Competition, Reputation and Compliance*

Paolo Vanin†

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Abstract

This paper displays a linear demand oligopoly model, in which firms endogenously decide whether to enter the market and whether to specialize on high or low quality products, and then repeatedly interact to sell experience goods. It shows that the intuition that low and rising prices grant compliance with quality promises extends to this setting, provided that high quality is sufficiently important to buyers.

JEL-Classification: L13, L14, L15

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†Department of Economics, Università di Bologna; e-mail: paolo.vanin@unibo.it

1 Introduction

Klein and Leffler (1981) suggest a mechanism through which reputation may provide the adequate incentive for sellers of experience goods to comply with promises (e.g., supply high quality). If buyers pay a price premium for high quality, if they are informed on past compliance and never buy from a seller who cheated on quality in the past, the present value of the stream of future profits granted by compliance may be higher than the one period deviation gain that can be obtained by cheating consumers, so that the seller is indeed induced to be trustworthy. Shapiro (1983) formally investigates this mechanism and shows that low and rising prices guarantee high quality in a competitive market, because premiums for high quality ensure that no firm has an incentive to cut on quality and cheat the market, but competition for such premiums induces firms to set initially low, loss-making prices, which correspond to an investment in reputation, to which later profits are the normal market return.

The degree of market competition may be fundamental to determine incentives for high quality provision. Competition may both lower monopoly rents, and thus reduce returns from promise compliance, and offer buyers more alternatives, and thus strengthen punishments for non compliance. Competition itself depends on entry and exit, which depend on expected profits and therefore on returns to reputation and on reputation building costs (besides standard entry and exit costs). If the number of entrants is limited, strategic interaction, which is assumed away in competitive models with infinitely many firms, is likely to play a major role.

This paper investigates how reputation works in a market in which the number of firms is endogenous, quality is a long lasting choice variable and low quality would not be bought (at profitable prices) if recognized as such. Specifically, I consider a game with four stages: entry, quality selection (of an experience good) and twice repeated market interaction (repetition allows for reputation accumulation). I show that the intuition that low and rising prices grant high quality provision extends to the present oligopolistic setting, provided that high quality is sufficiently important for buyers.[1]

This work is related to the literature on price signals of quality, in which

[1] In a companion paper (Vanin, 2009), to which I refer for a deeper discussion of the literature and of the model's details, I show that, if high quality is less important to buyers, then the reputation mechanism fails, giving rise to interesting market dynamics with equilibrium cheating.

market structure and quality are assumed as exogenously given.[2] Yet it is more closely related to the small recent literature that investigates reputation together with entry and quality choice.[3] In particular, Hörner (2002) presents a dynamic version of Klein and Leffler (1981), in which firms enter the market, choose quality every period and use prices to signal quality. Each firm's (quality guaranteeing) price rises over time (as its reputation increases), until bad luck drives it out of the market. Consumers' knowledge of a firm's customer base implies that it cannot raise its price to mimic higher reputation firms. My assumption that buyers would not purchase low quality at profitable prices under perfect information has the opposite implication that upwards price mimicry is feasible. Indeed, it is often the case that buyers ignore at the same time sellers' quality and their customer base. Besides for this aspect, the present work also differs from Hörner (2002) because it explicitly considers strategic interaction, rather than featuring a constant continuous mass of firms on the market. Toth (2008) presents a dynamic oligopoly model with stochastic entry and with investment in quality every period, and shows that market concentration may alleviate moral hazard. Yet his work is focused on firms' survival contest and does not present an explicit model of market interaction (in particular, prices are not used as signals of quality). My contribution consists precisely in providing an explicit analysis of dynamic market interaction, allowing prices to serve as signals of quality. Moreover, I also differ from these models in that I consider quality as a long-lasting choice variable, which increases marginal costs, rather than as a variable chosen made every period. To the extent that product quality depends on the skills of a firm's employees, as is the case in many service markets, this view appears plausible and worth investigating.

The remainder of the paper is organized as follows. Section 2 presents the model, Section 3 analyzes its equilibrium and Section 4 concludes. A technical lemma is presented in Appendix.

[2] See, among others, Milgrom and Roberts (1986) and Bagwell and Riordan (1991) for monopoly; Hertzendorf and Overgaard (2001), Fluet and Garella (2002), Yehezkel (2008) and Daughety and Reinganum (2007) for duopoly; Daughety and Reinganum (2008) for an n-firm oligopoly; and Allen (1984) and Cooper and Ross (1984) for competitive models with U-shaped average cost function (I maintain the assumption of constant returns to scale).

[3] Overgaard (1994) considers a monopolist with a potential entrant and Bester (1998) investigates a duopoly with both quality and location choice on a line. My work is complementary to the latter, because I endogenize the number of firms, taking as given the degree of horizontal differentiation.

2 Model

2.1 Structure

I consider a game with the following four-stage structure. At stage one an infinite number of potential entrants simultaneously decide whether to enter the market or not. Each entering firm pays a fixed entry cost $\zeta > 0$, which is sunk after entry, and chooses a different variety of an experience good. Varieties are imperfect substitutes. The number of firms who enter the market is denoted by n. At stage two the n firms on the market simultaneously choose whether to produce high or low quality. The result of these choices is a vector $\mathbf{z} \in \{0, 1\}^n$, with $z_j = 1$ meaning that firm j has chosen high quality. Denote $h = \sum_{j=1}^{n} z_j$ the number of high quality firms. Once decided, the quality level remains the same in the two following market stages. To simplify and concentrate only on asymmetric information on consumers' side, I assume that, once chosen, a firm's quality becomes known to all firms on the market, but not to consumers. Consumers may learn a firm's quality either through direct experience with its products or by information extraction from equilibrium price signals. At stage three firms and consumers interact on the market for the first time. They move sequentially: first, firms simultaneously choose prices, determining a price vector $\mathbf{p^1} \in \mathbb{R}^n_+$. Next, having observed $\mathbf{p^1}$, consumers (indeed, a representative consumer) decide how much to demand to each firm, determining the demand vector $\mathbf{q^1} \in \mathbb{R}^n_+$. Stage four is analogous to stage three, but consumers now have additional information: if they have consumed a positive quantity of a firm's product, they are fully informed about its quality. Again, first firms simultaneously choose prices and determine the new price vector $\mathbf{p^2} \in \mathbb{R}^n_+$ and then consumers choose the new demand vector $\mathbf{q^2} \in \mathbb{R}^n_+$.

2.2 Preferences and technology

Preferences are assumed in such a way as to generate a linear demand for each product:[4]

[4] The model first presented by Shubik and Levitan (1980) and more recently used by Motta (2004) is extended by allowing for imperfect observability and product-specific quality, yielding the following expected utility function: $U(\mathbf{q}, \mathbf{e}) = \sum_{j=1}^{n} \alpha(e_j) q_j - \frac{n}{2(1+\mu)} \left[\sum_{j=1}^{n} q_j^2 + \frac{\mu}{n} \left(\sum_{j=1}^{n} q_j \right)^2 \right]$. See Vanin (2009) for a derivation of (1).

$$q_j(\mathbf{p},\mathbf{e},n) = \frac{1}{n}\left\{\left[\frac{n+\mu(n-1)}{n}\right][\alpha(e_j)-p_j] - \frac{\mu}{n}\sum_{i\neq j}[\alpha(e_k)-p_k]\right\}, \quad (1)$$

or, in matrix notation, $\mathbf{q}(\mathbf{p},\mathbf{e},n) = E(n)\cdot[\alpha(\mathbf{e})-\mathbf{p}]$, where $E(n)$ is an $n\times n$ matrix with elements $E_{ii}(n) = \frac{n+\mu(n-1)}{n^2}$ and $E_{ik}(n) = -\frac{\mu}{n^2}$; $\mathbf{p}$ is the price vector; $\mathbf{e}\in[0,1]^n$ is a vector of beliefs, i.e., its elements are the probability attributed by the representative consumer to the fact that each good is of high quality, conditional on information about previous play of the game (which I omit to write for notational simplicity): $e_j = \Pr\{z_j = 1\}$; $\alpha(e_j)$ reflects the utility value attributed to good j's expected quality, defined as $\alpha(e_j) = \beta + e_j\gamma$, where $\beta \geq 0$ and $\gamma \geq 0$ are parameters: β captures the value attributed to a unit of a low quality good and γ the additional value of high over low quality; $\alpha(\mathbf{e})$ is the vector of $\alpha(e_j)$'s; $\mu\in[0,\infty)$ is a parameter capturing the degree of substitutability between different varieties; y is a perfectly competitive outside good, introduced only to make partial equilibrium analysis justified.

One feature of this model is that (at interior consumers' choices) market size, $Q(\mathbf{p},\mathbf{e},n) \equiv \sum_{j=1}^n q_j(\mathbf{p},\mathbf{e},n) = \frac{1}{n}\sum_{j=1}^n[\alpha(e_j)-p_j] = \bar{\alpha}-\bar{p}$, does not depend upon either the degree of substitutability or the number of products, but only upon average expected quality and average price.

In the special case in which all products are expected to be of the same quality $\alpha(e)$ and have the same price p, individual demands are simply $q_j = \frac{\alpha(e)-p}{n}$. Identical firms with constant returns to scale and marginal cost $c < \alpha(e)$ react to this demand by setting the Nash equilibrium price

$$p^E(n,e,c) = \frac{n\alpha(e)+[n+\mu(n-1)]c}{2n+\mu(n-1)}, \quad (2)$$

which is increasing in e and c, decreasing in n and μ, converges to c as $\mu\to\infty$ and further simplifies to the usual monopoly price $\frac{\alpha(e)+c}{2}$ if $n=1$.

To later consider deviations from equilibrium, notice that if firm j manages to convince consumers that it is the only one offering high quality, i.e., if $e_j = 1$ and $\forall i\neq j,\ e_i = 0$, then $\forall n > 0$ and $\forall\mathbf{p}$ such that $p_j < \alpha(1)$ and $p_i \geq \alpha(0)\ \forall i\neq j$, it holds that $q_j(\mathbf{p},\mathbf{e},n) = \frac{1+\mu}{n+\mu}[\alpha(1)-p_j]$ and $\forall i\neq j,\ q_i(\mathbf{p},\mathbf{e},n) = 0$[5].

[5]The reason why firm j's demand depends on n is that, although j is the only one

All goods are produced with a constant returns to scale technology, with higher quality being more expensive to produce. Marginal costs of low and high quality are $c_L \geq 0$ and $c_H > c_L$, respectively. Firms are assumed to exit the market whenever they expect non positive profits.

2.3 Equilibrium concept and parameter restrictions

I look for a pure strategy weak perfect Bayesian equilibrium (WPBE) of the entire game and restrict attention to equilibria that are symmetric, in the sense that all firms choosing the same quality also set the same price. Since several equilibria are possible, depending on how consumers form quality expectations based on observed prices, and on how firms use prices to signal (or hide) their quality, I restrict attention to a simple class of belief functions (specified below), characterized by the fact that consumers distrust price signals whenever they are easy to imitate, and to 'investment in reputation' introductory prices, by which high quality firms signal their quality through initially low, loss-making prices, which are too low to be profitably imitated by low quality firms. Two assumptions are maintained throughout the analysis and are introduced and discussed here. Let $\hat{\gamma} \equiv \left[\frac{\mu^2+6(1+\mu)+(2+\mu)\sqrt{\mu^2+8(1+\mu)}}{2(1+\mu)}\right](c_H - c_L)$.[6]

Assumption 1. $\alpha(0) = c_L$

Assumption 2. $\gamma \geq \hat{\gamma}$

Under perfect information, Assumption 1, which equalizes the intrinsic utility of low quality goods and their production cost, makes demand for low quality goods insufficient even for the profitable entry of a single low quality monopolist, since its demand would be positive only at prices strictly below

selling a positive quantity, it is not the only one initially on the market. Consumers are 'tempted' by the other goods, although they do not buy them: the presence of other firms posting prices and offering their products reduces the marginal utility derived from j's good, so that j is able to sell at p_j a lower quantity than it would, at the same price, if it were alone on the market (i.e., if $n = 1$). Technically, only j's FOC holds with equality, whereas all the other ones hold with strict inequality (see Vanin, 2009). Notice that, given $n > 1$ and $\mathbf{p}$, j's demand increases in μ, since a higher degree of substitutability reduces consumers' temptation from different goods.

[6] The technical origin of $\hat{\gamma}$ is made clear in the proof of Lemma 1 in the Appendix. Notice that $\hat{\gamma}$ is unboundedly increasing both in μ and in $(c_H - c_L)$.

marginal cost. This implies that, under imperfect information, firms can profitably produce goods only as long as they manage to convince consumers of their high quality (or count to recoup initial losses in the future). It also implies that separation (of high from low quality firms) through upward distorted prices is impossible, because, if any price above c_L were a credible signal of high quality, it would be imitated by low quality firms, thus losing its credibility.

Assumption 2 grants that, if high quality firms separate from low quality ones by setting initial prices at c_L, then future profits from repeated purchase always compensate initial losses, thus making full high quality provision possible.[7]

If recognized as such, low quality firms leave the market, whereas high quality firms stay on the market and price according to (2). This is reflected in the full information equilibrium at stage 4. By the same logic, it is impossible that at stage 3 both high and low quality firms stay on the market and set two different prices, thus being recognized as such. This justifies the focus on beliefs that support equilibria with pooling prices. If high quality firms are able to separate themselves from low quality ones through low prices, then the market dries up for low quality firms, and these are forced out of the market, implying that nobody at stage 2 would choose low quality. This is precisely what Assumption 2 grants. The effects of distrust and of investment in reputation on quality choice are discussed in Propositions 1 and 2. Entry costs then determine the number of entrants, thus closing the model and allowing to make comparative statics exercises (Proposition 3).

In what follows I make these ideas precise. I order firms on the market by assigning lower indices to high quality ones. I start solving the model by backward induction, establishing sequential rationality of strategies and deferring to the end the consistency requirement between beliefs and strategies along the equilibrium path of play.

[7] It also implies that $\gamma > c_H - c_L$, which, given Assumption 1, is equivalent to $\alpha(1) > c_H$, which ensures that high quality firms receive positive demand in equilibrium and also makes high quality provision socially efficient.

3 Analysis

3.1 Stage 4: second market interaction

When consumers choose demand in the last move before the game ends, they are fully informed about the quality of goods on the market.[8] All low quality firms exit the market. High quality firms set prices, sell quantities and make profits according to (3), (4) and (5), respectively:[9]

$$p^2(h) = \frac{h\alpha(1) + [h + \mu(h-1)]c_H}{2h + \mu(h-1)}, \tag{3}$$

$$q^2(h) = \frac{h + \mu(h-1)}{h[2h + \mu(h-1)]}[\alpha(1) - c_H], \tag{4}$$

$$\pi^2(h) = \frac{h + \mu(h-1)}{[2h + \mu(h-1)]^2}[\alpha(1) - c_H]^2. \tag{5}$$

3.2 Stage 3: first market interaction

At stage 3 (first market interaction) firms set prices $\mathbf{p^1}$, consumers observe them, formulate beliefs on each firm's quality and then choose demand.[10] There exists no pure strategy weak perfect Bayesian equilibrium in which, along the equilibrium path of play, at stage 3 both high and low quality firms are present on the market and set two different prices (one for each quality level). If it existed, consumers would infer each firm's quality and force low quality firms out of the market. I therefore look for equilibria with pooling introductory prices.

[8] Beliefs are $e_j^2(\mathbf{p^1}, \mathbf{q^1}, \mathbf{p^2}) = z_j$ if $q_j^1 > 0$ and I assume $e_j^2 = 0$ if $q_j^1 = 0$, to rule out the possibility that a firm finds it optimal to produce only at stage 4. The superscript 2 is due to the fact that beliefs are relevant only in the two stages of market interaction and stage 4 is the second one.

[9] Notice that $p^2(h)$, $q^2(h)$ and $\pi^2(h)$ are all decreasing functions of h.

[10] Strategies specify each firm's introductory price after any possible $n > 0$ and $\mathbf{z} \in \{0,1\}^n$, since this identifies any possible information set at which firms may be called to set prices. While at stage 4 any collection of previous histories of play identifies a proper subgame, this is not the case at stage 3, because, for any n, any price vector $\mathbf{p^1} \in \mathbb{R}_+^n$ identifies one information set for the representative consumer, independently of $\mathbf{z} \in \{0,1\}^n$.

Beliefs

In order to study equilibria with pooling introductory prices, I restrict attention to a specific simple class of beliefs, which is especially likely to support such equilibria: $\forall n > 0$, $\forall j \in \{1, ..., n\}$, $\forall \mathbf{p^1} \in \mathbb{R}^n_+$, $\forall e^0 \in [0, 1]$,

$$e^1_j(\mathbf{p^1}, e^0) = \begin{cases} e^0 & \text{, if } \exists p^1 \in (c_L, \alpha(1)) : \ \forall i,\ p^1_i = p^1 \\ 1 & \text{, if } p_j \leq c_L \\ 0 & \text{, otherwise} \end{cases} \tag{6}$$

This means that, upon observing a pooling introductory price (in the range of profitable prices for low quality firms), consumers receive no information from price signals and do not revise their prior.[11] If a firm's price is (weakly) lower than c_L, then consumers conclude it must be a high quality firm, since at such introductory price a low quality firm's expected profits would be (weakly) negative and it would exit the market. If different prices are observed on the market, consumers interpret any price, at which low quality firms could make profits, as a trial to cheat them, and hence expect low quality.

In summary, I am considering an environment in which consumers do not trust price signals, because they are too easy to imitate, unless they convey the information that a firm is indeed willing to incur losses to build a good reputation, losses that low quality firms could never recoup.

Introductory prices, quantities and profits

I consider each firm's strategy as specifying an introductory price function of the form $p^1(n, h, e^0)$. When all potential entrants adopt the same $p^1(n, h, e^0)$, I call it a pooling introductory price function.[12]

Given a pooling introductory price function $p^1(n, h, e^0)$, if at any stage 3 information set it happens that $p^1 < \alpha(e^1)$, where $p^1 = p^1(n, h, e^0)$ and

[11] I interpret the prior e^0 as the degree of consumers' initial trust in firms' product quality, which is assumed to be common knowledge. Although introducing a prior is not necessary (only equilibrium beliefs matter), it is useful to relate the present work to the existing literature.

[12] In this case consumers' quality expectations at stage 3 are the same for all firms and, using (6), can be written as $e^1(p^1, e^0)$, where $p^1 = p^1(n, h, e^0)$. Notice that to study equilibria with pooling introductory prices, it is enough to summarize $\mathbf{z}$ through h. This will imply some slight abuse of language when talking about firms' stage 3 information sets. Such imprecisions are immaterial to the analysis and simplify the exposition.

$e^1 = e^1(p^1, e^0)$, then each firm j on the market expects to receive demand $q^1(p^1, e^1, n)$ according to (1) , so that initial profits for low and high quality firms are $\pi^1_L(p^1, e^1, n)$ and $\pi^1_H(p^1, e^1, n)$, respectively, where

$$q^1(p^1, e^1, n) = \frac{\alpha(e^1) - p^1}{n}, \tag{7}$$
$$\pi^1_L(p^1, e^1, n) = (p^1 - c_L) \cdot q^1(p^1, e^1, n), \tag{8}$$
$$\pi^1_H(p^1, e^1, n) = (p^1 - c_H) \cdot q^1(p^1, e^1, n), \tag{9}$$

Overall profits

From equations (5), (6), (8), (9), letting again $p^1 = p^1(n, h, e^0)$ and $e^1 = e^1(p^1, e^0)$, define low and high quality firms' overall expected profits at a pooling price $p^1 < \alpha(e^1)$ as $\pi_L(n, h, p^1, e^1) \equiv \pi^1_L(p^1, e^1, n)$ and $\pi_H(n, h, p^1, e^1) \equiv \pi^1_H(p^1, e^1, n) + \pi^2(h)$, respectively.[13] If $n > 1$, denote a firm's overall deviation profits, if it sets $p \neq p^1$ when all other firms set p^1, as $\pi'_L(n, h, p, p^1, e')$ and $\pi'_H(n, h, p, p^1, e')$, for a low and a high quality firm respectively, where e' is still derived from (6), but taking into account that the deviating firm prices at p and all the other ones price at p^1. In particular, if $n \geq h > 0$, $e^0 > 0$ and $p^1 > c_L$, then by deviating to $p = c_L$, a high quality firm earns overall deviation profits $\pi'_H(n, h, c_L, p^1, 1) = -(c_H - c_L)\left(\frac{1+\mu}{n+\mu}\right)\gamma + \pi^2(1)$.

Sequential rationality

To be part of a WPBE, a pooling introductory price function must be sequentially rational. In particular, it must specify a sequentially rational price for a monopolist, but this restricts the initial levels of trust, for which we may find sequentially rational pooling introductory price functions, to $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$.[14] Given this, necessary and sufficient conditions for a pooling price function to be sequentially rational are stated in Lemma 1 in Appendix.

[13] At any price $p^1 \geq \alpha(e^1)$, any firm receives zero demand and expects zero overall profits: in this case, let $\pi_L(n, h, p^1, e^1) = 0$ and $\pi_H(n, h, p^1, e^1) = 0$.

[14] If a high quality monopolist sets $p^1 \geq \alpha(e^0)$, it receives zero demand and makes zero overall profits. This is not sequentially rational, because there exists an introductory price $p \in (c_L, \alpha(e^0))$, sufficiently close to $\alpha(e^0)$, which grants strictly positive overall profits. In turn, $\forall p^1 < \alpha(e^0)$, $\exists\, p \in (p^1, \alpha(e^0))$, which grants strictly lower initial losses and the same future profits, and hence strictly higher overall profits.

Under Assumptions 1 and 2, pricing at the initially loss making price c_L is profitable for high quality firms. Although any pooling introductory price $p^1 \in (c_L, \alpha(1))$ is strictly preferred by all firms to an equilibrium in which they initially pool on $p^1 = c_L$, it holds that, whenever h and n are high, the only sequentially rational pooling introductory price is precisely $p^1 = c_L$. The reason is that the temptation to monopolize future gains is so high, that any high quality firm would deviate from a profitable pooling introductory price and would rather incur the initial losses necessary to grant future monopoly, thus creating a sort of Prisoner's Dilemma situation.[15,16] I therefore focus on an 'investment in reputation' pooling introductory price function, denoted $p^1_R(n, h, e^0)$, and such that $\forall n > 1,\ \forall h \in \{0, \dots, n\},\quad p^1_R(n, h, e^0) = c_L$. From Lemma 1 in Appendix it then follows that this function is sequentially rational, given beliefs (6), if and only if $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$ and, in case of monopoly, it specifies $p^1_R(1, 1, 0) = c_L;\quad \forall e^0 \in (0, 1],\ p^1_R(1, 1, e^0) = p^E(1, e^0, c_H);\quad \forall e^0 \in [0, 1],\ p^1_R(1, 0, e^0) = p^E(1, e^0, c_L)$.

3.3 Stage 2: quality choice

At stage 2 the n firms on the market simultaneously choose whether to specialize their technology to produce high or low quality goods. This choice determines whether their marginal cost in the two subsequent periods will be c_H or c_L, respectively. Indeed, firms' strategies must specify such quality choice at any possible stage 2 information set. This yields a sequence of quality choice profiles $\mathbf{z}(n, e^0)$. The following two propositions show that in equilibrium such sequences imply that all firms on the market choose high quality if either consumers have zero trust ($e^0 = 0$) or if firms are going to invest in reputation (price according to $p^1_R(n, h, e^0)$).

Proposition 1. ***(distrust grants high quality with low and rising***

[15]Formally, $\forall h > 1,\ \forall e^0 \in [0, 1]$ and for any pooling introductory price function $p^1(n, h, e^0)$, $\exists\, \bar{n}(h, e^0) < \infty$ such that, if $p^1(n, h, e^0)$ is sequentially rational, then it must specify, $\forall n > \bar{n}(h, e^0),\ p^1(n, h, e^0) = c_L$. This follows from Lemma 1 in Appendix, since for any $h > 1$ and $e^0 \in [0, 1]$, as n diverges, the LHS in (10) converges to zero, whereas the RHS remains bounded away from zero.

[16]In Vanin (2007) I show that, if either γ or μ are sufficiently high, then the most intuitive pooling introductory price function (which has high quality firms setting the price that is most profitable for them, given that low quality firms mimic it) is sequentially rational only if oligopolists always invest in reputation (that is, for any $n > 0$ and $h \leq n$, and not just when h and n are high).

prices)
If $e^0 = 0$, then at any WPBE supported by beliefs (6), at which $n > 0$ firms enter the market, all of them choose high quality and subsequently set $p^1 = c_L$ and $p^2 = p^2(n) > c_L$.

Proof. Given Assumptions 1 and 2 and beliefs (6) with $e^0 = 0$, a low quality firm expects to make zero profits at any sequentially rational introductory price (i.e., at any $p^1 \geq c_L$); a high quality firm has a unique sequentially rational introductory price, $p^1 = c_L$, by setting which it grants itself strictly positive profits. Thus all firms choose high quality. Equilibrium pricing strategies must specify, $\forall (n,h) : \; n > 0, \; n \geq h \geq 0$ (i.e., both on and off the equilibrium path of play), $p^2 = p^2(n)$, $p^1 = c_L$ if $h > 0$, and $p^1 \geq c_L$ if $h = 0$. Along the equilibrium path of play, if $n > 0$, then $h = n$ implies $p^1 = c_L$ and $p^2 = p^2(n)$. □

Proposition 1 confirms the result obtained by Shapiro (1983), that consumer's initial distrust forces firms to offer high quality at prices that are initially below marginal cost, since they have to invest in reputation, in order to later price above marginal cost, when the initial investment pays off. Yet in Shapiro's equilibrium consumers are not fully rational, since their initial expectations turn out to be on average wrong. In contrast, under the present distinction between initial trust and posterior beliefs, consumers' distrust forces high quality firms to lower their initial price to a level that is unprofitable for low quality ones, thus convincing consumers of their high quality. So initial distrust is reconciled with correct equilibrium quality expectations.

Notice that for $e^0 = 0$ implies that for any $n > 0$ and $h \in \{0, \ldots, n\}$, it holds that $p^1_R(n, h, e^0) = c_L$. Therefore Proposition 1 can be read as stating that the strategy of investing in reputation succeeds in granting high quality when buyers distrust sellers' price signals. The next proposition extends the result to the case of higher initial trust.

Proposition 2. ***(full high quality with investment in reputation)***
If $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1 \right]$ and firms price according to $p^1_R(n, h, e^0)$ at stage 3 and play the Nash equilibrium at stage 4, then a sequence of quality choice profiles is sequentially rational, given beliefs (6), if and only if all firms entering the market choose high quality.

Proof. Denote overall expected profits for firms investing in reputation $\pi^R_i(n, h, e^0) \equiv \pi_i\left(n, h, p^1_R(n, h, e^0), e^1(p^1_R(n, h, e^0), e^0)\right)$, for $i \in \{L, H\}$. Let

$n > 1$. Since $\forall h \in \{0, \ldots, n\}$, $p^1_R(n,h,e^0) = c_L$, Assumptions 1 and 2 imply that for $h > 0$, $\pi^R_H(n,h,e^0) > \pi^R_L(n,h-1,e^0) = 0$; and for $h < n$, $\pi^R_L(n,h,e^0) = 0 < \pi^R_H(n,h+1,e^0)$. Now let $n = 1$. Since $p^1_R(1,1,e^0) = p^E(1,e^0,c_H)$ and $p^1_R(1,0,e^0) = p^E(1,e^0,c_L)$, we have that $\pi^R_H(1,1,e^0) > \pi^R_L(1,0,e^0) \iff e^0 < \frac{c_H - c_L}{\gamma} + \frac{[\alpha(1)-c_H]^2}{(c_H-c_L)\gamma}$, which always holds under Assumptions 1 and 2, because they imply $\frac{[\alpha(1)-c_H]^2}{(c_H-c_L)\gamma} > 1$. □

Not surprisingly, Proposition 2 shows that choosing high quality is the only sequentially rational choice for firms that are going to invest in reputation, since, anticipating that they are going to set low introductory prices, which are not profitable for low quality firms, it would be inconsistent to choose low quality.

3.4 Stage 1: entry and consistency

At stage 1 firms decide whether to enter the market or not. Since, overall expected profits are decreasing in n, the sequentially rational number of entrants is uniquely determined as the highest integer such that each firm's overall expected profits are higher than the fixed entry cost ζ.

At a WPBE beliefs must be consistent with strategies along the equilibrium path of play. Whenever equilibrium introductory pooling prices imply that posterior beliefs are determined by initial trust, consistency imposes equality in equilibrium between trust and average expected quality. Yet, if along the equilibrium path of play all firms choose high quality and set the introductory price $p^1 = c_L$, then consistency places no restrictions on the initial value of e^0.[17]

Proposition 3. ***(equilibrium and comparative statics for high*** γ***)***
The strategy profile $(n, h(n), p^1_R(n,h,e^0))$, together with the unique Nash equilibrium at stage 4 and with beliefs (6), is a WPBE if and only if (i) $e^0 \in \{0\} \cup \left(\frac{c_H-c_L}{\gamma}, 1\right]$, (ii) $h = n$, (iii) $p^1 = c_L$, (iv) $p^2 = p^2(n)$, and (v) n is the highest natural number such that equilibrium profits exceed entry costs, $\pi_H(n,n,c_L,1) = \frac{(c_L-c_H)\gamma}{n} + \frac{[n+\mu(n-1)][\alpha(1)-c_H]^2}{[2n+\mu(n-1)]^2} \geq \zeta$.[18]
The equilibrium number of firms has the following properties.

[17] The only restrictions on e^0 come from the requirement that introductory prices are sequentially rational at any information set, on and off the equilibrium path of play.

[18] To be precise, this statement must be qualified by specifying that $e^0 = 0$ if $\zeta \in \left(\frac{(c_L-c_H)\gamma}{2} + \frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}, (c_L - c_H)\gamma + \frac{[\alpha(1)-c_H]^2}{4}\right]$, whereas $e^0 = 1$ if $\zeta \in$

1. *If* $\zeta \leq \frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}$*, then* $n = \infty$.[19]

2. *If* $\zeta \in \left(\frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}, \frac{(c_L-c_H)\gamma}{2} + \frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}\right]$*, then* $2 \leq n < \infty$ *and* n *is increasing in* γ *and decreasing in* μ *and* $(c_H - c_L)$.

3. *If* $\zeta \in \left(\frac{(c_L-c_H)\gamma}{2} + \frac{(1+\mu)[\alpha(1)-c_H]^2}{(2+\mu)^2}, \frac{[\alpha(1)-c_H]^2}{2}\right]$*, then* $n = 1$.

4. *If* $\zeta > \frac{[\alpha(1)-c_H]^2}{2}$*, then all markets are closed.*

Proof. Property (i) follows from sequential rationality of a monopolist's introductory price; (ii) follows from Propositions 1 and 2; point (iii), with the appropriate qualification in footnote, from the definition of $p_R^1(n, h, e^0)$ and from Lemma 1 in Appendix; (iv) from stage 4 Nash equilibrium; point (v) and the subsequent results from simple algebra. $\square$

In words, the number of entrants is decreasing in entry costs, and it passes from infinity to zero as ζ rises. Moreover, it is increasing in the utility difference between high and low quality, γ, and decreasing in their cost difference, $(c_H - c_L)$, as well as in the degree of substitutability of different varieties, μ. Whenever markets are open, all firms choose high quality, and whenever they compete with other firms, they invest in reputation, initially pricing at low quality firms' marginal cost and then raising prices at their perfect information optimum. Given this pricing strategy, consistency of consumers' beliefs does not impose any restriction on their initial trust level, except for the particular case in which entry costs are so high that just a monopolist enters the market. In all other cases, firms' low introductory prices convince consumers that quality is high, independently of initial trust. This is in line with Shapiro (1983), who shows that at a competitive equilibrium firms signal product quality through low and rising prices.

4 Conclusion

I have displayed a linear demand oligopoly model, in which firms endogenously decide whether to enter the market and whether to specialize on high

$\left((c_L - c_H)\gamma + \frac{[\alpha(1)-c_H]^2}{4}, \frac{[\alpha(1)-c_H]^2}{2}\right]$, in which case $p^1 = p^2 = p^2(1)$.

[19] Then $p^2 = \lim_{n\to\infty} p^2(n) = \frac{\alpha(1)+(1+\mu)c_H}{2+\mu}$.

or low quality products, and then repeatedly interact to sell experience goods to consumers, who are able to precisely discover a firm's product quality only after the first purchase, but who are sufficiently rational to form correct expectations about average market quality. Although introductory prices may be used as signals of quality, consumers do not trust them if such signals are too easy to imitate. This creates a strong incentive for firms to pool on the same introductory price, independently of their quality. If high quality is sufficiently important to buyers, then all firms entering the market specialize on high quality and set initially low and rising prices. Profits from repeated purchase then more than compensate initial losses, and the reputation mechanism assures compliance with quality promises.

This result has been derived under the assumption that low quality products cannot be profitably sold under perfect information. This makes separation through high prices impossible, because for low quality sellers it is always profitable to mimic such prices.[20] Relaxing this assumption, equilibria in which high and decreasing prices grant high quality might emerge, as well as equilibria in which different qualities co-exist in the market and are recognized as such. Although conceptually straightforward, the analysis of such equilibria poses new technical subtleties, which require a separate work.

An analogous argument applies to the assumption that the utility difference between high and low quality is much higher than the cost difference, so that low introductory prices constitute a profitable investment in reputation, independently of the degree of market competition. In a companion paper (Vanin, 2009) I show that, when high quality is not much more valuable to buyers than more costly to firms, the reputation mechanism fails and the four stage game considered here yields interesting market dynamics with equilibrium cheating. The analysis of the intermediate case, in which investing in reputation may be profitable when competition is low but not when it is high, would make derivation and presentation of results unnecessarily cumbersome, without adding much to intuition. It is to be expected that, if entry costs are low and the equilibrium number of entrants is high, then the reputation mechanism would fail, yielding equilibrium cheating by some firms; in turn, if entry costs are high and the equilibrium number of entrants is low, results would resemble those obtained here.

[20] This, in turn, makes consumers skeptical when they observe different market prices, unless such prices are so low that they cannot be profitably imitated by low quality firms.

Appendix

Lemma 1. ***(seq. rational pooling introductory price functions)***
A pooling introductory price function $p^1(n,h,e^0)$ is sequentially rational given beliefs (6), with $e^0 \in \{0\} \cup \left(\frac{c_H - c_L}{\gamma}, 1\right]$, if and only if it satisfies the following conditions.[21]

1. *$\forall n > 0,\ \forall h \in \{0,\dots,n\},\ p^1(n,h,e^0) \ge c_L$ and,*
   if $h > 0$ and $e^0 = 0$, then $p^1(n,h,e^0) = c_L$.

2. *$\forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, $p^1(1,0,e^0) = p^E(1,e^0,c_L)$ and $p^1(1,1,e^0) = p^E(1,e^0,c_H)$.*

3. *$\forall n > 1,\ \forall h \in \{1,\dots,n\},\ \forall e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$, $p^1(n,h,e^0) < \alpha(e^0)$ and,*
   if $h > 1$ and $p^1(n,h,e^0) \in (c_L, \alpha(e^0))$, then

$$[p^1(n,h,e^0) - c_H]\left\{\frac{e^0\gamma - [p^1(n,h,e^0) - c_L]}{n}\right\} + (c_H - c_L)\left(\frac{1+\mu}{n+\mu}\right)\gamma$$
$$\ge \left\{\frac{1}{4} - \frac{h + \mu(h-1)}{[2h + \mu(h-1)]^2}\right\}[\gamma - (c_H - c_L)]^2. \tag{10}$$

Proof. Notice first that, given Assumption 1, Assumption 2 can be equivalently re-written in one of the following ways: $\gamma \ge \hat{\gamma} \iff \forall n > 0,\ \forall h \in \{1,\dots,n\},\ \pi_H(n,h,c_L,1) \ge 0 \iff \forall n > 0,\ \pi_H(n,n,c_L,1) \ge 0 \iff \lim_{n\to\infty} \pi_H(n,n,c_L,1) \ge 0$.[22] Assumptions 1 and 2, together with beliefs (6), imply that for any $n > 1$ and $h \in \{1,\dots,n\}$, a high quality firm's deviation from $p^1(n,h,e^0) > c_L$ to $p = c_L$ yields strictly positive overall expected profits $\pi'_H(n,h,c_L,p^1(n,h,e^0),1) > 0$.

1. If, for some $n > 0$ and $h \in \{0,\dots,n\}$, $p^1(n,h,e^0) < c_L$, then at the corresponding information set any firm would strictly gain by deviating to $p = c_L$. Under beliefs (6) and Assumption 1, $e^0 = 0$ implies that demand is positive if and only if $p^1(n,h,0) \le c_L$. Given Assumption 2, in turn, $\forall n > 0,\ \forall h \in \{1,\dots,n\},\ \pi_H(n,h,c_L,1) > 0$.

[21]In the cases not explicitly considered no additional constraints are imposed. See Vanin (2007) for a generalization of this lemma outside Assumption 2.
[22]The precise expression of $\hat{\gamma}$ comes from this last version.

2. Given Assumption 1, $e^0 \in \left(\frac{c_H - c_L}{\gamma}, 1\right]$ is equivalent to $\alpha(e^0) > c_H$ and therefore implies $\alpha(e^0) > c_L$. In this case, under beliefs (6) a low quality monopolist faces exogenous quality expectations e^0, whatever price it may choose in the interval $(c_L, \alpha(e^0))$. Only if it chooses its optimal monopoly price in this interval, no profitable deviations are possible. For a high quality monopolist, an analogous argument applies.

3. When several firms initially enter the market and a pooling introductory price $p^1(n, h, e^0) \geq c_L$ is expected, low quality ones have no profitable deviations. Any high quality firm ($h > 0$) may guarantee itself zero overall expected profits through a deviation to $p > c_L$; if it deviates from $p^1(n, h, e^0) > c_L$ to $p \leq c_L$, it monopolizes the market at both stages 3 and 4, but it makes initial losses (so that the best such deviation is to $p = c_L$). Assumptions 1 and 2, together with beliefs (6), imply that, given $e^0 > 0$, a pooling introductory price $p^1(n, h, e^0) \geq \alpha(e^0)$ is not sequentially rational, because it yields zero overall expected profits and high quality firms would gain by deviating to $p = c_L$. A pooling price $p^1(n, h, e^0) \in (c_L, \alpha(e^0))$, in turn, is sequentially rational if and only if $\pi_H(n, h, p^1(n, h, e^0), e^0) \geq 0$ and $\pi_H(n, h, p^1(n, h, e^0), e^0) \geq \pi'_H(n, h, c_L, p^1(n, h, e^0), 1)$. The former inequality holds for any $n > 1$ and $h > 0$, because $e^0 > 0$ and $p^1 \in (c_L, \alpha(e^0))$ imply that $\pi_H(n, h, p^1, e^0) > \pi_H(n, h, c_L, 1) \geq 0$. If $h = 1$, the second inequality also holds for any $n > 1$ and $e^0 > 0$, because by deviating from $p^1 \in (c_L, \alpha(e^0))$ to $p = c_L$, the high quality firm would simply worsen its initial losses (or start to make them) without any future benefit. In turn, if $h > 1$, then initial deviation losses might pay off in the future (in terms of reduced competition), so that sequential rationality requires $\pi_H(n, h, p^1(n, h, e^0), e^0) \geq \pi'_H(n, h, c_L, p^1(n, h, e^0), 1)$, which is is equivalent to condition (10).

□

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