

# A Note on Multiplicative Uncertainty and Partisan Policies<sup>\*</sup>

Massimiliano Castellani<sup>#</sup> and Antonello E. Scorcu<sup>§</sup>

<sup>#</sup> Dipartimento di Scienze Economiche  
Università di Bologna  
Piazza Scaravilli 2  
40125 Bologna, Italy  
fax ++39 051 221968  
castellani@rimini.unibo.it

<sup>§</sup> Dipartimento di Scienze Economiche  
Università di Bologna  
Strada Maggiore 45  
40125 Bologna, Italy  
fax ++39 051 2092664  
antonello.scorcu@unibo.it

December 2003

## Abstract

In this paper we consider the effects of «multiplicative» uncertainty about the structure of the economy in the standard partisan model. An increase in the uncertainty decreases of the inflation rate pursued by the liberal policymaker, but increases the inflation rate pursued by the conservative policymaker. For certain configurations of the parameters, an increase in the uncertainty reduces the expected loss of both parties.

JEL classification: E31, E61

Keywords: multiplicative uncertainty, partisan parties, inflation

---

<sup>\*</sup> We would like to thank A. Bucci, G. Candela, R. Cellini and P. Figini for helpful comments on a previous version of the paper. The usual disclaimer applies.

What is the optimal discretionary monetary policy in the partisan model when there is uncertainty about the structure of the economy?

In the partisan model augmented with «multiplicative» uncertainty, an increase in the degree of uncertainty leads to converging monetary policies because the inflation rates pursued, when in office, by a liberal and a conservative policymaker are, respectively, reduced and increased. Moreover, for certain configurations of the parameters, the expected loss in presence of uncertainty is lower than in the deterministic case.

## 1. The model

Let us consider the standard partisan model (see Alesina *et al.*, 1997) in which a liberal (left-wing) policymaker L and a conservative (right-wing) policymaker R attach different relative weights to output ( $y$ ) and inflation ( $\pi$ ). When in office, policymaker  $i = L, R$ , minimizes the expected value of the loss:

$$L_i = \frac{1}{2} (y - y^*)^2 + \theta_i (\pi_i)^2 / 2 \quad \theta_i > 0 \quad (1)$$

with respect to  $\pi_i$ , which is the only instrument at his disposal<sup>1</sup>.  $y^*$  is the desired output level for both policymakers, and the ideological differences between the two are expressed solely by  $\theta_R > \theta_L$ : the conservative policymaker R is relatively more concerned with inflation than output, whereas the liberal policymaker L is relatively more concerned with output than inflation.

The economy is described by an expectation-augmented Phillips curve:

$$y = y^n + b(\pi - \pi^e) \quad (2)$$

---

<sup>1</sup> We neglect the complications arising from the distinction between the rate of inflation and the rate of money growth or from the imperfect control of the inflation rate.

here  $\pi$  is the inflation rate,  $\pi^e$  the inflationary expectation,  $y^n > 0$  the equilibrium level of output and  $b > 0$  is the slope of the (short-run) Phillips curve. As usual in these models, in order to create an incentive towards surprise inflation, we assume  $y^* > y^n$ .

The only departure from the textbook partisan model is the existence of uncertainty upon the structure of the economy; the parameter  $b$  is therefore a i.i.d. random variable:

$$b \sim (\beta, \sigma^2) \quad (3)$$

with  $\beta > 0$ . Whereas the value of  $\beta$  is known from the beginning to both policymakers and private agents, the actual value of coefficient  $b$  is revealed only after inflationary expectations and the monetary policy have been set. Therefore, the policymaker in office has no informational advantage upon the private sector in predicting  $b$ , nor any strategic advantage to exploit. The uncertainty about  $b$  is of the «multiplicative» type and its effects have been analyzed in a large body of literature since Brainard (1967)<sup>2</sup>.

Inflationary expectations are rational and are described by:

$$\pi^e = p\pi_R + (1-p)\pi_L \quad (4)$$

$(1-p)$  and  $p$  are the exogenous, common knowledge, probabilities of policymakers L and R of being elected, respectively;  $\pi_L$  and  $\pi_R$  are the discretionary inflation rates set by L and R. Because of the previous assumption upon the preferences of the policymakers, it will be easy to show that  $\pi_L > \pi_R$ . At the beginning of each period, inflationary expectations  $\pi^e$  are set; then election occurs, the winner enters the office and sets the inflation rate; finally the actual slope of the Phillips curve,  $b$ , is revealed.

## 2. Policy activism and convergence under uncertainty

In the derivation of the main result of the paper, it is convenient to consider the case in which the two policymakers expect the same  $\beta$ , but have different expectations about the variability of  $b$ ,

---

<sup>2</sup> The case of an additive random disturbance can be analysed by the use of the Certainty Equivalence Principle and will not be considered.

possibly because of different evaluations about the importance of the short-run effects of the monetary policy. The variances  $\sigma_L^2$  and  $\sigma_R^2$  are expected by policymakers L and R, respectively. Hence the differences in the two policymakers concern both the loss functions ( $\theta_R > \theta_L$ ) and the structure of the economy ( $\sigma_L^2 \neq \sigma_R^2$ ).

The problem for policymaker R is the minimization of:

$$\begin{aligned} E[L_R] &= \frac{1}{2} E[(y - y^*)^2 + \theta_R \pi_R^2] = \\ &= \frac{1}{2} [(y^n)^2 + (\beta^2 + \sigma_R^2)(\pi_R - \pi^e)^2 + 2y^n\beta(\pi_R - \pi^e) - 2y^*\beta(\pi_R - \pi^e) - 2y^*y^n + (y^*)^2 + \theta_R \pi_R^2] \end{aligned} \quad (5)$$

and from the first order condition  $dE[L_R]/d\pi_R = 0$  we get:

$$(\beta^2 + \sigma_R^2)(\pi_R - \pi^e) + \theta_R \pi_R + \beta(y^n - y^*) = 0 \quad (6)$$

For policymaker L the problem is the minimization of:

$$E[L_L] = \frac{1}{2} E[(y - y^*)^2 + \theta_L \pi_L^2] \quad (7)$$

and the relevant first order condition is:

$$(\beta^2 + \sigma_L^2)(\pi_L - \pi^e) + \theta_L \pi_L + \beta(y^n - y^*) = 0 \quad (8)$$

Substituting the inflationary expectation (4) in equations (6) and (8), we obtain  $\pi_R, \pi_L$ :

$$\pi_R = h[\theta_L + p(\beta^2 + \sigma_L^2) + (1-p)(\beta^2 + \sigma_R^2)] \quad (9)$$

$$\pi_L = h[\theta_R + p(\beta^2 + \sigma_L^2) + (1-p)(\beta^2 + \sigma_R^2)] \quad (10)$$

where  $h = \beta(y^* - y^n) / \{\theta_L\theta_R + \theta_R p(\beta^2 + \sigma_L^2) + \theta_L(1-p)(\beta^2 + \sigma_R^2)\} > 0$ .

Note that  $\pi_L > \pi_R$  because  $\theta_R > \theta_L$ , that is, policymaker L pursues a more expansionary policy than policymaker R. Two different inflationary effects can be recognised in the optimal choice of  $\pi_i$ : a direct effect given by the inflationary incentives of policymaker  $i$ , and an indirect effect, given by the inflationary incentives of the other policymaker, that influences the choices of  $i$  through  $\pi^e$ .

Expressions (9) and (10) can be simplified in the case  $\sigma_R^2 = \sigma_L^2$ :

$$\pi^*_R = h^*(\theta_L + \beta^2 + \sigma^2) \quad (11)$$

$$\pi^*_L = h^*(\theta_R + \beta^2 + \sigma^2) \quad (12)$$

where  $h^* = \beta(y^* - y^n) / \{\theta_L \theta_R + (\beta^2 + \sigma^2) [\theta_R p + \theta_L(1-p)]\}$ .

However, the effect of a change in  $\sigma^2$  is different for  $\pi^*_L$  and  $\pi^*_R$ ; straightforward algebra gives us the following results:

$$d\pi^*_R/d\sigma^2 = k^* [(1-p)\theta_L(\theta_R - \theta_L)] > 0 \quad (13)$$

$$d\pi^*_L/d\sigma^2 = k^* [p\theta_R(\theta_L - \theta_R)] < 0 \quad (14)$$

where  $k^* = \beta(y^* - y^n) / \{\theta_L \theta_R + (\beta^2 + \sigma^2) [\theta_R p + \theta_L(1-p)]\}^2 > 0$ .

An increase in uncertainty implies some convergence in monetary policies, as it reduces the discretionary inflation rate of the (activist) policymaker L, and increases the discretionary inflation rate of the conservative policymaker R. This result qualifies the traditional conclusion of Friedman (1953) and Brainard (1967) of an attenuated optimal policy in the case of parameter uncertainty<sup>3</sup>; in fact, policymaker R reaction is stronger when uncertainty is higher. Obviously, under uncertainty the policymaker R inflation rate is always higher than in that the deterministic case; the opposite results holds for policymaker L.

However, the attenuating effect induced by the multiplicative uncertainty holds for the average (and expected) inflation rate  $E(\pi) = \pi^e = p\pi_R + (1-p)\pi_L$ :

$$dE(\pi)/d\sigma^2 = -p(1-p)k^* (\theta_R - \theta_L) < 0 \quad (15)$$

The previous results can be explained if we consider separately the effects of a change in  $\sigma^2_i$  on  $\pi_i$ , as computed from equations (9) and (10):

$$d\pi_R/d\sigma^2_R = k(1-p)(\theta_R - \theta_L)[\theta_L + p(\beta^2 + \sigma^2_L)] > 0 \quad (16)$$

$$d\pi_R/d\sigma^2_L = kp(1-p)(\theta_L - \theta_R)(\beta^2 + \sigma^2_R) < 0 \quad (17)$$

$$d\pi_L/d\sigma^2_L = kp(\theta_R - \theta_L)[\theta_R + (1-p)(\beta^2 + \sigma^2_R)] < 0 \quad (18)$$

$$d\pi_L/d\sigma^2_R = kp(1-p) (\theta_R - \theta_L) (\beta^2 + \sigma^2_L) > 0 \quad (19)$$

where  $k = \beta(y^* - y^n) / \{\theta_L \theta_R + \theta_R p(\beta^2 + \sigma^2_L) + \theta_L(1-p) (\beta^2 + \sigma^2_R)\}^2 > 0$ .

---

<sup>3</sup> In the original Brainard (1967) framework a similar result emerges with specific values of the covariances between parameters. Cf. Turnovsky (1977).

In equation (16), the increase in the uncertainty perceived by R leads to an increase of the inflation rate engineered by the policymaker R, when in office. An increase in  $\sigma^2_R$  weakens the direct inflationary incentive on  $\pi_R$ , whereas the indirect inflation increasing effect becomes more important; because  $\pi_L > \pi_R$ , this implies  $d\pi_R/d\sigma^2_R > 0$ . The indirect effect eventually predominates for  $\sigma^2_R \rightarrow \infty$ ; in fact from equation (9) we obtain:

$$\lim_{\sigma^2_R \rightarrow \infty} \pi_R = \lim_{\sigma^2_R \rightarrow \infty} \pi_L = \beta(y^* - y^n)/\theta_L \quad (20)$$

Unsurprisingly, the inflation rate in (20) corresponds to the case  $p = 0$ .

On the other hand, as shown in equation (17), an increase in  $\sigma^2_L$  makes more important the direct effect, which tends to lower  $\pi_R$ . When  $\sigma^2_L \rightarrow \infty$  the indirect effect vanishes and the behavior of R is the same to the one in which  $p = 1$ . From equation (10) we obtain:

$$\lim_{\sigma^2_L \rightarrow \infty} \pi_R = \lim_{\sigma^2_L \rightarrow \infty} \pi_L = \beta(y^* - y^n)/\theta_R \quad (21)$$

Because the direct effect in equation (16) is stronger than the indirect effect in equation (17), the overall result is the positive relationship between uncertainty and inflation shown in equation (13). The same reasoning can be applied to  $\pi_L$ .

The effect of an increase in the uncertainty in the structure of the economy tends to reduce the differences in the monetary policies of partisan policymakers<sup>4</sup>:

$$d(\pi_L - \pi_R)/d\sigma^2 < 0 \quad (22)$$

Unsurprisingly, when the uncertainty in the economy increases indefinitely ( $\sigma^2_L = \sigma^2_R = \sigma^2 \rightarrow \infty$ ) the attenuation effect predominates and the differences between the partisan policies disappear:

$$\lim_{\sigma^2 \rightarrow \infty} \pi_L = \lim_{\sigma^2 \rightarrow \infty} \pi_R = \beta(y^* - y^n)/[(1-p)\theta_L + p\theta_R]. \quad (23)$$

### 3. Inflation variance and expected losses

By substituting equations (11) and (12) into (4), (2) and (1) defines the (ex-ante) loss function of party  $i$  in terms of the parameters  $p$ ,  $\theta_L$ ,  $\theta_R$  and  $\sigma^2$ . Obviously, even if the inflation variance is not a policy instrument, because of the different effects it exerts on  $\pi^*_L$  and  $\pi^*_R$ , there is a link between  $\sigma^2$  and the ex-ante loss  $E[L_i]$  in equation (5). Provided that policymakers behave optimally (with  $\pi_i$  given by equations (11) and (12)), in certain circumstances  $dE[L_i]/d\sigma^2 < 0$ . In these cases policymaker  $i$  is better off in an uncertain rather than in a deterministic world, and his expected loss is lower the higher the degree of uncertainty on the structure of the economy.

In fact, straightforward but tedious algebra leads, for both policymakers, to the following result:

$$dE[L_i]/d\sigma^2 = (\beta^2 + \sigma^2)(\theta_R - \theta_L)p^2 + 2[\theta_L\theta_R + \theta_L(\beta^2 + \sigma^2)]p - [\theta_L\theta_R + (\beta^2 + \sigma^2)] \quad (24)$$

It is easily shown that  $dE[L_i]/d\sigma^2 < 0$  for  $p = 0$ , whereas  $dE[L_i]/d\sigma^2 > 0$  for  $p = 1$  and  $p = 1/2$ . Hence, when the liberal policymaker is very likely to win the elections, an increase in uncertainty reduces the losses of both policymakers, because it lowers  $\pi^*_L$  (equation (14)). The result is reversed if the conservative policymaker is more likely to win.

### 4. Conclusions

In this note we have introduced a multiplicative uncertainty effect on the structure of the economy in the basic partisan model.

We have shown that in the basic partisan model, an increase in the uncertainty leads: i) to an increase of the optimal inflation rate pursued by the conservative policymaker; ii) to a decrease of the liberal policymaker inflation rate; iii) to a convergence in the discretionary monetary policies pursued by the two policymakers.

---

<sup>4</sup> On the contrary, in Schultz (2002) an increase in the uncertainty leads to more extreme partisan policies.

The first result, in particular, differs from the conclusion, popularized by Friedman (1953) and Brainard (1967) of a more prudent behavior in the case of increased uncertainty.

Moreover, for certain configurations of the parameters, a rise in the degree of uncertainty reduces the expected losses of both policymakers.

## References

Alesina A., Roubini N. and Cohen G.D., 1997, *Political Cycles and the Macroeconomy*, MIT Press, Cambridge.

Ball L. and Cecchetti S. G., 1991, Wage Indexation and Discretionary Monetary Policy, *American Economic Review*, 81, 1310-19.

Brainard W.C., 1967, Uncertainty and the Effectiveness of Policy, *American Economic Review*, 57, 411-25.

Friedman, M., 1953, The Effect of Full-Employment Policy on Economic Stability, in: *Essays in Positive Economics*, Chicago University Press, Chicago.

Schultz C., 2002, Policy biases with voters' uncertainty about the economy and the government, *European Economic Review*, 46, 486-506.

Turnovsky S.J., 1977, *Macroeconomic analysis and stabilization policy*, Cambridge University Press, Cambridge.