A MULTISTAGE MECHANISM FOR MANAGING AGGRESSIVE FLOWS IN THE NEXT GENERATION INTERNET

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Abstract: In this note, we provide a multistage game form which may be used for managing aggressive ‡ows which may cause network congestion or monopolisation. The mechanism here presented attains economic e¢ciency, technical e¢ciency and other desirable properties.

1 Introduction

An information network can be seen as a facility used by heterogeneous subjects for communicating and/or creating immaterial added value. The public Internet and the other networks[1] are complex, open or closed, webs through which it is possible to exchange goods, services and information. Moreover, the growing commercial use of the Internet, has shown that a network can be also used as an e-market locus in which continuous time transactions between delocalized agents take place. Hence, access conditions are a crucial issue for any network's user as far as informative advantages, quick communications and real-time exchanges frequently mean higher welfare.

Independently by the kind of network we deal with, if network's traf…c is higher than its capacity problems of congestion may arise as well as the need of a precise charging mechanism for using the net. Whenever network resources are insu¢cient for the aggregate demand of transit, economic ef…ciency imposes to relate usage and paid price in order to give priority to those consumers which give more value to their ‡ows. In periods of congestion, some users have to be rationed (i.e. their ‡ows must be delayed) and an anonymous mechanism is necessary in order to decide which ‡ows will or not be postponed.

Economic analysis has mainly purposed two di¤erent kinds of charging mechanisms[2]. On the one hand, Generalized Vickrey Auctions [3], where consumers announce at a given time their willingness to pay by bidding for bandwidth. On the other, Dynamic volume pricing models where the network provider sets a base-price for bandwidth adjusting it according to the loan of the network. In the former case, bids lower than transportation costs are rejected and auction's winners pay the lower market-clearing price. In the latter, a well-de…ned function describes, at any node, the price increase

[1] See Odlyzlo (2000) for a nice contribution on utilisation of di¤erent information networks.

[2] See for a survery Fankhauser et al. (2000).

[3] Originally MacKie and Varian (1994) have suggested to use a Vickrey auction for pricing network usage. More recently some extensions of this idea have been studied by Fankhauser et al. (2000).

due to congestion. Both mechanisms are characterized by a unique stage in which users submit bids or receive prices for packets[4].

In what follows, we present a multistage mechanism which fulfills some desirable properties like economic efficiency (giving more bandwidth to more valued transits) or network efficiency (described in terms of full capacity utilization). Furthermore, as we will see, our mechanism is universal in the sense that it may be easily applied to any kind of network or inteconnection.

Our analytical set up is inspired to the well known literature on non-cooperative implementation under complete information[5] and, more precisely, to subgame perfect implementation[6]. As it will be argued, using the following game form, a unique subgame equilibrium will be reached with full attainment of some desirable properties.

The paper is organized as follows. In section 2 we define our set up and we introduce our mechanism. Section 3 contains the full characterisation of our multi-stage game form, while section 4 states some general properties of a mechanism aimed to manage network congestion and our central result. As usual, the last section is reserved to comments and conclusions.

2 The Network Model

The commercial use of an information, packet-based network arises some well-known issues concerning how charging, accounting and billing users' differentiated traffic (i.e. electronic commerce, marketplaces, e-mail etc.). The best architecture for protocols aimed to exploit these functions clearly depend on network structure. Network nodes are normally referred as hosts that exchange data directly (end-to-end connection) or indirectly through routers. A router is able to identify different network paths available for delivering packets. Data flows are normaly visible to hosts and routers by identification tags. As clearly stated by Fankhauser et al. (1999), some supports are necessary for hosts and routers in order to charge and account flows. These supports include an application programming interface (API) for collecting

4 For recent advancements in this field of research see MacKie-Mason et al. (1999).

5 See for an introduction to the theory of implementation Corchòn (1998).

6 See Moore and Repullo (1988).

data, accounting tra¢c and doing bandwith reservations. Furthermore, accounting protocols de…ne a fuction from a given resource usage into technical values which may be used for charging and scheduling packets.

Monitoring network tra¢c and, in particular, aggressive ‡ows is a fairly important issue if the network may be congested by eccessive usage. In absence of congestion a ‡at fee pricing scheme for covering access and set up costs is economically e¢cient given usage costs almost equal to zero. Nevetheless, in the case of congestion, a policy layer and a processing layer are needed. The former is used to implement a mechanism aimed to manage ‡ows, assigning bandwith and relative charges to any user. The latter detects packets conditions assinging them header information and works as a packet scheduler. These layers, located in hosts and routers, de…ne the access policy of a service provider based on network status (in terms of bandwith) and data volume.

In order to design a possible mechanism to control ‡ows, a precise network model must be speci…ed. Let we suppose to have a information network used by a service provider (for instance Internet service providers or Internet Backbones) to transit a given amount of bytes from one host to another. This network truncation has a given transit-capacity C. Two kinds of connections are possible between our hosts: an end-to end one (hosts to hosts) or a router mediated connection. Such a router is also linked to one or more additional hosts which may be used to send data trought alternative paths. The following schede illustrates our network components.

INSERT FIGURE 1

In principle and given a speci…ed network structure, a mechanism used to manage network resources must be able to determine e¢cient outcomes. Moreover, it might detect and prevent congestion periods, minimising delays and losses which may decrease users' utility. In what follows, we specify a mechanism mainly thought to deal with aggressive data ‡ows.

3 A Multistage Mechanism for Managing Aggressive Flows

Let us suppose that at time t a given set of users need to use network bandwith. Denote this set with $H(t)$[7]. Any agent submits an initial capacity request, in terms of bandwith, equal to ω_h for $\forall h \in H(t)$. For the sake of tractability, only two types of users are allowed: heavy and light ones[8]. An user is identified as heavy if and only if $\omega_h \geq \frac{C}{2}$. A given share α of network capacity is reserved for the latters and their aggressive flows. Obviously the complementary portion of C is available for light users. Any heavy user is direct to the router. Hence, only light users are admitted to end to end connection. Additionally, let us assume homogeneous value packets for each agent. In this case, normalizing a packet's value at one, we have that users' loss function will be equal to $l_h = v_h - e_h$ for any h, where $v_h - e_h = \varphi_h$ is the capacity shortage they face. Obviously, $l_h^{max} = v_h$ and $l_h^{min} = 0$. A three-stage mechnism is adopted to manage flows. It is composed by the following steps.

First Stage

A Scheduling Stage where, through identification tags, the two class of users are recognized. Among the light ones, a sequece of sets

$$\left(K_1(t); K_2(t+\varepsilon_1); \ldots; K_p\left(t + \sum_{s=1}^{p-1} \varepsilon_s \right) \right) \qquad (1)$$

is randomly built such that, for $\forall s$,

$$K_s(t) = \left\{ h \in H(t) : \sum_h \omega_h = (1-\alpha) C \right\} \qquad (2)$$

[7] Additionally suppose that at time t-1 the network is empty. In this way is possible to avoid problems of initialisation. See on the topic Akamatsu e Kuwahara (1997).

[8] See for this distinction Crèmer and Harinton (1999).

ε is a congestion delay which will be cumulated by last users. This variable may be used to assign priority to some agent, giving them an higher probability to be in the next set of users[9], or as a mean to identify packets which may be refused[10]. All these agents pay a flat fee equal to f_L sufficiently high to cover service provider's costs. Heavy users $R(t) = H(t) - \bigcup_s K_s(t)$ are randomly matched building a sequence of couple af agent (i, j) given by, if $Card(R(t))$ is an even numer,

$$\left\{ (i,j)_1^t, (i,j)_2^{t+\varepsilon_1}, ..., (i,j)_{\frac{|R(t)|}{2}}^{t+\sum_{s=1}^{\frac{|R(t)|}{2}-1}\varepsilon_s} \right\} \tag{3}$$

or, alternatively, with an odd number of users

$$\left\{ (i,j)_1^t, (i,j)_2^{t+\varepsilon_1}, ..., (i,j)_{\frac{|R(t)|-1}{2}}^{t+\sum_{s=1}^{\frac{|R(t)|-1}{2}-1}\varepsilon_s}, (i)^{t+\sum_{s=1}^{\frac{|R(t)|-1}{2}}\varepsilon_s} \right\} \tag{4}$$

Each agent receives a precise information about its position in the sequence (3) or (4) and his/her waiting time. Thus, they are free to decide whether or not to leave the queue, dropping the connection. Let $I(t)$ the set of agents that remain in the queue and f_H (with $f_H \geq f_L$) the flat fee paid by these agents independently by any congestion delay. Remaining users $h \in H(t) - \left[\bigcup_s K_s(t) \cup I(t)\right]$ is switched to an alternative route.

Second Stage

[9] This becomes crucially important if new agents ask for bandwith at time $t+1$. In the last case, users form period t will have absolute priority in the composition of the new sequence of sets $K_1(t+1), K_2(t+1+\varepsilon_1), ..., K_p\left(t+1+\sum_{s=1}^{p-1}\varepsilon_s\right)$.

[10] Any service provider may, for instance, fix a maximum admissible delay for post-poned users, deciding, in this way, how many flows refuse directly at time t.

For any $(i; j) \in I(t)$ a new Reservation Stage asks for new bandwith request (v) given the possibility of congestion. Call $\bar{C} = \alpha C$ the network capacity between our router and the receiving host.

If $v_i + v_j \le \bar{C}$, no congestion occurs. In this case effective transits e are equal to capacity reservations and the price paid, t; for any sent packet is equal to its marginal cost $\bar{c}$; reasonably near to zero. Let we call this solution no congestion outcome (NCO).

Contrary if $v_i + v_j > \bar{C}$; there is effective congestion and the problem of how to allocate capacity does emerge. Following precise instructions received by the service provider, each agent cannot bid a capacity reservation higher than $\bar{C}$.

Let we suppose, without loss of generality, that $v_i > v_j$ and call b the market value[11] of v in periods of congestion. Once the service provider has verified that effective congestion occurs, it sends to any user information about the market value of their capacity reservations.

Third Stage

Finally, the Bids Stage where agents submit bids for priority, p; revealing their willingness to pay for reserved transits. Using, last bids the service provider decides how to allocate network capacity among users.

Our last two stages may be describe more precisely as follows: any user $h \in (i; j)$ can choose a certain capacity reservation $v_h \in V_h \equiv \left\{ v_h | v_h \in \left[0; \bar{C}\right] \right\}$ for $h = i; j$ and a preference consistent bid for priority $p_h \in P \equiv [0; +\infty\}$. Hence, users' strategy sets are defined as $(S_h)_{h=i;j} \equiv (V_h)_{h=i;j} \times (P_h)_{h=i;j}$. An outcome function $g : S_i \times S_j \longrightarrow E$ decides effective transits for any user, where $E = \left\{ (e_i; e_j) \mid e_i + e_j \le \bar{C} \right\}$. At time t; the unique reachable solution is what we have call NCO. If this is not the case, the outcome function may lead to three alternative results described below.

Definition 1 In the case of congestion, if $\sum_h p_h \le \sum_h b_h$ a Capacity Splitting Rule (henceforth CSR) equally divides network capacity between users, i.e. $e_h = \bar{C}/2$ for $\forall h$:

[11] This will be rekoned, following Muller (1997), by the service provider as any h

$$b_h = a_h t_h$$

where a_h is the application price and t_h a traffic factor.

Under a CSR, both agents are asked to share network capacity since what they can pay for transits is lower than what the service provider expects to earn. Both users pay a price equal to the usage marginal cost $t_h = \Phi$ for $\forall h$ and they face a capacity shortage (or excess) equal to $\varphi_h = v_h - \Phi/2$ for $\forall h$.

If $\sum_h p_h > \sum_h b_h$ we can have two possibilities:

(i) $p_h - b_h > 0 \geq p_{-h} - b_{-h}$ for $h = i, j$, that is, a user h bids for his/her transit a value higher than b_h and the other consumer a value lower than b_j

(ii) $p_h > b_h$ for $\forall h$.

Consistently, we define for (i)

Definition 2 In the case of congestion, if $p_h - b_h > 0 \geq p_{-h} - b_{-h}$ for $h = i, j$ a Bid Differential Rule (henceforth BDR) decides for $v_h = e_h$ and $e_{-h} = \Phi - e_h$.

Under a BDR, agent h pays exactly his/her bid ($t_h = p_h$) with $\varphi_h = 0$. The other agent obtains residual transit given a certain network capacity, paying a tariff equal to $t_{-h} = \frac{e_{-h}}{v_{-h}} p_{-h}$ and getting $\varphi_{-h} = v_{-h} - e_{-h} = v_{-h} + v_h - \Phi$. Hence, insufficient network capacity penalizes only who values less transits. As it is straightforward t_{-h} decreases the lower is agent $-h$'s effective transit with respect his starting capacity reservation and it increases the higher is his second stage bid.

Finally, in the case (ii), we use the following rule

Definition 3 If $p_h > b_h$ for $\forall h$ a Relative Bids Ratio Rule (henceforth RBRR) assigns effective transit through a relative bids ratio B; equal to

$$B = \frac{p_h/v_h}{p_{-h}/v_{-h}}$$

By construction, effective transits, applying a RBRR, are given by:

$$e_h = \left(\frac{B}{1+B}\right)\Phi \tag{5}$$

$$e_{-h} = \left(1 - \frac{B}{1+B}\right)\Phi = \left(\frac{1}{1+B}\right)\Phi \tag{6}$$

In this outcome, each agent pays his second stage bid and capacity shortage are shared between network's users.

Manipulating expression (1), we can easily write $e_h = \dfrac{p_h v_{-h}\Phi}{v_h p_{-h} + v_{-h} p_h}$ and this allow us to show a basic property of this rule: relatively more valued transit obtains higher priority. To see this take the following first order partial derivatives:

$$\begin{aligned} \frac{\partial e_h}{\partial v_h} &= \frac{-p_h v_{-h}\Phi p_{-h}}{[v_h p_{-h} + v_{-h} p_h]^2} < 0 \\ \frac{\partial e_h}{\partial p_h} &= \frac{v_h v_{-h}\Phi p_{-h}}{[v_h p_{-h} + v_{-h} p_h]^2} > 0 \end{aligned} \tag{7}$$

Intuitively, (2) says that agent h will get higher effective transit with respect to what he needs, the lower his capacity reservation and the higher his relative second stage bids are. The rule works symmetrically for agent $-h$.

Summing up, we can describe our two-stage mechanism using the following tree structure[12]

INSERT FIGURE 2

In the next section we will show how our proposal satisfies some nice properties in managing networks' usage during congestions.

[12] For a similar mechanism see Jackson and Moulin (1992).

4 The Properties and The Theorem

As stressed in the introduction, we can draw some nice properties for a mechanism aimed to reduce ine¢ciencies due to networks congestion. It seems reasonable to require that: …rstly such a mechanism ensures allocative and technical e¢ciency, secondly that it may be easily applied at any level of a information network[13], …nally that it might be able to prevent periods of congestion to phase out. More precisely:

Property EE (Economic E¢ciency) A mechanism satis…es economic e¢ciency if under congestion assigns priority to more valued transit; i.e. $e_h > e_{¡h}$ if $p_h{=}b_h > p_{¡h}{=}b_{¡h}$.

Property U (Universality): A mechanism is universal if it can be applied to any network truncation..

Property NE (Network E¢ciency): A mechanism satis…es network e¢ciency if under congestion it ensures full utilization of network capacity; i.e. $e_h + e_{¡h} = \bar{c}$:

Property NCEn (No Congestion Enforcing): A mechanism is no congestion enforcing if $v_h \quad \frac{\bar{c}}{2}$ for 8h.

We are now able to prove the following:

Theorem: For any (i; j) 2 I (t) the mechanism proposed satis…es EE, NE, NCEn and U.

Proof. See Appendix ■

[13] For the Internet this means to Local Access Network as well as to Regional Service Provider Networks as well as to End to End Backbone Networks.

5 Discussion

The mechanism here presented has some appealing features in managing network congestions due to aggressive ‡ows. First of all, it is relatively simple and universal. Secondly it enforces full utilization of network resources and absence of congestion. This multistage structure allows full control of aggressive ‡ows. It also sistematically reserve network resources to light data ‡ows avoiding a network monopolisation by heavy users.

Di¤erently by many existing pricing model for internet applications[14], it avoids congestion, at least between heavy users, since they decide to share network capacity. If some agent needs more than his/her share of capacity, he/she may decide to congest the network, trying to get more transit at an higher price. Nevertheless, this will be an ine¢cient strategy. In opposition, it will be preferable paying a price equal to usage marginal costs (almost equal to zero) and employing saved resources to interconnect with a new service provider (i.e. multihoming).

A precise normative viewpoint is implicit in such a solution. Any network is a globalized pure or impure public good[15]. Hence, it is socially preferable not to have few almost-congested service providers, used by well endowed auction winners (at a price higher than usage marginal cost) and in which losers always wait Contrary, it might be better to ensure equal access to existing network resources, simply using excess demand to multiply provides and, hence, increasing facility-based competition. From an individual perspective, since any bid-based rationing is anonymous, it might be possible to be the relatively poor heavy user and thus the auction loser. Her/his transits will be post-poned with the consequent maximum loss.

Obviously, our mechanism must be linked with ‡at-fee pricing schemes aimed to cover service provider's access and set up costs. However, these schemes may easily co-exist in real networks. Finally, our mechanism deal with only transport prices, while a pricing scheme may be necessary also for content delivery[16].

[14] For a survey see Leinen et al. (2001).

[15] For a de…nition and discussion of global public good see Grunberg et al. (1999).

[16] See Fankhauser et al. (1998)

Appendix

Proof of the Theorem:

For property U:

By construction our set of rules is not dependent on the kind of connection and on the hierachical position of network's users (i.e. IB-IB, IB-ISP, ISP-ISP, ISP-user). Hence, it can be applied to any network ramification.

For properties NCEn and NE:

Let we consider the second stage. For agent $h \in (i; j)$:

Claim 1) if $p_{-h} < b_{-h}$ then:

(i) if $p_h \leq b_h$ then $\sum_h p_h < \sum_h b_h$ thus the CSR is applied with $t_h = \Phi$ and $\varphi_h^{CSR} = v_h - \frac{\Phi}{2}$

(ii) if $p_h > b_h$ then with probability, say, equal to q we will have that $p_h - b_h < b_{-h} - p_{-h}$ and the CSR works. Hence, $t_h = p_h$ and $\varphi_h^{CSR} = v_h - \frac{\Phi}{2}$. With the complementary probability $(1 - q)$; we may have that $p_h - b_h > b_{-h} - p_{-h}$: Then the FBDA is applied. For agent h we have that $t_h = p_h$ and $\varphi_h^{BDR} = 0$. Since it is always true that $q\varphi_h^{CSR} + (1 - q)\varphi_h^{BDR} < \varphi_h^{CSR}$, then agent h's best reply to $p_{-h} < b_{-h}$ will be to bid $p_h > b_h$.

Claim 2) if $p_{-h} = b_{-h}$ then:

(i) a CSR is used for $p_h \leq b_h$ with $t_h = \Phi$ and $\varphi_h^{CSR} = v_h - \frac{\Phi}{2}$

(ii) if $p_h > b_h$ then $\sum_h p_h > \sum_h b_h$ a FBDA works. Consequently, $t_h = p_h$ and $\varphi_h^{FBDA} = 0$. It is straightforward to prove that even in this case $p_h > b_h$ is a best reply.

Claim 3) if $p_{-h} > b_{-h}$ then:

(i) if $p_h = b_h$ then a BDR is applied and $t_h = \frac{e_h}{v} p_h$ and $\varphi_h^{BDR} = v_h + v_{-h} - \Phi$

(ii) if $p_h > b_h$ a RBRR is used to manage congestion and $t_h = p_h$, $\varphi_h^{RBRR} = v_h - \left(\frac{B}{1+B}\right)\Phi$

(iii) finally, if $p_h < b_h$ we can get, with probability q, $\sum_h p_h > \sum_h b_h$ thus applying a BDR with $t_h = \frac{e_h}{v} p_h$ and $\varphi_h^{BDR} = v_h + v_{-h} - \Phi$ or, with the complementary probability, we may have $\sum_h p_h \geq \sum_h b_h$. In the latter case a CSR works and $t_h = \Phi$ and $\varphi_h^{CSR} = v_h - \frac{\Phi}{2}$. Agent h knows that increasing his second stage bids he/she may obtain, if $p_{-h} > b_{-h}$ and he/she bids a price p_h sufficiently high, a larger share of network capacity. To show this, we can take the following limit given a certain $\bar{p}_{-h}$

$$\lim_{p_h \to \infty} e_h = \lim_{p_h \to \infty} \left(\frac{B}{1+B} \right) \Phi = \lim_{p_h \to \infty} \frac{\left(\frac{p_h}{\bar{p}_{-h}}\right)\left(\frac{v_{-h}}{v_h}\right)}{1 + \left(\frac{p_h}{\bar{p}_{-h}}\right)\left(\frac{v_{-h}}{v_h}\right)} \Phi =$$

$$\overset{H}{=} \lim_{p_h \to \infty} \frac{\left(\frac{1}{\bar{p}_{-h}}\right)\left(\frac{v_{-h}}{v_h}\right)}{\frac{1}{p_h} + \left(\frac{1}{\bar{p}_{-h}}\right)\left(\frac{v_{-h}}{v_h}\right)} \Phi = \Phi$$

In this way, $B > 1$ and it will be true that $\varphi_h^{RBRR} < \varphi_h^{BDR}$, $\varphi_h^{RBRR} < \varphi_h^{CSR}$. Hence in case (iii), each agent will bid a price higher than b. In some sense, this rule is provider's revenue maximizing[17].

Joining claims 1-3, we may conclude that $p_h > b_h$ is a dominant strategy for each agent. Hence, in the second stage a RBRR always works.

Now we can move backward to the first stage.

If $\sum_h v_h \leq \Phi$ no congestion occurs and $\varphi_h = 0$ for $\forall h$. In the remaining case, a RBRR works with $\varphi_i = v_i - \left(\frac{B}{1+B} \right) \Phi$ and $\varphi_j = v_j - \left(\frac{1}{1+B} \right) \Phi$.

Rationally any agent will

$$\min_{v_h \in [0;\Phi]} \varphi_h \equiv \max_{v_h \in [0;\Phi]} l_h$$

or equivalently given second stage's outcomes:

[17] For a discussion of the so called revenue efficiency see Fankhauser et al. (1998).

$$\min_{v_i} \left(v_i - \frac{p_i v_j}{v_i p_j + v_j p_i} \Phi \right) \tag{8}$$

$$s.t.\ v_i \leq \Phi$$

and

$$\min_{v_j} \left(v_j - \frac{p_j v_i}{v_i p_j + v_j p_i} \Phi \right) \tag{9}$$

$$s.t.\ v_j \leq \Phi$$

Thus, the two Lagrangians are equal to:

$$\begin{aligned} L_i &= v_i - \frac{p_i v_j}{v_i p_j + v_j p_i} \Phi - \lambda \left[\Phi - v_i \right] \\ L_j &= v_j - \frac{p_j v_i}{v_i p_j + v_j p_i} \Phi - \lambda \left[\Phi - v_j \right] \end{aligned} \tag{10}$$

Applying Kuhn-Tucker conditions to expressions (6), we easily get that if $\lambda = 0$ than it must be that $(v_i p_j + v_j p_i)^2 = -p_i v_j p_j \Phi$ and $(v_i p_j + v_j p_i)^2 = -p_i v_i p_j \Phi$. Clearly a contradiction.

In opposition, if $\lambda \neq 0$, hence the constraints in (3) and (4) are bidding and $v_i = v_j = \Phi$.

Under a RBRR both agents will ask for all network capacity, inducing congestion and getting

$$\begin{aligned} \varphi_i^{RBRR} &= \left(\frac{p_j}{p_i + p_j} \right) \Phi \\ \varphi_j^{RBRR} &= \left(\frac{p_i}{p_i + p_j} \right) \Phi \end{aligned} \tag{11}$$

However, both network users knows that if they avoid congestion, bidding $v_i = v_j = \frac{\Phi}{2}$ they get $\varphi_i^{N\Phi O} = \varphi_j^{N\Phi O} = 0$. In the first stage simultaneous game, four possible outcomes arise : both agents bid for the whole capacity $\begin{pmatrix} v_i \\ v_j \end{pmatrix} = \begin{pmatrix} \Phi \\ \Phi \end{pmatrix}$, both decide to avoid congestion $\begin{pmatrix} v_i \\ v_j \end{pmatrix} = \begin{pmatrix} \frac{\Phi}{2} \\ \frac{\Phi}{2} \end{pmatrix}$ or mixed outcomes $\begin{pmatrix} v_h \\ v_{-h} \end{pmatrix} = \begin{pmatrix} \Phi \\ \frac{\Phi}{2} \end{pmatrix}$ for $h = i; j$. The following matrix summaries the first stage game:

	$v_j = \Phi$	$v_j = \frac{\Phi}{2}$
$v_i = \Phi$	$\frac{p_j}{p_i + p_j}\Phi; \frac{p_i}{p_i + p_j}\Phi$	$\frac{2p_j}{p_i + 2p_j}\Phi; \frac{p_i - 2p_j}{2p_i + 4p_j}\Phi$
$v_i = \frac{\Phi}{2}$	$\frac{p_j - 2p_i}{4p_i + 2p_j}\Phi; \frac{2p_i}{p_j + 2p_i}\Phi$	$0; 0$

As it is possible to notice, $\frac{\Phi}{2}$ is a dominant strategy for both players. Hence, in the subgame perfect equilibrium we will have that:

$$\begin{aligned} v_i^* &= v_j^* = \frac{\Phi}{2} \\ \sum_h v_h^* &= \Phi \end{aligned}$$

This does ensure NE and NCEc.

For property EE:

Trivially, without congestion no problems of priority to higher-valued traffic emerge. In the case that one or both agents decide to bid for the whole network capacity, because of some trembling hand deviations, economic efficiency does require that, under a RBRR, if $\frac{p_i}{v_i} > \frac{p_j}{v_j}$ then $e_i > e_j$. Using (1) we can easily check that this holds.

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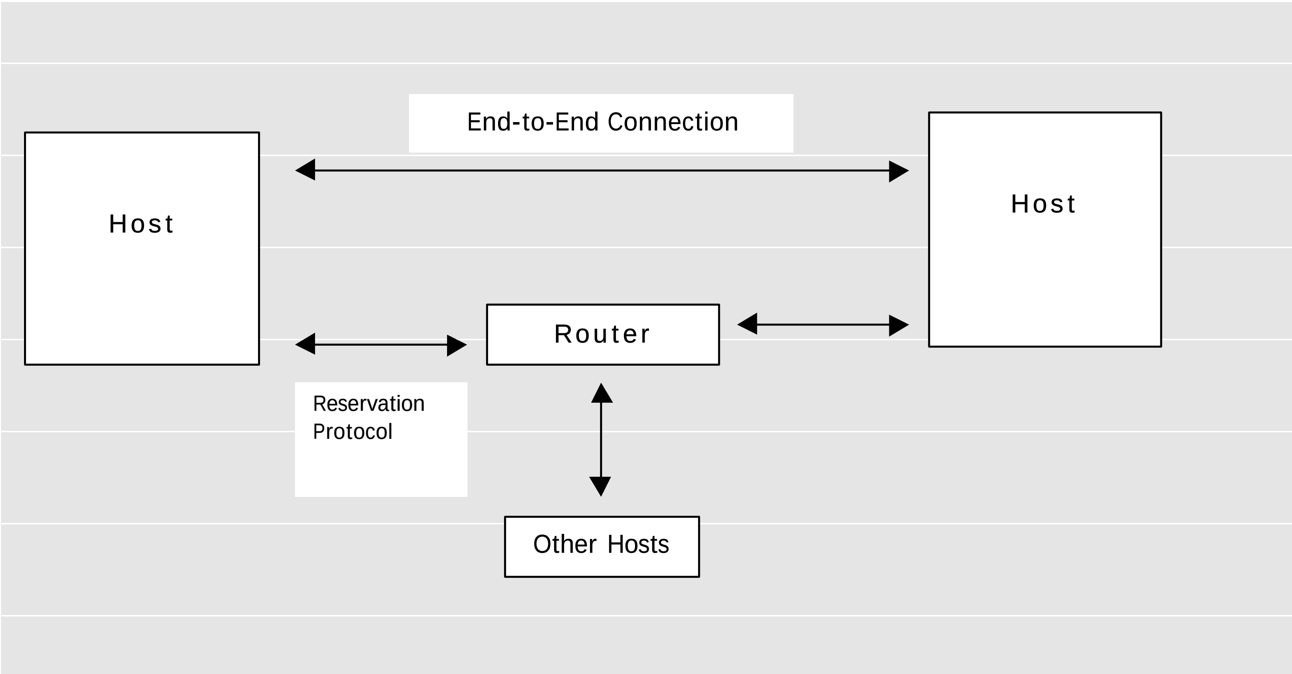

Figure 1
Network components

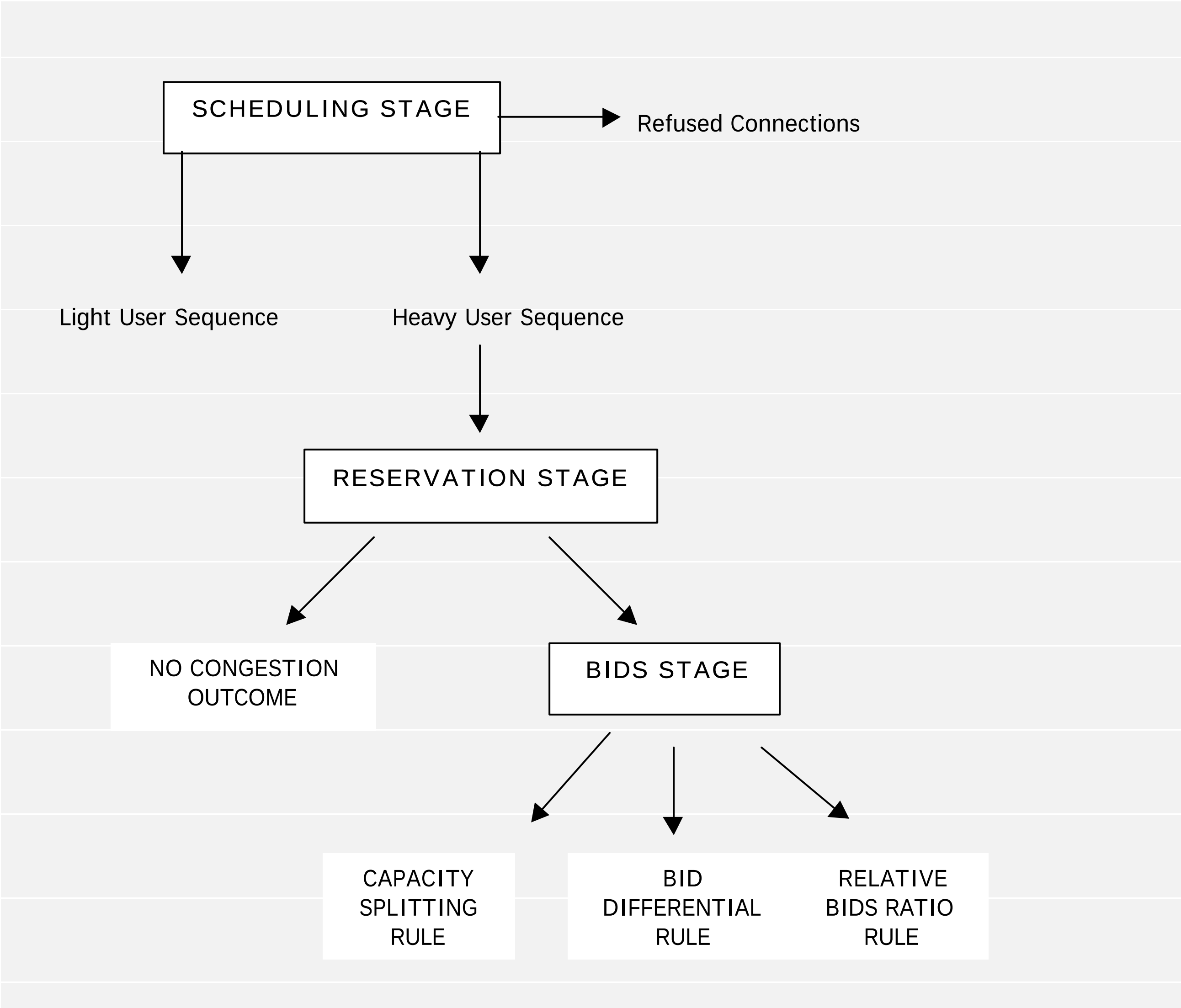

Figure 2
The Game Form