The Effects of Minimum Quality Standards: Better or Worse Products?

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Abstract

In a duopoly where two firms’ products are differentiated both, horizontally and vertically, introduction of a quality standard affects equilibrium quality levels of both firms. The effects, furthermore, depend upon consumers being or not perfectly informed about qualities. Qualities are strategic substitutes and under perfect information only non-innocuous standards, i.e. above the lowest quality in an unregulated equilibrium, change the equilibrium. However, the average quality in the market may go down due to the standard, because the high quality firm will lower its own quality, and total consumers welfare may decrease. Under uncertainty, even innocuous standards, below the lowest unregulated equilibrium quality, may alter the equilibrium quality choices.

JEL: L0, L5

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1. Introduction

The aim of this article is to explore the impact of Minimum Quality Standards on the quality level chosen by firms in oligopoly. The existing literature is based on models of pure vertical differentiation, following the work by Ronnen (1991), while I use a model where products are differentiated both, horizontally and vertically. A Minimum Quality Standard obliges all firms to set their vertical quality dimension above a given threshold. The pure vertical differentiation models for unregulated markets pioneered by Gabszewicz and Thisse (1979), in the case of a duopoly, obtain equilibria where firms design products with maximum quality differentiation. Qualities are, for relevant ranges of parameters, strategic complements. Even under the assumption that quality can be increased at no cost, one of the two firms will produce a product with the lowest possible quality. In such a scenario the possibility that regulatory intervention improves upon the market outcome seems most likely. A summary of the main results by Ronnen (1991), however, will help introduce the problems: (i) Under pure vertical differentiation, imposing an MQS that is intermediate between the qualities of the two firms will lead to higher qualities for both firms and more consumers entering the market. (ii) Provided the standard is not too high, the utility of consumers is increased by the MQS, the high quality seller’s profit is decreased and the low quality seller’s profit is increased. (iii) Finally, only if the standard is not too high, total welfare is increased (Theorems 2 to 5 in Ronnen 1991). It is useful to stress how the welfare results indicate that excessive standards may lead to lower welfare, while all consumers always gain by the introduction of a standard. As expected, quality regulation mainly serves the interests of consumers.

The negative effects on welfare are further stressed in Scarpa (1998), who introduces a third firm in the pure vertical differentiation model. Then, quality may be a strategic substitute for some firms. Consumers, overall, gain by the imposition of a MQS, although the profits of all firms decrease. Crampes and Hollander (1995) analyses the case where variable costs depend upon the quality choice; they too find that a mild standard, i.e. one that is not too high will increase both qualities and social welfare, although the profits of the high quality firm are reduced. Consumers gains are also obtained by Boom (1995), where standards are applied in an international trade model with two countries. More recently, the pure vertical differentiation model has been used to study the possible manipulation of the regulatory agency’s choice by leading firms in an industry (Lutz et al., 2000), while Ecchia and Lambertini (1997), analyzing the effects of standards upon the incentives for collusion among firms, find that they are increased. Moraga-Gonzalez and Padron-Fumero (2002) use the model of perfect vertical differentiation to analyze anti-pollution emission standards. In their model consumers’ preferences are such that lower emissions are translated in higher individual utility from consumption of a unit of the good. A standard reduces per-unit emissions, increases price competition, and more production after the standard may lead to more pollution overall.

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In the present paper I assume that products are differentiated not only vertically, but also horizontally, as when design and other features matter for consumers, or when consumers differ in their preferences upon brand names. The model is a modified Hotelling linear city of unit length, where two firms are located at the two opposite endpoints of the spectrum of horizontal characteristics. The vertical dimension is represented by a one-to-one relation between a parameter, $\theta_i$, and the gross consumers' surplus from the purchase of product $i$. Following the literature, an increase in quality is assumed to be costly in that it implies a fixed investment that is raising in function of the quality produced. If firms had identical costs of improving their quality they would end up with identical qualities at equilibrium, producing a rather special case. In order to get two different qualities it shall be assumed that firms may differ in their ability to obtain a quality improvement. The game played by the two firms is the following. At the first stage firms simultaneously choose their vertical quality level and incur the corresponding fixed cost. At the second stage firms choose prices and obtain their final payoffs.

I shall consider two alternative scenarios, one of perfect information and one where consumers receive an imperfect signal about the products' qualities. In both scenarios if the regulator chooses a level for an MQS, firms make their quality choice knowing the regulatory requirement. I shall consider MQS that are higher than both qualities, intermediate between the quality levels of the two firms, and lower than both qualities. The latter type of MQS shall be defined an "innocuous" MQS as one would expect it to leave firms' choices unaffected.

Under perfect information I take up the issues of quality determination and consumers' welfare. The results point to an even less satisfactory performance of MQS regulation than that in pure vertical differentiation. In particular, I show that where firms' products are differentiated both horizontally and vertically, then (i) an MQS that is intermediate between the two qualities will not lead to higher qualities for both firms, and lead to a lowering of the quality for the high quality firm, (ii) as a result the average quality may go down for some parameter ranges. This is reminiscent of particular results by Scarpa (1998) with three firms. However, the result that no consumer is harmed by MQS's does not carry through as the consumers who purchase the high quality prior to regulation suffer a welfare loss. Total consumers' surplus may decrease due to the introduction of an MQS.

The reason of the lowering of quality by the high quality firm is that qualities are strategic substitutes in the present context.

Under consumers' imperfect information, the analysis is more complex. Consumers' perceptions of qualities can be affected by the presence of a standard. To make the point I use two examples. In the first, firm 2 is a new entrant and consumers only have noisy signal about the quality of this firm. In this simple set up, the imposition of an "innocuous" standard prevents consumers from believing hat the quality of the entrant could fall below the standard and therefore the expected quality is higher than without the standard, for whatever true underlying quality level chosen by the entrant. Since qualities are strategic substitutes this leads to a lower equilibrium value for the high quality and to a higher one for the low quality firm.

Finally, I analyze the case of a duopoly without new entrants. There, an "innocu-
ous” standard can lead to an increase in both qualities when firms have symmetric costs and consumers are uncertain about the quality of both firms. The causality running from consumers’ expectations to firms quality choices is again the force underlying the quality changes. In some sense this may constitute a vindication of the use of MQS under asymmetric information. The results, however still seem rather special. In a companion paper (Garella and Petrakis forthcoming) we study symmetric uncertainty under a more general formulation and provide some more robust results concerning this case. In the concluding section I shall comment on the use of quality regulation under imperfect information and on the limits of the present paper and of the existing literature.

2. Unregulated industry equilibrium

There are two firms, indexed 1 and 2. Products are horizontally differentiated and, by hypothesis, the two firms are located at the opposite endpoints of a Hotelling linear city (Hotelling 1929). Each product is also characterized by a vertical quality dimension, \( \theta \), which is the result of independent technological efforts by firms towards quality improvements. Firms are asymmetric, in the sense that firm 1 is assumed to be more efficient than firm 2 in improving the vertical dimension of its quality. The production cost for the quantity of output \( q_1 \) for firm 1 is
\[
C_1(q_1, \theta_1) = cq_1 + \frac{\theta_1^2}{2},
\]
where \( c > 0 \) is a constant marginal cost independent of \( \theta \). The cost for firm 2 is
\[
C_2(q_2, \theta_2) = cq_2 + \alpha \frac{\theta_2^2}{2},
\]
where \( \alpha > 1 \) is a cost parameter that distinguishes firm 2 from 1. As it appears from these cost functions, the quality \( \theta \) only affects fixed costs, as in much of the existing literature.

Consumers have an address \( x \in [0, 1] \), and the distribution of consumers is uniform with unit density. When buying at location \( z \), for \( z = 0, 1 \) a consumer bears a “transportation cost”, or a lower utility than if it was buying her own preferred brand. This cost is \( t|x - z| \), where \( |\cdot| \) denotes absolute value and \( t \) is a parameter describing the importance of horizontal dimension in the consumers’ utility function (the parameter \( t \) is known as “unit transportation cost” in the spatial interpretation of the model). Then, given the prices \( p_1 \) and \( p_2 \), the utility derived from consumption of good 1 and 2 is assumed to be equal to, respectively,
\[
u_1(x, \theta_1) = v + \theta_1 - tx - p_1
\]
\[
u_2(x, \theta_2) = v + \theta_2 - t(1 - x) - p_2.
\]

Firms compete in two stages: at the first stage they simultaneously choose their quality levels \( \theta_1 \) and \( \theta_2 \), and pay the relative costs of achieving those qualities; at the second and final stage firms simultaneously choose their prices. It is also possible to think of the \( \theta \)'s as of improvements that firms realize over a basic product.

For convenience it is assumed that \( t \) is high enough, or

**Assumption 1.** \( t > 2/9 \).

Furthermore, it shall be assumed that \( v \) is large enough so that the market is always entirely served. In particular, the following assumption will guarantee also that the market shares of the two firms overlap at an equilibrium.
Assumption 2. $v > 2t + c$.

Solving the game by backward induction shall provide the desired results. First, note that the consumer indifferent between buying at one or the other firm has address denoted $\bar{x}$ such that

$$\bar{x}(p_1, p_2) = \max \left\{ 0, \min \left\{ 1, \frac{1}{2} + \frac{(p_2 - p_1) + (\theta_1 - \theta_2)}{2t} \right\} \right\}. $$

Accordingly, the demand functions at the second stage of the game are defined as $D_1(p_1, p_2; \theta_1, \theta_2) = \bar{x}$ and $D_2(p_1, p_2; \theta_1, \theta_2) = 1 - \bar{x}$. Then, when $\bar{x}(p_1, p_2)$ is not equal to either 0 or 1, the profit maximization problem for firm 1 at the second stage can be written as

$$\max_{p_1} (p_1 - c) \left[ \frac{1}{2} + \frac{(p_2 - p_1) + (\theta_1 - \theta_2)}{2t} \right] - \frac{\theta_1^2}{2}. $$

This provides the best reply function for firm 1 at the second stage

$$\hat{p}_1(p_2) = \frac{p_2}{2} + \frac{(\theta_1 - \theta_2) + t + c}{2}. $$

Similarly one gets the best reply for firm 2 as

$$\hat{p}_2(p_1) = \max \left\{ \frac{p_1}{2} + \frac{(\theta_2 - \theta_1) + t + c}{2}, 0 \right\}. $$

![Figure 1: price best replies](image)

Define $p_1^0$ as the intercept of the best reply function for firm 1, namely $p_1^0 = \frac{(\theta_1 - \theta_2) + t + c}{2}$ (respectively, $p_2^0 = \frac{(\theta_2 - \theta_1) + t + c}{2}$ as the intercept for $\hat{p}_2(p_1)$). Notice that if $\hat{p}_2(p_1^0) \leq 0$, then firm 2 is priced out of the market because its quality is too low; otherwise stated, the two best reply functions would cross at a value for $p_2$ less than or equal to zero. This obtains when

$$\frac{(\theta_1 - \theta_2) + t + c}{4} - \frac{\theta_1 - \theta_2}{2} + \frac{t + c}{2} \leq 0, $$

5
\[
\theta_2 \leq \theta_1 - 3(t + c) \equiv \theta_2^0.
\]

Then, if \(\theta_2 < \theta_2^0\) the Nash prices are \(p_1^*(\theta_1, \theta_2) = 0\) and \(p_2^*(\theta_1, \theta_2) = \max \{p_1^m, p_2^m\}\), where \(p_1^m\) is the monopoly price for firm 1, or \(p_1^m = \max \{(v + \theta_1 - t) / 2, v + \theta_1 - t\}\) because at prices \(p_1\) lower than \(v + \theta_1 - t\) all consumers would buy from the monopolist firm 1. A similar reasoning goes for the case when the quality of firm 1 is too low, namely when \(\theta_1 \leq \theta_2 - 3(t + c) \equiv \theta_1^0\). We shall exclude that either firm uses a quality so low as to be priced out, so that the analysis of the case where \(\theta_2 \leq \theta_2^0\) (or \(\theta_1 \leq \theta_1^0\)) shall not be further pursued. Obviously, this implies that the cost of quality improvements cannot be too different, or that \(\alpha\) cannot be too much higher than 1.

Then, if \(\theta_2 > \theta_2^0\), the Nash prices at the second stage as functions of the values for the \(\theta_i\)'s are
\[
\begin{align*}
p_1^*(\theta_1, \theta_2) &= t + c + (\theta_1 - \theta_2)/3 \\
p_2^*(\theta_1, \theta_2) &= t + c - (\theta_1 - \theta_2)/3.
\end{align*}
\]

Notice that \(p_1^*(\theta_1, \theta_2) - p_2^*(\theta_1, \theta_2) = (2/3)(\theta_1 - \theta_2)\) is positive if the quality of firm 1 is higher than that of firm 2. The equilibrium demand functions are
\[
\begin{align*}
D_1'(\theta_1, \theta_2) &= (1/2) + (\theta_1 - \theta_2)/(6t) \\
D_2'(\theta_1, \theta_2) &= (1/2) - (\theta_1 - \theta_2)/(6t).
\end{align*}
\]

The reduced form profits, that shall be used to solve the first stage of the game, are
\[
\begin{align*}
\pi_1^*(\theta_1, \theta_2) &= \left\{ (3t + (\theta_1 - \theta_2))^2 / (18t) - (\theta_1)^2 / 2 \right\} \\
\pi_2^*(\theta_1, \theta_2) &= \left\{ (3t - (\theta_1 - \theta_2))^2 / (18t) - \alpha(\theta_2)^2 / 2 \right\}.
\end{align*}
\]

It is predictable that if the strict inequality \(\alpha > 1\) holds, then at an equilibrium firm 1 will have a higher demand, a higher price, and a higher profit than firm 2, thanks to its lower cost of increasing the level of \(\theta\) for its product. Now it is possible to solve for the values of \(\theta_1\) and \(\theta_2\) at the first stage. Firm 1 and firm 2 maximization program at the first stage give the best reply functions
\[
\begin{align*}
\theta_1(\theta_2) &= \frac{3t - \theta_2}{3t - \theta_1} \\
\theta_2(\theta_1) &= \max \left\{ \frac{3t - \theta_1}{3t - \theta_2}, \theta_1 - 3(t + c) \right\}.
\end{align*}
\]

The two functions are represented in figure 2 as two downward sloping straight lines assuming \(t > 2/9\). Vertical qualities are strategic substitutes. Note that firm 2 cannot choose a quality lower than \(\theta_1 - 3(t + c)\) otherwise it is priced out of the market at the second stage, this explains the V-shape of its best reply function. The same holds true for firm 1, although, for illustrative purposes, the figure is drawn as if this firm were not concerned with being priced out. If the best reply functions cross where they are both downward sloping then both firms shall enjoy a positive market share.
at equilibrium, with positive prices (irrespective of profits). One obtains, then, the Nash equilibrium values for the qualities of the two firms as,

\[
\begin{align*}
\theta_1^* &= \frac{9\alpha t - 2}{3(9\alpha t - \alpha - 1)} \\
\theta_2^* &= \frac{9t - 2}{3(9\alpha t - \alpha - 1)}.
\end{align*}
\]

In order to have \(\theta_2^* > 0\) it must be \(t > 2/9\), as assumed, and \(9\alpha t - \alpha - 1 > 0\) or \(\alpha > 1/(9t - 1) \equiv \alpha(t)\). Let \(A(t)\) define the set of values for \(\alpha\) such that \(\alpha \geq \max\{1, \alpha(t)\}\); then the following analysis proceeds upon the restriction that \(t > 2/9\) and \(\alpha \in A(t)\).

As expected, \(\theta_1^*\) is not lower than \(\theta_2^*\). Indeed \(\theta_1^* - \theta_2^* = 3t [(\alpha - 1) / (9\alpha t - \alpha - 1)] \geq 0\). This situation is interesting for the following analysis of quality regulation. Note that the parameter \(t\), that is related to consumers’ attachment to their preferred quality and that confers market power to firms, also affects the quality levels.

\[\begin{array}{c}
\theta_2 \\
\theta_1^* \quad \theta_2^* \\
0 \\
\theta_1
\end{array}\]

Figure 2: quality best replies.

**Remark 1.** Under \(A.1\) and if \(\alpha \in A(t)\), then \(1 > \theta_1^* \geq \theta_2^* > 0\), with strict inequality \(\theta_1^* > \theta_2^*\) if \(\alpha > 1\).

**Proof:** Note that \(\theta_2^*\) and \(\theta_1^*\) belong to the open interval \((0, 1)\) for \(t > 2/9\). Indeed, one has \(3(1 - \theta_1^*) = (18\alpha t - 3\alpha - 1) / (9\alpha t - \alpha - 1)\) and one can check that this is positive for \(t = 2/9\); furthermore, the first derivative of \(\theta_1^*\) with respect to \(t\) is negative\(^2\) for all values of \(t\), the second derivative is positive, while \(\lim_{t \to \infty} \theta_1^* = 1/3\). The derivative of \(\theta_2^*\) with respect to \(t\) is positive for all values of \(t\) and equal to \(3(\alpha - 1) / (9\alpha t - \alpha - 1)^2\), while the second derivative is negative, and \(\lim_{t \to \infty} \theta_2^* = 1/(3\alpha)\).

\(^2\)Indeed \(\frac{d}{dt} \left( \frac{9\alpha t - 2}{9\alpha t^2 - \alpha - 1} \right) = 3\alpha - \frac{1-\alpha}{(9\alpha t - \alpha - 1)^2}\).
The equilibrium quality difference is \( \mu_1 - \mu_2 = 3t[(\alpha - 1)/(9at - \alpha - 1)] \). The derivative of \( \mu_1 - \mu_2 \) with respect to \( t \) is obviously negative: if firms have more market power because consumers attach more and more importance to the horizontal dimension (increases in \( t \)), firms’ qualities become more and more alike. One can take the limit of the equilibrium quality difference for \( t \) going to infinity, even if this makes no economic sense because when \( t \) is above a threshold level firms’ market shares are identified by non-overlapping intervals over \([0,1]\). In any case, \( \lim_{t \to \infty} (\theta_1^* - \theta_2^*) = (\alpha - 1)/(3\alpha) \), so that the quality levels as functions of \( t \) never cross.

Prices and demands at equilibrium are

\[
p_1^* = t + c + \frac{t(\alpha - 1)}{9at - \alpha - 1}, \quad \text{and} \quad p_2^* = t + c - \frac{t(\alpha - 1)}{9at - \alpha - 1}
\]

\[
D_1^* = \frac{1}{2} \left[ 1 + \frac{\alpha - 1}{9at - \alpha - 1} \right], \quad \text{and} \quad D_2^* = \frac{1}{2} \left[ 1 - \frac{\alpha - 1}{9at - \alpha - 1} \right].
\]

It is possible that if \( t \) is too high, then the market shares of the two firms do not overlap and they behave as separate monopolists. To exclude this possibility it is assumed that even for consumer with address \( x = 0 \) one has \( u_2 > 0 \) at an equilibrium, so that consumers buy from firm 1, eventually, because it gives a higher utility and not because they have no positive utility from the alternative offered by firm 2. This is true if the inequality \( v + \theta_2^* - p_2^* > t \) holds. This is guaranteed by A.2, as it can be easily checked\(^3\).

Further, it can be checked that \( D_2^* > 0 \) and that \( p_2^* > 0 \) for \( t > 2/9 \). Therefore, under the assumption that \( t \geq 2/9 \) the best reply function in qualities of the two firms cross where they are both downward sloping. The equilibrium demand for firm 2, which can also be written as \( \alpha(9t - 2)/(9at - \alpha - 1) \), is positive for \( \alpha \in A(t) \).

The equilibrium profits for the unregulated industry are,

\[
\pi_1^u = \frac{9t - 1}{18} \left( \frac{9at - 2}{9at - \alpha - 1} \right)^2 \quad \text{and} \quad \pi_2^u = \alpha \left( \frac{9t - 1}{18} \right) \left( \frac{9t - 2}{9at - \alpha - 1} \right)^2
\]

Under the assumptions that \( \alpha \in A(t) \) and A.1 both profits are nonnegative and that of firm 1 is always larger than that of firm 2 in the case \( \alpha > 1 \).

### 3. Regulation via a Minimum Quality Standard

If there exist a regulating agency that imposes a MQS, denoted \( \Theta \), we can have three different situations. (a) The first is when \( \Theta \) exceeds both quality levels in an unregulated environment. (b) The second is when the standard is in between the higher and the lower quality, (c) the third is when the standard is below both. Case (a) is clearly a case where the effect on qualities is easy to predict. So it seems for case (c), that shall be termed the “Innocuous Standard Case”. Case (b) is the one

\(^3\)The desired inequality can be fully written as \( v + [(9t - 2)/(3T)] - 2t - c + t [(\alpha - 1)/T] \) where \( T = 9at - \alpha - 1 \).
on which the literature has so far concentrated and the one on which this section is
mainly devoted. We shall see in the section on asymmetric information that case (c)
is not obvious either.

It is unlikely that the regulatory agency be so informed about the firms’ cost
functions as to choose the quality standard that maximizes social welfare under the
constraint that firms choose their Nash equilibrium prices given the standard. More
likely is the case that the agency adopts some standard in response to concern by
consumers associations or public opinion in general. We shall analyze in turn the
three possible cases.

3.1. High standards

Case (a) is quite trivial: both firms are obliged to raise their quality to the level
denoted, for ease of classification, \( \Theta_a \). Firms will sell the same quality: the standard
in this case eliminates the game in quality choices. The symmetric equilibrium price
shall be \( p = t+c \), unaffected by the particular quality standard chosen. This depends
crucially on the assumption that quality only affects fixed costs. The consumers’
welfare is raised as far as the utility of a consumer depends positively on quality and
as far as the price of firm 1 decreases, while the price of firm 2 increases less than the
utility derived from the increase in quality:

\[
 u_2 = \mu_2 - t(1-x) - t - c - (\theta_1 - \theta_2)/3
\]
is increasing in \( \mu_2 \) and actually it raises to

\[
 u_2(\Theta_a) = \Theta_a - t(1-x) - t - c, \text{ so that the utility gain for any consumer who would be purchasing good 2 in an unregulated industry is given by the positive amount (} \theta_1^* - \theta_2^*/3).
\]

The price of firm 2 is raised from \( p_2^* \) by the amount

\[
 \frac{t(\alpha - 1)}{9\alpha \theta - \alpha - 1}
\]
The profits of the two firms will decrease as far as they end up on lower isoprofit
curves in the \( \theta_1 - \theta_2 \) plane.

In particular, \( \pi_2(\Theta_a) = t/2 - \alpha \Theta_2^* \) is negative if \( \Theta_a > \sqrt{t/(2\alpha)} \). It then follows,

**Proposition 1.** If the regulator sets a quality standard \( \Theta_a \), such that \( \theta_1^* < \Theta_a < \sqrt{t/(2\alpha)} \), then both qualities are raised and all consumers gain. The firms’ profits are lower than without the standard. If the standard is such that \( \Theta_a > \sqrt{t/(2\alpha)} \) then firm 2 will exit the market and firm 1 remains a monopoly.

3.2. Intermediate standards

Turn now to Case (b). Let the standard be denoted by \( \Theta_b \), where \( \theta_2^* > \Theta_b > \theta_1^* \). Since firm 2 must set its quality so that \( \theta_2 = \Theta_b \), the reaction function of firm 2 will exhibit a flat segment in correspondence of \( \Theta_b \). Firm 1, then, will choose according to its

\[
 \theta_1^b = \theta_1(\Theta_b) = \frac{3t - \Theta_b}{3t - 1} \text{ and } \theta_2^b = \Theta_b.
\]

It is clear that:
Proposition 2. If the regulator sets a standard $\Theta_b$ such that $\theta^*_1 > \Theta_b > \theta^*_2$, then the quality levels under the standard regulation, $\theta^*_1, \theta^*_2$ are such that $\theta^*_1 > \theta^*_1 > \Theta_b = \theta^*_2 > \theta^*_2$; the high quality under the standard is lower than without it.

Then, the quality difference is lowered to $(\theta^*_1 - \Theta_b) = 3t \frac{1 - 3\Theta_b}{\theta^*_1 - \theta^*_2}$ and the corresponding prices will be

$$p^*_1 = t + c + t ((1 - 3\Theta_b)/(9t - 1)) \text{ and } p^*_2 = t + c - t ((1 - 3\Theta_b)/(9t - 1));$$
clearly, the price of firm 1 goes down and that of firm 2 goes up as the standard is raised. The demand addressed to firm 1 and 2 shall be respectively

$$D^*_1(\theta^*_1, \theta^*_2) = \frac{1}{2} \left(1 + \frac{1 - 3\Theta}{9t - 1}\right) \text{ and } D^*_2(\theta^*_1, \theta^*_2) = \frac{1}{2} \left(1 - \frac{1 - 3\Theta}{9t - 1}\right).$$

The profits of the two firms shall be

$$\pi^*_1(\Theta) = \left[\frac{3t - \Theta}{2(9t - 1)}\right] - \Theta^2 \left[\frac{1}{2}ight] \left(1 - \frac{1 - 3\Theta}{9t - 1}\right)^2 - \alpha \Theta^2 \left[\frac{1}{2}\right] \left(1 - \frac{1 - 3\Theta}{9t - 1}\right).$$

It is clear that the profit of firm 1 will decrease after the imposition of the standard, since it will choose a lower quality, corresponding to an isoprofit curve corresponding to a lower value of profits than the one reached at an unregulated equilibrium. On the other hand, firm 2 may gain from the imposition of a standard if this is not too high with respect to its unrestricted equilibrium quality. In particular, one can make reference to figure 2 to see that the unrestricted equilibrium isoprofit curve for firm 2 may lie below or above that obtained after the imposition of a standard, depending upon the standard position on the graph. Indeed one has that the derivative $(d\pi_2/d\Theta) = t \left(\frac{2t^2 + 3\Theta}{9t - 1}\right) - \alpha \Theta$ can be negative or positive.\(^4\)

Remarkably, average quality in the market may go down. This is most likely when the high quality has a high market share prior to the imposition of the standard.

Remark 2. When the standard $\Theta_b$ is such that $\theta^*_1 > \Theta_b > \theta^*_2$, average quality with the standard may be higher or lower than without.

Average quality is $\bar{\Theta} = \frac{D^*_1(\theta^*_1, \theta^*_2) \left(\frac{3t - \Theta}{9t - 1}\right) + D^*_2(\theta^*_1, \theta^*_2) \left(\frac{9t - 2 + 3\Theta}{9t - 1}\right)}{\theta^*_1 - \theta^*_2}$ which is given by:

$$\bar{\Theta} = \frac{3}{2} \left(\frac{3t - \Theta}{9t - 1}\right)^2 + \Theta \left(\frac{9t - 2 + 3\Theta}{9t - 1}\right)$$

And $d\bar{\Theta}/d\Theta^b = (-45t + 81t^2 + 2 + 54\Theta^b t)/(2(9t - 1)^2)$. This derivative is increasing with the standard, so that when the regulator sets a standard that is close

\(^4\)If $\Theta = \theta^*_2$ this derivative is equal to:\[\frac{3t(9t - 2)}{(9t - 1)^2} + \left(\frac{t}{(9t - 1)^2}\right)\left(\frac{9t - 2}{9at - \alpha - 1}\right) - \frac{9t - 2}{3(9at - \alpha - 1)}\] which can be positive for low values of $\alpha$.\]
enough to the low quality it is likely that the average quality decreases. Indeed
the numerator of $d\Theta/d\Theta^b$ is lowest when one takes $\Theta^b$ equal to the minimum level $\theta^*_2 = (9t - 2)/[3(9at - \alpha - 1)]$. For such a level, the numerator of $d\Theta/d\Theta^b$ can be shown to be strictly negative for all values of $\alpha$, provided $t > 1/9$.

For instance, for $t = 3/9$ and $\alpha = 2$ one has that the unregulated qualities are $\theta^*_2 = 1/9$ and $\theta^*_1 = 4/9$; then, suppose the regulator sets $\Theta = 1/6$ as an intermediate level of quality: one finds the numerator of $d\Theta/d\Theta^b$ to be equal to $-1$.

Summarizing, for $\Theta^b$ higher than, but sufficiently close to $\theta^*_2$, and for low values of $t$, the average quality in the market can be decreased after the imposition of a standard.

Furthermore, as a consequence of the quality adjustments one has that:

**Proposition 3.** If the regulator sets a standard $\Theta_b$ such that $\theta^*_1 > \Theta_b > \theta^*_2$, the welfare of the consumers that prior to the standard were purchasing the high quality may decrease, the welfare of the other consumers is increased. Total consumers’ welfare may decrease. As $\Theta_b$ is raised, total consumers welfare increases are more likely. Total welfare in the industry may also decrease.

That total consumers’ welfare may decrease can be proved as follows. Let $u_1(x, \Theta_b) = v + \theta_1(\Theta_b) - p_1(\Theta_b) - tx$ and $u_1(x, \Theta_b) = v + \Theta_b - p_2(\Theta_b) - t(1 - x)$.

Then define $U_1(\Theta_b) = \int_0^x u_1(x, \Theta_b) dx$, and $U_2(\Theta_b) = \int_0^1 u_2(x, \Theta_b) dx$ as the welfare of consumers purchasing good 1 and good 2 respectively.

Then the derivative with respect to $\Theta_b$ of the sum $U^c(\Theta_b) = U_1(\Theta_b) + U_1(\Theta_b)$ is equal to

\[
(dU^c(\Theta_b)/d\Theta_b) \left[ \int_0^{\bar{x}(\Theta_b)} du_1/\Theta_b dx + \int_{\bar{x}(\Theta_b)}^1 du_2/d\Theta_b dx \right],
\]

or

\[
dU^c(\Theta_b)/d\Theta_b = 3/\left[2(9t - 1)^2\right] [3t\bar{x}(\Theta_b) - 6t + 1].
\]

This derivative is increasing with $\Theta_b$ and increasing in $t$, while it does not depend upon $\alpha$ because consumers’ welfare does not. It can be seen, by substituting for $\bar{x}(\Theta_b)$ that $dU^c(\Theta_b)/d\Theta_b$ is negative if $(9t)(9t - \Theta_b - (10/3)) + 2 > 0$, an inequality that holds for several parameter values, for instance take $t = 1/4$ and $\alpha = 2$, then $\theta^*_1 = 5/9$ and $\theta^*_2 = 1/18$. If the regulator sets $\Theta_b$ halfway between these two values, namely if $\Theta_b = 11/36$ then $(9t)(9t - \Theta_b - (10/3)) + 2 = .25$ and it is positive, so that welfare is decreasing in $\Theta_b$. Instead, if the regulator sets a high level of standard, then total consumers welfare increases are more likely. Indeed, for $\Theta_b = \theta^*_1 = 5/9$ the derivative has a strictly positive value.

That total welfare may decrease can be shown by taking again the case where total consumers welfare is decreased, with $t = 1/4$, $\alpha = 2$ and $\Theta_b$ taken to be halfway between the two unregulated quality levels, namely $\Theta_b = 11/36$. Then, if the sum of firms profits is decreased for this quality standard, total welfare is a fortiori decreased. It turns out to be the case that the sum of profits without the standard is $\left(\frac{9t-1}{18}\right) \left(\frac{9at-2}{9at-\alpha-1}\right)^2 + \alpha \left(\frac{9t-2}{9at-\alpha-1}\right)^2 = (85/432)$. While the sum with
the standard $\frac{[3t - \Theta]^2}{2(9t - 1)} + \left(\frac{t}{2}\right) \left[1 - \frac{1-3\Theta}{9t-1}\right]^2 - \alpha \frac{\Theta^2}{2}$. When the standard is equal to 11/36 this is equal to $\frac{1021}{10800}$, which is lower than 85/432. In general, the derivative of the sum of profits with respect to the standard under regulation is equal to $\left(\frac{1}{\Theta}\right) \left[3t \left(\frac{3t-1}{9t-1}\right) + \Theta(1 - \alpha)\right]$, which can clearly be negative also for low values of the standard.

These results contrast with previous results by Ronnen (1991) and Scarpa (1998), since there all consumers gain from a standard. Furthermore, the result that average quality may decrease is rather striking. This means that if there is some positive externality associated with high quality, that the single consumer does not take into account when making a purchase (like when one buys a car with a better airbag system than average), the imposition of a MQS may lead to a deterioration in welfare due to the externality.\footnote{A similar point is raised in Moraga-Gonzalez and Padron-Fumero (2002), who study a model of perfect vertical differentiation, although the increase in the external effect, there, is due to an increase in total sales.}

### 3.3. Innocuous standards

As for Case (c), one has that $\theta_1^* > \theta_2^* > \Theta$. Let the level chosen by the regulator be denoted as $\Theta^c$. Such a regulation may be introduced for reassuring public opinion, or to make sure that if cost or demand conditions change, or if new entrants enter the market, quality of any firm does not fall below a given threshold. As the regulator might be induced to think, a low level of quality standard should not interfere with market outcome. This, however, is true only in the framework so far considered.

### 4. Consumers’ uncertainty about qualities

In the present section the scenario in which firms compete shall be modified to allow for uncertainty. In order to keep the model as simple as possible it shall be assumed that firms and the regulator observe qualities without noise and that there is no uncertainty in the determination of quality. By contrast, consumers are uncertain about the success rate of products. This may be due to asymmetric information, so that consumers are less informed than firms and the regulator. Consumers can be thought of receiving an imperfect signal about the quality of a product. Such a description of consumers’ beliefs has been often invoked to model situations when they cannot try the product before purchase or use, like in the consumption of pharmaceutical drugs, or when the true enjoyment of the product characteristics comes by over time, like with dietary products, some kind of electronic durables, tires or components in vehicles, energy saving improvements in domestic appliances, and safety enhancing devices.

We shall analyze, in turn, the case where only the improvement made by firm 2 is uncertain and the case where both firms’ improvements are uncertain. An improvement is uncertain when consumers believe that the true value of the parameter $\theta_i$ of a product is distributed according to some distribution function $F(\theta_i)$ over a
nondegenerate support. The support can be an interval or a discrete set of points. The main idea is most simply conveyed first by analyzing the case where consumers are uncertain only about the product of one firm (say firm 2) and the support of the distribution is the set of two points \( \{ \theta_0, \theta_2 \} \). Consumers then attribute probability \( \mu \) to the event that \( \theta_2 \) is the true value that the firm actually has obtained and \( (1 - \mu) \) to the event that the value is \( \theta_0 \).

Since firms and the regulator perfectly observe the improvement they know that it will lead to a higher gross surplus for all concerned consumers. The model could be adapted to allow for two-sided uncertainty, but the greater generality so acquired has a cost in complexity and in interpretation. The model with one-sided uncertainty is suited to reflect in a simple way the existence of some information asymmetry between consumers and producers-regulators. Furthermore, as it shall become apparent, there exists a causality running from the effect of regulation on the consumers’ perceptions of quality and the strategic interplay between firms. This is the causality that we want to isolate.

4.1. Uncertainty about quality of an entrant

Assume that consumers are risk neutral and that they are uncertain about the quality of firm 2’s product only. This would be a fair description of the case where firm 2 is a new entrant in a market, or a foreign firm entering a domestic market. Consumers assign probability \((1 - \mu)\) to the event that the gross utility they derive from consuming good 2 is \( v + \theta_0 \), where \( \theta_0 < \theta_2 \) is a quality level on which no restrictions are imposed for the time being. Then the expected quality improvement from good 2 is denoted

\[
E(\theta_2) = \mu \theta_2 + (1 - \mu) \theta_0.
\]

Before proceeding note the obvious fact that the first partial of \( E(\theta_2) \) with respect to \( \theta_0 \) and \( \mu \) have the same sign and are both positive: an increase in consumers’ confidence can be represented by an increase in either one or both these variables.

The consumer indifferent between buying quality 1 or 2 is given by

\[
\bar{x} = \frac{1}{2} + \frac{(p_2 - p_1) + (\theta_1 - E(\theta_2))}{2t},
\]

and the demand and profit functions at the second stage can be accordingly rewritten. Define for convenience \( H \equiv (3t)^{-1}(1 - \mu)\theta_0 \). Then, at the first stage the profit function of firm 2 writes as

\[
\pi_2 = \left( \frac{(3t - \theta_1 + \mu \theta_2 + 3tH)^2}{18t} - \alpha(\theta_2)^2 \right)
\]

Taking the first derivative and setting it equal to zero gives the best reply of firm 2 as:

\[
\theta_2 = \mu \frac{3t(1 + H) - \theta_1}{9at - \mu^2}.
\]
As for the profit function of firm 1 it is simply
\[ \pi_1 = \left\{ \frac{[3t + (\theta_1 - E(\theta_2))]^2}{18t} \right\} - (\theta_1)^2/2 \]. So that the best reply function is the same as in the previous section, with \( E(\theta_2) \) replacing \( \theta_2 \). Note that a change in \( E(\theta_2) \) will lead to a downward shift in the reaction function of firm 1. Then, letting \( 9t(9at - \mu^2 - \alpha) - \mu(1 - \mu) \equiv \phi \) one has:
\[ \theta_1^* = (3t/\phi) \left[ 9at - \mu(1 + \mu + H) \right], \]
and
\[ \theta_2^* = \frac{\mu(3t + 3tH)}{9t\alpha - \mu^2} - \frac{3t}{9t\alpha - \mu^2} \theta_1^*. \]

To study the impact of a minimum quality standard, suppose that the government chooses a level \( \Theta \) such that \( \theta_0 < \Theta < \theta_2^* \) obtains. Then it is possible to state the following:

**Proposition 1.** If consumers are uncertain only about the quality of product 2 and the MQS is such that the standard is lower than the lowest quality in the unregulated equilibrium, then both equilibrium qualities are affected by the presence of regulation. In particular, the higher quality will be decreased and the lower quality increased.

**Proof:** In this simple case, if \( \Theta > \theta_0 \), the support of the distribution representing consumers’ beliefs is changed from the set \( \{\theta_0, \theta_2\} \) to the set \( \{\Theta, \theta_2\} \). To study the impact of the standard, note that the derivative of \( \theta_1^* \) with respect to \( H \) is equal to the derivative with respect to \( \theta_0 \) (or \( \Theta \)) multiplied by \( (1 - \mu)/3t \), and therefore both have the same sign. This derivative is: \( d\theta_1^*/dH = -(3t\mu)/\phi \), which is easy to sign as negative given the assumption that \( t > 2/9 \). Then, since qualities are strategic substitutes, if \( \theta_1^* \) decreases with \( H \) then \( \theta_2^* \) will increase.

Both qualities are changed and in the same direction as in the case (b) with certainty analyzed above, where the standard is intermediate between the two qualities. The apparently ”innocuous” standard works by modifying the consumers’ perceptions of firm 2’s quality. This, in turn, shifts down the best reply function of firm 1, since for any true quality chosen by the rival, the perceived quality by consumers is higher with the standard than without it. The changes in the equilibrium quality values are then obvious.

One may notice also that the same type of change can be obtained through regulations that improve the value of \( \mu \), that can represent a measure of the level of consumers’ confidence in the product.

**4.2. Symmetric uncertainty**

The case where consumers’ beliefs about the quality of both firms are symmetric can be built on the results so far obtained and it reveals again the striking feature of the effect of a MQS that is lower than the equilibrium value of qualities in an unregulated set up. The direction of the changes in actual qualities is not uniquely determined,
However, and not general, as it can be easily seen by noting that if, under symmetry of beliefs $E(\theta_1) = \mu_1 + (1 - \mu)\theta_0$ then $E(\theta_1) - E(\theta_2) = \mu(\theta_1 - \theta_2)$: then at the first stage of the game one has that $\pi_2 = \left[(3\mu - \mu(\theta_1 - \theta_2))^2 \right] / (18t - (\alpha/2)(\theta_2)^2$. This profit function, as well as that for firm 1, is independent of the lower bound of the distribution, $\theta_0$, and the impact of a MQS that is lower than the unregulated qualities is nihil. However, for different distribution functions of beliefs the non-neutrality of an apparently innocuous standard is obtained again, although the effect on qualities is difficult to predict in general.

To simplify, assume that firms are cost-symmetric, namely $\alpha = 1$.

In order to understand the source of the difficulties in signing the effects of a standard, take the case where the value of $\theta_i$ is believed to belong to the interval of values $[h_0, t_i]$, where $h_0$ can be any real number and $t_i \geq \theta_i$, so that the interval always contains the true value for $\theta_i$. Then, application of a standard at a level, say, $h > h_0$ implies that the expected value for $\theta_i$ is modified to $E(\theta_i | h) = \int_h^{t_i} \frac{1}{1-F_i(h)} f_i(x)dx$, where $f_i(.)$ is the density of $\theta_i$ over $[h_0, t_i]$ and, as the support for the two qualities may be different, even if the distributions for the two firms have the same form they will take different values. In general, the expected value of the quality difference $E(\theta_1) - E(\theta_2)$ depends upon the lower bounds of the supports of the two distributions.

The effect of the amelioration of the consumers’ beliefs on the difference $E(\theta_1) - E(\theta_2)$, cannot be signed without reference to specific examples. Since it is this difference that enters the expression for the profit functions of the two firms the effect on equilibrium qualities cannot be signed in general either. Indeed it can be either positive or negative.

It is then possible to state the following:

**Proposition 2.** If firms’ costs are symmetric (namely if $\alpha = 1$) and consumers’ uncertainty about the two products is symmetric, a standard below the quality of both firms may lead to a change in both quality levels. The change may be an increase or a decrease.

Proof: Since $E(\theta_1) - E(\theta_2)$ enters the profit functions of both firms and since the first derivative of $E(\theta_1 | h) - E(\theta_2 | h)$ can take zero values only in particular cases, the best reply functions of both firms shall be affected by an increase in $h$.

As an example consider the case with a discrete support for both qualities. Namely assume that $\theta_i$ is a priori believed to take with equal probability three possible values, $-\beta \theta_i, \theta_i, \lambda \theta_i$, with $\beta > 0, \lambda > 1$ (the improvement may be negative in the eyes of consumers).

Then, suppose the standard is such that $h = 0$, or that the regulation prevents selling a quality with negative $\theta_i$. The expected quality without a standard for firm $i$ is $(1/3)(1+\lambda-\beta)\theta_i$ while with the standard the quality is expected to be $(1/3)(1+\lambda)\theta_i$. Letting $z^u = (1/3)(1+\lambda-\beta)$ and $z(h) = (1/3)(1+\lambda)$ one has that $E(\theta_1 | \mu) - E(\theta_2 | \mu)$ reduces to the expression $z(h)(\theta_1 - \theta_2)$. One can then calculate the equilibrium qualities as

$$\theta^*(z) = 3tz \left( \frac{9t - z - z^2}{z^4 - 18z^2t + 81t^2 - z^2} \right)$$
and check, for instance for the case $\lambda = \beta = 4/3$, then when $z = z(h)$ the equilibrium qualities are higher with the standard than they are for $z = z^u$.

The proposition shows that under asymmetric information the imposition of an innocuous standard may not be innocuous indeed. Similarly, eliminating an MQS policy because firms have been complying with it may not be innocuous either, as consumers expectations may change and equilibrium qualities also.

5. Conclusion and discussion

The theoretical underpinning of policy intervention on product quality through quality standards seems, to date, to be rather weak. Spence (1975) had already shown that a monopoly can produce a quality level that can be lower or higher than the socially optimal level. according to this result, without knowledge of the way in which the demand function is affected by quality improvements it is impossible to determine if a minimum quality standard is desirable or not in case of a monopoly. In the case of oligopolistic rivalry, the determination of equilibrium qualities is first studied in the pure vertical differentiation model of Gabszewicz and Thisse (1979) that obtains the result that one of the two firms in a duopoly always selects the lowest possible quality. Whether this result is believed to be special or general, it raises the problem, again, of regulatory intervention. The results by Ronnen (1991) have been discussed in the Introduction above. The possibility of welfare improvements and therefore of a justification for imposition of MQS is the main achievement for practical purposes of that paper. However, MQS can have adverse effect on firms’ profits and, therefore, on total welfare (as also Scarpa (1998) has also pointed out for a market with three firms). In the present paper, abandoning the hypothesis that the products are differentiated only in the vertical dimension and allowing for both, vertical and horizontal differentiation, I have shown that under perfect information the imposition of an MQS can lead to welfare gain or to losses for consumers, for firms, and for the industry as well. Again, therefore, the application of an MQS is difficult to justify in general in context of this kind.

It seems worthwhile noting at this point how, so far, the discussion on MQS policy in oligopoly has been cast in terms of models with perfect information. However, as casual observation reveals, the political debate moves upon the hypothesis that some market failure may occur, either due to imperfect information or due to external effects like, for instance, polluting emissions. I have not considered here the existence of externalities, but I have introduced imperfect information. I have shown how that, when consumers are less informed than firms and the regulator about the products’ qualities, the imposition of an MQS can push firms to increase their quality levels. This is true even for MQS that set standards below both the quality levels at an unregulated equilibrium. Rather surprisingly, innocuous standards turn out to be not innocuous indeed. The basic intuition is easily gained in the case where consumers are uncertain only about the quality level of one firm, like in the case of a new entrant. There, the imposition of an MQS cuts the lower tail of the support for the distribution representing consumers’ expectations about this quality level. The result is that the expected quality of the entrant is, for any level of quality actually chosen, higher with
the MQS than without it. This changes the equilibrium quality of the firm that is not subject to uncertainty as well. In this case, again, the level of the high quality is lowered and that of the low quality raised at equilibrium when a standard is imposed. The welfare consequences for this case, therefore, replicate those of the case under perfect information, leading again to the conclusion that MQS can have positive or negative effects on welfare.

Finally, however, some vindication for the use of MQS may come out of the analysis of the case where consumers are uncertain about the qualities of both firms, although one has to be prudent in this case also. It is easy to see that the consumers’ perceptions about both qualities may be changed and equilibrium qualities raised by a standard that is set below the unregulated quality levels. Not only an innocuous MQS is not innocuous, but it may be beneficial to all consumers. Unfortunately however, this result is not general either, as one can produce examples where the standard has adverse effects on the quality levels produced at equilibrium.

REFERENCES


