Endogenous Attention Costs and Intertemporal Decision-Making

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Abstract

Recent contributions on intertemporal decision-making and self-control have focused on the impact of cognitive constraints on the way people behave over time. In particular there is evidence on the fact that exerting cognitive effort on the job, adhering to some specific behavioral plan and self-regulating behavior is fatiguing so that, unless sufficient rest is allowed for, the performance on these tasks degrades over time.

In this paper I propose an intertemporal decision-making model to determine the optimal path of effort that a worker should exert on a cognitively demanding task. In this environment the worker trades-off current performance with the endogenous accumulation of fatigue; consequently multi-tasking or exogenous cognitively demanding factors (like stress or noise) play a critical role, and they can induce the decision-maker to optimally take rest-breaks in order to save on the cognitive resources to be used in the future. In the model multiple equilibria and thresholds can emerge, with the consequence that the long-term outcome toward which the agent converges critically hinges on the initial condition of fatigue of the worker. When this is the case, it can be optimal to force the agent to take a rest break, or a holiday, in order to allow her to recover and to converge toward the desirable long-term outcome in which she is more productive and more rested.

These results highlight the importance of cognitive constraints in the study of intertemporal behavior and they suggest an alternative explanation for the evidence of preferences for improving consumption profiles and the evidence of (apparent) time-inconsistent behavior. More generally, this paper shows that the assessment of how people evaluate intertemporal utility profiles, on which the economic literature has mainly focused, should be complemented by considering also cognitive constraints, since they may limit the set of feasible paths of behavior that people can implement over time much in a way as a budget constraint limits the set of feasible alternatives in a standard decision-making model.

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1. Introduction

In the economic literature the human mind is generally treated as a black box that transforms inputs (information and economic constraints) into outputs (decisions and solutions to a given problem) that are to be selected according to some ranking of the possible outcomes. In general no explicit attention is devoted to how such transformation occurs: the economic agent is assumed to be able to solve any kind of problem, whatever the complexity of the task to be performed and whatever the amount of information to be collected and processed. Moreover no explicit cost is involved in the decision-making process, since no time is required to provide the correct answer and no cognitive limit affects the computation and the implementation of the optimal choice.

With some notable exceptions such approach has been (and still is) the dominant theoretical framework in the microeconomic literature, but it is not unique. Simon (1955), for example, observed that decision-making is not instantaneous and costless, and that there exist internal constraints that are intrinsic to the process of decision-making and thus significantly influence how people behave in real-life. Starting from such a point of view different economists have tried to explicitly include the computational and cognitive limits of the human mind in decision-making and problem-solving[3]. In this direction a part of the literature has focused on high-level cognition, a broad category that includes mental activities such as thinking, solving problems and focusing on long-term goals, as well as consciously suppressing thoughts and emotions or exerting self-control. Low-level cognition, on the contrary, is largely subconscious. It involves the perception of external and internal stimuli and it is generally associated to automatic mental activities. Such a dichotomy parallels the neuroscientific classification between controlled and automatic processing of information within the brain. Similar distinctions are also common in social psychology and psychoanalysis and they have been variously labeled[4].

A remarkable feature of this dichotomy is that low-level cognition is generally considered to be effortless, in the sense that it can potentially be borne *ad libitum* and that multiple simultaneous low-level tasks do not interfere significantly one with the other. On the contrary, high-level mental processes are considered to be costly, since they cannot be performed simultaneously and they require the exertion of attention effort[5]. In economics this is typically taken into account by postulating that exerting effort yields disutility for the decision-maker[6]. An alternative research approach has focused on the techniques that people adopt to substitute for their deficient computational abilities. From this perspective, for example, Gigerenzer and his colleagues (1999) have focused on the "cognitive shortcuts", such as rules-of-thumb and heuristics, that people use to save on the time, information and computational load that is required to provide a solution to a given problem[7]. An alternative direction is taken by Gabaix

[3] See, for example, Conlisk (1996) and references therein for a survey motivating the introduction of bounded rationality arguments within the standard economic literature.

[4] According to a quite rough classification, high-level cognition occurs in the neocortex, which is the most evolutionary advanced part of human brain and it is considered to be the "thinking center". It involves conscious and intentional mental activity that can eventually be stopped, and it governs the human ability to solve problems, speak, focus on broader goals and plan actions to implement them. Low-level cognition, on the other hand, is located in the occipital, parietal and temporal parts of the brain, with the amygdala -located below the cortex- having a major role in automatic affective responses. For an overview, see Camerer, Loewenstein and Prelec (2005) and Glimcher (2003); see also Strack and Deutsch (2004) for a psychological model in which social behaviour is the result of the interaction between high- and low-level cognitive processes.

[5] In this paper the expressions “attention effort” and “cognitive effort” are used as synonyms, the same holds for “attention costs” and “cognitive costs”.

[6] See, for example, Mas-Colell et al. (1995:479).

[7] Though not guaranteeing global optimality, they have been shown to be locally (ecologically) optimal, in the sense that they provide the best solution given the specific context in which the decision is to be made; see also Gigerenzer and Goldstein (1996).

et al. (2003)[8]. Their starting point is the evidence that people have finite mental processing speeds, so that paying attention to one specific task prevents one from paying attention to another one. This can be interpreted by saying that attention-demanding tasks compete on the cognitive resources that are required to perform them successfully, and that every time people are focusing on some task, they are implicitly leaving less time for alternative activities. Since time is a scarce resource, the (shadow) cost of cognition would be given by the opportunity cost of the time it takes to perform a certain cognitive task. Building on this observation, the authors develop and test a model to explain how a decision-maker should allocate her time endowment on different attention-demanding tasks.

Instead of focusing on the fact that there are computational limits for the decision-maker or that cognition takes time, this paper studies the cost of decision-making in terms of *cognitive fatigue*. This statement does not refer to the customary economic assumption that providing effort yields direct utility costs[9]. My point here is that exerting cognitive effort, besides the shadow cost considered by Gabaix et al. (2003), has two further indirect costs:

1. It contracts the span of time in which a person can exert positive effort;
2. It can impair performance on both simultaneous and subsequent tasks requiring high-level cognition.

Concerning the first point, different experiments on cognitive fatigue have shown that people can successfully exert cognitive effort only for a limited amount of time[10]. Such evidence has been interpreted by suggesting that high-level cognition consumes some kind of a limited stock of cognitive resources and that depletion of such a stock prevents high-level cognition. The additional experimental finding that cognitive overload induced by a very demanding cognitive task reduces the span of time in which cognitive effort can be exerted[11] is consistent with the cognitive resource depletion hypothesis. Note that this argument is different from the "time as a scarce resource" which assumes a fixed interval of time that needs to be divided between tasks, because it says that, as cognitive fatigue endogenously sets, an agent can exert decreasing levels of cognitive effort until, when the individual is completely exhausted, she cannot perform complex cognitive tasks.

Concerning the second statement, the cognitive literature reports that the fatigue due to a complex cognitive activity negatively affects the performance on both simultaneous and sequential cognitive tasks[12]. Interestingly, this result holds even if performance is measured on quite different tasks involving, for example, self-regulation, choice-making and problem-solving[13]. Baumeister and colleagues suggest a possible explanation: all these tasks involve high-level cognition and therefore they compete with each other by drawing on the same stock of cognitive resources. As a consequence exerting effort on a certain cognitively demanding task does not simply consume time, but it literally consumes part of the stock of cognitive resources that are necessary for performing alternative tasks.

In the economic literature the role of cognitive fatigue has been scarcely investigated. Ozdenoren et al. (2005) formalize the cognitive depletion hypothesis by studying a cake-eating problem in which a limited stock of cognitive resources, labeled as "willpower", is

[8] See also Gifford (2001a, 2001b), De Shazo and Fermo (2004).
[9] If the consumers' valuation of a good is malleable, whether a familiar activity or experience is pleasant or unpleasant is not a self-evident matter (see, for example, Ariely, Loewenstein and Prelec, 2006). From this perspective, whether exerting cognitive effort directly yields utility or disutility is just a matter of individual preferences.
[10] See, for example, Dorrian et al. (2000), Lorist et al. (2000), Bourne, Yaroush (2003), Tucker (2003).
[11] See, for example, Tucker (2003) and references therein.
[12] See, for example, Dorriant et al. (2000), Haines et al. (2001), French (2002).
[13] Vohs, Faber (2004), Baumeister et al. (1998).

depleted whenever consumption is below a given level. Thus the decision-maker must trade-off consumption with the availability of willpower over time because, when willpower is exhausted, no more self-control is possible. In this paper I describe a different environment and I study the intertemporal trade-off between performance and cognitive fatigue by considering the optimal pattern of attention/cognitive effort that a worker should devote to a given job over time.

In the following section I first present the basic version of the model in order to provide the rationale that drives the optimal management of effort over time as a function of fatigue. I then provide two extensions. In the first one I consider how the decision-making environment affects the intertemporal management of cognitive resources. It turns out that multitasking and cognitive overload induce a reduction in the optimal effort to exert, eventually forcing the decision-maker to take rest-breaks to optimally deal with the accumulation of fatigue. More generally, there are long lasting effects both on the optimal path of effort and on the utility profile that the decision-maker can get. In the second extension I consider the case in which the decision-maker has preferences on the availability of cognitive resources, a condition that I interpret as a preference for being rested. This extension allows showing that, in such a case, multiple, rankable equilibria can emerge, so that a policy intervention would be advisable in order to change the initial conditions and enhance the most desirable equilibrium. For example, if the best equilibrium were reachable only when the agent is sufficiently rested, the legislator/employee could impose a period of total rest in case the worker were too fatigued. In other words, she could impose holidays in the working schedule so that the worker can "recharge the battery" and converge to the desired steady state[14].

2. Attention Effort and Endogenous Accumulation of Fatigue

2.1 The Basic Model

In this section I study the optimal pattern of cognitive effort that a worker, given her condition of fatigue, should devote to a cognitively demanding job over an infinite-time horizon. Exerting effort is positively related to instantaneous productivity on the task and, by assumption, to instantaneous consumption; but it also has an indirect, endogenous cost due to the depletion of a limited stock of cognitive resources that are necessary to perform cognitively demanding tasks[15]. Consistent with the empirical evidence, total depletion of this stock (or, equivalently, the overcoming of a given threshold of fatigue) is assumed to prevent high-level cognition[16]. To recover cognitive resources (i.e. to rest) the worker can exert little or no cognitive effort, but this reduces her productivity and, consequently, her consumption.

The problem is solved from the point of view of an external observer[17]. Such external observer can be thought of as a benevolent planner, or an external agency, that studies the optimal pattern of cognitive effort that a worker should exert over time, given her condition of

[14] In this paper I refer to holidays as periods in which the worker is forced, either by the law or by the employee, to rest in order to recover from fatigue and to be more productive; I do not refer to those days in which no work is allowed for other reasons, such as religious or celebrative ones.

[15] On the terminology: I will indifferently use the expressions “large stock of cognitive/attention resources" and “being rested” (interpreted as antonyms of “low stock condition” or “being fatigued”) to identify the cognitive condition of the worker at a given point in time.

[16] See Ozdenoren et al. (2005) for a similar assumption.

[17] The role of the external planner is introduced to avoid the regress problem, a critique that potentially affects all models in which cognitive costs are explicitly introduced (see Conlisk, 1996, and references therein). To avoid this critique, I assume that the problem is solved by an external agent whose computational and cognitive costs are not take into account.

cognitive fatigue, in order to maximize her intertemporal utility function. I also rule out risk, ambiguity and information constraints. Formally the maximization problem can be formulated as follows:

$$
\begin{aligned}
& Max_{\{x(t)\}} \int_0^{\infty} e^{-rt} u(x(t))dt \\
\text{s.t. } & \dot{s}(t) = s(t)[\bar{s} - s(t)] - f(x(t)) \qquad (1) \\
& s(0) = s_0 > 0, \bar{s} > 0, s(t) \geq 0, x(t) \geq 0 \quad \forall t.
\end{aligned}
$$

where the first expression indicates the intertemporal utility profile that is to be maximized over an infinite-time horizon, with $x(t)$ being the instantaneous effort that is exerted, $u(x(t))$ the instantaneous utility and r a fixed discount rate. The second expression is the law of motion of the stock of cognitive resources: it depends on the existing stock $s(t)$ and, according to the depletion function $f(x(t))$, on the exerted effort; s_0 is the initial stock of cognitive resources and $\bar{s}$ can be thought of as the "holiday steady state", i.e. the homeostatic condition of (non) fatigue that the decision-maker would get if she didn't exert any cognitive effort at all. Neither effort nor the stock of cognitive resources can be negative[18].

The phase diagram in Fig. 1 shows how the optimal level of effort and the condition of fatigue of the worker evolve over time to satisfy the planner's intertemporal problem (1), given an initial condition of fatigue s_0. Graphically the solution can be represented by the trajectory (in bold) leading to point *S*, the unique steady state in which positive effort is exerted in the long run. Such a trajectory is unique, meaning that for any level of available resources there exists a unique optimal level of effort that allows reaching (as time goes to infinite) the internal steady state and to maximize the objective function.

The solution to the intertemporal cognitive problem (1) shows that there exists a need of optimally managing the endogenous costs of cognition to avoid suboptimal outcomes. More precisely, on the optimal path the decision-maker should exert a high effort when she is rested, and she should exert a low attention effort when she is fatigued. This means that, when cognitive resources are abundant, it is optimal to exert high effort and to be very productive and, when they are scarce, it is optimal to work with low cognitive intensity in order to recover from fatigue. Such a pattern of effort choices endogenously induces a variation in the stock of available cognitive resources and it makes the dynamic system converging toward the internal steady state. If, on the contrary, the worker didn't adhere to such a path of choices, she would get suboptimal results and the dynamic system would be led toward the two corner solutions (0,0) and $(\bar{s},0)$, in which no effort is exerted and, consequently, no utility can be enjoyed[19].

By requiring a fatigued worker to exert a low, but increasing path of effort (which implies an increasing path of productivity and consumption) until the steady state is reached, the solution to problem (1) has a further implication. In the context of the revealed-preference theory, in fact, observing that an individual is experiencing an improving consumption profile when she is fatigued is interpreted *as if* she had a preference for sequences that improve over time (when she is fatigued). Such a conclusion contradicts the standard economic result in

[18] I further assume that, for the utility function, $u(x) \geq 0, u(0)=0, u'>0, u''<0$ and, for the depletion function, $f(x) \geq 0, f(0)=0, f'>0$ for $x>0, f'(0)=0, f'' \geq 0$. Note that, to simplify the discussion, I have not imposed the condition $x(t) \leq s(t)$; consequently $s(t)$ should be interpreted, in this decision-making environment, as an indicator of cognitive fatigue and not as the maximum amount of cognitive resources that can be used at time t (i.e. in terms of cognitive capacity).

[19] For details, see Dragone (2006).

intertemporal decision-making according to which impatient agents with decreasing marginal returns prefer, ceteris paribus, (smooth) decreasing profiles of consumption to increasing ones. Nevertheless there exists experimental evidence according to which "[t]o most persons, a deteriorating series of utility levels is a rather close approximation to the least attractive of all possible patterns, regardless of the nature of events that are being ordered"[20]. The evidence on preferences for improving profiles has been explained in the behavioral literature by invoking, for example, savouring or dread of future utilities and negative time preferences (Loewenstein, Prelec, 1991) or reference-dependent utility functions (Loewenstein, 1987). Without modifying the standard assumptions of the discounted utility model, the model with endogenous attention costs that has just been presented provides an alternative explanation by considering the existence of endogenous cognitive constraints in the intertemporal maximization problem[21].

An additional feature of the optimal trajectory leading to the steady state is that, for very low levels of fatigue, it is optimal to exert no effort at all (see Fig. 1). In other words, along the optimal trajectory there exists a "*Warm-Up Zone*" in which it is optimal for the worker to exert no effort until enough cognitive capacity has been collected. This conclusion is due to the fact that, when the agent is very fatigued it is better not to work at all and to recover cognitive capacity for future use (provided the agent is not completely exhausted), than paying some attention on the task and getting more and more fatigued

2.2 Multi-tasking and cognitive overload

The model proposed in the section 2.1 illustrates the role of fatigue and scarce cognitive resources when some cognitively demanding task is to be performed. The general result is that a worker should balance the benefits of exerting effort with the endogenous costs due to the depletion of cognitive resources. Over an infinite horizon, problem (1) admits a unique, internal steady state (a saddle point) that is always reachable for any positive stock of resources. If such a stock is very low, it can be the case that the agent had better provide no effort until she is sufficiently rested.

Clearly there are many features of the model that do not correspond to a realistic description of a decision-making environment. For example the task to be performed can demand peaks of attention, or the worker may be required to simultaneously perform multiple complex tasks. On this point the empirical evidence shows that stress, disturbing noises and attention-demanding stimuli negatively affect the performance of people both in terms of the ability to stay concentrated on a task in a specific moment and in terms of the ability to maintain attention over a prolonged time horizon. The literature in Cognitive Science and Psychology explains such evidence by arguing that a very demanding cognitive task or multiple tasks can induce overload because they compete for the same, limited stock of attention resources. This results in a reduction in short-term memory which, in turn, may negatively affect both instantaneous and future performance on cognitive tasks. The negative relation between task demand (also called mental workload) and task performance is particularly evident when there is cognitive *over*load[22]. For example, cognitive overload interferes with self-regulating behaviour, as it is shown by the evidence of those people that

[20] Loewenstein, Prelec (1991:347). See also Loewenstein and Sicherman (1991), Frederick and Loewenstein (2002), Ariely and Carmon (2003).
[21] See also Ozdenoren et al. (2005).
[22] For a review of the literature on mental workload, see Gopher and Donchin (1986) and O'Donnell and Eggmeier (1986). For research on mental workload and mental overload (called also the workload redline) see De Waard (1996) and references therein.

deviate from a diet (or a well-intentioned saving program) when they are experiencing stress[23] or they are cognitively fatigued[24].

Extending the model to allow for mild cognitive load and cognitive overload, it is possible to justify the observed evidence on the reduction in both the performance and the span of time in which positive effort can be exerted. Moreover, the model predicts that, in very cognitively demanding environments, people should optimally take rest-breaks, while exerting prolonged effort, in order to be able to save on cognitive resources to be used afterwards. This conclusion suggests that the evidence of people taking temporary breaks and postponing the completion of a task is not necessarily a cue of dynamic inconsistent preferences. Rather it can be due to an optimal tactic for saving on cognitive resources in case of excessive cognitive load. Indeed, if the agent did not take a rest-break, the exhaustion of the available resources would be too quick and the decision-maker would not be maximizing her performance over the relevant time horizon.

Graphically the effect of both cognitive load and overload can be represented in a phase diagram similar to the one proposed in the previous section, with the difference that the new steady state and the optimal streamlines shift downward (see Fig. 2 and Fig. 3).

In Figure (2) I show the effect of a permanent, but mild source of cognitive load, as it is the case of mild, chronic forms of acoustic pollution in a workplace[25]. Given the mild load on the worker, the new steady state *S'* is still in the feasible area, meaning that it is possible to reach it. Nevertheless, two remarkable features should be noted. First, along the new optimal path the effort that should be exerted is less than the previous situation in which no cognitive load occurs, with the consequence that the intertemporal profile in utility, productivity and consumption is lower. Second, the new warm-up zone shifts to the right, meaning that the new steady state *S'* can only be reached if the available resources are above a minimal condition of rest S_C (see Fig. 2). Accordingly, if the worker is fatigued (i.e. $s(t)<S_C$) the mild cognitively demanding task cannot be borne *ad infinitum* and the worker will be lead, at some point, to total exhaustion. The existence of the threshold S_C justifies the intervention of a policy-maker in order to make the worker converging toward the desired long-term outcome, an issue I will discuss in the next subsection.

Figure (3) shows the case in which cognitive overload is particularly demanding, so that there is no possibility for the decision-maker to recover from fatigue. This would be the case, for example, of a very stressful and noisy workplace in which the worker gets more and more fatigued, eventually until exhaustion. In such a case the optimal streamline to be selected critically depends on the period of time during which the individual is required to work and on the final condition of fatigue to achieve. Fig. (3) shows the case in which the worker must work until she is exhausted (i.e. her cognitive stock is completely depleted). Specifically, in the figure I show two different finite-time horizons: streamline (1) represents the optimal schedule of effort that should be exerted over a relatively short working day, while streamline (2) concerns a relatively longer span of time.

It is easy to see that, in the latter case, the worker should work with less cognitive intensity. This reasonable conclusion is due to the need to save on cognitive energies in order to make them "last" until the prescribed long time horizon of the problem. Accordingly there exists a

[23] For a review on the connections between stress and cognition, see Bourne and Yaroush (2003). Stress can often lead to relapse in abstinent addicts, as showed in Shiffman and Waters (2004); analogously Herman and Polivy (2003) report that dieters are more likely to deviate from their diets when they are under stress. For an interesting experiment that links self-regulatory ability and cognitive overload, see Shiv and Fedorikhin (1999).
[24] See the empirical contributions on the effects of cognitive fatigue on impulsive buying (Vohs, Faber, 2004) and the negative influence that cognitive depletion has on self-regulation activities, as documented experimentally by Muraven et al. (1998), Baumeister et al. (1998) and Vohs, Heatherton (2000).
[25] See, for example, Haines et al. (2001).

range of conditions of fatigue (corresponding to the cases in which the stock of cognitive resources is between *Sa* e *Sb*) in which it is optimal for the worker to exert no effort and take a rest-break during the working day. The existence of a *Rest-Break Zone* means that, if the cognitive load induced by the environment is, over a prolonged span of time, very demanding, the best advice the external planner can give to the worker is, at some level of fatigue, to stop working and provide no effort. This result provides a rationale for the need of "taking a break" in case of a prolonged work overload, showing that unavoidable events that cognitively overload the agent (such as stress) can optimally lead her to temporarily interrupt a cognitively task that is currently performed (say to pay prolonged attention on the job, to adhere to a saving plan or to a diet), but that can be postponed in order to save on fatigue. In particular, this implies that the observation that people sometimes temporarily interrupt their saving plans, their working routines or their diets can have an alternative explanation with respect to the idea that people possess hyper-myopic preferences (as in the hyperbolic discounting literature). Indeed, in a decision-making environment in which a prolonged attention effort is required, temporarily postponing an avoidable demanding task can turn out to be the best tactic to avoid premature exhaustion of cognitive capacity and to enhance the likelihood of successfully completing the cognitive task over the required time horizon.

2.3 Sensitivity to fatigue and path-dependence

In section 2.1, unless the agent gets totally exhausted the worker can always manage to converge to the equilibrium point *S*. Thus, in such environment the external planner turns out to have no policy role: if the agent were given a schedule with the optimal path of effort choices to exert, she could reach the equilibrium without any external intervention. A different situation would emerge if the internal equilibria were many. In such a condition, when the welfare properties associated to each steady state are different, some steady states can be preferred to others and, if the external observer had policy power, she could take actions to make the agent converging toward the best equilibrium.

In Dragone (2006) multiple, internal equilibria are obtained within the cognitive model just presented by introducing the possibility that the worker enjoys being rested. This means that the problem can be formally represented as before, with the difference that now the worker doesn't simply enjoy consumption (*via* production and effort exertion), but she also has a preference on the availability of cognitive resources. Formally, the problem can be written as:

$$\begin{aligned} &Max_{\{x(t)\}} \int_0^\infty e^{-rt}[u(x(t)) + \beta s(t)]dt \\ \text{s.t. } &\dot{s}(t) = s(t)[\bar{s} - s(t)] - f(x(t)) \qquad (2) \\ &s(0) = s_0 > 0, \bar{s} > 0, s(t) \geq 0, x(t) \geq 0 \quad \forall t. \end{aligned}$$

where $\beta \geq 0$ represents the individual preference for being rested. According to this exogenous, fixed sensitivity parameter we can distinguish three classes of optimal solutions. In Fig. 4 I show the three solution paths together: the dashed trajectory corresponds to a worker with a low preference (or sensitivity) for being rested, the bold path corresponds to a worker that is mildly sensitive and the dotted path to a worker that has a strong preference for being rested.

The figure shows that, in the first and third case (the agent has either a low or a high sensitivity to fatigue), there is a unique equilibrium (respectively: *L* or *H*) that can be reached according to an optimal saddle path with the features that have been described in section 2.1. Comparing the two saddle paths corresponding to a low- and a highly-sensitive individual, note that the worker with a low sensitivity for being rested exerts more effort, for any level of fatigue, than the highly sensitive worker. Moreover, by considering the warm-up zone, it is easy to see that the low-sensitivity worker is also the one that waits less before starting to work. However, as a consequence of so much effort exertion, over a long time horizon she turns out to be the less productive and the more fatigued worker. Additionally, she is also the most "fragile" worker to exogenous sources of overload, since relatively little cognitive shocks are enough to move the steady state in the unfeasible area (below the s axis) eventually leading the worker, according to the previous discussion on permanent cognitive overload, to the exhaustion of all available cognitive resources.

For a worker with a high sensitivity to fatigue there is a unique equilibrium, H, and the opposite conclusions hold. The steady state H is, in fact, associated to a relatively high level of effort exertion and little fatigue because the agent cares so much about being rested that she saves on effort all along the path. In other words, this kind of worker "takes it easy": she waits more before start working and she produces less at any condition of fatigue. However, over the infinite-time horizon under examination, she can get more satisfaction, compared to the low-sensitive worker, both in terms of consumption (via effort exertion) and in terms of the stock of cognitive resources she enjoys. An additional consequence of the high preference for being rested is that this kind of agent is less sensitive to cognitive overload than the low-type worker. In other words, whenever she is providing positive effort, exogenous sources of stress or cognitive overload do not impact on performance as they do on the low-type agent because the accumulated cognitive resources buffer the depletion induced by the additional cognitive demand. Finally, note that these results are not a consequence of the way the two kinds of agent evaluate utilities over time, since both workers are assumed to be exponential discounters and the discount rate r is the same for both: the different pattern of choices in the short-run and the different performance in the long-run is only due to their different sensitivity to fatigue.

For the worker with mild preferences, instead of a unique, internal global saddle point, multiple internal equilibria emerge: s_L, s_M and s_H. Points s_L and s_H (two saddle points) can be reached if the worker follows the optimal streamlines indicated in Fig. 4, while s_M (an unstable node) tends to repel all trajectories away from it[26].

In terms of welfare, reaching either the equilibrium s_L or s_H is not irrelevant. In fact, s_H is strictly preferred to s_L because both consumption and the availability of cognitive resources are higher. However, despite its desirability, s_H cannot be optimally reached for any condition of fatigue. Indeed, when the worker is on the left of point s_M, the solution to problem (2) requires a pattern of effort choices that will optimally lead her to s_L in which she is more fatigued and less productive than in s_H (see Fig. 4). The emergence of multiple equilibria and the path-dependence of suboptimal outcomes is consistent with the real life evidence of those workers that, though fatigued, fail to move their initial conditions beyond the threshold s_M and keep on working, even when this is less and less productive. For example, pilots and drivers often find it optimal to keep on working even when they are so fatigued that they lose the necessary attention to prevent accidents to occur[27]. In a similar vein, the night before an exam some students find it better to keep on studying instead of going to sleep, with the consequence that they end up being very fatigued and little able to be cognitively productive at the exam. According to the model just presented, this behavior is optimal (even if suboptimal is the outcome) and it is neither due to a lapse in rationality, nor to the way the agent evaluates future utility flows[28]: the suboptimal outcome is simply due to the fact that

[26] See Dragone (2006).
[27] See Tucker (2001).
[28] Note that in economics an agent is rational if she possesses complete and transitive preferences; so that she is able to evaluate and compare all the possible alternatives. In other words rationality is a technical concept that

point s_L is a local maximum and, when the agent is fatigued (i.e. $s(t) < s_M$), for the agent the best thing to do is to approach it.

Typically, when path-dependence emerges and the outcomes are rankable, the intervention of a policy-maker would be desirable in order to allow the system to reach the best long run equilibrium. In the present environment, this means that the policy-maker should take some action aimed at shifting the initial condition of the mildly-sensitive worker beyond the threshold level s_M.[29] Here I mention three possible policy interventions involving the introduction of rest-breaks within the optimal pattern of effort to exert[30]. One possibility is that the planner forces the worker to rest until she has recovered enough resources to be able to reach the desirable long-term outcome. This is the case, for example, of those bosses that force their fatigued and stressed collaborators to take breaks and holidays. Alternatively, instead of requiring an external planner (or the boss) to check the condition of fatigue of the agent, the worker could monitor herself and take rest-breaks whenever she feels she is beyond the threshold of fatigue s_M[31]. This option would require the worker to be able to correctly monitor her condition of fatigue in order to detect if she is beyond the threshold, a feature that we can conjecture to be mildly-demanding and, consequently, it would require combining the sensitivity of the worker for being fatigued with the extension on mental workload presented in section 2.2. Alternatively, the worker may follow some rule-of-thumb that requires to stop working according to some predetermined schedule. This option does not require much additional cognitive effort to the worker and it seems to be particularly recommendable when coordination between several workers is required.

3. Conclusions

This paper introduces the role of cognitive fatigue in a dynamic model of intertemporal decision-making. The idea is that performing complex cognitive activities such as solving problems, thinking and paying attention to some intellectual activity is costly because it requires the exertion of effort. In the economic literature, effort is generally taken into account by assuming that it yields disutility to the decision-maker. In a short time horizon this is a plausible way to take cognitive effort into account. Nevertheless, in many situations people must exert effort for prolonged periods of time and the empirical evidence shows that, when performance depends on their level of attention, this tends to decrease over time: students get distracted, workers lose concentration and efficiency, drivers and pilots incur in a higher probability of incurring in an accident. Similar degrading patterns in performance when prolonged cognitive effort is required are common in many complex cognitive activities people do. Moreover, the accumulation of fatigue gets faster when multiple cognitively demanding tasks are to be performed at the same time, as it is the case of people trying to ignore stress or noise in a workplace.

I propose an intertemporal decision-making model in which cognitive constraints are explicitly introduced in order to study how the accumulation of fatigue should be managed

has nothing to do with a folk interpretation that associates rationality with being smart and/or far-sighted. Furthermore, to rule out the emergence of any dynamic inconsistency, I have used a standard exponential discounting utility function that does not allow for any change of taste as time goes on (see Strotz, 1955/6).

[29] Or, when considering a mild cognitive load, to shift $s(t)$ beyond s_C.

[30] Note that the pattern and the optimal duration of rest breaks are still a matter of investigation in the psychological literature; see Tucker (2001).

[31] Tucker (2001:130) observes that "[I]t is worth noting that the European Working Time Directive (EC Working Hour Directive, 1990) entitles adults who work for more than 6 h at a stretch to a 20-min rest break (workers under 18 years are entitled to 30 min rest if they work more for than 4.5 h). The timing of breaks is at the discretion of employers [...]. Employers are only required to ensure that workers can take a rest, but they do not have to ensure that a rest is taken".

over time. The general result is that, in order to provide good performance and to avoid a complete exhaustion of cognitive resources to devote to cognitive tasks, there exists a need to optimally manage fatigue over time so that it is optimal to provide high effort when the agent is rested and low effort when she is fatigued. Moreover the model shows the existence of a warm-up zone, i.e. a range of conditions in which the decision-maker is extremely fatigued and it is optimal to exert no effort at all and wait until enough cognitive resources have been collected.

The model is extended in two directions. First the role of cognitive load and multitasking are considered, showing that it can indeed be optimal to take rest-breaks as an effective means of maintaining good performance results, managing fatigue and controlling the accumulation of risk over time. The second extension studies how the individual preferences on being rested affect the optimal path of effort to exert. The main result is that it is possible that multiple equilibria emerge. As a consequence it can be the case that people find themselves stuck in situations in which they are very fatigued and yet they optimally keep on working, with the undesirable result that they get more and more fatigued while being little productive. In the psychological literature on mental fatigue this is frequently observed in drivers and pilots that keep on working even when their reactivity and vigilance is so degraded that they seriously risk accidents. In such cases I claim that an external intervention would be advisable in order to move the initial conditions below the threshold of fatigue.

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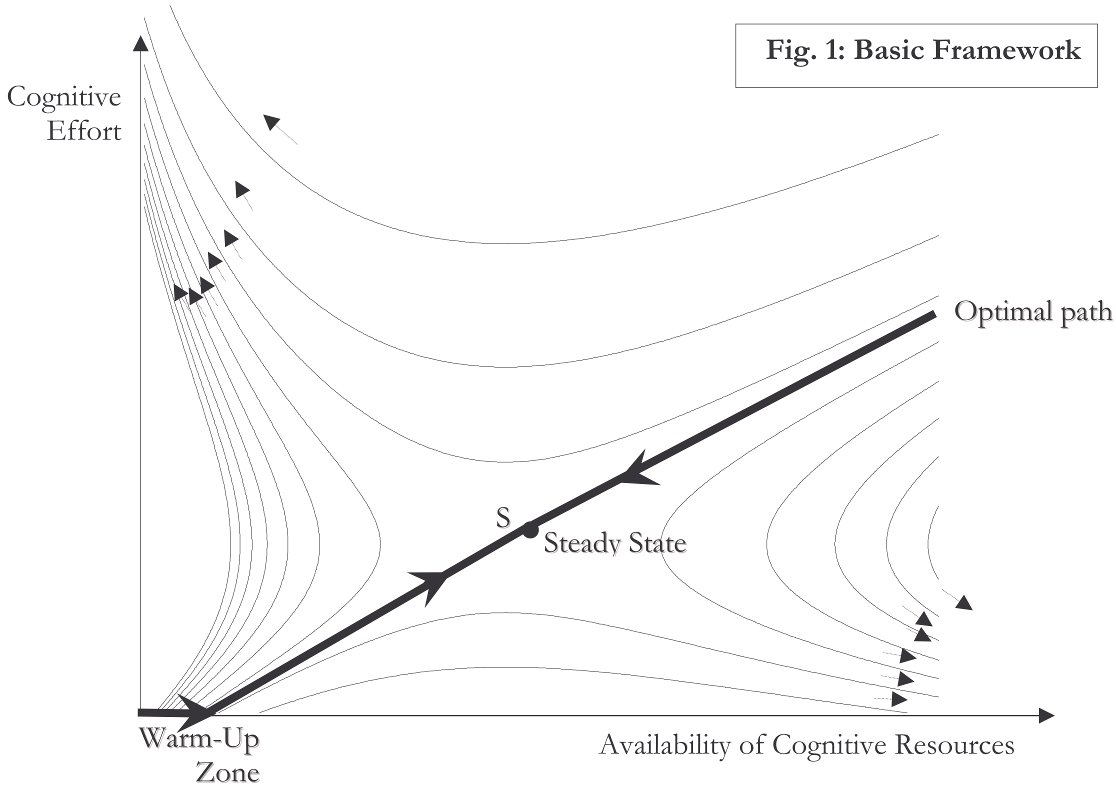
Fig. 1: Basic Framework
Cognitive Effort
Optimal path
S
Steady State
Warm-Up Zone
Availability of Cognitive Resources

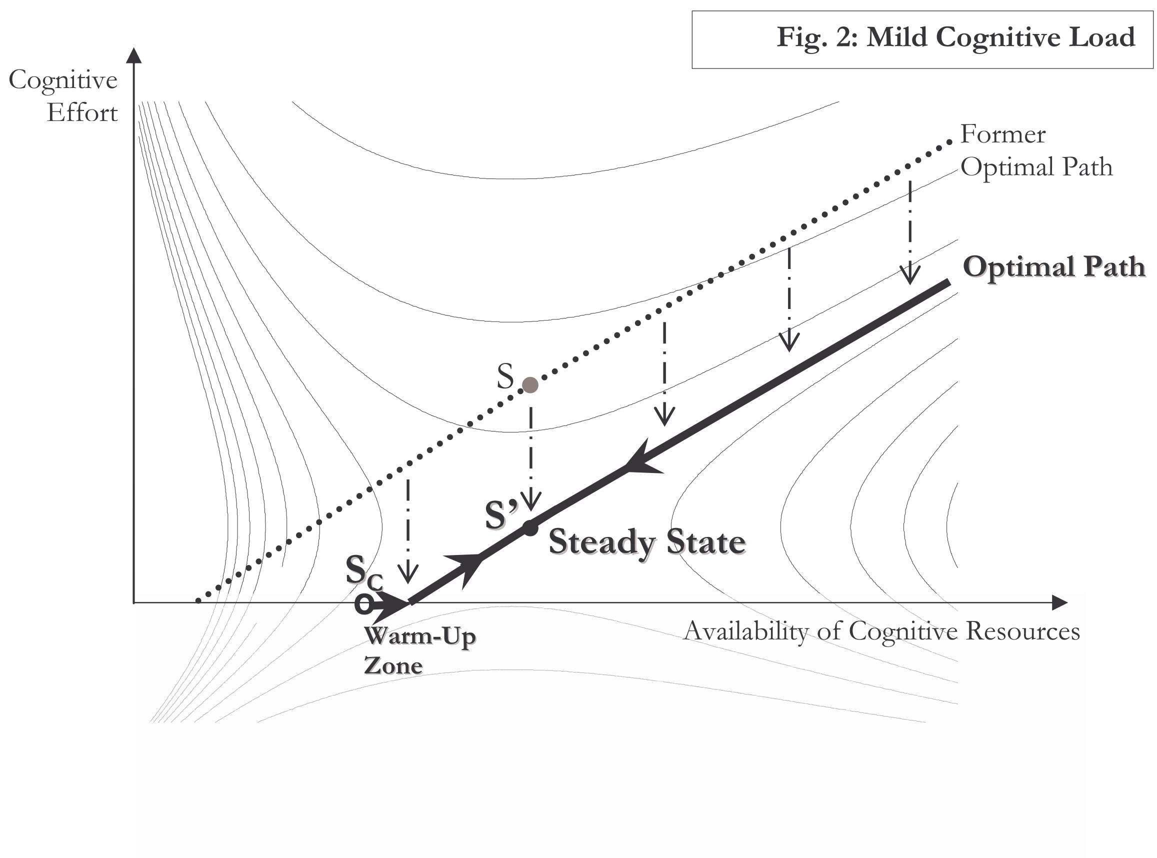
Fig. 2: Mild Cognitive Load
Cognitive Effort
Former Optimal Path
Optimal Path
S
S'
Steady State
S_C
Warm-Up Zone
Availability of Cognitive Resources

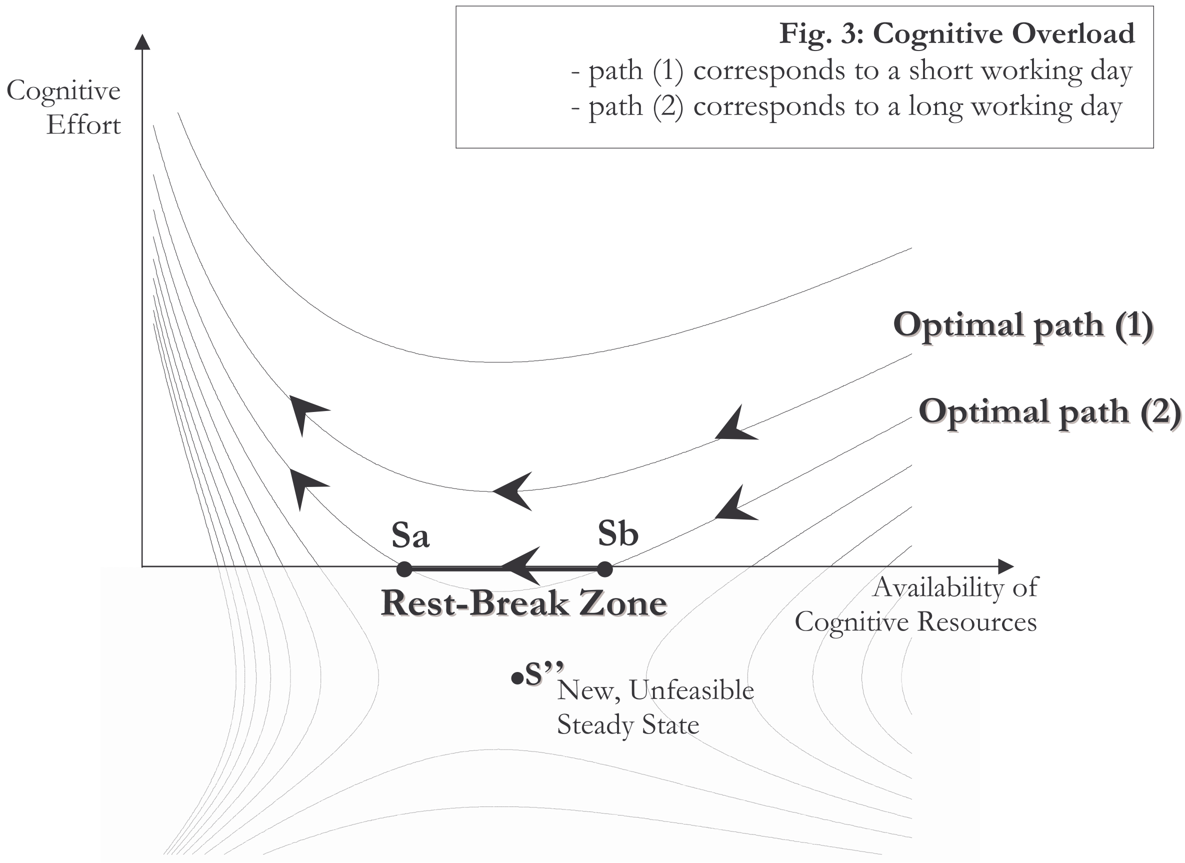

Fig. 3: Cognitive Overload
- path (1) corresponds to a short working day
- path (2) corresponds to a long working day
Cognitive Effort
Optimal path (1)
Optimal path (2)
Sa
Sb
Rest-Break Zone
Availability of Cognitive Resources
S''
New, Unfeasible Steady State

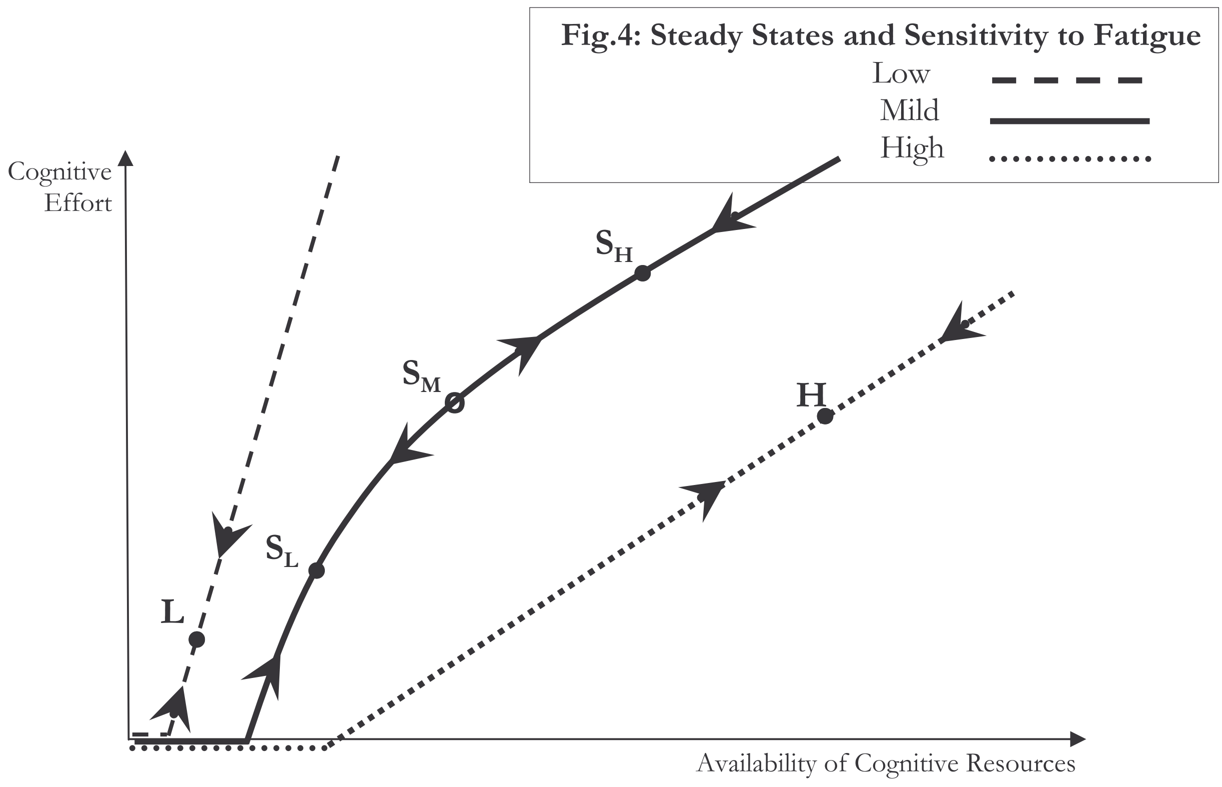

Fig.4: Steady States and Sensitivity to Fatigue
Low
Mild
High
Cognitive Effort
S_H
S_M
H
S_L
L
Availability of Cognitive Resources