Brain Drain and Fiscal Competition.
A theoretical model for the Europe.

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Abstract
In this paper we study Brain Drain (BD) and Fiscal Competition (FC) in a unified framework for the European Union (EU) specific context. Potential mobility of educated workers can increase the degree of FC through taxation or the provision of public education. An increase in FC can be caused by competition among different jurisdictions that aim to attract educated workers. When the importance of FC increases, then the European States may employ FC as a new policy tool. First, we analyze FC and BD with reference to EU regions. In this instance, the EU may find incentive to control the interactions between BD and FC in order to coordinate fiscal policies and/or the provision of public goods as education. Second, we furthermore consider the entry of new state inside the EU. The absence of coordination implies that, in addition to the FC, a “migration competition” may be generated in EU, where the region inside the union try to attract educated workers of the new entry. We derive the conditions which BD leads to a decrease (increase) in welfare and growth for new entry country.

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1 Introduction

In the European Union (EU hereafter) context, mobility of European citizens is free of institutional constraints so that cultural integration increases the probability to migrate inside the Union. For this reason, workers flows acquired a relevant position in the EU research agenda. The study of the Brain Drain (BD hereafter) is linked with the choice of education for both workers and/or by governments. If education is a public good, educated workers are free to migrate, as a side effect Fiscal Competition (FC hereafter) can arise. If governments do not coordinate taxation and provision of public goods, then the economy may suffer strong negative externalities. In fact, if the growth of the economies is associated with educated workers, we may record lower taxation, worse income redistribution, and lower provision of public goods. Furthermore, the system may record lower growth.

Although BD and FC are connected thorough agents mobility, the literature studied them separately due to the complexity of a joint analysis. In particular, previous studies developed two separate branches for BD and FC. The first one focuses on BD in a macroeconomic setup and studies its impact on the growth of different economies. The second one analyses FC using microeconomics tools and focuses on competitive interactions between workers and Jurisdictions. Several studies are focused on externalities stemming out from human capital migration but all these studies analyze only indirectly the interaction between BD and FC and so they are not adequate to simulate the new European framework. In fact, in the past this BD was an unidirectional flow of highly skilled labor from third-world countries and so the literature has explained the lower provision of human capital as a “negative fiscal externalities” due to migration\(^1\). More recently, increased integration in the labor markets, especially within the EU, has drawn attention to problems that arise from bidirectional movement of skilled labor between similarly developed countries. It is so necessary to define a new BD typology specific to the European context where the FC can be used as a “new policy tools” by the regions. Furthermore, when we analyze the enlarged EU then we can distinguish two different “clubs” of region, the former, with higher growth, and the new entries with lower growth and labour productivity. In this new context a new specification of BD and FC can arise. The Former regions can compete to attract the educate workers of the new regions by use the FC tool. This “migration competition”, in the absence

\(^1\)Berry and Soligo (1969), for example, show that as far as the production of human capital (i.e. schooling and professional or academic education) is subsidized, the emigration country loses human capital when people with human capital leave their origin. Consequently (and according to the theory of public goods) the production of human capital in the emigration countries is too low in comparison to a world without migration. Bhagwaty (1976a) shows the existence of a negative fiscal externality on the emigration country, if education is publicly subsidized. If the economy wide education is expanded in response to emigration the governmental deficit increases ceteris paribus. This effect is accelerated, if the newly educated are less gifted. Furthermore educational subsidies can be regarded as an investment of the old generation into their pension which is lost in case of permanent emigration (Grubel & Scott, 1977).
of a specific coordination inside the EU can generate strong negative externalities to the new entry. There exist numerous examples of the FC and the “migration competition” in EU.

For example, much of the discussion of the migration of highly skilled has focused on the potential BD from east to west. Statistics\(^2\) show a migration of scientific personal from Eastern Europe and the former Soviet Union to Western Europe.

Tackling financial barriers to researcher mobility could be done by following the Scandinavian example. A recent Swedish policy of reducing the tax burden for high level researchers going to Sweden for three years. Similar initiatives are being implemented in Denmark. Furthermore, scrutiny of the work permit system of most European states indicates clearly that professional, managerial and technical constitute the bulk of those accepted: in UK for example, they are accounted for around 80 per cent of all work permit issues. Within Western Europe, a complex series of “Brain exchanges” has developed, superimposed upon free movement system inherent in the operation of the European Economic Area (EEA).

In the last years, several countries in Western Europe have taken steps to increase their immigration of skilled workers. Germany has introduced a “Green Card” system to attract 20,000 IT workers to fill shortages, although there are still difficulties in finding enough potential migrants with the necessary skills. The UK government has also adopted a more positive attitude towards skilled labor migration, making changes to the work permit system which are designed to increase the inflow of a range of skilled occupations, including IT and medical personnel\(^3\).

This paper is organized as follows. In Section 2 we describe the structure of the model. In Section 3 we solve the model in an “autarchic context” where there is no migration. In Section 4 we describe the “restricted mobility case” where there is migration of educated worker inside the EU. In Section 5 we describe the “enlarged mobility case”, where we analyze the entrance of a new state into the EU.

\(^2\)See, for example, Wolburg (1996, 1997) and Wolburg & Wolter (1997)

1.1 Survey of the literature on the Brain Drain

Brain Drain is an expression of British origin commonly used to describe one of the most sensitive areas in the transfer of technology. It refers to skilled professionals who leave their native lands in order to seek more promising opportunities elsewhere.

Causes Migration of this type has been linked to several possible causes. The most frequently cited are the lack of employment opportunities for returning graduates, lower salary levels in the indigenous country, preference of graduates to live abroad, asymmetric information in the labor market\(^4\), different fiscal and social packages\(^5\) and the incentive to finance education\(^6\).

\(^4\)Kwok Viem (1982) suggests as cause for the exodus of foreign-trained students: asymmetric information in the labor market. That is, employers in the country training the students have a more accurate (but not necessarily more optimistic) judgment of the true productivity of students than have employers in the students' native country. This asymmetry results from foreign employers' familiarity with their own academic system and with the curricula offered by different schools; their past experience in hiring large numbers of both foreign and domestic graduates of their universities; and the in-depth interviews which are a regular part of the employment process in many Western countries, and particularly in the United States. He also shows that the graduates who do return tend to be those of lesser productivity than those who remain abroad.

\(^5\)When the choice is among countries, rather than among municipalities, mobility is much less, and the fiscal and social packages can be, and are, much more different. But the basic point [based on the model of Charles Tiebout's (1956) which explains how political jurisdictions can offer quite different packages of services and tax rates, and where individuals vote with their feet to find the packages most suiting their tastes and values] remains those who move face, not only different taxes rates but different patterns and types of public services, as well. Perhaps even more relevant to the study of migration of the well-educated and well-off countries differ, not only in their average taxes rates and in the size and efficiency of their public services and transfer payments, but also in the distribution of costs and benefits among different groups of taxpayers and beneficiaries. Among those who do migrate whether domestically or abroad, the highly educated are over-represented, partly because they are more likely to possess skills that are in demand, but also because they are more likely to have contacts in and knowledge about possible places to move. To extent that migration of the highly skilled may to be triggered by different factors, survey data reported by Grubel and Scott (1966, 1976) suggests that job opportunities and challenges are even more important to the highly educated. It is also true that for many such workers, particularly in health care, education, and government-supported fundamental research, the 1990s have seen large cuts in government spending induced by budget pressures. For example the pre tax and post tax distributions of the income have become more unequal in the US relative to Canada. All of these factors may have increased the net attraction of migration for the better-educated. [Helliwell 1999]

\(^6\)Beyond the overall package of taxes and public services, special attention has been given, especially in the context of BD discussions, to the structure of education finance. Many commentators have argued that because BD migrants take their taxpayer-supported educational capital with them, they should face an exit tax or an educational loan that is forgiven only for those who stay and work where they acquired their subsidized education.
Welfare and growth effects  The BD literature is linked to the concept of "human capital" and its measurement has been developed by Schultz (1960) and Becker (1964). Positive technological externalities of immigration arise by the additional capital that is available to the host economy. The theoretical argument goes back to the development literature of 50's (Hirschman, Myrdal, Perroux, Wallerstein) They have seen a revival in the mid-1980's with the birth of the so called New Growth Theory. Starting with Paul Romer (1986, 1987, 1990) and Robert Lucas (1988) the immigration of skilled migrants has been evaluated as stimulating for the dynamics of economic growth.

The possibility that the welfare of those remaining in the LDCs could be reduced by an outflow of educated manpower had been recognized in the literature as well. From the work of Grubel and Scott, Berry and Soligo, and Harry Johnson in the 1960s, the main conclusion was that welfare of non-migrants would fall only if the migrants' contribution to national output were greater than their income (or consumption in a static model). For a number of reasons the literature believes that the conditions for a BD to be welfare-deteriorating are often verified. Differently from the standard results, Mountford (1997) find some conditions in which BD generates positive externalities for the regions where some educated workers migrate.

Furthermore there are different studies about the BD and considerable attention has been given to a proposal of Bhagwati's for a "brain drain tax" which would reduce the incentives for such a migration to take place. Finally there are different methodologies to compute these benefits and costs. For example Usher (1977) suggests that "an assessment of the costs and benefits of migration need take account of the fact that a large portion of a country's property is publicly owned, so that a migrant on going from one country to another must as a rule abandon his share of publicly owned property of origin and acquire a share of publicly owned property in his country of destination. The emigrant exchange his right to send his children to school in his country of origin for the right to send his children in his country of destination, reducing the need for new school building in the former country and increasing it according in the latter". Grubel and Scott (1976) point out that "since our concern is with the gains to the United States, it is appropriate to use U.S. prices, so that our computations amount to estimating what it would have cost to bring a native American to the level of education held by the average immigrant at the time he arrives".

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7 He shows that when migration is not a certainty, a BD may increase average productivity and equality in the source economy even though average productivity is a positive function of the past average levels of human capital in an economy.

8 See Bhagwati and Hamada (1974); Bhagwati and Rodriguez (1975a; 1975b); McCulloch and Yellen (1975); Blomevits (1986); Bodenhofer (1967); Sjaastad (1962); Rodriguez (1975); Romans (1974); Edding and Bodenhofer (1966); Johnson (1965); Kesselman (2000).

Public good provision effects If one assumes that the allocation to human capital investments made by the region (e.g., local expenditures or state support for education in the national framework and national investment outlays in the international setting) depends on the returns expected to accrue internally (as the individual investment decisions are assumed to be determined by expected private returns), the existence of external benefits from investments made by a region will cause suboptimal allocation judged from marginal productivity rules.\(^{10}\)

The cost of education would be irrelevant to the assignment of gains and losses from migration if each man paid the full cost of his education, but it becomes important when education is subsidized or provided free of charge by the state. It is sometimes supposed that there is an implicit contract between the student and the state in which the latter supplies education at less than cost on the understanding that the net income of educated labor will one way another, be less than its marginal product. The immigration of educated labor generates the benefits of this arrangement without the cost.\(^{11}\)

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10 There is a large literature on the efficiency properties of a system of competing regional jurisdictions. One strand is the fiscal externality literature. The standard conclusion in this literature is that there is an externality associated with an individual’s migration that generally leads to an inefficient distribution of population across region.

11 Education in general accounts for as much as of 5% of GNP, and 10% or more of public spending in advanced industrialized countries, with public funding covering, on average, almost 90% of education costs in these countries. Higher education typically accounts for 15-20% of overall education expenditures. Migration of skilled labor implies that those who pay the bill for public higher education may find it difficult to fully capture its benefits.
2 The model

The model we analyze in this paper is based on Mountford (1997)\textsuperscript{12}.

The literature on BD identifies a negative externality of BD on regions growth. Differently from standard results, the Mountford’s model finds some conditions in which BD generates positive externalities for the regions where some educated workers migrate\textsuperscript{13}. This interesting result opens the way for a better identification of the negative effect generated by the mutual interaction of FC and BD. We extend the Mountford’s model in different directions. First, we introduce a role for the government in the educational decisions of agents though the introduction of educational subsidies and taxation. Second, we study the specific case in which the region analyzed is a member of the European Union where the mobility of workers is freely allowed\textsuperscript{14}.

The model analyses a small open economy, under perfect capital mobility, with only one good produced under constant returns to scale by two factors, capital and efficiency units of labor. There is a continuum of agents within each generation\textsuperscript{15}. The education decision is assumed to be a discrete choice: agents can choose either to be educated or not be educated.

Productive sector Let us define $K_t$ to be the total amount of capital in time period $t$ and $L_t$ to be the efficiency units of labor. The productivity of labor (or the state of technology) in period $t$ is given by $\lambda_t$. Production is generated by a constant returns to scale production functions. The output produced at time $t$, $Y_t$, is

\[ Y_t = F(K_t, \lambda_t L_t) = f(k_t)\lambda_t L_t \]

where

\[ k_t = \frac{K_t}{\lambda_t L_t} \]

We make the standard assumptions about this function, namely

\textsuperscript{12}This model is a simple version of Miyagiwa (1991) studing of the model of the brain drain and human capital formation.

\textsuperscript{13}See note (7)

\textsuperscript{14}In this analysis we do not take in account redistribution policies of the governments. If we take in account the redistribution policies we accentuate the negative effects of the FC. According to the literature we will obtain less redistribution and less provision of public good with respect to the efficient value (which could be obtained in the absence of mobility or in the presence of coordination among jurisdictions). In Giannoccolo and Marchand (mimeo) we have analyzed the negative externalities due to FC and to educated migration and we have analyzed their effect on the redistribution policies and on the supply of education as public good.

\textsuperscript{15}For simplicity we normalize the population in each generation to unity.
\[ f(k) > 0, f'(k) > 0, f''(k) < 0, \forall k > 0 \quad (3) \]

and the “Inada conditions”

\[ \lim_{k \to 0} f(k) = 0, \lim_{k \to 0} f'(k) = \infty, \lim_{k \to \infty} f(k) = 0 \quad (4) \]

Factor prices are determined in the standard way by the factor’s marginal product, thus the net return to capital \( r_t \) is equal to \( f'(k_t) + (1 - \delta) \), where \( \delta \) is the capital stock depreciation rate. Let us assume for simplicity that the world is in a steady state equilibrium and thus that the world net rate of return, \( r^* \), is constant. Due to the perfect capital mobility and the narrow dimension of the economy, this fixes the domestic net rate of return to capital, \( r_t \), equal to \( r^* \) and thus fixes the domestic capital to efficiency labor ratio, \( k_t \), as well. Thus \( k_t = k \ \forall t \) where \( k \) is a constant. Given the level of technology \( \lambda_t \), \( k \) determines the wage rate per efficiency unit of labor, \( w(k) \).

\[ w_t = \lambda_t [f(k) - kf'(k)] = \lambda_t w(k) \quad (5) \]

**The distribution of ability** Individuals possess different levels of latent ability, where \( e^i \) denotes the latent ability level of individual \( i \). These latent abilities are assumed to be distributed over the closed interval \([0, E]\) according to the density function \( g(e^i) \), where, by definition

\[ \int_0^E g(e^i) \, de^i = 1; \quad g(e^i) > 0 \ \forall e^i \in [0, E] \quad (6) \]

Let us assume that all generations have latent abilities which are picked up from the same distribution and that the abilities of children are independent from the abilities of their parents\(^{16}\).

**The growth externality** Let us assume that there is an economy wide growth externality related to the proportion of educated workers in the economy in the previous period \( s_{t-1} \). Thus we model \( \lambda_t \) to be a positive function of the proportion of educated workers in the previous period, that is

\[ \lambda_t = \lambda(s_{t-1}) \text{where} \ s_{t-1} = \int_{e^*_{t-1}}^E g(e^i) \, de^i \quad (7) \]

Let’s also assume that \( \lambda(0) = 1 \) and that \( \lambda(1) \) is finite.

\(^{16}\)In a second step this assumption could be substituted by the assumption of externalities of the education of the precedent education.
Region’s gain from education  Let us define $\Omega^J_t$ a measure of the welfare of region $J^{17}$.

$$\Omega^J_t \equiv s^J_t \left[ \lambda \left( s^J_{t-1} \right) w(k) \int_{\epsilon^{-1}}^{E} d\epsilon_{i,t} \right] + s^J_t \left[ \lambda \left( s^J_{t-1} \right) w(k) \int_{\epsilon^{-1}}^{E} d\epsilon_{i,t} \right] +$$

$$- s^J_t \left[ c \left( 1 + r^* \right) \right] + Z \left( s^J_t + s^J_{I,t} \right) \text{ with } J, I = A, B \quad (8)$$

where $s^J_t$ is the number of agents which are educated in region $J$ at time $t$ and work in region $J$ at $t + 1$; $s^J_{I,t}$ is the number of agents which are educated in region $I$ at time $t$ and work in region $J$ at time $t + 1$. The first and the second term on the right hand side of (8) denote the total production of region $J$ due to the presence of educated workers. The third term corresponds to the education costs in region $J$. The fourth term is an increasing function of the number of educated agents and denotes all the other gains (not only pecuniary) that the presence of educated gives to region $J$.

Moreover, let us assume that $Z(\cdot) \geq 0$, $Z(0) = 0$ and $Z'(\cdot) > 0$

Education’s role of the government: taxes and subsidies  Let us now assume that the government subsidizes part of the educational costs sustained $^{18}$. The government influences the education decision of the agents by taxing the educated workers and covering part of their education costs $^{19}$. Let us define the education subsidy, $\gamma_t$, as the unit of output reimbursed to educated agents in generation $t$. Thus, the effective individual cost of being educated becomes $(c - \gamma_t)(1 + r^*)$ with $\gamma_t \in [0, c]$.

Let us define $T_t$ to be the marginal rate of taxation of educated workers in generation $t$. Introducing taxation, the wage rate per efficiency unit of labor becomes

$$w_t = w(k) \left[ 1 - T_t \right].$$

For each time $t$ the government $J$ maximizes the $\Omega^J_t$ subject to a balance constraint for each generation $t$. Let us, furthermore, assume that the government decides independently by the positive externality of education of generation $t$ for the future generations and that the balance constraint is binding.

So we have

$$[\gamma_t (1 + r^*)] s^J_t = [\lambda_t w(k) T_t] h^J_t \quad (9)$$

$^{17}$This is a non-standard function of social welfare. It is a gross measure of the region’s gain derived from education and it allows to compare the different scenarios analyzed in this model.

$^{18}$These subsidies are given directly to educated. The analysis does not change if we consider an equivalent average education investment of the government (academic and research infrastructures, school places, teachers, etc.).

$^{19}$We assume that only the educated workers are taxed. Then we focus our analysis on a particular quota of the taxation reserved to pay the education’s subsidies.
where \( s_t^J \) is the number of educated workers of generation \( t \) born in region \( J \) and \( \bar{h}_t^J \) is the number of educated agents of generation \( t \) those work in region \( J \).

The individual’s decision to be educated Agents live in a overlapping generations world and live for three periods, deriving utility only from the third period consumption\(^{20} \). In their first period of life agents can invest resources in education. They have not resources of their own, so they must borrow from the capital market at the world’s rate of interest, \( r^* \). Let us assume the cost of education to be fixed at \( c \) units of output. Agents that invest in education obtain \( e_i \) efficiency units of labor in their second period of life, where \( e_i \) is the level of the latent ability of agent \( i \). Furthermore let’s assume that the agents who do not invest in education have only one efficiency unit of labor in their second period of the life. Agents can only work in their second period of life and in this period the agent must repay the debt of the first period. In the third period they are retired and use their savings to consume. All agents have the same preferences and access to the same technology, although they do not have the same levels of latent ability.

The optimal decision for agent \( i \) will be to invest in education if

\[
[1 - T_t] \lambda_t w(k) e_i > \lambda_t w(k) + (c - \gamma_t) (1 + r^*)
\]

In each period \( t \) we assume that \( s_{t-1}^J \) is given, then

\[
\lambda \left( s_{t-1}^J \right) w^J (k) = a
\]

Thus, all agents with a latent ability greater than \( e^* \) will invest in education, were \( e^* \) is uniquely defined by the following equality:

\[
e^* = \frac{a + (1 + r^*) \left[ c - \gamma_t^J \right]}{(1 - T_t^J) a}
\]

Let us assume that the model is such that \( e^*_t \in [0 + \varepsilon, E - \varepsilon] \), where \( 0 < \varepsilon < \frac{E}{2} \).

Dynamics and steady state productivity The only dynamics in the model derive from the growth externality. From equation (7) it is clear that the proportion of workers who are educated at time \( t \) is an increasing function of the proportion of workers who were educated at time \( t - 1 \), that is

\[
s_t = \psi (s_{t-1})
\]

\(^{20}\)The introduction of three periods is necessary because agents borrow to finance their first period of life and they can evidently not borrow from agents who will not be alive to be repaid in the next period.
Since

$$\frac{\partial e_t^*}{\partial s_{t-1}} = \frac{\lambda (s^j_{t-1}) (1 + r^*) (c - \gamma^j_t)}{\lambda^2 (s^j_{t-1}) w(k) [1 - T^j_t]} \quad (14)$$

Thus

$$\frac{\partial s_t}{\partial s_{t-1}} = g (e^*_j) \left( \frac{\lambda (s^j_{t-1}) (1 + r^*) (c - \gamma^j_t)}{\lambda^2 (s^j_{t-1}) w(k) [1 - T^j_t]} \right) \quad (15)$$

Let us assume that \( E \) is high enough so that the most able worker will always chooses to be educated even if no one was educated in the previous period. Since we know that agent \( i \) with \( e^i = 0 \) will never choose to be educated, then this implies that there must exist at least one steady state equilibrium for \( s_t \), which we denote as \( \bar{s} \). Whether this is a unique steady state depends on the properties of the function \( \lambda_t = \lambda (s_{t-1}) \). If this function has convex regions, representing “critical masses” of educated people in the economy, then there my be multiple steady states. The unique Steady State case is depicted in figure 1.

![Diagram](image-url)

**Figure 1:** \( \Psi^0 (s_t) \) indicates the proportion of educated agents in autarkic case, when there is not migration and there are government’s subsidies. \( \Psi^m (s_t) \) indicates the proportion of educated agents in autarkic case, when there is not migration and there are not government’s subsidies to educated.
3 Autarchic case

In this section we solve the model in an “autarchic context” where there is not migration.

Let us recover the timing of the model.

Time $t$ The government decides $T_i$ and $\gamma_i$ and each agent $i$ decides whether to invest in education or not according to their latent ability $e_i$. Agents who invest in education receive $\gamma_i$ and borrow $c - \gamma_i$ from the capital market.

Time $t + 1$ The educated agents pay $T_i$ to their government and repay the debt of the first period of the life.

Time $t + 2$ All agents are retired and use their savings to consume.

It is possible to solve the maximization problem through the Backward Induction method (BI hereafter). First we solve the maximization problem of the agents at time $t+1$ and then we solve the maximization problem of government.

The agent’s decision is given by equation (12).

Let us define $\Omega_{1,t}^J$ a measure of the welfare of region $J$ in the autarchic case when the government introduces taxes and subsidies. Equation (8) becomes

$$\Omega_{1,t}^J \equiv s_{t}^g,J \left[ \lambda \left( s_{t-1}^g,J \right) w \left( k \right) \int_{e_i^g,J}^E d e_i,t - c \left( 1 + r^* \right) \right] + Z \left( s_{t}^g,J \right) \forall t \text{ with } J = A, B$$

(16)

Furthermore, in an autarchic case we have $s_{t}^J = h_{t}^J$ so the equation (9) becomes

$$\gamma_{t}^J = \frac{a}{(1 + r^*)} T_{t}^J$$

(17)

The maximizations for the government is

$$\max_{T_{t}^J} s_{t}^g,J \left[ a \int_{e_i^g,J}^E d e_i,t - c \left( 1 + r^* \right) \right] + Z \left( s_{t}^g,J \right)$$

(18)

The First Order Condition is

$$\text{Foc} \left( T_{t}^J \right) \frac{\partial s_{t}^g,J}{\partial T_{t}^J} \left[ a \int_{e_i^g,J}^E d e_i,t - c \left( 1 + r^* \right) \right] + s_{t}^g,J \left( -a \frac{\partial e_i^g,J}{\partial T_{t}^J} \right) + Z' \left( \cdot \right) \frac{\partial s_{t}^g,J}{\partial T_{t}^J} = 0$$

(19)
By \( \frac{\partial s_{i}^{g,J}}{\partial t_{i}} = - \frac{\partial s_{i}^{g,J}}{\partial t_{i}} g ( e_{i}^{g,J} ) \) and by the optimal value of \( e^{g,J} \) equation (19) becomes

\[
ag \left( e_{i}^{g,J} \right) \int_{e_{i}^{g,J}(T_{i}^{J})}^{E} dc_{i,t} - \left[ (1 - T_{i}^{J}) a \left( e^{g,J} - 1 \right) \right] g \left( e_{i}^{g,J} \right) + as_{i}^{g,J} + \\
+ Z'(\cdot) g \left( e_{i}^{g,J} \right) = 0 \quad (20)
\]

The optimal value of the taxation \( T_{i}^{J} \) (and indirectly, by the equation (17), the optimal value of the subsidies to educated) is

\[
T_{i}^{J} = 1 - \left[ E - e_{i}^{g,J} \right] + \frac{1}{g(e_{i}^{g,J})} \int_{e_{i}^{g,J}}^{E} g ( e' ) dc_{i,t} + Z'(\cdot) \quad (21)
\]

### 3.1 Role of government on the region growth

If we analyze the model in absence of government subsidies (Mountford case) equation (12) becomes

\[
e^{*J} = \frac{a + (1 + r^{*}) c}{a} \equiv e^{*m} \quad (22)
\]

Comparing expression (12) and (22) we have that

\[
\begin{cases}
if \; T_{i}^{J} = 0 & \text{then} \; e_{i}^{*J} = e_{i}^{*m} \\
if \; T_{i}^{J\text{Max}} > T_{i}^{J} > 0 & \text{then} \; e_{i}^{*J} < e_{i}^{*m} \\
if \; T_{i}^{J} \geq T_{i}^{J\text{Max}} & \text{then} \; e_{i}^{*J} \geq e_{i}^{*m}
\end{cases} \quad (23)
\]

where

\[
T_{i}^{J\text{Max}} = \frac{\gamma_{i}^{J} (1 + r^{*})}{a + (1 + r^{*}) c} \quad (24)
\]

Let us define \( \Omega_{J,t}^{J} \) as a measure of the welfare of region \( J \) in autarchic context and in absence of government subsidies and taxations.

\[
\Omega_{J,t}^{J} \equiv s_{i}^{m,J} \left[ \lambda \left( s_{i-1}^{m,J} \right) w \left( k \right) \int_{c_{i}^{m}}^{E} dc_{i,t} - c (1 + r^{*}) \right] + Z(s_{i}^{m,J}) \forall t \text{ with } J = A, B \quad (25)
\]

Summing up, \( \forall T_{i}^{J} \in [0, T_{i}^{J\text{Max}}] \) we have that \( \int_{c_{i}^{m}}^{E} g ( e' ) dc_{i,t} \geq \int_{c_{i}^{m}}^{E} g ( e' ) de' \).

In presence of subsidies to educated workers, there is an increase in the number
of the educated in region \( J \). This increase implies a positive externality on the growth of the economy \( J \). Comparing equation (16) and (25) it is straightforward to verify that \( \Omega^1_J > \Omega^0_J > 0 \ \forall T^J_t \in [0, T^{J_{\text{Max}}}_t] \), with \( J = A, B \).

The only difference between expression (16) and (25) is the number of educated workers. In figure (2) is shown the effect of taxation on the welfare function. The government increase the number of educated workers by decreasing the education cost of the agents with lower latent ability and financing these subsidies by taxing more the agent with higher latent ability. When the number of educated workers increase two welfare and growth effects arise. First, if there are multiple steady state equilibria then the economy can move from a low to a high education steady state. Second, there is a positive effect on welfare which is show by function \( Z(\cdot) \). If we assume that in \( Z(\cdot) \) explains also the more future growth of a region which increases the number of their educated workers then it is straightforward to verify that the introduction of subsidies implies more growth.

The results obtained in this section can be summarized by the following proposition.

**Proposition 1** In an autarchic context, the introduction of government’s subsidies to educated increases the number of educated agents and has positive effect on the growth of the economy if and only if \( 0 < T^J_t \leq \frac{\gamma^J_t(1+r^*)}{(1+r^*)+\gamma^J_t} \equiv T^{J_{\text{Max}}}_t \). In this situation, the presence of the government moves the economy in a new context which is PO with respect to the Mountford case. The more \( T^J_t \) is closed to \( T^{J_{\text{Max}}}_t \), the more the number of educated agents is closed to the maximum and so the more is the growth of the economy.

The steady state of the two cases is shown in figure 1.

![Figure 2: Taxation effects](image-url)
4 Mobility case (restricted case)

Let us introduce in the model workers mobility. We examine the case in which only the educated agents can migrate (BD)\(^{21}\). Let us assume that a small economy is member of an economic union like the EU so that the educated workers can migrate inside the union without impediments. We assume that there are only two regions A and B\(^{22}\). In this section, we also assume that for the region inside the Union there is a total impediment to migrate outside the Union.

The timing of the model is the following:

**Time t** Government decides \(T_t\) and \(\gamma_t\) and each agent \(i\) decides whether to invest in education or not according to their latent ability \(e^i\). Agents investing in education receive \(\gamma_t\) and borrow \(c - \gamma_t\) from the capital market.

**Time \(t + 1\)** Educated agents decides whether to migrate or not. They pay \(T_t\) to the government of the region in which they work and they repay the debt of the first period of the life.

**Time \(t + 2\)** All agents are retired and use savings to consume.

It is possible to solve the maximization problem through the BI method. First, we solve the maximization problem of the agents at time \(t + 1\) and then we solve the maximization problem of government. In period \(t\), the agent \(i\) chooses whether if educate himself or not given the government decisions about \(T_t^J\) and \(\gamma_t^J\) with \(J = A, B\). The optimal decision for agent \(i\), born in region \(J\), is to invest in education if

\[
\arg \max \left\{ \left[1 - T_t^J\right] \lambda_t^J w^J(k); \left[1 - T_t^J\right] \lambda_t^J w^J(k) \right\} e^i > \lambda_t^J w^J(k) + (c - \gamma_t^J) (1 + r^*) \tag{26}
\]

In each period \(t\) we assume that \(s_{t-1}^{a,J}\) and \(s_{t-1}^{a,J}\) are given then we can write

\[
\lambda \left( s_{t-1}^{a,J} \right) w^J(k) \equiv a \tag{27}
\]

\[
\lambda \left( s_{t-1}^{a,J} \right) w^J(k) \equiv b \tag{28}
\]

Thus, all agents with a latent ability greater than \(e^{**}_t\) invest in education, were \(e^{**}\) is uniquely defined by the following equality:

\(^{21}\)This hypothesis is compatible with the assumption that there are not mobility costs. The results do not change if we assume that the costs of mobility (transfers' costs, social costs, integration's costs, etc...) are very small for educated workers (closed to zero) and very high for non educated. It is furthermore possible extend the analysis to the case in which there are not educational requirement for emigration but becomes hard distinguish the BD aspects of the workers migration.

\(^{22}\)It is possible, without changing the results, assume that the region B represents the rest of the Union and so the assumption that the region it is a small open economy is verified.
\[ e_{gJ}^* = \frac{a + (c - \gamma_J) (1 + r^*)}{\arg \max \left\{ \left[ 1 - T_J^* \right] a; \left[ 1 - T_I^* \right] b \right\}} \] (29)

The same result follows for agent \( i \), born in region \( I \).

\[ e_{gI}^* = \frac{b + (c - \gamma_I) (1 + r^*)}{\arg \max \left\{ \left[ 1 - T_J^* \right] a; \left[ 1 - T_I^* \right] b \right\}} \] (30)

In this model we assume that for the educated agents there is not mobility costs so that educated workers decide whether migrate or not in response to different net wage that they receive. Their future wage is related to the taxation/subsidies policies of the governments and to the differences of technology between regions. It is straightforward to verify that the educated workers will prefer stay in region \( J \) if

\[ T_J^* < \frac{a - b + bT_I^*}{a} \] (31)

We can therefore distinguish three different states of the world:

Case (1) \( T_J^* < \frac{a - b + bT_I^*}{a} \) all educated migrate in region \( J \).

Case (2) \( T_I^* > \frac{a - b + bT_I^*}{a} \) all educated migrate in region \( I \).

Case (3) \( T_I^* = \frac{a - b + bT_I^*}{a} \) there is no migration.

In each state of the world, the government maximizes \( \Omega^J_t \) subject to a balance constraint and decides the optimal value of taxation independently by the positive externality of education of generation \( t \) for the future generations.

Let us define \( \Omega^{J,I}_{2,t} \) a measure of the welfare of region \( J \) in mobility context and in presence of government subsidies and taxations.

\[ \Omega^{J,I}_{2,t} \equiv s_{t+1}^{g,J,I} \left[ a \int_{e_{g,J}^{*,t}}^{E} d\epsilon_{i,t} \right] + s_{t+1}^{g,I,J} \left[ b \int_{e_{g,I}^{*,t}}^{E} d\epsilon_{i,t} \right] - s_{t}^{g,J} \left[ c \left( 1 + r^* \right) \right] + Z(s_{t}^{g,J,I} + s_{t}^{g,I,J}) \] with \( J, I = A, B \) (32)

Let us define for regions \( J \) and \( I \)

\[ A = a \frac{s_{t}^{g,J,I} + s_{t}^{g,I,J}}{s_{t}^{g,J}} \] (33)

\[ B = b \frac{s_{t}^{g,J,I} + s_{t}^{g,I,J}}{s_{t}^{g,I}} \] (34)
The balance constraint of government $J$ is

$$[\gamma_t^J (1 + r^*)] s_t^{g,J} = (aT_t) [s_t^{g,J,J} + s_t^{g,I,J}]$$  \hspace{1cm} (35)$$

By the assumption that it is binding in each period, we obtain the following equality

$$\gamma_t^J = \frac{A}{(1 + r^*)} T_t^J$$  \hspace{1cm} (36)$$

The government maximization problem is for the region $J$

$$\text{Max}_{T_t^J} : s_t^{g,J,J} \left[ a \int_{e_t^{g,J}} \text{de}_{i,t} \right] + s_t^{g,I,J} \left[ b \int_{e_t^{g,I}} \text{de}_{i,t} \right] - s_t^{g,J} [c (1 + r^*)] + Z(s_t^{g,J,J} + s_t^{g,I,J})$$

subject to:  \hspace{1cm} (1 + r^*) \gamma_t^J s_t^{g,J} = T_t^J a \left( s_t^{g,J,J} + s_t^{g,I,J} \right)$$  \hspace{1cm} (37)$$

Let us analyze the case in which $a > b$.

Then, equation (36) can be written

$$T_t^J = \eta + (1 - \eta) T_t^J$$  \hspace{1cm} (39)$$

The three cases, depicted in figure (3), become

Case (1) $T_t^J < \eta + (1 - \eta) T_t^J$

Case (2) $T_t^J > \eta + (1 - \eta) T_t^J$

Case (3) $T_t^J = \eta + (1 - \eta) T_t^J$

Let us now analyze the government’s decision by using the BI for each different states of the world.

Case (1): $T_t^J < \eta + (1 - \eta) T_t^J$ We have that $s_t^{g,J,J} = s_t^{g,J}$; $s_t^{g,I,J} = s_t^{g,J}$ and $s_t^{g,I,I} = s_t^{g,I,J} = 0$

The maximization problem of the government becomes

$$\text{Max}_{T_t^J} : s_t^{g,J} \left[ a \int_{e_t^{g,J}} \text{de}_{i,t} - c (1 + r^*) \right] + s_t^{g,I} \left[ a \int_{e_t^{g,I}} \text{de}_{i,t} \right] + Z(s_t^{g,J})$$

subject to: \hspace{1cm} (1 + r^*) \gamma_t^J s_t^{g,J} = T_t^J a \left( s_t^{g,J,J} + s_t^{g,I,J} \right)$$  \hspace{1cm} (40)$$
Figure 3: The three cases for the "different technology scenario"

From $A = a \left[ 1 + \frac{s^I_J}{s^I_x} \right]; B = 0$, the (29) becomes

$$e^{**gJ}_t = \frac{a + (1 + r^*) c - a \left[ 1 + \frac{s^I_J}{s^I_x} \right] T^J_t}{[1 - T^J_t] a} \leq e^{*gJ}_t \leq e^{*J}_t$$

(41)

The (30) becomes

$$e^{**gI}_t = \frac{b + (1 + r^*) c}{[1 - T^I_t] a} > e^{*gI}_t$$

(42)

It is furthermore straightforward to verify that

$$e^{*I}_t \geq e^{**gI}_t > e^{*gI}_t$$

(43)

The First Order Condition is

$$\text{Foc}(T^J_t): \frac{\partial s^I_J}{\partial T^J_t} \left[ a \int_{e^{**gJ}_t(T^J_t)}^{E} d e_{i,t} - c (1 + r^*) \right] + s^I_J \left( -a \frac{\partial e^{**gJ}_t}{\partial T^J_t} \right) + Z'() \frac{\partial s^I_J}{\partial T^J_t} = 0$$

(44)

or

$$a g \left( e^{**gJ}_t \right) \int_{e^{**gJ}(T^J_t)}^{E} d e_{i,t} - c (1 + r^*) g \left( e^{**gJ}_t \right) + a s^I_J + Z'() g \left( e^{**gJ}_t \right) = 0$$

(45)
from $e_t^{*gJ}$ we compute $[1 - T_t^J] a \left( e_t^{*gJ} - 1 \right) + a_s^{gJ} T_t^J = (1 + r^*) c$

We then found the optimal value of the taxation $T_t^{**J}$ (and, indirectly, the value of the subsidies to educated).

\[
T_t^J = 1 - \frac{E - e_t^{*gJ}}{g(e_t^{*gJ})} \int_{e_t^{*gJ}}^E g(e^J) de + Z'(\cdot) \equiv T_t^{**J}
\]  

Comparing equation (46) and (21), it is straightforward to verify that from (41) derives

\[ T_t^{**J} < T_t^J \]  

For region $I$, the maximization problem in case (1) becomes

\[
\max_{T_t^I} : s_t^{gJ} [\gamma_t J] 
\text{sub to} : (1 + r^*) \gamma_t J = 0 \text{ or } \gamma_t = 0
\]

It is easy to verify that the maximization is solved by

\[ T_t^{**I} = 0 \]

where $T_t^{**I} < T_t^I$ and where the subsidies are zero.

Comparing the region gains we have that

\[
\begin{align*}
\Omega_{2, t}^{J} &> \Omega_{1, t}^{J} > \Omega_{0, t}^{J} > 0 \\
\Omega_{1, t}^{I} &> \Omega_{0, t}^{I} > \Omega_{2, t}^{I}
\end{align*}
\]

Case (2): $T_t^J > \eta + (1 - \eta) T_t^I$ It is symmetric to the case (1), so we have $s_t^{g^{J,I}} = s_t^{g^{J,I}}$ and $s_t^{g^{J,J}} = s_t^{g^{J,J}} = 0$

From $A = 0$ ; $B = b \left[ 1 + \frac{s_t^{g^{J}}} {s_t^{*gJ}} \right] \geq b$

we have

\[
\begin{align*}
e_t^{*gI} &\leq b \left[ 1 + r^* \right] c - b \left[ 1 + \frac{s_t^{g^{J,I}}} {s_t^{*gJ}} \right] \leq e_t^{*gI} \leq e_t^{*gI} \\
e_t^{*gJ} &\leq \frac{a + (1 + r^*) c}{1 - T_t^J} \leq e_t^{*gJ}
\end{align*}
\]
it is straightforward to verify that
\[ e_t^{g_J} \geq e_t^{**g_J} > e_t^{g_J} \]  \hfill (54)

\[ T_t^{**g_I} = 1 - \frac{\left[ E - e_t^{**g_J} \right] + \frac{1}{\varphi(e_t^{**g_J})} \int_{e_t^{**g_J}}^E \varphi \left( e^i \right) \, de^i + Z' \left( \cdot \right) - e_t^{**g_J} \left( -1 \right) - e_t^{**g_J}}{e_t^{**g_J}} \]  \hfill (55)

Comparing the region gains we have
\[ \Omega_{I,t}^{J^*} > \Omega_{J,0,t}^{J^*} > \Omega_{J,2,t}^{J^*} \]  \hfill (56)
\[ \Omega_{2,t}^{I^*} > \Omega_{I,0,t}^{I^*} > 0 \]  \hfill (57)

Case (3): \( T_t^J = \eta + (1 - \eta) T_t^I \) This is the autarchic case. The solutions are
\[ s_t^{g_I,J} = s_t^{g_J} ; s_t^{g_J,J} = s_t^{g_J} \] and \( s_t^{g_I,J} = s_t^{g_J} \)
\[ \varepsilon = a ; B = b \]
\[ e_t^{**g_J} = e_t^{**g_J} ; e_t^{**g_J} = e_t^{**g_J} \]
\[ e_t^{*g_I} = e_t^{*g_I} ; e_t^{*g_I} = e_t^{*g_I} \]
\[ T_t^{*g_I} = T_t^{*g_I} ; T_t^{*g_I} = T_t^{*g_I} \]  \hfill (58)

We have the same results of the autarchic case.

\[ \Omega_{0,t}^{J^*} = \Omega_{2,t}^{I^*} = \Omega_{I,t}^{I^*} = \Omega_{J,t}^{J^*} = 0 \]  \hfill (59)

We can resume the results of the BI in the following game in compact form\footnote{The true compact form is}

| \( T_t^{g_I} = 0 \) | \( T_t^{g_J} = \eta - \varepsilon \) | \( T_t^{g_I} = T_t^{*g_J} = T_t^{g_I} \) |
| \( T_t^{g_I} = T_t^{*g_J} = T_t^{g_I} \) | \( T_t^{g_I} = \Omega_{2,t}^{I^*} ; \Omega_{2,t}^{J^*} \) | \( T_t^{g_I} = \Omega_{I,t}^{I^*} ; \Omega_{I,t}^{J^*} \) |
| \( \varepsilon > 0 \) and \( \varepsilon \to 0 \) |

were \( \varepsilon > 0 \) and \( \varepsilon \to 0 \)

The only NE sub game perfect equilibrium is \( T_t^{g_J} = \eta + \varepsilon \) and \( T_t^{g_I} = 0 \). This NE is not PO. The coordinated solution, \( T_t^{g_I} = T_t^{*g_J} = T_t^{g_I} \) and \( T_t^{g_I} = T_t^{*g_I} = T_t^{g_I} \) is the only PO solution of this game.

Each value of \( 0 < T_t^{g_J} < T_t^{*g_J} \) and \( 0 > T_t^{g_I} < T_t^{*g_I} \) is strongly dominated by \( T_t^{g_I} = \eta \) and \( T_t^{g_I} = 0 \) which are sub game perfect equilibrium.
The following propositions summarize the results obtained in this section.

**Proposition 2** Let us assume that (i) the educated workers can migrate without restrictions inside a union and there is not migration outside the union; (ii) the regions inside are not symmetric (different initial technology); (iii) in the previous period there is not taxation and there is not education subsidies (or there is the same fiscal policy); (iv) there is a positive role for the government in the decisions of education for agents. The absence of coordination on fiscal policies and the opportunistic behavior of the jurisdictions imply a mechanism of FC which generates the Prisoner Dilemma’s. The only NE sub-game perfect equilibrium is zero taxation and zero subsidies for the weaker region and positive taxation/subsidies otherwise. Taxation for the stronger region is \( T^J_g = \eta - \varepsilon \) (with \( \varepsilon \to 0 \)) where \( \eta \) it is a measure of the technology gap among regions. Furthermore, if \( \eta \leq T^J_{g\max} \), then this NE is not PO because it is strongly dominated in Pareto’s mean by the NE obtained if there is coordination and where there is a positive value of taxes and subsidies [prop. (1)].

Let us now analyze different coordination policies\(^{24}\).

1) **Optimal level of subsidies and taxes.** The presence of a Central Authority which imposes to each region the optimal level of subsidies and taxes for each region generates the optimal solution. If the CA imposes \( T^J_{I\max} \) and \( T^J_{I\max} \), the maximum level of taxation/subsidies for each region, then we can have the maximum level of welfare for each region independently if there are different technologies. It is the optimal solution because all the negative effect of the fiscal competition is eliminated.

2) **Minimum level of taxation/subsidies** (\( T^\min \)) Let us assume that the CA cannot impose to the governments of the union the optimal level of taxation but it is only able to impose a minimum level of subsidies and so of taxation. Let us assume that \( T^\min \) is lower of \( T^J_{I\max} \). Two different cases can be now distinguished when \( a > b \).

2.a) \( T^\min \) is equal in all the Union.

In this case the government’s final decisions can be resumed in the following game in compact form

<table>
<thead>
<tr>
<th>( T^J_g )</th>
<th>( T^\min )</th>
<th>( T^J_g = T^\min )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Omega^J_{2,t} : \Omega^J_{2,t} ) (case1)</td>
<td>( \Omega^J_{2,t} : \Omega^J_{2,t} ) (case1)</td>
<td>( \Omega^J_{1,t} : \Omega^J_{1,t} ) (case1)</td>
</tr>
<tr>
<td>( \Omega^J_{2,t} : \Omega^J_{2,t} ) (case2)</td>
<td>( \Omega^J_{2,t} : \Omega^J_{2,t} ) (case2)</td>
<td>( \Omega^J_{1,t} : \Omega^J_{1,t} ) (case2)</td>
</tr>
</tbody>
</table>

\(^{24}\)It is important to specify that in the following paragraphs when we indicate the different typology of coordination we do not refer only to the taxation policies. In fact, by the assumptions of the model, taxes and subsidies are linked by a direct proportionality function.
The only NE sub game perfect equilibrium is \( T^g_J = T^\text{min} \) and \( T^g_I = T^\text{min} \). if \( T^{\text{min}} > \eta \) or it is \( T^g_J = \eta \) and \( T^g_I = T^\text{min} \) if \( T^{\text{min}} < \eta \). This Equilibrium is not PO respect the optimal solution and does not Pareto improve respect to the NE achieved without coordination. If we impose a coordination that do not take in account the different level of technology damages the weakest region. This region not only looses all its educated workers but must supply a minimum subsidy (linked to the \( T^{\text{min}} \)) to its workers whiteout receiving a gain from this investment\(^{25}\).

2.b) \( T^{\text{min}} \) is different between regions.

In this case the government’s final decisions can be resumed in the following game in compact form

<table>
<thead>
<tr>
<th>( T^g_I = T^{\text{min}} = \frac{T^g_J - \eta}{1-\eta} )</th>
<th>( T^g_I = T^{\text{min}} = \frac{T^g_J - \eta}{1-\eta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T^g_I = T^{\text{max}} = T^g_J )</td>
<td>( T^g_I = T^{\text{max}} = T^g_J )</td>
</tr>
<tr>
<td>( \Omega_{1,t}^J : \Omega_{2,t}^J ) (case 1) (case 3)</td>
<td>( \Omega_{1,t}^J : \Omega_{2,t}^J ) (case 1) (case 3)</td>
</tr>
<tr>
<td>( \Omega_{1,t}^I : \Omega_{2,t}^I ) (case 2) (case 2)</td>
<td>( \Omega_{1,t}^I : \Omega_{2,t}^I ) (case 2) (case 2)</td>
</tr>
</tbody>
</table>

The only NE sub game perfect equilibrium is \( T^g_J = T^{\text{min}} \) and \( T^g_I = T^{\text{min}} = \frac{T^g_J - \eta}{1-\eta} \). This Equilibrium is not PO respect the optimal solution. If \( T^{\text{min}} > \eta \) then this NE is Pareto improvement respect the NE achieved without coordination. This is the “second optimal” solution. More \( T^{\text{min}} \) is closed to \( T^g_J \), more the second optimal is closed to the optimal solution.

These cases are depicted in figure (4)

The results obtained in this section can be summarized by the following propositions.

**Proposition 3** Let us assume that the CA is able to impose the optimal level of subsidies and taxes for each region, it follows that the coordination between jurisdictions implies a new NE which is Pareto improvement respect the non coordination NE. The CA imposes the maximum level of taxation/subsidies for each region \( T^J_{\text{Max}} \) and \( T^I_{\text{Max}} \). This Equilibrium is PO and it can be defined as the “optimal solution” for the CA.

**Proposition 4** Let us assume that (i) the CA is not able to impose the optimal level of subsidies and taxes for each region but only to impose a minimum level of taxation (or subsidies to educated); (ii) this minimum is the same for all the regions; (iii) the regions are asymmetric; it follows that the coordination between jurisdictions implies a new NE which is not Pareto improvement respect the non coordination NE. In this asymmetric case, the coordination which do not take in account the different level of technologies damages the weakest region.

\(^{25}\)In this asymmetric case, it is possible obtain the “second optimal” solution (case 2.b) only by use a transfer from the strongest region in way to refund of the subsidies that aren’t covered by the tax.
Figure 4: "Different technology scenario" with minimum tax specific for each region.

**Proposition 5** Let us assume that (i) the CA is not able to impose the optimal level of subsidies and of taxes for each region but it can only impose a minimum level of taxation (or subsidies to educated); (ii) this minimum is not the same for all the regions but is defined by this relationship $T_{J}^{\text{min}} = T_{I}^{\text{min}} - \eta$. Then follows that if $\eta < T_{J}^{\text{min}}$ the coordination between jurisdictions implies a new NE which is Pareto improvement respect the non coordination NE. This Equilibrium is PO and can be defined as the "second optimal" solution for the CA.

4.1 Mobility case in the “same technology scenario”

Let us now analyze the scenario in which $a = b$, i.e., initially the regions have symmetric characteristics. The individual’s optimal decision do not change respect the asymmetric scenario. Their future wage is related to the taxation/subsidies policies of the governments and to differences of technology between regions. It is straightforward to verify that the educated workers prefer to stay in region $J$ if

$$T_{i}^{J} < T_{i}^{I}$$ (60)

We can therefore distinguish three different states of the world. Using the BI we can analyze the government decisions for each different state of the world.
Case (1): $T^j_t < T^I_t$ Comparing the region gains we have

$$\Omega^*_t > \Omega^0_t > 0 \quad (61)$$

$$\Omega^*_t > \Omega^0_t > 0 \quad (62)$$

Case (2): $T^I_t > T^j_t$ It’s symmetric to the case (1), so we have

$$\Omega^*_t > \Omega^0_t > 0 \quad (63)$$

$$\Omega^*_t > \Omega^0_t > 0 \quad (64)$$

Case (3): $T^j_t = T^I_t$ We have the same results of the autarchic case.

$$\Omega^*_t = \Omega^0_t = \Omega^2_t > 0 \quad (65)$$

We can resume this three cases in the following game in compact form\textsuperscript{26}

<table>
<thead>
<tr>
<th>$T^*_t = 0$</th>
<th>$T^<em>_t = T^{</em>+g}_t = T^{*g}_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T^*_t = 0$</td>
<td>$\Omega^j_t ; \Omega^I_t$ (case3/noGvt) (case3/noGvt)</td>
</tr>
<tr>
<td>$T^<em>_t = T^{</em>+g}_t = T^{*g}_t$</td>
<td>$\Omega^j_t ; \Omega^I_t$ (case1) (case1)</td>
</tr>
</tbody>
</table>

The only NE sub-game perfect is $T^*_t = 0$ and $T^*_t = 0$. This Equilibrium is not PO. The coordinated solution, $T^*_t = T^{*+g}_t = T^{*g}_t$ and $T^*_t = T^{*+g}_t = T^{*g}_t$ it is the only PO solution of this game. The results obtained in this section can be summarized by the following propositions.

**Proposition 6** Let us assume that (i) the educated workers can migrate without restrictions inside a union and there is not migration outside this union; (ii) regions inside are symmetric (same initial technology); (iii) in the previous period there is not taxation and there is not educational subsidies (or there is the same fiscal policy); (iv) there is a positive role for the government in the decisions of education for agents. It follows that the absence of coordination on fiscal policies and the opportunistic behavior of the jurisdictions imply a mechanism of FC which results in a Prisoner Dilemma’s situation. The only NE sub-game perfect is zero taxation and zero subsidies for both regions. This NE is not PO because it is strongly dominated in Pareto’s mean by the NE obtained if there is coordination and where there is a positive value of taxes and subsidies [prop. (1)].

\textsuperscript{26}Each value of $0 > T^*_t < T^{*+g}_t$ and $0 > T^*_t < T^{*g}_t$ is strongly dominated by $T^*_t = 0$ and $T^*_t = 0$ which is sub-game perfect equilibrium.
In the analysis of different coordination policies it is straightforward to see that the only difference between the symmetric and asymmetric cases are when the CA can impose only a $T^\text{min}$.

5 Mobility case (enlarged case)

Let us assume that a new State enters inside the union$^{27}$. Let us assume that the wage per efficiency unit of labor of this economy is always lower than the wage inside the union independently of the specific fiscal policy and technology characteristics of each region inside the union.

In the following sections we analyze the model in presence and in absence of a central coordination.

5.1 New entry in a union with central coordination

Let us assume that when the new region ($N$ hereafter) is admitted inside the union the mobility of its educated workers is not perfect but there is a probability of a successful emigration in the region $J$, $\pi^J$ with $J = A, B$, that is independent of the number of workers who are eligible to migrate$^{28}$. Furthermore, we assume that emigration policy is fully anticipated. In accordance with the previous section, we assume that $\pi = 1$ for educated workers of the union. We assume that $\pi < 1$ and that it is very small for workers of region $N$. This assumption can be justified by the presence of strong mobility costs (pecuniary and social). Each region of the union can influence $\pi$ by migration policies that remove this costs. The CA collects the migration policies of the region inside the union and decides the optimal value of $\pi^U$ for the region $N$.

When there is a probability of emigrating and earning a higher wage, the agent's educational decision becomes an expected utility problem. For simplicity, we assume that agents are risk neutral, that only the educated workers can migrate and that all the other assumptions of the previous section are verified$^{29}$.

Let us define

$$\lambda \left(s_{t-1}^N\right) w^N (k) \equiv w^N_k$$

as the wage rate per efficiency unit of labor for the educated of region $N$.

By assumption, in each period we have that

$$w^U > w^N$$

$^{27}$This analysis can be extended without changing the results to the new entry of most regions.

$^{28}$This assumption follows from the small country hypothesis.

$^{29}$Same distribution of ability, same education costs, etc.
where

\[ w^U \equiv \arg \max \{ \lambda (s_{i-1}^l) w^l (k) [1 - T_i^l] ; \lambda (s_{i-1}^l) w^l (k) [1 - T_i^l] \} \]  \tag{68} \]

the best wage rate per efficiency unit of labor available inside the union.

The optimal decision for agent \( i \) born in \( N \) will be to invest in education if

\[ \left[ \pi^U w^U + (1 - \pi^U) (1 - T_i^N) w^N \right] e^i > w^N + c (1 + r^*) \]  \tag{69} \]

Thus, all agents with a latent ability greater than \( e^*_N \) will invest in education, were \( e^*_N \) is uniquely defined by the following equality:

\[ e^*_N = \frac{w^N + (1 + r^*) [c - \gamma^N_i]}{\pi^U w^U + (1 - \pi^U) (1 - T_i^N) w^N} \]  \tag{70} \]

As in the previous analysis it is possible to identify a positive value of taxes \( 0 < T_i^N \leq T_i^{\text{Max}} \) such that \( e^*_N \) is lower than the case whiteout government subsidies. Where

\[ T_i^{\text{Max}} = \frac{\gamma^N_i (1 + r^*) \left[ \pi^U w^U + (1 - \pi^U) (1 - T_i^N) w^N \right]}{(1 - \pi^U) \left[ w^N + (1 + r^*) c \right]} \]  \tag{71} \]

The average proportion of educated people in the economy \( N \) is given by the following,

\[ s_t^N = \frac{(1 - \pi^U) \int_{e^*_N}^{E} g (e^i) \, de^i}{1 - \pi^U \int_{e^*_N}^{E} g (e^i) \, de} \]  \tag{72} \]

If \( \pi = 1 \) then the source economy loses all his educated workers and \( s_t^N = 0 \). If \( \pi = 0 \) then there is no migration inside the union. Thus, a sufficient condition for the existence of a positive level of BD such that the source economy benefits in terms of productivity is that \( \frac{ds_t^N}{d\pi} > 0 \) when \( \pi = 0 \). The optimal level of \( \pi \) will be given where \( \frac{ds_t^N}{d\pi} = 0 \). Differentiating equation (72) we obtain

\[ \frac{ds_t^N}{d\pi} = \frac{\partial s_t^N}{\partial \pi} + \frac{\partial s_t^N}{\partial e^*_N} \frac{\partial e^*_N}{\partial \pi} \]  \tag{73} \]

where

\[ \frac{\partial s_t^N}{\partial \pi} = - \left[ \int_{e^*_N}^{E} g (e^i) \, de^i \right] \left[ 1 - \pi^U \int_{e^*_N}^{E} g (e^i) \, de \right] < 0 \]  \tag{74} \]

\[ \frac{\partial s_t^N}{\partial e^*_N} = - \frac{(1 - \pi^U) g (e^*_N)}{\left[ 1 - \pi^U \int_{e^*_N}^{E} g (e^i) \, de \right]^2} \]  \tag{75} \]

\[ \frac{\partial e^*_N}{\partial \pi} = - \frac{\left( w^N + (1 + r^*) [c - \gamma^N_i] \right) \left[ w^U - (1 - T_i^N) w^N \right]}{\left[ \pi^U w^U + (1 - \pi^U) (1 - T_i^N) w^N \right]^2} < 0 \]  \tag{76} \]
Setting \( \pi^u = 0 \) and noting that \( \int_{e^{-\gamma N}}^E g(e^i) \, de^i \left[ 1 - \int_{e^{-\gamma N}}^E g(e^i) \, de^i \right] \) is at most a quarter, we obtain the results summarized by the following proposition.

**Proposition 7** In presence of CA inside the union which decides the optimal value of \( \pi^U \), it is possible have a positive optimal level of BD emigration if

\[
g(e^N) \left[ w^U + (1 + \tau) \left( c - \gamma N \right) w^U - \left( 1 - T^N \right) w^N \right] > \frac{1}{4} \text{ and } 0 < T^N \leq T^N_{\text{Max}}.
\]

This proposition states that the source economy can benefit from the BD in the extent that there are a sufficient number of people who would be entitled to invest in education. The introduction of taxes and subsidies changes the sufficient condition found by Mountford. There are two different results. The subsidies increase the number of educated workers and so decrease the probability for the new entry to be in the “optimal BD conditions”. Furthermore, the taxes increase the wage differentials between the entry region and the others and so increase the probability to gain from the BD.

Let us consider the case of uniformly distributed abilities

\[
g(e^i) = \frac{1}{E} \quad \text{ (77)}
\]

\[
\int_{e^{-\gamma N}}^E g(e^i) \, de^i = 1 - \frac{e^N}{E} \quad \text{ (78)}
\]

\[
\frac{ds^N}{dx} > 0 \text{ iff } \left( 1 - \pi^U \right) \frac{w^U - \left( 1 - T^N \right) w^N}{\pi^U \left( 1 - \pi^U \right) \left( 1 - T^N \right) w^N} > \left( 1 - \frac{e^N}{E} \right) \quad \text{ (79)}
\]

Thus, a BD will increase the proportion of educated people in the economy if \( \pi \) is low, if \( w^U \) is very high relative to \( \pi^U \) and if the proportion of educated people in the economy was previously low.

Equation (79) implies that when abilities are distributed uniformly, if \( w^U \) is large enough there is a positive level of \( \pi^U \) such that next period productivity increases in the source economy.

As in Mountford (1997), in presence of an optimal migration policy under a BD, the return function \( s_t = \psi(s_{t-1}) \) is everywhere above the return function compared with the case of no emigration. Thus clearly an optimal emigration policy will increase the short and long run productivity in the source economy. Finally, if there are multiple steady state equilibria then a temporary emigration policy might lift a source economy from a low to a high education steady state.

The figure (5) depicts these results.
Figure 5: depicts the dynamics of the economy when there is a unique steady state equilibrium for the case where there is not migration and when there is optimal emigration ($\Psi^0$ is the case with optimal taxation and $\Psi^{0m}$ is the case without taxation).

5.2 New entry in a union without central coordination

Let us assume that there is not a CA and that each region of the union decided independently the value of $\pi$ maximizing his own welfare and do not take in account the welfare of the region $N$.

Let us assume that $0 < \pi < \pi^{\text{max}}$ with $\pi^{\text{max}} < 1$.

Let us define $\Psi_J$ a measure of the welfare of region $J$.

$$\Psi_J \equiv \Omega_J + \Gamma(\pi_J) \quad \text{with } J, I = A, B$$

(80)

were $\Omega_J$ is the same function defined in (8) and $\Gamma(\pi)$ is the “brain drain gain” for the region $J$ deriving by the attraction of educated workers of region $N$. We define

$$\Gamma(\pi_J) \equiv \pi_J s_{N,J}^{t+1} \left[ k \left( s_{t-1}^e \right) w(k) \int_{S_{t-1}}^E \right] + Z(\pi_J s_{t}^{N,J})$$

(81)

Where $\pi_J s_{N,J}^{t+1}$ is the number of agents which are educated in region $N$ at time $t$ and work in region $J$ at $t+1$. The first expression on the right hand side of (81) denotes the total production of region $J$ due to the presence of educated workers coming from region $N$. The second term is an increasing function of the number of educated agents and denotes all the other gains (not only pecuniary) that the presence of new educated workers gives to region $J$. Let us assume that $\Gamma(\cdot) \geq 0$, $\Gamma(0) = 0$ and $\Gamma'(\cdot) > 0$. 

28
It is straightforward to show that

If $\pi^J > \pi^I$ then $\Gamma(\pi^J) = \pi^J s_t^N \left[ a \int_{e_i^N}^E de_{i,t} \right] + Z(\pi^J s_t^N)$ (82)
and $\Gamma(\pi^I) = 0$ (83)

If $\pi^J < \pi^I$ then $\Gamma(\pi^J) = 0$ (84)
and $\Gamma(\pi^I) = \pi^I s_t^N \left[ b \int_{e_i^N}^E de_{i,t} \right] + Z(\pi^I s_t^N)$ (85)

we also assume that

If $\pi^J = \pi^I$ then $\Gamma(\pi^J) = \frac{1}{2} \pi s_t^N \left[ a \int_{e_i^N}^E de_{i,t} \right] + \frac{1}{2} Z(\pi s_t^N)$
and $\Gamma(\pi^I) = \frac{1}{2} \pi s_t^N \left[ b \int_{e_i^N}^E de_{i,t} \right] + \frac{1}{2} Z(\pi s_t^N)$ (86)

Let us define the timing of the model.

Time $t$ The government $N$ decides $T_t$ and $\gamma_t$ and each agent $i$ decides whether to invest in education or not according to their latent ability $e_i$. Agents investing in education receive $\gamma_t$ and borrow $c - \gamma_t$ from capital markets. Governments $J$ and $I$ decide $\pi$.

Time $t + 1$ The educated agents decides whether to migrate or not. They pay $T_t$ to the government of the region in which he works and they repay the debt of the first period of the life.

Time $t + 2$ All agents retire and use their savings to consume.

It is possible to solve the maximization problem through the BI method.

The optimal decision for agent $i$ born in $N$ is uniquely defined by $e^{*N}$ (eq. 70)

For simplicity, we analyze only the optimal decision of government $J$ and $I$ about the value of $\pi$. Hence we focus our attention on the “migration competition” inside the union. It is straightforward see that all these analysis can be extended to the fiscal competition between the two regions without changing the results.

Each government of the union maximizes the value of $\Gamma(\pi)$ and it is straightforward see that, without coordination the only NE sub game perfect of this “migration competition” is

$$\pi^J = \pi^I = \pi^{max} > \pi^U$$ (87)
The government $N$ decides $T_t$ and $\gamma_t$ according to the optimal decision of the agents (70) and of the region inside the union (87). In the previous section, the CA collects migration policies of the region inside the union and decides the optimal value of $\pi^U$ for the region $N$. Therefore, we have a “positive Brain Drain” if we are in the condition delineated in the proposition (7). When there is not coordination, then there is a “migration competition” between the governments inside the union which involves in a value of $\pi > \pi^U$, the value of $\pi$ is too high to have positive externalities from BS also for the region $N$. In this case the BD had negative effect on the growth of the region $N$ and their optimal decision is to have zero subsidies.

6 Conclusion

In this paper we introduce a role for the government in the educational decisions of the agents through the introduction of educational subsidies and taxation. This make it possible to study BD and FC in a unified framework and analyze the impact of the absence of coordination inside the EU. In Section 3, we solve the model in an “autarchic context” and we obtain the optimal level of taxation and subsidies [Proposition 1]. In Section 4, we solve the model in a “restricted mobility context” where there is perfect migration of the educated worker inside the EU. The FC among the regions destroyed the positive externalities due to the subsidies. According to the literature, the FC causes a fall in the provision of public goods. [Proposition 2]. For the presence of a CA which coordinates the fiscal policies, it is necessary to obtain the welfare and growth gains due to the subsidies. [Proposition 3, 4 and 5]. In Section 5, we solve the model in a “enlarged mobility case” where there is a new entry inside the EU. If the mobility of the educated workers of the new entry are not perfect and can be influenced by the “migration policies” of the former members of the EU, then the absence of coordination implies a “migration competition” where the region inside the union tries to attract educated workers of the new entry. This competition increases welfare and growth loss for the new entry due the BD even if there are the condition to have a “gain from the BD” [Proposition 6].

In this paper, we have analyzed a simple model in which we show that, if we analyze the BD and the FC, the absence of coordination implies two negative externalities. First, the FC decreases taxation and causes a fall in the provision of public goods in all the regions of the EU (either for the former region either the new entry). Second, the FC can lead to in a “migration competition” which increases the “welfare and growth loss” due to BD.
Extensions of the model

The model presented in this paper can be extended in order to analyze different economic and political analyses.

1. We can introduce a “mobility cost” for the educated workers. This cost can be not only referred to the pecuniary costs directly linked to the migration (transport, new house, etc.) but also it can be referred to the “non pecuniary cost” indirectly linked to the migration (live in a new nation, different language, etc.). The introduction of these costs does not change the main results obtained in this paper but there are some important results:

   - The more are the “mobility cost”, the less is the role of FC.
   - While the “pecuniary cost” are normally similar between the different regions, on the contrary the “non pecuniary costs” can be very different and they can be directly influenced by the policies of the government. These differences may increase or decrease eventually technology’s differences and so the FC and BD externalities. Furthermore, by decreasing these costs, the government of the former region inside the EU can try to attract the educated workers of the new entry (migration competition).

2. We can introduce an “enlarged role of the government”. In this paper we have analyzed a government which do not take into account redistribution income policies. If we consider a new version of the social welfare function that the government wants to maximize then we have other important results:

   - The FC implies not only less provision of public good but also less income redistribution. This results, in accordance to the FC literature, is due to the fact that each government decreases the tax in order to attract the educated worker and so it must decrease the income redistribution.
   - If we analyze the redistribution policies, then we must take into account also the “non educated” migrations. The risk to attract many non educated workers implies less income redistribution and so increase the negative externalities of the FC.

3. We have analyzed the impact of the FC and BD when the new entry region has just decided to be in the EU. It is also possible to enlarge this analysis by studying a new step in which the new entry decides even if it is convenient to be a member of the EU.

4. We can substitute the assumption that the abilities of children are independent from the abilities of their parents with the assumption of externalities of education of the precedent education. In this case we obtain a more realistic model with more rich dynamics and we increase the negative externalities due to the FC. Otherwise, the main results showed before do not change.
References


