Rossini (2006) extend the analysis to a two-country model focusing on the market size role. Their analysis is further investigated in a dynamic context by Colombo et al. (2009). With respect to the existing literature, our model simultaneously features vertically differentiated firms and heterogeneous consumers with ex-ante asymmetries in transport cost.

The paper is organized as follows. The next section describes the model. In Section 3 the game is solved, with particular focus on the equilibrium investment of each firm. Section 4 provides comparative-static analysis and the economic intuition of the results. Section 5 performs a welfare analysis under two alternative scenarios. In Section 6 we extend the model by endogenizing the quality levels. Section 7 concludes.
2 The Model

Consider two firms, and label them $h$ and $l$. Firms are located in two different regions, also labeled $h$ and $l$. Firms are immobile, so notational confusion is avoided. Each firm offers a good, whose quality is defined in the classical terms of vertical product differentiation. Therefore the quality of the good of firms is defined by the variables $u_h$ and $u_l$, with $u_h > u_l$, so that firm $h$ is the high-quality firm and firm $l$ is the low-quality firm. For the ease of analysis, we normalize to zero the production cost borne by firms. Finally, the prices charged by firms for one unit of their good are $p_h$ and $p_l$.

Assume that, in a market external to regions $h$ and $l$, a continuum of consumers is distributed over the interval $[0, \bar{\theta}]$ according to their quality appreciation. Each consumer may purchase either one unit of the good or abstaining from consumption. In addition to price, consumers bear a transport cost. Let $t_h$ be the cost borne by a consumer buying from firm $h$, similarly, label $t_l$ the cost for buying from firm $l$.

We define consumers’ utility à la Mussa and Rosen (1978), so that consumer $\theta \in [0, \bar{\theta}]$ enjoys a utility

$$U(\theta, u_i) = \begin{cases} 
U_0 + \theta u_h - p_h - t_h & \text{if } \theta \text{ buys from } h \\
U_0 + \theta u_l - p_l - t_l & \text{if } \theta \text{ buys from } l \\
0 & \text{if } \theta \text{ does not buy any good.}
\end{cases}$$

(1)

In (1), $U_0 > 0$ is the utility derived from buying the good, whatever is its quality. We define $u_h - u_l \equiv \Delta u > 0$ as the quality premium of firm $h$ relative to firm $l$. The standard marginal consumer approach allows to identify the consumers that are indifferent between choosing $h$ and $l$; indeed, by solving for $\theta$ the equation $\theta u_h - p_h - t_h = \theta u_l - p_l - t_l$ we obtain that

$$\theta_{h,l} = \frac{p_h + t_h - p_l + t_l}{\Delta u}.$$  

(2)

Similarly by solving $\theta u_l - p_l - t_l = 0$ we identify the consumer indifferent between $l$ and not buying:

$$\theta_{l,0} = \frac{p_l + t_2 - U_0}{u_l}.$$  

(3)

\footnote{The assumption that transport costs are borne by consumers is interchangeable with that of transport costs borne by firms, as long as they are per unit shipped.}
From (2) and (3) the demands for firms $h$ and $l$ respectively are respectively:

\[
D_h = \frac{\theta - \theta_{h,l}}{\theta} \tag{4}
\]
\[
D_l = \frac{\theta_{h,l} - \theta_{l,0}}{\theta} \tag{5}
\]

Notice that the demand system defined by (4) and (5) differs from that obtained under the standard vertical differentiation assumptions by the terms $t_h$ and $t_l$, the transport costs. Furthermore, if the term $U_0$ in (1) is high enough (say $U_0 > \bar{U}_0$), $\theta_{l,0}$ is negative, and the demand for the low-quality reduces to

\[
D_l = \frac{\theta_{h,l}}{\theta} . \tag{6}
\]

In the rest of the paper we retain the simplifying assumption that $U_0 > \bar{U}_0$.

We start by assuming that firms compete by playing a two-stage game. At the first stage firms invest in increasing their market accessibility while at the second stage they set prices. An alternative interpretation is that the investment (say, a public infrastructure) is made by the corresponding regional government. Since a single firm operates in each region, and consumer are located outside the regions, firms and regional authorities would share the same objective function, i.e. the firm’s profit. To solve our model we apply the standard concept of subgame-perfect Nash equilibrium (SPNE).

### 3 Equilibrium

We tackle the price-setting stage first. The profit of firm $i = h, l$ at this stage is

\[
\pi_i = D_i p_i, \quad i = h, l. \tag{7}
\]

The solution to the system

\[
\begin{align*}
\frac{\partial \pi_h}{\partial p_h} &= 0 \\
\frac{\partial \pi_l}{\partial p_l} &= 0
\end{align*}
\]
yields the profit-maximizing prices at the first stage (second-order conditions are satisfied). In particular they are:

\[
\hat{p}_h(t_h, t_l) = \frac{1}{3}(2\bar{\theta}\Delta u - t_h + t_l), \tag{8}
\]

\[
\hat{p}_l(t_h, t_l) = \frac{1}{3}(\bar{\theta}\Delta u - t_h + t_l). \tag{9}
\]

By plugging (8) and (9) back into the expression of profits we obtain

\[
\hat{\pi}_h(t_h, t_l) = \frac{(2\bar{\theta}\Delta u - t_h + t_l)^2}{9\bar{\theta}\Delta u}, \tag{10}
\]

\[
\hat{\pi}_l(t_h, t_l) = \frac{(\bar{\theta}\Delta u - t_l + t_h)^2}{9\bar{\theta}\Delta u}. \tag{11}
\]

We next move to the transport cost reduction stage. To proceed we need to describe more in detail the process that allows firms to reduce the transport cost borne by consumers. We assume that without any investment the transport cost paid by consumers is equal to \(T_i > 0, i = h, l\); we will refer to \(T_i\) as the ex-ante transport cost for firm \(i\), and to the difference \(T_h - T_l \equiv \Delta T \geq 0\) as the relative ex-ante accessibility for firm \(l\) (region \(l\)) relative to firm \(h\) (region \(h\)).\(^4\) We assume that the transport cost for buying from firm \(l\) is non higher than the transport cost for buying from firm \(h\), and an increase in the relative ex-ante accessibility implies that, all else equal, the cost for buying from \(h\) increases with respect to the costs from buying from \(l\). Furthermore, we assume that the investment required to reduce by an amount \(r_i\) the transport cost requires an investment equal to \(R_i(r_i) = \gamma r_i^2/2\). Stated differently, by investing \(R_i\) firm \(i\) reduces the transport cost paid by consumers to \(t_i(r_i) = T_i - r_i\), \(i = h, l\). Notice that the technology governing transport cost reduction displays decreasing returns to scale, and that the parameter \(\gamma\) is an inverse measure of the efficiency of the transport cost reduction technology. The new problem each firm faces is

\[
\max_{r_i} \hat{\pi}_i(t_h(r_h), t_l(r_l)) - R_i(r_i), \text{ for } i = h, l. \tag{12}
\]

Problem (12) can be solved through to the first-order condition approach. The

\(^4\)This simplifying assumption can be relaxed without affecting our results by allowing \(\Delta T\) to be negative but larger than a certain value. For \(\Delta T\) smaller than this threshold, demands and prices for the low-quality firm would be negative, invalidating our analysis.
solution to the system\textsuperscript{5}

\[
\begin{align*}
\frac{\partial \hat{\pi}_h() - R_h(\cdot)}{\partial r_h} &= 0 \\
\frac{\partial \hat{\pi}_l(\cdot) - R_l(\cdot)}{\partial r_l} &= 0
\end{align*}
\]  

(13)

yields the optimal investment levels:

\[
\begin{align*}
r^*_h &= \frac{2[3\gamma(2\bar{\theta}\Delta u - \Delta T) - 2]}{3\gamma(9\theta \gamma \Delta u - 4)}, \\
r^*_l &= \frac{2[3\gamma(\bar{\theta}\Delta u + \Delta T) - 2]}{3\gamma(9\theta \gamma \Delta u - 4)}.
\end{align*}
\]  

(14), (15)

It is then a matter of algebraic manipulations to obtain the expression of equilibrium prices, demands and profits. In particular equilibrium prices are:

\[
\begin{align*}
p^*_h &= \frac{\bar{\theta}\Delta u [3\gamma(2\bar{\theta}\Delta u - \Delta T) - 2]}{9\gamma \theta \Delta u - 4}, \\
p^*_l &= \frac{\bar{\theta}\Delta u [3\gamma(\bar{\theta}\Delta u + \Delta T) - 2]}{9\gamma \theta \Delta u - 4}.
\end{align*}
\]  

(16), (17)

By substituting (16) and (17) back into (4) and (6) we obtain equilibrium demands for firm $h$ and $l$ respectively:

\[
\begin{align*}
D^*_h &= \frac{3\gamma(2\bar{\theta}\Delta u - \Delta T - 2)}{9\gamma \theta \Delta u - 4}, \\
D^*_l &= \frac{3\gamma(\bar{\theta}\Delta u + \Delta T - 2)}{9\gamma \theta \Delta u - 4}.
\end{align*}
\]  

(18), (19)

To guarantee that prices and demands in (16)-(19) are positive we assume that\textsuperscript{6}

\[
\Delta u > \max\left\{ \frac{2 + 3\gamma \Delta T}{6\gamma \bar{\theta}}, \frac{4}{9\gamma \bar{\theta}} \right\}.
\]  

(20)

Finally equilibrium profits are:

\[
\begin{align*}
\pi^*_h &= \frac{(9\gamma \bar{\theta}\Delta u - 2)[3\gamma(2\bar{\theta}\Delta u - \Delta T) - 2]^2}{9\gamma (9\gamma \theta \Delta u - 4)^2}, \\
\pi^*_l &= \frac{(9\gamma \bar{\theta}\Delta u - 2)[3\gamma(\bar{\theta}\Delta u + \Delta T) - 2]^2}{9\gamma (9\gamma \theta \Delta u - 4)^2}.
\end{align*}
\]  

(21), (22)

\textsuperscript{5}Second order conditions require that $\gamma > \frac{2}{6\bar{\theta} \gamma}$.  
\textsuperscript{6}Condition 20 implies that second order conditions are satisfied.
4 Accessibility investments

Let us now discuss more in detail the accessibility investment behavior of firms.

4.1 Profits and investments: high quality vs low quality firm

We begin by comparing profits and investments of the high quality and low quality firm. A first result is that the high-quality firm can earn lower profits than the low-quality one. Indeed comparison of (21) and (22) reveals that $\pi_h^* \leq \pi_l^* \Leftrightarrow \Delta T \geq \frac{\Delta u}{2}$. The expression $\frac{\Delta u}{2}$ is the hedonic willingness to pay for the quality premium ($\Delta u$) of the median consumer ($\hat{\theta}_2$). When the (relative) accessibility for firm $l$ is higher than the willingness to pay for the quality premium, the $h$-firm earns lower profits than its rival.

Relative accessibility investment is defined as

$$r_h^* - r_l^* \equiv \Delta r^* = \frac{2\hat{\theta} \Delta u - 4\Delta T}{9\gamma \hat{\theta} \Delta u - 4} \quad (23)$$

Let us consider the role of the relative ex-ante accessibility on the relative investments. It is easy to ascertain that the sign of $\frac{\partial \Delta r^*}{\partial \Delta T}$ is negative. An increase of the relative cost of serving consumers for firm $h$ reduces the relative investment of firm $h$. Let us now expand on the relative size of the optimal investment themselves. The sign of the difference $\Delta r^*$ depends upon the sign of its numerator, which in turn depends the relative sizes of the willingness to pay of the median consumer for the quality premium and the relative gross accessibility for firm $l$. The high-quality firm invests more than the low-quality if and only if the relative gross accessibility for firm $l$ is smaller than the willingness to pay for the quality premium of the median consumer. This is exactly the same condition as for profit comparison. When $\Delta T < \frac{\Delta u}{2}$, the high-quality firm fully exploits its quality advantage and invests more resources in transport cost reduction than its rival, and at equilibrium this results in higher profits for the $h$-firm than for the $l$-one. The contrary holds when $\Delta T$ is higher than $\frac{\Delta u}{2}$. In this case the high-quality firm suffers an accessibility disadvantage which lowers its investment in transport cost reduction and makes it earn lower profits than its low-quality rival.$^7$

$^7$The result that in models of vertical differentiation the high-quality producer may end up to earn lower profits than its low-quality competitor is not novel to the literature (see, e.g. Bacchiega (2007)).
All these observations are summarized in the following Proposition.

**Proposition 1** (i) When \( \Delta T < \frac{\theta_\Delta u}{\gamma} \), \( \pi^*_h > \pi^*_l \) and the high-quality firm invests more in transport cost abatement than the low-quality one; (ii) when \( \Delta T > \frac{\theta_\Delta u}{\gamma} \), \( \pi^*_h < \pi^*_l \) and the high quality firm invests less than the low-quality one.

### 4.2 Optimal accessibility investment levels

Let us start by briefly considering the impact of transport costs on first-stage prices (8) and (9), which will prove to be useful in the following discussion. Simple calculations show that

\[
\frac{\partial \hat{p}_h}{\partial T_h} = \frac{\partial \hat{p}_l}{\partial T_l} = \frac{1}{3},
\]

\[
\frac{\partial \hat{p}_h}{\partial T_l} = \frac{\partial \hat{p}_l}{\partial T_h} = \frac{1}{3}.
\]

The higher the cost borne to buy from a firm in a region, the lower has to be the price charged by the firm operating there. Symmetrically, the higher the cost borne to buy from the rival firm, the higher can be the price charged by the firm. As expected, higher transport costs increase the overall price paid by consumers and therefore lessen competition.

We now move to the structure of optimal investment levels. First, we analyze the role of firm’s quality level on the investment effort exerted. By inspecting (14) and (15), it can easily ascertain that

\[
\frac{\partial r^*_h}{\partial u_h} = \frac{\partial r^*_l}{\partial u_l} = \frac{2(9\gamma \Delta T - 2)}{(9\gamma \theta_\Delta u - 4)^2},
\]

\[
\frac{\partial r^*_h}{\partial u_l} = \frac{\partial r^*_l}{\partial u_h} = \frac{2(2 - 9\gamma \Delta T)}{(9\gamma \theta_\Delta u - 4)^2}.
\]

A preliminary remark is necessary, which clarifies the discussion below. All else equal, an exogenous increase in \( u_h \) widens the quality gap between firms, thus differentiating products more and relaxing competition. By contrast, an increase in \( u_l \) makes products more homogeneous, thus increasing competition. Consider the situation \( \Delta T < \frac{2}{9\gamma} \equiv \bar{\Delta T} \). In this case an increase in the quality of the own variant reduces the optimal investments in accessibility, while an increase in the rival’s quality has the opposite effect. If the ex-ante difference in transport costs is relatively small, the quality difference and prices play a major role in determining firms’ demands and profits. Furthermore, since pro-
duction costs are nil for both firms, the high-quality producer is in a better competitive position than its low-quality rival. Keeping this in mind, consider the high-quality producer. An increase in $u_h$—all else equal—increases the price this firm can charge on the consumers, which translates into a higher mark-up on every unit sold. As a consequence, this firm may increase its profits by saving on accessibility investments (recall that $\Delta T$ is “small”), thus reducing $r_h^*$. An increase in $u_l$ reduces the firm’s price (and thus the mark-up), and pushes the high-quality firm to look for a competitive advantage by enhancing its accessibility. Move now to the low-quality producer, and remember that since $\Delta T$ is “small” this firm is relatively disadvantaged. An increase in $u_l$ harshens competition and reduces the firm’s mark-up. The firm compensates this decrease by improving its accessibility. By contrast, when $u_h$ increases, the margin for the low-quality producer widens, resulting in savings in accessibility investments.

We consider now the case in which $\Delta T > \bar{\Delta} T$: transport costs are relatively asymmetric and harm firm $h$ more than firm $l$. In this case an increase in the quality level of the own variant increases own accessibility investments while an increase in the rival’s one reduces them. Again consider the high-quality firm first. This firm is relatively disadvantaged by transport costs. An increase in $u_h$ increases the mark up for the firm, and consequently the incentives to improve its accessibility to take full advantage of higher quality. By contrast, an increase in $u_l$ makes competition fiercer, and leads the $h$-firm to reduce investments. Consider the low-quality firm now. If $u_l$ increases, competition is tougher and the low-quality firm preserves its relative accessibility advantage by increasing investments. Finally, if $u_h$ increases, competition is looser and the $l$-firm can save on investment costs without losing profits. We summarize the foregoing remarks in the following.

**Proposition 2** Let $\Delta T = \frac{2}{\gamma}$

(i) when $\Delta T < \bar{\Delta} T$, an increase in one firm’s quality level decreases its own investment and increases its rival’s one;

(ii) when $\Delta T > \bar{\Delta} T$, an increase in one firm’s quality level increases its own investment and decreases its rival’s one.

### 4.3 Investments and ex-ante accessibility

A natural question is about the effect of an increase in the ex-ante accessibility cost ($T_i$ with $i = h, l$) on the optimal investment levels. Computations show
that:
\[
\frac{\partial r_h^*}{\partial T_h} = \frac{\partial r_l^*}{\partial T_l} = \frac{2}{9\gamma \theta \Delta u - 4} < 0; \quad (28)
\]
\[
\frac{\partial r_l^*}{\partial T_l} = \frac{\partial r_h^*}{\partial T_h} = \frac{2}{9\gamma \theta \Delta u - 4} > 0. \quad (29)
\]
Assumption (20) together with \(\Delta T \geq 0\) determine the sign of (28) and (29). Each firm (i) decreases its optimal investment in accessibility when own gross accessibility worsens while (ii) a deterioration in the rival’s accessibility increases the investment of the firm. The intuition for these results relies on the interaction of demand and strategic effects. An increase in own transport costs lowers profits due to both a reduction in consumers served (which now prefer the other firm) and a decrease in the price charged. These effects reduce (expected) profits and thus the incentives for transport cost abatement. Symmetrically, if the rival’s transport cost increases, more consumers choose the firm’s good and the price competition is milder. Both effects increase profits and increase incentives for reduction in transport costs. The following proposition summarizes this discussion.

**Proposition 3** (i) An increase in the ex-ante transport costs for one firm’s own region reduces the firm’s optimal investments in accessibility improvement; (ii) an increase in the ex-ante transport costs for the rival’s region increases the firm’s optimal investments in accessibility improvement.

5 Welfare

Welfare is usually defined as the sum of consumer surplus and firm profits minus transport costs in the location in which policy authorities rule. Since in our model firms and consumers are located in different regions, the definition of the territorial unit of policy intervention defines the form of the welfare function itself. If we consider the high- and low-quality regions as independent from one another then welfare in each region coincides with firms’ profits, and regional policymakers simply compete in attracting customers. Thus the welfare-maximizing accessibility investments coincide to those described in the previous sections, and all the analysis carries through unchanged. Let us now cope with the situation where regions belong to the same policy unit. In particular, in what follows we tackle the two cases of (i) one single policy authority for re-
regions $h$ and $l$, which hence does not take into account the consumers’ surplus, and (ii) the case in which consumers and firms belong to a single policy unit. We denote case (i) “Aggregate Profit Maximization” (PM) and case (ii) “Social welfare maximization” (SWM).

In case (i) (aggregate profit maximization), the two regions $h$ and $l$ are subject to the authority of a single policymaker. The policymaker aims at maximizing the total welfare of the two regions. As discussed above, this translates into the following problem.

$$\max_{r_h, r_l} \hat{\pi}_h(\cdot) + \hat{\pi}_l(\cdot) - R_h(\cdot) - R_l(\cdot),$$

(30)

where $\hat{\pi}_i$, with $i = h, l$ are as in (10) and (11). In the Appendix we prove that the accessibility investments that maximize total profits are

$$r_{APM}^h = \frac{2\hat{\theta}\Delta u - 4\Delta T}{9\gamma\hat{u} - 9}, \quad r_{APM}^l = 0.$$  

(31)

The policy prescription suggested by this result is to concentrate all the accessibility investment on the high-quality region. This result is counterintuitive at a first sight only. Indeed the purpose of this policy guideline is to maximize the number of consumers selecting the high-quality firm. The high-quality firm charges a higher price on consumers and thus extracts a higher surplus from them. Therefore improving the accessibility for region $h$ relative to region $l$ results in an increase in the surplus extracted because of the higher number of consumers selecting the high-quality firm.

Let us now move to the case (ii) in which both the firms and the set of consumers reside in the same administrative region. In this case the policy authority faces a different problem with regard to the previous one, because now policymakers should take into account the consumer surplus as well. In this case the relevant welfare function is of the more usual form

$$W^{SWM}(r_h, r_l) = \frac{1}{\beta} \left[ \int_{0}^{\hat{\theta}_{h,l}} (U_0 + \theta u_l - t_l)d\theta + \int_{\hat{\theta}_{h,l}}^{\hat{\theta}} (U_0 + \theta u_h - t_h)d\theta \right] - R_h(\cdot) - R_l(\cdot),$$

(32)

where $\hat{\theta}_{h,l} \equiv \theta_{h,l}(\hat{\phi}_h(\cdot), \hat{\phi}_l(\cdot))$. As in the previous case, the policymaker’s problem is

$$\max_{r_h, r_l} W^{SWM}(r_h, r_l)$$

(33)

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In the Appendix we show that the unique solution to problem (33) is
\[ r^{SWM} = \frac{7\theta \Delta u - 5\Delta T}{9\gamma \theta \Delta u - 5}, \quad r^{SWM}_l = 0. \] (34)

All investments should be made in the enhancement of the high-quality firm’s accessibility, as in the previous case. The surplus-extraction intuition previously provided is still valid here, yet it should be integrated as follows. By abating the transport cost for the high-quality region, the number of consumers selecting the high-quality firm grows, which, in turn, further increases the overall surplus generated. The following proposition summarizes these results.

**Proposition 4** Social welfare maximization requires to foster accessibility investments of the high-quality firm’s region only.

The prescriptions contained in proposition 4 have some interesting consequences. In particular they entail that the accessibility investments under duopolistic competition to region \( l \) are always larger than those socially optimal, and thus define a clearcut guideline of policy intervention towards this firm.

### 6 Extension: endogenous qualities

In this section we extend our model to allow firms to endogenously choose the quality level of the good they supply. This allows us to investigate the influence of the level of ex-ante transport costs on the quality level selected by firms.

In order to introduce the choice of quality in our model, we follow Moorothy (1988) by assuming that the quality level of a good directly influences its marginal production cost. In particular, let the marginal production cost of good \( i \in \{h, l\} \) be quadratic in the quality level:
\[ C'_i(u_i) = u_i^2. \]

Profits (see eq. 7) are thus re-defined as follows (no fixed costs are borne by firms)
\[ \Pi(p_h, p_l, u_h, u_l) = D_i(.) (p_i - u_i^2). \] (35)

Firms play now a three-stage game. At the first stage they select the investment in accessibility improvement, at the second stage they select the quality level of
their product, and finally they set price. Moves at each stage are simultaneous. The solution concept is that of subgame perfection.

The third stage is solved through standard maximization techniques, yielding the following prices

\[ \hat{p}_h = \frac{1}{6}[2u_h^2 + u_t^2 + 4\theta(\Delta u) - 2(t_h - t_t)], \tag{36} \]
\[ \hat{p}_l = \frac{1}{6}[u_h^2 + 2u_t^2 + 2\theta(\Delta u) + 2(t_h - t_t)]. \tag{37} \]

Notice that the first three terms inside the square brackets of (36) and (37) are positive, while the last one can be positive or negative, depending on the relative size of \( t_h \) and \( t_t \), the ex-post accessibility costs. In the Appendix we prove that \( t_h > t_t \) only is compatible with the existence of optimal strategies at the quality stage.\(^8\) As usual, (36) and (37) can be plugged back into (35), for \( i = h, l \). The obtained equations are then used to derive the optimal values for qualities. In the Appendix we show that the optimal qualities selected at this stage are unique and they are

\[ \hat{u}_h = \frac{2(t_h - t_t)}{3\theta} + \frac{5}{4}\bar{\theta}; \tag{38} \]
\[ \hat{u}_l = \frac{2(t_h - t_t)}{3\theta} - \frac{1}{4}\bar{\theta}. \tag{39} \]

To complete the characterization of the SPNE of the game we need now to determine the optimal investments in accessibility by firms, after having substituted \( \hat{u}_h \) and \( \hat{u}_l \) back into (35). We stick to the previous section’s modeling strategy and we assume that the accessibility improving technology displays decreasing returns to scale. A reduction in the ex-ante accessibility cost \( T_i \) by an amount \( r_i \) requires an investment equal to \( \gamma r_i^2/2 \), \( i = h, l \). We do not report the analytical details leading to the solution of the first stage, but only the main outcomes.

The optimal investment levels at the first stage are given by:

\[ r_h^* = \frac{2}{3\gamma} - \frac{16\Delta T}{27\gamma\theta^2 - 32}, \tag{40} \]
\[ r_l^* = \frac{2}{3\gamma} + \frac{16\Delta T}{27\gamma\theta^2 - 32}. \tag{41} \]

\(^8\)Since \( t_i = T_i - r_i \), this condition involves the values of optimal investments, which are determined at the first stage of the game. In the following we prove that the condition holds at equilibrium.
By plugging (40) and (41) back into eqs. (36) to (39) we obtain the equilibrium prices and qualities, which we report hereafter:

\[
p^*_h = \frac{\bar{\theta}^2}{32} \left\{ 49 + \frac{144\gamma \Delta T [9\gamma (4\Delta T + 3\bar{\theta}^2)]}{(32 - 27\gamma \bar{\theta}^2)^2} \right\}, \tag{42}
\]

\[
p^*_l = \frac{\bar{\theta}^2}{32} \left\{ 25 + \frac{432\gamma \Delta T [3\gamma (4\Delta T + 9\bar{\theta}^2)]}{(32 - 27\gamma \bar{\theta}^2)^2} \right\}; \tag{43}
\]

and

\[
u^*_h = \frac{18\gamma \Delta T \bar{\theta}}{27\gamma \bar{\theta}^2 - 32} + \frac{5}{4} \bar{\theta}, \tag{44}
\]

\[
u^*_l = \frac{18\gamma \Delta T \bar{\theta}}{27\gamma \bar{\theta}^2 - 32} - \frac{1}{4} \bar{\theta}. \tag{45}
\]

We assume that (45) and (44) are positive, which requires that \(\gamma > \max\{\frac{32}{27\gamma \bar{\theta}^2 - 32}, \frac{32}{27\gamma \bar{\theta}^2 - 27\bar{\theta}^2}\}\).

Together with our initial hypothesis \(\Delta T \geq 0\), this guarantees the positivity of equilibrium prices (42) and (43), and the fulfilling of second order conditions at the accessibility investment stage.\(^9\) A first result is summarized in the following

**Proposition 5** The high-quality firm invests in accessibility improvement always less than the low-quality one.

The high-quality firm exploits the higher willingness to pay of consumers for the good it sells, and thus is less keen on devoting resources in accessibility improvement. Furthermore, an increase in the relative ex-ante accessibility for region \(l\) (i.e. an increase in \(\Delta T\)) entails the same increase in the quality level of the firms, but a reduction in the accessibility investment of firm \(h\) and an increase in that of firm \(l\). An increase in \(\Delta T\) harms firm \(h\) by lowering its accessibility. This firm reacts by increasing its quality level and reducing the accessibility investment. By contrast, the low-quality firm is advantaged by an increase in \(\Delta T\), and reacts to an improvement in its relative accessibility by fostering its investments both in accessibility and quality. Finally, notice that variations in \(\Delta T\) do not affect the equilibrium level of product differentiation, which is always equal to \(\frac{3}{2}\bar{\theta}\).

\(^9\)A direct consequence of our assumption \(\Delta T > 0\) and the result that \(r^*_h < r^*_l\) is that the ex post accessibility costs difference \(t^*_h - t^*_l \equiv T_h - T_l + r^*_l > 0\), which satisfies the condition for the existence of optimal qualities at the second stage of the game.
sured by their equilibrium profits, that are:

\[ \Pi_h^* = \frac{3[8\bar{\theta}(3\gamma \Delta T + 4) - 27\gamma \bar{\theta}^3]^2}{8(27\gamma \bar{\theta}^2)^2}; \]  \hspace{1cm} (46) \\

\[ \Pi_l^* = \frac{3[8\bar{\theta}(3\gamma \Delta T - 4) + 27\gamma \bar{\theta}^3]^2}{8(27\gamma \bar{\theta}^2)^2}. \]  \hspace{1cm} (47)

Direct comparison of (46) and (47) yields

**Proposition 6** The low-quality firm earns higher profits than the high-quality one.

This outcome can be explained by resorting to the results by Cremer and Thisse (1991) on the relationship between models of vertical and horizontal differentiation. Abstracting from the accessibility improvement stage, the present model is isomorphic to an Hotelling (1929) model of horizontal product differentiation with symmetric firms, quadratic transport costs and uniformly distributed consumer tastes. This type of model has a symmetric Nash Equilibrium at which firms select maximum product differentiation and earn equal profits which exists, therefore, in the corresponding vertical differentiation model. The accessibility improvement stage breaks the symmetry of the model (as long as \( \Delta T \neq 0 \)). In particular the assumption \( \Delta T \geq 0 \) charges an initial burden on firm \( h \) which results in lower equilibrium profits for this firm. It is easily ascertained that when there is no asymmetry in the ex-ante transport costs (\( \Delta T = 0 \)) the symmetry of Nash Equilibrium of the model is restored.

This extension highlights the importance of proximity to consumers to determine the profitability of a firm. Being the producer of high-quality is neither necessary nor sufficient to become the market leader in terms of profits. Indeed the low-quality firm invests more than the high-quality one and ends up with earning higher profits than its higher-quality rival.

### 7 Conclusion

In this paper, we model the competition between firms engaging in a strategic investment in market accessibility. Firms are differentiated in terms of quality and, before competing on prices, they invest in market accessibility enhancement.

Our results can be summarized as follows. First, high-quality firms can earn lower equilibrium profits than the low-quality one. This happens when the
market accessibility for the products of the low quality firm is higher than the willingness to pay for the quality premium. Interestingly, this result parallels the condition for investments: the firm investing more is the firm earning higher profits. Second, optimal investments depend both on quality and ex-ante transport costs. In particular, an increase in quality for high-quality firm increases the investment only if the relative accessibility of the low-quality firm is high enough. Symmetrically, an increase in the quality for low quality firm increases the investment only if its own relative market accessibility is low. Moreover the level of investment is positively affected by improved ex-ante accessibility conditions. Third, the low quality firm has incentives toward transport cost reduction that are unambiguously larger than those maximizing total profits or social welfare. This suggests a clearcut intervention guideline for policymakers. Last, the model is extended allowing firms to endogenously select the quality level of the good they produce. In this case, the high quality firm invests in accessibility improvement always less that the low-quality one, and this results in lower profit.

A Welfare: calculus

A.1 Case (i): Aggregate Profit Maximization (APM)

The welfare function in this case is:

\[ W_{APM}(r_h, r_l) = \hat{\pi}_h(\cdot) + \hat{\pi}_l(\cdot) - R_h(\cdot) - R_l(\cdot) = \]
\[ = \frac{1}{18} \left[ 4(r_h - r_l + \Delta T) + 10\theta\Delta u + \frac{4(r_h - r_l - \Delta T)^2}{\theta\Delta u} - 9\gamma(r_h^2 + r_l^2) \right] \]

The government maximizes (48) with respect to \( r_h \) and \( r_l \). To solve this problem we use the first-order condition approach. The solution to the following system gives the candidate maximizers.

\[ \begin{align*}
\frac{\partial W_{APM}(r_h, r_l)}{\partial r_h} &= 0 \\
\frac{\partial W_{APM}(r_h, r_l)}{\partial r_l} &= 0
\end{align*} \] (49)

The system (49) has three solutions; label them \( (r_{APM1}^h, r_{APM1}^l) \), \( (r_{APM2}^h, r_{APM2}^l) \) and \( (r_{APM3}^h, r_{APM3}^l) \).\(^{10}\) By substitution of \( (r_{APM1}^h, r_{APM1}^l) \) and \( (r_{APM3}^h, r_{APM3}^l) \) the expressions for the candidate maximizers are available upon request.
into (48), we obtain that $W_{APM}(r_{APM}, r_{APM}) = 0$, while $W_{APM}(r_{APM}, r_{APM}) \neq 0$. Thus the set of candidate maximizers of $W_{APM}(.)$ reduces to the singleton $(r_{APM}, r_{APM})$. To complete the proof of Proposition 3, we shall show that this candidate is indeed a maximum. To this end, we evaluate the concavity of the welfare function. Define the Hessian matrix of the maximization problem as

$$
H_{W_{APM}} = \begin{vmatrix}
\frac{\partial^2 W_{APM}(\cdot)}{\partial r^2 h} & \frac{\partial^2 W_{APM}(\cdot)}{\partial r^2 h \partial r_l} \\
\frac{\partial^2 W_{APM}(\cdot)}{\partial r^2 h \partial r_l} & \frac{\partial^2 W_{APM}(\cdot)}{\partial r^2 l}
\end{vmatrix}
$$

Calculations show that $\frac{\partial^2 W_{APM}(\cdot)}{\partial r^2 h} > 0$ and $\det(H_{W_{APM}}) < 0$. We conclude that $(r_{APM}, r_{APM})$ maximizes $W_{APM}(\cdot)$. The maximizers are given by:

$$
r_{APM} = \frac{4\Delta T - 2\bar{\theta} \Delta u}{4 - 9\gamma \bar{\theta} \Delta u},
$$

$$
r_{APM} = 0.
$$

To avoid cumbersome notation in the text we have suppressed the identifier 1 in the indexes.

### A.2 Case (ii): Social Welfare Maximization (SWM)

The analysis in this case develops along the same lines as before, so it will only be sketched. The first-order condition approach leads to three candidate maximizers. By analogy with the previous case, we label them $(r_{SWM}^{SWM1}, r_{SWM}^{SWM1})$, $(r_{SWM}^{SWM2}, r_{SWM}^{SWM2})$ and $(r_{SWM}^{SWM3}, r_{SWM}^{SWM3})$. It can be proved that the last two candidate make can be discarded. The analysis of the Hessian matrix associated to this problem confirms that $(r_{SWM}^{SWM1}, r_{SWM}^{SWM1})$ are indeed the desired maximizers. Their value is as follows (identifiers have been suppressed, the expressions for the other candidate are available upon request).

$$
r_{SWM} = \frac{5\Delta T - 7\bar{\theta} \Delta u}{5 - 9\gamma \bar{\theta} \Delta u},
$$

$$
r_{SWM} = 0.
$$

### B Equilibrium qualities

In this Appendix we derive the optimal qualities at the second stage of the game. We are interested in pure-strategy equilibria with $u_h > u_l$. 

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By plugging (36) and (37) back into (35) for \( i = h, l \), we obtain the following firm profits

\[
\hat{\Pi}_h(\cdot) = \frac{[2(t_h - t_l) + \Delta u(u_h + u_l)]^2}{36\theta \Delta u}, \tag{54}
\]

\[
\hat{\Pi}_l(\cdot) = \frac{[2(t_h - t_l) + \Delta u(u_h + u_l)]^2}{36\theta \Delta u}; \tag{55}
\]

where \( \Delta u \equiv u_h - u_l \). The candidate equilibria at the quality stage are the solutions of the following system

\[
\begin{align*}
\frac{\partial \hat{\Pi}_h(\cdot)}{\partial u_h} &= 0, \\
\frac{\partial \hat{\Pi}_h(\cdot)}{\partial u_l} &= 0. \\
\end{align*}
\tag{56}
\]

Five pairs \((u_h, u_l)\) satisfy (56), namely

\[
\begin{align*}
(\bar{\theta} \quad \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \quad \sqrt{8(t_h - t_l) + 9\bar{\theta}^2 - 2\bar{\theta}}) &\equiv (u^1_h, u^1_l); \\
\left(\frac{2}{3} \frac{t_h - t_l}{\theta} + \frac{5}{4} \frac{2}{3} \frac{t_h - t_l}{\theta} - \frac{1}{4} \frac{\bar{\theta}}{\theta}\right) &\equiv (u^2_h, u^2_l); \\
\left(2\bar{\theta} - \frac{1}{2} \sqrt{-8(t_h - t_l) + 9\bar{\theta}^2 - \bar{\theta}}\right) &\equiv (u^3_h, u^3_l); \\
\left(\frac{\bar{\theta}}{2} - \frac{1}{2} \sqrt{8(t_h - t_l) + 9\bar{\theta}^2 - \bar{\theta}}\right) &\equiv (u^4_h, u^4_l); \\
\left(2\bar{\theta} + \frac{1}{2} \sqrt{-8(t_h - t_l) + 9\bar{\theta}^2 - \bar{\theta}}\right) &\equiv (u^5_h, u^5_l);
\end{align*}
\]

Two cases have to be considered depending on the sign of \( t_h - t_l \).

(i) Consider the case \( t_h - t_l < 0 \). Candidates \((u^1_h, u^1_l)\) and \((u^3_h, u^3_l)\) are excluded because \( u_h < u_l \). Candidate \((u^2_h, u^2_l)\) is excluded because \( u^2_l < 0 \). The fourth candidate is disregarded because \( u_l \) is either not real or negative. At the last candidate firm-\( h \)'s profits are equal to zero, and it can be easily proved that there exists a profitable deviation for this firm at the quality stage. This allows us to conclude that there is no pure-strategy equilibrium at the quality-choice stage with \( t_h < t_l \).

(ii) Move to case \( t_h > t_l \). The pair \((u^3_h, u^3_l)\) can again be excluded because it involves a negative value for \( u_l \). Candidates \((u^1_h, u^1_l)\) and \((u^5_h, u^5_l)\) can be disregarded because firm-\( h \)'s profits \( \Pi_h(u^1_h, u^1_l) \leq 0 \), \( \forall t_h > t_l \). It can be
proven that this firm has a profitable deviation in one of the stages of the game. Candidate \((u_3^h, u_3^l)\) is excluded because it fails to fulfill (local) second order conditions. The only candidate left under scrutiny is \((u_2^h, u_2^l)\). This candidate satisfies (local) second order conditions and it can be checked that there are no profitable deviations in the whole strategy space for the two firms. This allows us to conclude that the only qualities that can be a part of a SPNE of the game are

\[(u_2^h, u_2^l) \equiv (\hat{u}_h, \hat{u}_l)\]

with \(t_h > t_l\).

References


