Product Innovation and Transportation Technology in a Cournot Duopoly*

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Abstract

In this paper I will evaluate the stategic behaviour of two firms which can activate R&D investments either to process or to product innovation. I will consider a particular kind of process R&D activity, which I will call Transport and Communication R&D and which aims at increasing the net amount of the product that reaches consumers. I will limit my study to a Cournot duopoly setting. The strategic interaction will be therefore expressed in terms of a two-stage three strategy game, where firms first decide whether to invest in one of the two types of R&D and then they compete in the market by setting quantities. As a result, I will obtain both symmetric and asymmetric equilibria, depending on the relative efficiency of the R&D effort.

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1 Introduction

The information revolution which has recently emerged strongly affected many fields of industrial organization and led to the revision of standard economic theories. In the era of information technology the exploitation of communication networks like Internet may allow firms to enlarge the linkages to their consumers. As Shapiro and Varian (1999) appreciate, *"Information technology is about information and the associated technology".* By investing in computers and more advanced logistics the firm can then manipulate informations and reach a kind of one-to-one relation with consumers. In this way she may be able to ship the product to those consumers really interested in it and/or she may design the product in such a way to match consumers' requirements. Starting from this twofold consideration, this work aims at linking aspects of product differentiation with the analysis of transportation technologies.

Modern theories of product differentiation have been very much influenced by Hotelling (1929), who described product and price competition through a spatial framework. In his model, he distributed consumers in space and considered the different transportation costs which they have to sustain in order to reach the closest firm. Furthermore, he also provided a precise analysis of consumers' tastes for differentiated products. In the famous example of the cider, the transportation cost was conceived in terms of a decrease of utility which a consumer suffers if he is situated far from the ideal product.

> Instead of sellers of an identical commodity separated geographically we might have considered two competing cider merchants side by side, one selling a sweeter liquid than the other...The measure of sourness now replaces distance, while instead of transportation costs there are the degrees of disutility resulting form a consumer getting cider more or less different from what he wants (p.454).

Hotelling's model was fundamentally based on the concept of horizontal differentiation. On the other hand, it is only recently that vertical differentiation has been analysed. After the work of Lancaster (1979), it became clear that products can be identified by two different interpretations of their position in the space of characteristics. Two products are said to be *horizontally differentiated* when they own different characteristics so that, if supplied at the same price, they both obtain a positive market share. It is not possible to rank unambiguously the products because consumers may have different tastes. On the other hand, products are said to be *vertically differentiated* when they own different amounts of the same characteristics so that, if offered at the same price, only one product is sold. In this case each consumer recognizes that there are products of of higher quality. Gabszewicz and Thisse (1986) introduced elements of vertical differentiation within the spatial framework. A very interesting approach was also used by Launhardt (1885), whose contribution has been recently acknowledged. In fact, Launhardt proposed a simple spatial duopoly model which considered both horizontal and vertical product differentiation. Furthermore, he paid attention to the influence of differences in transportation costs. He thus recognized the possibility of different form of heterogeneity among firms, associated either to location or to transportation technology. Recently Thisse and Dos Santos Ferreira (1996) expanded Launhardt model by allowing firms to choose their transportation cost technologies.

Product differentiation and transport costs constitute then a fundamental feature of the strategic interaction among firms. So far, however, the topic of strategic investment to reduce the burden of TC has been rather neglected. This is quite surprising given the role played by new technologies, which can make transport costs less expensive. Firms spend substantial amounts on research and development (R&D). Investments in R&D are generally classified into two types, process innovation and product innovation. The activity of product innovation consists in the development of technologies for producing new products or for increasing the quality of the existing ones. As a consequence, in most of the cases, it decreases the degree of substitutability between rival products in oligopolies. No matter which firm engages in product innovation, there might be a beneficial effect also on rivals that find competing products less close. Recently Lambertini and Rossini (1998) analysed a noncooperative two-stage game of duopolistic interaction, where firms are first required to set the reciprocal degree of differentiation through R&D efforts aiming at product innovation, and then compete in the market either in quantities or in prices. They have shown that firms may end up choosing no heterogeneity as a result of a prisoner's dilemma, no matter whether Bertrand or Cournot competition is used. This appears to be quite consistent with the externality brought about by product innovation through its effect on substitutability. As the effectiveness of the R&D expenditure increases, however, both firms decide to invest in product innovation, thus maximizing their aggregate payoff.

As far as process innovation is concerned, it aims at decreasing the costs of producing existing products. Literature has considered the different degree of efficiency of process innovating R&D between the Cournot and the Bertrand setting. An established result states that there is an excess of process-innovating R&D under Cournot competition (Brander & Spencer, 1983), while the opposite holds under Bertrand competition (Dixon, 1985). Delbono and Denicolò (1990), on the other hand, study homogeneous products and show that the incentive to introduce a cost-reducing innovation is greater for a Bertrand competitor than for a Cournot competitor. Bester and Petrakis (1993) consider vertical differentiated products instead and they obtain a mixed result: the incentive to invest in process innovation is higher for the Cournot (Bertrand) competitor if the degree of product differentiation is large (small).

In this paper, however, I will consider a particular type of process innovation, which can be generally thought to affect transport and communication costs. These costs may be interpreted in terms of the distance between the consumer and the producer. My choice is based on different reasons: first of all, in many circumstances, the costs associated to process innovation may be very high and firms may be discouraged from pursuing such an activity. Firms can thus think of investing in transport and communication technology in order to enlarge their market by serving more consumers. Secondly, as it was introduced at the very beginning, the exploitation of network technologies increases the linkages with the potential costumers. The striking advance of information and communication technology accelerated the growth of networking and the economic advantages for those firms and individuals who have access to such networks. Finally, several works (Krugman and Venables, 1990; Martin and Rogers, 1995) emphasized the impact of different types of infrastructures on trade patterns and industrial competition. These infrastructures can be interpreted in a broad sense as encompassing any facility, service or good that can facilitate the juncture between producers and consumers. Poor infrastructures impose high shipping costs and then a large portion of the goods can be "lost on the way". By investing in communication and transport specific R&D, a firm may then increase this fraction and enlarge her market. The introduction of a transport technology choice constitutes one of the main difference between my approach and the standard Hotelling model, where the demand was assumed to be fixed and the transportation costs exogenous.

I will develop therefore a model where firms may invest either in product innovation or in transport technology innovation. Such a specification of different ways to invest in R&D will give me the opportunity to consider a broader scenario which can be related to the present situation, where Information Technology plays a very important role. Lambertini, Mantovani and Rossini (2001) analyse R&D activity in transport and communication technology in a Cournot duopoly. This paper represents an attempt to expand their results by allowing firms to choose between two different kinds of R&D strategies.

The paper is organized as follows. The next section considers the most recent contributions devoted to the issue of product vs. process innovation. Section 3 provides the basic setting of my model and analyses the choice among investing in TC R&D, investing in PI R&D and not investing at all. In section 4 I will use the reduced form of the game played by firms to find the subgame perfect Nash equilibria. Section 5 presents a two-strategy game, with firms facing the binary choice between investing in TC R&D or not investing. Conclusions and final remarks are in section 6.

2 Product vs. process innovation

Even if there is a vast literature on the economic aspects of innovative activity, the topic of product vs. process innovation has been rather neglected. In particular, there have been only few attempts to explain what factors might be important in directing R&D expenditure towards product or process innovation. In fact, most models on innovation dealt primarily with overall innovative activity (that is the sum of product and process innovation) or with one specific type of innovative activity (either process or product). Rainganum (1989) and De Bondt (1997) provide comprehensive surveys on this field. Empirical evidence indicates that firms usually have a portfolio of R&D projects, some aiming at process innovation, some at product innovation. In many markets, however, budget constraints may force managers to choose whether to employ the advances in information technology to produce a new good, thus enlarging the choice for the consumers, or to ensure a higher rate of return by exploiting the gains deriving from a lower unit cost. This fact has been rather ignored in industrial organization theory, while it has been recently investigated in the business literature. Abernathy and Utterback (1978) proposed a "technological life-cycle" model in which firms initially tend to direct most of their R&D resources to product innovation, because the market potential is large. As the proliferation of brand new products becomes excessive, R&D profitability fades and firms shift from product to process innovation. The concept of product life-cycle was recently translated into a formal model by Klepper (1996), who highlighted the role of firm innovative capabilities and size in conditioning R&D spending. This issue raised interesting questions also in cross-country comparisons. It is frequently argued that Japanese firms are more oriented toward process innovation, while Western countries concentrate more on product innovation[1]. Several explanations have been given to such a phenomenon. Albach (1994) distinguishes between a process-orientation in Japan deriving from the Samurai tradition and the result-orientation in America and Europe deriving from Calvinistic moral values.

In this paper I will limit my analysis to a single market where only two firms operate. The aim is to study the causes that drive firms' strategies in deciding which kind of R&D to undertake. A useful reference can be found in Rosenkranz (1996), which considers a two-stage Cournot duopoly model with horizontal differentiation. Product innovation is then considered as a reduction of product substitutability. In the first stage firms simultaneously choose the product characteristic and the unit cost, while in the second stage they choose outputs. Demand is characterized by preference for product variety. The main result consists in the fact that firms tend to allocate more resources in product innovation when: (i) consumers' willingness to pay increases, if the R&D efficiencies of the two kinds of innovation are similar; (ii) market size increases. In both circumstances, in fact, firms can reach a higher profit by differentiating the products than by reducing the costs. As the number of potential consumers increases, and/or when these consumers are more willing to spend for brand new products, firms prefer to enlarge the variety of the goods offered in the market, thus dampening price competition. Furthermore, in her study Rosenkranz also investigates the effects of R&D coordination on the optimal proportion of innovative activities. It is shown that the creation of cooperative agreements drives firms' investment to product innovation. The increase in efficiency due to the elimination of wasteful duplication effects is larger then the negative impact on firms' investment incentive. In my analysis I will follow the same specification

[1] Mansfield (1988), in a comparative study, argues that Japanese firms have traditionally engaged more in process innovation than American firms.

of product innovation as a reduction of product substitutability, but I will use a less conventional type of process innovation. I will in fact consider an investment in transport and communications, as I explained in the previous section. Moreover, Rosenkranz allows each firm to invest in both types of innovation simultaneously, while in my approach the choice between process or product innovation will be mutually exclusive.

In the field of horizontal product differentiation, another interesting work is due to Eswaran and Gallini (1996), who examine the role of patent policy in redirecting the mix of product and process innovation towards a more efficient technological change. Firms will in fact engage in these innovations to different degrees, depending on the incentives offered by the market in terms of patent protection. In their model R&D is undertaken by two firms, in sequence. The pioneer patronizes a new market with a product innovation, using a new or an old process, while the entrant develops a variant of its rival's innovation. In absence of patent policy, it can be shown that there is an excessive investment in product innovation, relative to the social optimum. The reason lies in the fact that product innovation relaxes competition because it ensures the presence of relatively high prices in a market where goods are more differentiated. The government can rectify this distortion toward product innovation by granting wider patent breadths on process innovation.

Other contributions present weaker links to my work, mainly because they consider the decision whether to direct R&D expenditure toward product or process innovation in a vertically differentiated market[2]. Process innovation is still defined as a reduction in the firm's production costs, but vertical product innovation is conceived as an improvement in the quality of a firm's product. In a recent article, Bonanno and Haworth (1998) provide an interesting duopoly model which explains the R&D choice according to the type of competitive regime where firms operate (Cournot vs. Bertrand). There are two firms in the market, an innovator and a competitor and they have to decide whether to pursue product or process innovation. The cost is supposed to be the same for both types of innovation and the investment opportunity is therefore profitable if the expected increase in profits deriving from the adoption of the chosen innovation is greater then the cost. Two cases are considered, according to the kind of innovator. When the innovator is a high quality firm, three solutions can arise: (i) both the Cournot and the Bertrand competitor invest in process innovation; or (ii) both invest in product innovation; or (iii) they make different choices, the Cournot competitor opts for process innovation while the Bertrand competitor will favor product innovation. A Bertrand competitor is then more prone to choose product innovation than the Cournot competitor, which favors process innovation. The opposite holds when the innovator is a low quality firm and the Bertrand competitor is therefore more willing to adopt process innovation. Battagion and Tedeschi (1998) consider a Bertrand duopoly and study the effects of the different types of innovation on the degree of vertical

[2] The basic reference for models of vertical differentiation is given by Mussa and Rosen (1978).

differentiation. The main result is that a symmetric adoption of a process innovation increases vertical differentiation, while the opposite holds in case of a symmetric adoption of product innovation. However, both contributions do not explicitly take into account the possibility of strategic interaction between firms. The former confines its analysis to the decision of each firm separately, while the latter is limited to symmetric cases where firms undertake the same type of R&D investment.

A very interesting attempt to model the strategic interaction among firms deciding simultaneously which type of innovation to adopt is given by Filippini and Martini (2000). They consider a three-stage duopoly model where firms sell a differentiated good. In the first stage firms decide whether to invest in process or in product innovation, in the second stage they select the quality and in the last stage they play a Bertrand game in the market. Three subgame perfect Nash equilibria may arise, two symmetric, where firms select the same type of innovation, and one asymmetric, where the high quality firm chooses a product innovation, while the low quality firm chooses a process innovation. The determinant of such equilibria is what they call an impact index, defined as the ratio between the *cost saving effect* due to process innovation and the *quality effect* due to product innovation. The smaller the value of the index (i.e the greater is the quality effect), the more likely is that both firms adopt a product innovation and viceversa. The asymmetric equilibrium deserves a further explanation. The high quality firm has more incentives to sell higher quality goods and then it is the first to adopt a product innovation. This justifies the presence of an interval of the impact index where firms choose asymmetrically. The above equilibria have also different impacts on the degree of vertical differentiation, which increases only if firms adopt different types of innovation. As a consequence, the intensity of competition is not relaxed by the symmetric investment of each type of innovation but through the creation of an efficiency gap.

Finally, Lambertini and Orsini (2000) analyse the incentive to introduce process and product innovation in a vertical differentiated monopoly. By evaluating the social planning against the monopoly optimum, the authors show that the social incentive towards both kinds of innovation is always larger than the monopolists' private incentives. This result has some relevant implications on the assessment of the social inefficiency of monopoly in a way that has been overlooked so far, given that the existing literature has performed such an evaluation for a given technology.

3 The setting

I analyse a duopoly where two firms, i and j, compete non-cooperatively in a two-stage framework in a Cournot setting. In the first stage they may decide to invest either in transport and communication (TC) or in product innovation (PI) R&D. Product innovation is conceived in an horizontal sense as a reduction of product substitutability. The second stage is the market stage and firms set quantities. I then use backward induction to solve the game and get subgame perfection. In other words, firms may priorly commit themselves to an investment in R&D and then they compete in the market by choosing the profit-maximizing quantity. The basic feature is that at the very beginning both firms have to choose between undertaking R&D or not. In the former case, they have also to decide whether devoting the R&D expenditure to process or to product innovation. In my model, they can thus either invest in PI or in TC. I assume in fact that, given to budget constraints, they can undertake only one type of R&D investment, with capital expenditure represented by $k > 0$ in both cases.

In order to build my model, I adopt a modified version of the linear duopoly model used by Dixit (1979), Singh and Vives (1984) and many others. Without loss of generality, I assume unitary reservation price and constant marginal costs equal to c. Demand functions are then given by:

$$p_i = 1 - t_i\, q_i - \gamma\, t_j\, q_j \tag{1}$$

$$p_j = 1 - t_j\, q_j - \gamma\, t_i\, q_i \tag{2}$$

where q_i (q_j) is the quantity produced by firm *i (j),* while t_i $(t_j)\in\ ]0,1]$ indicates the fraction of the product shipped by firm *i (j)* which arrives at destination. Given that I consider two symmetric single-product firms, the analysis can be simplified by assuming that $t_i = t_j = t$. The presence of transport costs constitutes the main departure from standard duopoly models. I will come back later to this point. Finally, the parameter $\gamma \in [0,1]$ represents product substitutability as perceived by consumers. The degree of product differentiation decreases with the parameter γ: as $\gamma \to 1$, products become perfect substitutes, while as $\gamma \to 0$ firms effectively become monopolists and product differentiation is at highest.

Let me briefly explain the basic consequences associated to the two kinds of innovations considered. On the one hand, as I mentioned above, by investing in TC a firm can improve her efficiency in reaching the consumers. Transport and communication costs are assumed to be of the 'iceberg' form invented by Samuelson (1954) and widely used in trade theory (Helpman and Krugman, 1985; Krugman, 1990). Different methods have been suggested to formalize such costs. In many international trade models they are included in the cost function; in Brander and Krugman (1983), for example, domestic marginal cost is a constant c, while the marginal cost of export is c/t, $t \in [0,1]$. I think that such an approach would be appropriate if I considered a pure process-innovating R&D investment. But in my model I will treat a particular kind of process innovation, which influences the quantity *effectively* shipped. As a consequence, when a quantity q_i (q_j) is produced, only a fraction $t \in\ ]0,1]$ of the product reaches the consumer[3]. This is the way in which I formalized the transport costs in the above demand functions. For the sake of simplicity, I assume that an investment in TC R&D of a fixed amount k enables the firm to deliver the entire product to her customers, thus no portion is lost in the way ($t = 1$). In the case where the firm does not invest in TC, she will deliver only a portion $t \in \left]0,1\right[$ of the product. I might think of $(1-t)$ as the "waste" of product during the freight process. The initial t is then exogenously given and it can be conceived in terms of the state of public facilities and infrastructures, which is common for both firms. Nonetheless, through a private investment in TC, each firm can improve her ability in reaching the consumers.

On the other hand, in the case of investment in PI, a firm can decrease the degree of substitutability between her own product and the rival's one. Starting from a situation of homogeneity ($\gamma = 1$), I assume that the level of the susbstitution parameter γ decreases from one to a lower level $\gamma_1 \in]0,1[$ if only one firm invests in product innovation and from one to $\gamma_2 < \gamma_1$, $\gamma_2 \in]0,1[$, if both firms invest. This reflects the externality brought about by product innovation. In other words, products become more differentiated in the case of both firms investing in PI than in the case where only one firm does that.

Given the framework above, I therefore consider different cases according to the investment choice of firms. This will cover all the possible combinations of the Cournot market stage. Demand functions will be obtained from 1 and 2 by taking into account the kind of R&D undertaken by firms.

3.1 None invests in R&D (case A)

In this case products are homogeneous ($\gamma = 1$) and transport costs affect both firms' shipping ability. Demand functions are given by:

$$p_i = (1 - tq_i - tq_j) \tag{3}$$

$$p_j = (1 - tq_j - tq_i) \,. \tag{4}$$

Profit functions are then:

$$\pi_i = (1 - tq_i - tq_j)\, tq_i - cq_i$$

$$\pi_j = (1 - tq_j - tq_i)\, tq_j - cq_j.$$

[3] Another way to model the iceberg technology is that when q_i is shipped, only a fraction q_i/t arrives at destination, with $t > 1$.

It can be noticed that this is a symmetric case which simplifies the calculations. From market stage first order conditions (FOCs)[4] I get the following equilibrium quantities:

$$q_i^* = q_j^* = \frac{(t-c)}{3t^2}.$$

Non-negativity constraints on quantities imply that $q_i^* = q_j^* \geq 0$ if $t \geq c$. By substituting the values of $q_i^* = q_j^*$ into the profit functions, I get the equilibrium total profits:

$$\pi_i^A = \pi_j^A = \frac{(c-t)^2}{9t^2} \tag{5}$$

where the superscript A indicates the case under consideration.

3.2 Only one firm invests in TC (cases B and D)

Suppose that firm j invests in TC while firm i does not invest at all (case B). Hence, in this case, there is only one firm that undertakes R&D investments. More precisely, firm j decides to allocate resources to the transport and communication innovative activity with the aim of shipping the entire product to her consumers. On the contrary, firm i does not undertake any type of R&D investment.

Demand functions can then be written as:

$$p_i = (1 - tq_i - q_j) \tag{6}$$

$$p_j = (1 - q_j - tq_i). \tag{7}$$

Profit functions are then:

$$\pi_i = (1 - tq_i - q_j)\, tq_i - cq_i$$

$$\pi_j = (1 - q_j - tq_i)\, q_j - cq_j - k.$$

[4]First Order Conditions are:

$$\frac{\partial \pi_i}{\partial q_i} = 0, \ \frac{\partial \pi_j}{\partial q_j} = 0.$$

Second order conditions are always satisfied, as it can be easily checked in this and subsequent cases.

By applying the same procedure as before, I calculate the equilibrium quantities:

$$q_i^* = \frac{c\,(t-2)+t}{3t^2}$$

$$q_j^* = \frac{c+t-2ct}{3t}$$

Non-negativity constraints on quantities imply that $q_i^* \geq 0$ if $t \geq \frac{2c}{1+c}$ and $q_j^* \geq 0$ if $t \geq \frac{c}{2c-1}$. By comparing the two threshold values it can be easily shown that $\frac{2c}{1+c} > \frac{c}{2c-1}$ and therefore the binding constraint is $t \geq \frac{2c}{1+c}$. Finally, equilibrium total profits are given by:

$$\pi_i^B = \frac{(ct-2c+t)^2}{9t^2} \tag{8}$$

$$\pi_j^B = \frac{c^2\,(1-2t)^2 + 2ct\,(1-2t) + t^2}{9t^2} - k. \tag{9}$$

In case D, firm j invests in TC while firm i does not invest at all. It can easily recognized that case D is the reverse of case B. By reversing the payoffs I get: $\pi_i^D = \pi_j^B$ and $\pi_j^D = \pi_i^B$[5].

3.3 Only one firm invests in PI (cases C and G)

Suppose that firm j invests in R&D to decrease the degree of substitutability, while firm i does not invest at all (case C). Demand functions are then:

$$p_i = (1 - t\,q_i - t\,\gamma_1\,q_j) \tag{10}$$

$$p_j = (1 - t\,q_j - t\,\gamma_1\,q_i) \tag{11}$$

Total profits are:

[5] In this case q_i^* and q_j^* as well as the consequent non-negativity constraints are the reverse of the ones in case B, as it can be easily checked.

$$\pi_i = (1 - t\, q_i - t\, \gamma_1\, q_j)\ t\, q_i - c\, q_i$$

$$\pi_j = (1 - t\, q_j - t\, \gamma_1\, q_i)\ t\, q_j - c\, q_j - k$$

Equilibrium quantities are:

$$q_i^* = q_j^* = \frac{t - c}{(2 + \gamma_1)\, t^2}$$

Non-negativity constraints on quantities imply that $q_i^* = q_j^* \geq 0$ if $t \geq c$. Equilibrium total profits are:

$$\pi_i^C = \frac{(c - t)^2}{(2 + \gamma_1)^2\ t^2} \tag{12}$$

$$\pi_j^C = \frac{(c - t)^2}{(2 + \gamma_1)^2\ t^2} - k \tag{13}$$

As it can be noticed, in this case profits are larger for the firm which does not invest in product innovation, because she exploits the positive externalities brought about by the investment of the other firm without bearing any additional cost.

In case G, firm i invests in PI while firm j does not invest at all. By simply reversing the payoffs with respect to Case C, I obtain $\pi_i^G = \pi_j^C$ and $\pi_j^G = \pi_i^C$[6].

3.4 Both firms invest in TC (case E)

This is the symmetric case where both firms undertake TC R&D. Transport costs are completely eliminated by both firms and the demand functions are then given by:

$$p_i = (1 - q_i - q_j) \tag{14}$$

$$p_j = (1 - q_j - q_i)\,. \tag{15}$$

[6] In this case q_i^* and q_j^* are the reverses of the ones in case C. Furthermore, non-negativity constraints on quantities imply that $q_i^* = q_j^* \geq 0$ if $t \geq c$, as in case C.

Total profits are:

$$\pi_i = (1 - q_i - q_j) \; q_i - c \, q_i - k$$

$$\pi_j = (1 - q_j - q_i) \; q_j - c \, q_j - k.$$

Equilibrium quantities are given by:

$$q_i^* = q_j^* = \frac{1 - c}{3}$$

which are non-negative for $c \leq 1$. Equilibrium profits are then:

$$\pi_i^E = \pi_j^E = \frac{(1 - c)^2}{9} - k. \tag{16}$$

3.5 One firm invests in TC and the other in PI (cases F and H)

Suppose firm i invests in TC while firm j invests in PI (case F). In this asymmetric case firm i eliminates her transport costs, while firm j introduces product differentiation. Demand functions are given by:

$$p_i = (1 - q_i - t \; \gamma_1 \; q_j) \tag{17}$$

$$p_j = (1 - t \; q_j - \gamma_1 \; q_i). \tag{18}$$

Profit functions are then:

$$\pi_i = (1 - q_i - t \, \gamma_1 \, q_j) \; q_i - c \, q_i - k$$

$$\pi_j = (1 - t \, q_j - \gamma_1 \, q_i) \; t \, q_j - c \, q_j - k.$$

By applying the same procedure as before, I calculate the equilibrium quantities

$$q_i^* = \frac{2\,c\,t - 2\,t + \gamma_1\,t - c\,\gamma_1}{\left(\gamma_1^2 - 4\right)\;t}$$

$$q_j^* = \frac{2t - 2c - \gamma_1 t + c\,\gamma_1\,t}{\left(\gamma_1^2 - 4\right)\;t}.$$

It can be easily proved that $q_i^* \geq 0$ in the admissable region of parameters[7] and $q_j^* \geq 0$ for $t \geq \frac{2c}{2-\gamma_1+c\gamma_1}$. As a consequence, the constraint is $t \geq \frac{2c}{2-\gamma_1+c\gamma_1}$.

Finally, I find the equilibrium total profits:

$$\pi_i^F = \frac{\left(2\,c\,t - 2\,t + \gamma_1\,t - c\,\gamma_1\right)^2}{\left(\gamma_1^2 - 4\right)^2\;t^2} - k \tag{19}$$

$$\pi_j^F = \frac{\left(2\,t - 2\,c - \gamma_1 t + c\,\gamma_1\,t\right)^2}{\left(\gamma_1^2 - 4\right)^2\;t^2} - k. \tag{20}$$

In case H, firm i invests in PI while firm j invests in TC. By simply inverting the roles with respect to Case F, I get $\pi_i^H = \pi_j^F$ and $\pi_j^H = \pi_i^F$ [8].

3.6 Both firms invest in PI (case I)

This is the symmetric case where both firms invest in product innovation. Notice that, due to an externality effect, the degree of substitutability is $\gamma_2 < \gamma_1$, i.e. products are more differentiated than in the case where only one firm invests in PI.

Demand functions are thus:

$$p_i = (1 - t\,q_i - t\,\gamma_2\,q_j) \tag{21}$$

$$p_j = (1 - t\,q_j - t\,\gamma_2\,q_i) \tag{22}$$

[7] This has been proved for different values of $t \in (0,1]$ by using plots in three dimensions. The vertical axis measured the value of q_i, while the horizontal axes reported the values of the parameters $c \in (0,1]$ and $\gamma_1 \in (0,1)$. Simulation analysis covers a sufficiently large range of parameters to generalize the results.

[8] Equilibrium quantities and consequent non-negativity constraints are the reverses of the ones in case F.

Total profits are:

$$\pi_i = (1 - t\,q_i - t\,\gamma_2\,q_j)\;t\,q_i - c\,q_i - k$$

$$\pi_j = (1 - t\,q_j - t\,\gamma_2\,q_i)\;t\,q_j - c\,q_j - k$$

Equilibrium quantities are given by:

$$q_i^* = q_j^* = \frac{t - c}{(2 + \gamma_2)\;t^2}$$

which are non-negative for $t \geq c$. The equilibrium total profits are then given by:

$$\pi_i^I = \pi_j^I = \frac{(c-t)^2}{\left(2+\gamma_2\right)^2\,t^2} - k \tag{23}$$

4 The solution of the 3-strategy game

In the previous section I determined the equilibrium profits resulting from the different combinations of the R&D strategies undertaken by the two firms. The reduced form of two stage 3-strategy game is represented in normal form in matrix 1 below. Before comparing the profit levels to find the Nash-equilibrium of the game, I will consider the feasibility conditions deriving from non-negativity constraints on quantities. The aim is to reach a unique threshold value which ensures the sustainability of the game under consideration. Equilibrium quantities are non-negative for $c \leq 1$ (case E), $t \geq c$ (cases A, C, G and I)[9], $t \geq \frac{2c}{1+c}$ (cases B and D) and $t \geq \frac{2c}{2-\gamma_1+c\gamma_1}$ (cases F and H). Simple computations allow me to demonstrate that, in the admissable region of parameters, $\frac{2c}{2-\gamma_1+c\gamma_1} > \frac{2c}{1+c} > c$. As a consequence, the binding threshold value becomes $t > \bar{t} = \frac{2c}{2-\gamma_1+c\gamma_1}$, which is obviously subject to the condition $t < 1$.

[9] This condition may be interpreted as the non-negativity of the marginal revenues for the product shipped.

Let me now analyse the game represented below:

		firm j		
		0	TC	PI
firm i	0	$\pi_i^A = \pi_j^A$	$\pi_i^B \quad \pi_j^B$	$\pi_i^C \quad \pi_j^C$
	TC	$\pi_i^D \quad \pi_j^D$	$\pi_i^E = \pi_j^E$	$\pi_i^F \quad \pi_j^F$
	PI	$\pi_i^G \quad \pi_j^G$	$\pi_i^H \quad \pi_j^H$	$\pi_i^I = \pi_j^I$

Matrix 1 - the reduced form of the 3-strategy game

In what follows, I will simplify the calculations by fixing the values of the parameter γ. I will in fact assume that an investment level k in Product Innovation brings about a product differentiation equal to 1/4 over 1. As a consequence, when only one firm invests in PI, $\gamma_1 = 3/4$, while it becomes $\gamma_2 = 1/2$ when both firms invest in PI. The threshold value becomes then $\bar{t} = \frac{2c}{5/4+3/4c}$. Furthermore, given to the complexity of the calculations involved, I will resort to graphical simulation in order to compare the profits within the admissable region of the parameters considered[10].

In a Cournot duopoly game where firms face different kinds of R&D investments, a variety of Nash equilibria arises, depending on the relative efficiency of the R&D expenditure. Given that I fixed the substitutability parameter γ, such an efficiency can me measured by combining the initial value of the percentage of product shipped (measured by t) with the cost of the R&D investment (measured by the level of k). I will limit my analysis to the study of Nash equilibria in pure strategies. The following proposition summarizes the main results:

Proposition 1 *(i) For relatively small values of t, the game has a NE in dominant strategies where both firms invest in TC if the cost k is low. For very low levels of k, furthermore, such an equilibrium turns out to be also Pareto efficient. As k slightly increases, the game shows a Prisoner's Dilemma, because the aggregate payoff would be maximized by both firms not investing at all. For intermediate values of k, the game becomes a Chicken Game and there are two asymmetric equilibria off the principal diagonal in which only one firm invests in TC. Finally, for high values of k, the game shows a NE in dominant strategies where both firms decide not to invest at all in R&D.*

(ii) For intermediate values of t, the game presents again a NE where both firms invest in TC if the cost k is low. However, such an equilibrium is always not Pareto efficient and the game shows then a Prisoner's Dilemma. As t slightly increases, given a low k, there are two NE along the principal diagonal. Both firms either invest in TC or in PI, thus giving rise to a Coordination

[10] Calculations and simulations will be computed by using the program *Mathematica* (Wolfram, *1991)*.

Game. When k increases, there is a Chicken game where only one firm alternatively invests in PI. Moreover, there exists a very small interval with three Nash equilibria, two asymmetric, (PI, 0) and (0, PI), and one symmetric, where both firms invest in TC. Finally, for high values of k there is an equilibrium in dominant strategies where firms do not invest in R&D. Such equilibrium is Pareto-efficient only for very high levels of k, otherwise firms would maximize their aggregate profit by investing in PI.

(iii) For relatively high values of t, there is a NE in dominant strategies where both firms invest in PI for low levels of k. Such an equilibrium is Pareto-efficient from the firms' standpoint. As k rises, there is again a Chicken game where only one firm alternatively undertakes PI. Finally, for high values of k, the game shows a NE in dominant strategies where both firms decide not to invest at all in R&D. Such a NE turns out to be Pareto-efficient only for very high values of k, otherwise it generates a Prisoner's Dilemma, given that the aggregate payoff of firms would be still maximized by investing in PI.

4.1 Formal Proof

The formal proof is divided in 3 parts: in the first part I will compare the profits appearing in the principal diagonal; in the second part I will consider firms' best responses given the choice of the rival, while in the third part I will look for the Nash equilibria of the game. As I will show, depending on the relation between the initial value of the percentage of product shipped (measured by t) and the cost of investment (measured by the level of k), a variety of Nash equilibria will arise. I will in fact find equilibria in which both firms invest in TC, both invest in PI, both decide not to invest at all or where only one undertakes either TC or PI and the other does not invest. Let me now proceed with the comparison of the profits of Matrix 1.

4.1.1 The principal diagonal

In this preliminary step I will compare the profits accruing to firms when they undertake the same strategy. This part will turn out to be particularly useful in the *ex-post* evaluation of the strategic behaviour taken by firms. In particular, I will be able to compare the symmetric choices that are excluded as equilibria of the noncooperative game.

The main results appearing in this first part are:

$$\pi^A_{i,j} < \pi^E_{i,j} \;\; if \;\; k < k_1, \tag{24}$$

$$\pi^A_{i,j} < \pi^I_{i,j} \;\; if \;\; k < k_2, \tag{25}$$

$$\pi^E_{i,j} > \pi^I_{i,j} \;\; if \;\; \bar{t} < t < t_1 \; and \; \pi^E_{i,j} < \pi^I_{i,j} \;\; if \;\; t_1 < t < 1. \tag{26}$$

Where $\pi_i^A = \pi_j^A = \pi_{i,j}^A$, $\pi_i^E = \pi_j^E = \pi_{i,j}^E$ and $\pi_i^I = \pi_j^I = \pi_{i,j}^I$ to simplify the notation. Furthermore, it can be easily proved that the same threshold value of t can be used to discriminate between k_1 and k_2. As a consequence:

$$k_1 > k_2 \ \ if \ \ \overline{t} < t < t_1 \ \ and \ \ k_1 < k_2 \ \ if \ \ t_1 < t < 1 \tag{27}$$

The calculations and the precise values of the threshold levels of k and t are reported in Appendix 1.A. As it can be easily noticed, the comparison between $\pi_{i,j}^A$ and either $\pi_{i,j}^E$ or $\pi_{i,j}^I$ is not very difficult. It is in fact sufficient to find the threshold level of k which discriminates between the profits considered. For low values of k both firms would gain an higher profit if they invest in one of the two types of R&D, while for higher values of k they prefer not to invest at all. As for the symmetric profits associated to different types of R&D investments ($\pi_{i,j}^E$ vs. $\pi_{i,j}^I$), the level of k is useless in the comparison between the two payoffs. I assumed in fact that both the investment in TC and the one in PI require the same amount of resources. I have then to resort to the analysis of the effectiveness of such investments and this can be done by looking at the value of the initial t. For low values of t ($t < t_1$), firms would prefer to invest in TC, thus gaining from a large increase in the percentage which arrives at destination. For any fixed level k, the lower the initial t, the more efficient the TC R&D is, given that I assumed that such investment enables the firm to eliminate the freight costs, thus delivering the entire product ($t = 1$). On the contrary, when t is high ($t > t_1$), they would gain an higher profit by allocating resources to PI. By spending k on TC they would in fact only increase t by a smaller percentage than before and this is the reason why they prefer to differentiate their products, thus relaxing the competition on the market. As for the role played by the marginal cost c, numerical simulations show that both the value of $\overline{t}$ and the value of t_1 increase when c increase. In particular, when this happens, the feasibility interval shrinks and the value above which $\pi_{i,j}^E > \pi_{i,j}^I$ becomes very high.

In this and in the following cases the comparison among the payoffs of the matrix requires then a careful analysis of the relation between the values of t and k. By combining the two intervals of t with different levels of k, in fact, I can rank the symmetric payoffs appearing along the principal diagonal:

$$if \ \ \overline{t} < t < t_1 \left\{ \begin{array}{c} 0 < k < k_2 \Longrightarrow \pi_{i,j}^E > \pi_{i,j}^I > \pi_{i,j}^A \\ k_2 < k < k_1 \Longrightarrow \pi_{i,j}^E > \pi_{i,j}^A > \pi_{i,j}^I \\ k_2 < k_1 < k \Longrightarrow \ \pi_{i,j}^A > \pi_{i,j}^E > \pi_{i,j}^I \end{array} \right. \tag{28}$$

and

$$if \ t_1 < t < 1 \left\{ \begin{array}{c} 0 < k < k_1 \Longrightarrow \pi_{i,j}^I > \pi_{i,j}^E > \pi_{i,j}^A \\ k_1 < k < k_2 \Longrightarrow \pi_{i,j}^I > \pi_{i,j}^A > \pi_{i,j}^E \\ k_1 < k_2 < k \Longrightarrow \ \pi_{i,j}^A > \pi_{i,j}^I > \pi_{i,j}^E \end{array} \right. . \tag{29}$$

The symmetric case where both firms invest in TC yields the highest profit when TC R&D productivity is very high, i.e. when a low initial value of t ($t < t_1$) is associated with a low level of k ($k < k_1$). For higher levels of t ($t > t_1$) and low values of k ($k < k_2$), profits are at the highest for the symmetric case where both firms invest in PI instead. Finally, in both intervals of t, when the R&D cost is relatively high the largest symmetric profit is the one in which no firm invests in R&D.

4.1.2 The best responses of the firms

Let me now consider firms' best responses, given the choice of the rival. In what follows I will analyse the best responses of firm i given the strategy chosen by firm j. I will consider three cases, depending on the bahaviour of firm j, which can decide to invest in TC, to invest in PI or not to invest at all. The symmetric structure of the game ensures that the same results will hold for firm j, given the action taken by firm i.

- Profits for i (j) when j (i) does not invest at all:

in this case firm i (j) faces three alternatives, given the rival's choice of not investing at all. I then compare π_i^A, π_i^D and π_i^G (or π_j^A, π_j^B and π_j^C if I consider firm j's choice when the rival does not invest):

$$\pi_i^A(\pi_j^A) < \pi_i^D\ (\pi_j^B)\ if\ \ k < k_3, \tag{30}$$

$$\pi_i^A(\pi_j^A) < \pi_i^G\ (\pi_j^C)\ \ if\ \ k < k_4, \tag{31}$$

$$\pi_i^D(\pi_j^B) > \pi_i^G\ (\pi_j^C)\ if\ \ \bar{t} < t < t_2\ and\ \pi_i^D(\pi_j^B) < \pi_i^G\ (\pi_j^C)\ if\ \ t_2 < t < 1. \tag{32}$$

Furthermore, similarly to what I found before,

$$k_3 > k_4\ \ if\ \ \bar{t} < t < t_2\ \ and\ \ k_3 < k_4\ \ if\ \ t_2 < t < 1 \tag{33}$$

By simple computations it is also possible to demonstrate that, in the admissable region of parameters:

$$t_1 < t_2. \tag{34}$$

In Appendix 1.B it is possible to find all the exact calculations. As before, I need to combine different threshold levels of k with the discriminating value of t appearing in this part, hence t_2. I then get two subintervals, in which:

$$if\ \ \bar{t} < t < t_2 \begin{cases} 0 < k < k_4 \Longrightarrow \pi_i^D > \pi_i^G > \pi_i^A \ \left(\pi_j^B > \pi_j^C > \pi_j^A\right) \\ k_4 < k < k_3 \Longrightarrow \pi_i^D > \pi_i^A > \pi_i^G \ \left(\pi_j^B > \pi_j^A > \pi_j^C\right) \\ k_4 < k_3 < k \Longrightarrow \ \pi_i^A > \pi_i^D > \pi_i^G \ \left(\pi_j^A > \pi_j^B > \pi_j^C\right) \end{cases} \tag{35}$$

and

$$if\ \ t_2 < t < 1 \begin{cases} 0 < k < k_3 \Longrightarrow \pi_i^G > \pi_i^D > \pi_i^A \ \left(\pi_j^C > \pi_j^B > \pi_j^A\right) \\ k_3 < k < k_4 \Longrightarrow \pi_i^G > \pi_i^A > \pi_i^D \ \left(\pi_j^C > \pi_j^A > \pi_j^B\right) \\ k_3 < k_4 < k \Longrightarrow \ \pi_i^A > \pi_i^G > \pi_i^D \ \left(\pi_j^A > \pi_j^C > \pi_j^B\right) \end{cases} . \tag{36}$$

As a consequence, for low values of t ($t < t_2$) and small and intermediate levels of k ($k < k_3$), firm i (j), given the choice of the rival not to invest at all, decides to invest in TC, while she prefers to invest in PI for higher values of t ($t > t_2$) and not excessive levels of k ($k < k_4$). In other words, she will invest in TC when the R&D expenditure k is very efficient, leading to a strong increase in the percentage shipped from a low t to 1. On the contrary, she will opt for PI when the starting value of t is higher and then the amount spent in TC R&D would only increase t itself by a negligible percentage. In both intervals, furthermore, firms prefer not to invest when the efficiency of the R&D productivity fades, i.e. for high levels of k ($k > k_3$ in the first subinterval and $k > k_4$ in the second one).

As far as the parameter c is concerned, when its value increases it is necessary a t higher and higher to make firms willing to invest in PI instead of TC. In other words, for high values of the marginal cost firms are likely to invest in TC and this is due to the fact that such an investment can be conceived as a form of process innovation.

- Profits for i (j) when j (i) invests in TC:
  in this second case I have to compare π_i^B, π_i^E and π_i^H (or π_j^D, π_j^E and π_j^F):

$$\pi_i^B(\pi_j^D) < \ \pi_i^E \ (\pi_j^E)\ if\ \ k < k_5, \tag{37}$$

$$\pi_i^B(\pi_j^D) < \ \pi_i^H \ (\pi_j^F)\ \ if\ \ k < k_6, \tag{38}$$

$$\pi_i^E(\pi_j^E) > \ \pi_i^H \ (\pi_j^F)\ if\ \ \bar{t} < t < t_3\ and\ \pi_i^E(\pi_j^E) < \ \pi_i^H \ (\pi_j^F)\ if\ \ t_3 < t < 1. \tag{39}$$

By comparing the threshold values of k one gets:

$$k_5 > k_6\ \ if\ \ \bar{t} < t < t_3\ \ and\ \ k_5 < k_6\ \ if\ \ t_3 < t < 1 \tag{40}$$

For future reference, by simple computations it is also possible to demonstrate that, in the admissable region of parameters:

$$t_2 < t_3. \tag{41}$$

Detailed calculations are reported in Appendix 1.B. By combining the threshold values of k and t, I have:

$$if\ \ \bar{t} < t < t_3 \begin{cases} 0 < k < k_6 \Longrightarrow \pi_i^E > \pi_i^H > \pi_i^B \ \left(\pi_j^E > \pi_j^F > \pi_j^D\right) \\ k_6 < k < k_5 \Longrightarrow \pi_i^E > \pi_i^B > \pi_i^H \ \left(\pi_j^E > \pi_j^D > \pi_j^F\right) \\ k_4 < k_3 < k \Longrightarrow \ \pi_i^B > \pi_i^E > \pi_i^H \ \left(\pi_j^D > \pi_j^E > \pi_j^F\right) \end{cases} \tag{42}$$

and

$$if\ \ t_3 < t < 1 \begin{cases} 0 < k < k_5 \Longrightarrow \pi_i^H > \pi_i^E > \pi_i^B \ \left(\pi_j^F > \pi_j^E > \pi_j^D\right) \\ k_5 < k < k_6 \Longrightarrow \pi_i^H > \pi_i^B > \pi_i^E \ \left(\pi_j^F > \pi_j^D > \pi_j^E\right) \\ k_5 < k_6 < k \Longrightarrow \ \pi_i^B > \pi_i^H > \pi_i^E \ \left(\pi_j^D > \pi_j^F > \pi_j^E\right) \end{cases}. \tag{43}$$

I obtain then similar results as compared to the previous case. An interesting difference is that the interval in which firm i (j) prefers to invest in TC instead of PI increases. In fact, the threshold level which discriminates between π_i^E (π_j^E) and π_i^H (π_j^F) is given by $t_3 > t_2$. Suppose that firm j invests in TC; firm i chooses more often than before to adopt the same strategy, thus increasing the competition in the market. If firm i had decided to invest in PI, in fact, the effects would have been beneficial also for the rival, which could have shipped to the market an higher amount of an horizontally differentiated product (which can be sold at a higher price than before).

- Profits for i (j) when j (i) invests in PI:
  in this third case I will compare π_i^C, π_i^F and π_i^I (or π_j^G, π_j^H and π_j^I):

$$\pi_i^C(\pi_j^G) < \ \pi_i^F\ (\pi_j^H)\ if\ \ k < k_7, \tag{44}$$

$$\pi_i^C(\pi_j^G) < \ \pi_i^I\,(\pi_j^I)\ \ if\ \ k < k_8, \tag{45}$$

$$\pi_i^F(\pi_j^H) > \ \pi_i^I\,(\pi_j^I)\ if\ \ \bar{t} < t < t_4\ and\ \pi_i^F(\pi_j^H) < \ \pi_i^I\,(\pi_j^I)\ if\ \ t_4 < t < 1, \tag{46}$$

$$k_7 > k_8\ \ if\ \ \bar{t} < t < t_4\ \ and\ \ k_7 < k_8\ \ if\ \ t_4 < t < 1. \tag{47}$$

It is also easy to show that, in the admissable region of parameters:

$$t_2 < t_4 < t_3. \tag{48}$$

As usual, by combining the threshold values of k and t, I get[11]:

[11] More precise calculations are reported in Appendix 1.B.

$$
if\ \ \bar{t} < t < t_4 \begin{cases} 0 < k < k_8 \Longrightarrow \pi_i^F > \pi_i^I > \pi_i^C \ \left(\pi_j^H > \pi_j^I > \pi_j^G\right) \\ k_8 < k < k_7 \Longrightarrow \pi_i^F > \pi_i^C > \pi_i^I \ \left(\pi_j^H > \pi_j^G > \pi_j^I\right) \\ k_8 < k_7 < k \Longrightarrow \ \pi_i^C > \pi_i^F > \pi_i^I \ \left(\pi_j^G > \pi_j^H > \pi_j^I\right) \end{cases} \tag{49}
$$

and

$$
if\ \ t_4 < t < 1 \begin{cases} 0 < k < k_7 \Longrightarrow \pi_i^I > \pi_i^F > \pi_i^C \ \left(\pi_j^I > \pi_j^H > \pi_j^G\right) \\ k_7 < k < k_8 \Longrightarrow \pi_i^I > \pi_i^C > \pi_i^F \ \left(\pi_j^I > \pi_j^G > \pi_j^H\right) \\ k_7 < k_8 < k \Longrightarrow \ \pi_i^C > \pi_i^F > \pi_i^I \ \left(\pi_j^G > \pi_j^H > \pi_j^I\right) \end{cases} . \tag{50}
$$

In this case firm i (j), given the choice of the rival to adopt PI, decides to invest in TC for low values of t ($t < t_4$) and relatively low levels of k ($k < k_7$), while it chooses PI for higher values of t ($t > t_4$) and $k < k_8$. As k raises ($k > k_7$ in the first subinterval and $k > k_8$ in the second one) both firms decide not to invest. In practise I got similar results as compared to the previous cases. However, given that $t_2 < t_4 < t_3$, the interval in which firm i (j) prefers to invests in TC instead of PI increases as compared to the first case, where firm *j (i)* did not invest, but decreases as compared to the previous case, where firm *j (i)* invested in TC. This is very interesting because it shows that a firm finds more profitable to invest in PI when the rival does the same than in the case where the rival undertakes TC R&D. When the rival invests in PI, in fact, firm i (j) is more prone to invest in PI as well, leading in this way to a higher degree of product differentiation which is beneficial for both firms.

4.1.3 The Nash equilibria of the game

In the previous part I focused on firms' best responses and I found that they depend on the combination between the level of k and the value of the parameter t. By combining 34, 41, 48 and knowing that $\bar{t} < t < 1$, it is immediate to have the following result:

$$
\bar{t} < t_1 < t_2 < t_4 < t_3 < 1 \tag{51}
$$

In order to determine the subgame perfect Nash equilibria of the game represented in Matrix 1, I have to compare all the threshold levels of k discovered in the analysis of firms' best responses. To simplify the analysis I will proceed by considering separately the intervals of t as they appear in 51. In each interval I will then evaluate only the values of k which are necessary to discriminate among the best responses and I will find the Nash equilibria. I will also compare the resulting equilibria with the Pareto-efficient solutions.

- Let me start by considering a value of t such that $\bar{t} < t < t_1$. In this case firm i (j)' best response is:

- invest in TC for $k < k_3$, do not invest at all otherwise, when the rival does not invest in R&D (see 35);

- invest in TC for $k < k_5$, do not invest at all otherwise, when the rival invests in TC (see 42);

- invest in TC for $k < k_7$, do not invest at all otherwise, when the rival invests in PI (see 49).

Furthermore, along the principal diagonal the highest profit is given by (TC, TC) for $k < k_1$ and by $(0,0)$ otherwise (see 28).

By comparing the threshold values of k, it can be proved that, in the admissable region of parameters:

$$k_1 < k_5 < k_3 \tag{52}$$

and

$$k_7 < k_3. \tag{53}$$

Furthermore, in the interval considered, it is always true that $k_7 < k_5$. All these results are verified in Appendix 1.C. The following rank holds then in $\bar{t} < t < t_1$:

$$k_1 < k_5 < k_7 < k_3 \tag{54}$$

As a consequence, different cases have to be analysed, depending on the level of k:

$$\bar{t} < t < t_1 \begin{cases} 0 < k < k_5 \implies \text{one NE, } \mathit{(TC,TC)} \\ k_5 < k < k_3 \implies \text{two NE, } \mathit{(TC,0)} \text{ and } \mathit{(0,TC)} \\ k_3 < k \implies \text{one NE, } \mathit{(0,0)} \end{cases}. \tag{55}$$

As it can be noticed, for low values of k $(k < k_5)$, the game shows a NE in dominant strategies, where both firms invest in TC. However, when $k_1 < k < k_5$ the game is a *Prisoner's Dilemma*, because firms would gain an higher profit by deciding not to invest at all[12]. It is interesting to notice that only for very low values of k $(k < k_1)$ the simultaneous choice of both firms to invest in TC leads to the highest profit. In this case, in fact, the R&D expenditure turns out to be very efficient, because it is sufficient a small investment to increase the percentage of the product shipped from a very low t $(t < t_1)$ to 1. The benefits of such a strategy are then larger then the drawbacks associated to a stronger competition in the market.

Moreover, when the cost k increases $(k_5 < k < k_3)$ the game allows for two asymmetric NE, where only one firm invests in TC, while the other prefer not to

[12]The NE (TC, TC) is then Pareto dominated by $(0,0)$.

invest. The situation presents then a *Chicken Game* and it can be demonstrated that such a situation holds even if the option to invest in PI is eliminated from the analysis. I will investigate in more details this case in the next section. For the moment it is interesting to notice that, for intermediate levels of k, the commitment taken by one firm to invest in TC leads to a situation where the rival prefers not to invest in R&D. *A priori*, however, it is not possible to determine which firm will invest in R&D (and then which equilibrium will be reached)[13].

Finally, as the cost k increases further ($k > k_3$), the game shows a unique NE in dominant strategies where the aggregate payoff of the firms is maximized by not investing in R&D and this solution yields the Pareto efficiency from firms' standpoint.

- Let me now take t such that $t_1 < t < t_2$. Firm i (j)' best response is:

  - invest in TC for $k < k_3$, do not invest at all otherwise, when the rival does not invest in R&D (see 35);

  - invest in TC for $k < k_5$, do not invest at all otherwise, when the rival invests in TC (see 42);

  - invest in TC for $k < k_7$, do not invest at all otherwise, when the rival invests in PI (see 49).

  Furthermore, along the principal diagonal the highest profit is given by (PI, PI) for $k < k_2$ and by $(0, 0)$ otherwise (see 29). This constitutes the only difference as compared to the previous case.

The comparison of the threshold values of k is not so immediate as before. In Appendix 1.C it is demonstrated that the interval $t_1 < t < t_2$ can be separated into three subintervals:

$$\begin{array}{rl} t_1 < t < t_5, \text{ where} & k_5 < k_2 < k_7 < k_3 \\ t_5 < t < t_6, \text{ where} & k_5 < k_7 < k_2 < k_3 \\ t_6 < t < t_2, \text{ where} & k_5 < k_7 < k_3 < k_2 \end{array} \tag{56}$$

Without entering into details[14], it is important to notice that:

$$t_1 < t < t_2 \left\{ \begin{array}{rl} 0 < k < k_5 \Longrightarrow & \text{one NE, } \textit{(TC, TC)} \\ k_5 < k < k_3 \Longrightarrow & \text{two NE, } \textit{(TC, 0)} \text{ and } \textit{(0, TC)} \\ k_3 < k \ \Longrightarrow & \text{one NE, } \textit{(0, 0)} \end{array} \right. . \tag{57}$$

Hence I get results very similar to those obtained in the previous interval. The only difference, as I introduced before, is due to the fact that in the principal diagonal the highest profit is given by $(PI,\ PI)$ for $k < k_2$ and by $(0, 0)$

[13] Considerations of goodwill, reputations and so on can be used as *refinement* to select between the two possible Nash equilibria.

[14] Differences among the three subintervals are negligible and I prefer not to burden the analysis with too many subcases.

otherwise. In the interval $0 < k < k_5$, as a consequence, the NE in dominant strategies *(TC, TC)* never yields the highest profit and the game shows a *Prisoner's Dilemma.* Again, as k increases ($k_5 < k < k_3$), there is a *Chicken Game* with two asymmetric NE. For higher levels of k ($k > k_3$), finally, there is a NE in dominant strategies where firms do not invest at all. The NE *(0,0)* is always Pareto optimal in the interval $t_1 < t < t_6$, (where $k_3 > k_2$, and then $k > k_3$ implies that $k > k_2$) and in $t_6 < t < t_2$, for $k > k_2 > k_3$. However, in $t_6 < t < t_2$, when $k_3 < k < k_2$ the NE *(0, 0)* is Pareto dominated by *(PI, PI)* and this constitutes another *Prisoner's Dilemma.*

- Let me consider the interval $t_2 < t < t_4$. Firm i (j)' best response is:

  - invest in PI for $k < k_4$, do not invest at all otherwise, when the rival does not invest in R&D (see 36);

  - invest in TC for $k < k_5$, do not invest at all otherwise, when the rival invests in TC (see 42);

  - invest in TC for $k < k_7$, do not invest at all otherwise, when the rival invests in PI (see 49).

  Moreover, along the principal diagonal the highest profit is given by (PI, PI) for $k < k_2$ and by $(0, 0)$ otherwise (see 29).

The comparison of the threshold values of k leads to the determination of two subintervals (see Appendix 1.C):

$$\begin{array}{l} t_2 < t < t_7, \text{ where } \; k_5 < k_7 < k_4 < k_2 \\ t_7 < t < t_4, \text{ where } \; k_7 < k_5 < k_4 < k_2 \end{array} \tag{58}$$

In this case it is necessary to examine both subintervals, because they lead to different Nash equilibria:

$$t_2 < t < t_7 \begin{cases} 0 < k < k_5 \Longrightarrow \text{ one NE, } \; \textit{(TC, TC)} \\ k_5 < k < k_7 \Longrightarrow \text{no NE} \\ k_7 < k < k_4 \text{ two NE, } \textit{(PI, 0)} \text{ and } \textit{(0, PI)} \\ k_4 < k \Longrightarrow \text{ one NE, } \; \textit{(0, 0)} \end{cases}, \tag{59}$$

$$t_7 < t < t_4 \begin{cases} 0 < k < k_7 \Longrightarrow \text{ one NE, } \; \textit{(TC, TC)} \\ k_7 < k < k_5 \Longrightarrow \text{three NE, } \textit{(TC, TC), (PI, 0)} \text{ and } \textit{(0, PI)} \\ k_5 < k < k_4 \text{ two NE, } \textit{(PI, 0)} \text{ and } \textit{(0, PI)} \\ k_4 < k \Longrightarrow \text{ one NE, } \; \textit{(0, 0)} \end{cases}. \tag{60}$$

In the two subintervals it is worth to notice what happens for intermediate values of k: the first subinterval (for $k_5 < k < k_7$) shows no NE, while the second one (for $k_7 < k < k_5$) is characterized by the presence of three NE . As before, for low levels of k [15] the game is a *Prisoner's Dilemma,* because *(TC, TC)* is Pareto dominated by *(PI, PI)*. Notice that in this case (TC, TC) is not an equilibrium in dominant strategies; the best reply for firm i (j), when the rival does not invest, consists in an investment in PI. For higher values of k[16], furthermore, there are two asymmetric NE in which only one firm invests in PI while the other does not invest. Hence I get another *Chicken Game,* even if it is different from the one found in the previous interval. Here, in fact, the firm which undertakes R&D devotes resources to PI instead of TC and she ends up obtaining always a lower profit of the rival, as it can be easily checked by comparing 12 with 13. In fact, the firm which finances a product-differentiating R&D project creates positive externalities for the rival, which may decide not to invest at all, thus gaining from the degree of product differentiation introduced in the market, without bearing any additional cost. The R&D activity is in fact quite expensive in the interval of k considered and the presence of one firm in the market which adopts yet a product innovation is sufficient to discourage the other from pursuing any kind of R&D project.

Finally, for very high values of k ($k > k_4$), neither firm i nor firm j invests in R&D and the resulting NE in dominant strategies *(0,0)* turns out to be Pareto dominant only for $k > k_2 > k_4$, while it generates a *Prisoner's Dilemma* for $k_4 < k < k_2$[17]. This result holds in both the subintervals considered.

- Let me now pass to the interval $t_4 < t < t_3$. Firm i (j)' best response is:

  - invest in PI for $k < k_4$, do not invest at all otherwise, when the rival does not invest in R&D (see 36);

  - invest in TC for $k < k_5$, do not invest at all otherwise, when the rival invests in TC (see 42);

  - invest in PI for $k < k_8$, do not invest at all otherwise, when the rival invests in PI (see 50).

  Again, along the principal diagonal the highest profit is given by (PI, PI) for $k < k_2$ and by $(0,0)$ otherwise (see 29).

The comparison of the threshold levels of k leads to the determination of two subintervals (see Appendix 1.C for details):

$$\begin{array}{rl} t_4 < t < t_8, \text{ where} & k_8 < k_5 < k_4 < k_2 \\ t_8 < t < t_3, \text{ where} & k_5 < k_8 < k_4 < k_2 \end{array} \tag{61}$$

As before, I have to examine both subintervals separately, because they lead to different Nash equilibria:

[15] $0 < k < k_5$ for the first subinterval and $0 < k < k_7$ for the second one.
[16] $k_7 < k < k_4$ in the first subinterval, $k_5 < k < k_4$ in the second one.
[17] In the interval $k_4 < k < k_2$, in fact, the NE *(0,0)* is Pareto dominated by *(PI,PI)*.

$$t_4 < t < t_8 \begin{cases} 0 < k < k_8 \Longrightarrow \text{ two NE, } \textit{(TC, TC)} \text{ and } \textit{(PI, PI)} \\ k_8 < k < k_5 \Longrightarrow \text{three NE, } \textit{(TC, TC), (PI, 0)} \text{ and } \textit{(0, PI)} \\ k_5 < k < k_4 \Longrightarrow \text{ two NE, } \textit{(PI, 0)} \text{ and } \textit{(0, PI)} \\ k_4 < k \Longrightarrow \text{ one NE, } \textit{(0, 0)} \end{cases}, \tag{62}$$

$$t_8 < t < t_3 \begin{cases} 0 < k < k_5 \Longrightarrow \text{two NE, } \textit{(TC, TC)} \text{ and } \textit{(PI, PI)} \\ k_5 < k < k_8 \Longrightarrow \text{one NE, } \textit{(PI, PI)} \\ k_8 < k < k_4 \Longrightarrow \text{two NE, } \textit{(PI, 0)} \text{ and } \textit{(0, PI)} \\ k_4 < k \Longrightarrow \text{ one NE, } \textit{(0, 0)} \end{cases}. \tag{63}$$

The distinguishing characteristic of these two subintervals is the appearance of Nash equilibria in which both firms invest in PI and this happens for low values of k ($k < k_8$). For all $k < k_8$ in the first subinterval and $k < k_5$ in the second one, there is a *Coordination Game* with two NE, *(TC, TC)* and *(PI, PI),* along the principal diagonal[18]. It is important to notice that *(TC, TC)* is Pareto dominated by *(PI, PI).* There is indeed a region in the second subinterval ($k_5 < k < k_8$) in which *(PI, PI)* is the only Nash equilibrium. This confirms one of the intuition of the model: the higher the value of t, the more willing are firms to invest in PI, as shown before. For higher values of k[19], furthermore, the situation is in practise the same as before; there are two asymmetric NE, where only one firm invests in PI and the other does not invest, thus giving rise to a *Chicken Game.* Finally, for very high values of k ($k > k_4$), the game shows a NE in dominant strategies where firms do not invest at all. The resulting NE *(0,0)* is Pareto-efficient from the firms' standpoint for $k > k_2 > k_4$, while it generates a *Prisoner's Dilemma* for $k_4 < k < k_2$.

- Finally, I consider the interval $t_3 < t < 1$. Firm i (j)' best response is:

  - invest in PI for $k < k_4$, do not invest at all otherwise, when the rival does not invest in R&D (see 36);

  - invest in PI for $k < k_6$, do not invest at all otherwise, when the rival invests in TC (see 43);

  - invest in PI for $k < k_8$, do not invest at all otherwise, when the rival invests in PI (see 50).

  As before, the highest profit in the principal diagonal is given by (PI, PI) for $k < k_2$ and by $(0, 0)$ otherwise (see 29).

In Appendix 1.C it is shown that the following rank of the R&D expenditure levels k holds for $t_3 < t < 1$:

[18] An example of a *Coordination Game* is given by Matrix *2.1.f* in the previuos section.

[19] $k_5 < k < k_4$ in the first subinterval and $k_8 < k < k_4$ in the second one.

$$k_6 < k_8 < k_4 < k_2 \tag{64}$$

Depending on the value taken by k it is possible to find different Nash equilibria:

$$t_3 < t < 1 \begin{cases} 0 < k < k_8 \Longrightarrow & \text{one NE, } \textit{(PI, PI)} \\ k_8 < k < k_4 \Longrightarrow & \text{two NE, } \textit{(PI, 0)} \text{ and } \textit{(0, PI)} \\ k_4 < k \Longrightarrow & \text{one NE, } \textit{(0,0)} \end{cases} . \tag{65}$$

For high values of t, then, there is a wider interval ($k < k_8$) as compared to before in which both firms maximize the aggregate profit by investing in PI, thus giving rise to an equilibrium in dominant strategies *(PI, PI)* which is also Pareto-efficient. The other results are in practise the same as in the previous interval: for $k_8 < k < k_4$ the game presents two asymmetric equilibria off the principal diagonal where only one firm invests in PI. Moreover, there is a Nash equilibrium in dominant strategies (0, 0) for $k > k_4$ which is Pareto-efficient for $k > k_2 > k_4$, while it generates a *Prisoner's Dilemma* for $k_4 < k < k_2$. This last case simply confirms what I suggested before: as the value of t rises, firms would prefer to invest in PI only for reasonable costs associated to such R&D project, otherwise they would reach an higher profit by not investing at all.

4.2 Comments and remarks

In the previous parts I analysed matrix 1 of payoffs in order to determine the subgame perfect Nash equilibria. In the formal proof, I first considered the best responses of the firms, given the strategy chosen by the rival, and I then found the Nash equilibria of the game. The presence of many parameters required some simplifications: I assumed in fact that an investment in TC increased t up to 1 and I fixed the value of γ. Nonetheless, I was able to discover interesting results, due to the relationship between k, the cost of doing R&D and t, the initial percentage of product which reaches the consumers. By partitioning the admissable sets of t in several intervals and by considering different levels of k, I derived the results appearing in Proposition 1.

It is worth to notice that the game allows for both symmetric and asymmetric equilibria. Let me start by considering the symmetric equilibria when both firms invest in the same type of R&D. The aim is to give some intuitions behind the results. As for the case where both firms invest in TC, the initial t is low and an investment in TC turns out to be very efficient. However, the investment in TC tends to increase the competition within the market, because firms can deliver a higher quantity of the product to the same consumers, thus lowering the price. This consideration could be useful in explaining why the region in which such an equilibrium is Pareto-dominant is limited to the case of a very efficient R&D expenditure, i.e when a very low k is sufficient to increase the quantity shipped from a very small percentage t to 1. In all the other circumstances, with the exception of very high levels of k, firms would yield a higher aggregate profit by investing in PI. This can be easily explained by the fact that product innovation tends to weaken the competition within the market, by making products less close. If firms could coordinate their activity, therefore, they would prefer a simultaneous commitment to PI R&D. This is quite consistent with the analysis of cooperative agreements made by Rosenkranz (1996), which confirms the tendency for firms to shift the innovative activity toward product innovation in presence of R&D coordination. Almost surprisingly, however, in my analysis the intervals where firms simultaneously decide to invest in PI is limited to the case of very high values of t associated to low values of k. Two explanations can be given to justify this result: first, the presence of strong positive externalities associated to product innovation may dampen the incentive of investing in such an activity, given that it creates benefits also for the rival. The second explanation is related to the simplifications introduced in my analysis. In fact, I assumed that an investment in TC increases the percentage shipped from $t \in]0,1[$ to 1, while an investment of the same amount k in PI 'only' brings a differentiation equal to 1/4[20]. Numerical simulations show that the region where (*PI, PI*) is a unique Nash equilibrium increases with product differentiation, i.e. it increases with a drop of the parameter γ[21].

The last symmetric NE consists in both firms not undertaking any kind of R&D activity. This occurs for levels of k particularly high, both for low and for high values of t. It is still interesting to notice that such an equilibrium, for k not excessive, is still Pareto dominated by the case in which both firms invest in PI, while it becomes the best solution from firms' standpoint when R&D expenditure becomes too expensive.

Let me now examine the asymmetric equilibria which appear in the matrix. As shown before, for intermediate levels of k the game shows a *Chicken Game* and there exist two asymmetric equilibria in which only one firm invests either in TC (for low values of t) or in PI (for higher values of t). The cases where one firm alternatively invests in PI can be easily understood by considering the positive externality arriving also to the firm which does not undertake R&D. This firm, in other words, gains from the product differentiation brought about by the other firm and it does not invest in PI, being such an activity quite expensive in the intervals of k considered. A bit more difficult is the explanation of the asymmetric cases in which only one firm invests in TC and the other does not invest at all. In the next section I will study in detail such equilibria in a game where the possibility of investing in PI is not available. As I will demonstrate, for intermediate values of the parameter k the game still show the same asymmetric equilibria. The reason why one firm may decide not to invest in TC, when the other does it, relies then on strategic considerations.

As it can be noticed, furthermore, there are no asymmetric equilibria in the game where firms undertake different forms of R&D strategies. In fact, in all the

[20] Remember that I assumed that when only one firm invests in PI the parameter measuring the product differentiation goes from 1 to 3/4, while when both invest in PI it goes from 1 to 1/2.

[21] In fact I analysed the same matrix but I used $\gamma_1 = 2/3$ and $\gamma_2 = 1/3$, i.e. the impact of the PI activity is equal to 1/3.

intervals of t considered, a firm would never yield an higher profit by investing in PI when the rival does TC. An investment in PI would in fact give a very strong advantage to a rival which adopts TC. The firm doing TC, in fact, would deliver to the market an higher amount of a product which has become differentiated and which could then be sold at an higher price. The only exception takes place for very high values of t ($t_3 < t < 1$); the firm is willing to invest in PI when the rival invests in TC (see 43). However, this does not give rise to a Nash equilibrium, because every firm has an incentive to invest in PI, given the choice of the other to invest in PI (see 50). Filippini and Martini (2000), as I reported in Section 2, found asymmetric equilibria where firms adopt different innovative activities. However, they considered a vertically differentiated duopoly with a high quality firm and a low quality firm and this played a fundamental role in explaining the different incentives towards the two forms of R&D.

Finally, I would like to spend some words on the role played by the marginal cost c. Numerical simulations show that all threshold values of t increase as c rises. In this case the feasibility interval shrinks, thus reducing the values for which I can analyse matrix 1. Moreover, it is necessary to take numerical values of t higher and higher to switch from one interval to the other. As c increases, then, a firm can yield an higher profit by investing in transport and communication rather than by investing in product innovation. This is consistent with the nature of the TC innovative activity, which can be considered as a form of process innovation activity.

5 A particular case

In the previous section I analysed a matrix which summarized a three-strategy game in two stages. Different subgame perfect Nash equilibria have been discovered, depending on the relation between the parameters t, k and c. A very interesting case revealed the presence of asymmetric Nash equilibria where only one firm invested in TC, while the other preferred not to invest at all. This occurred for intermediate levels of k associated with low values of t. In this section I will investigate the nature of such equilibria by considering what would happen if firms faced only a binary choice between undertaking TC R&D or doing nothing. The reduced form of two stage 2-strategy game is represented in normal form in matrix 2 below. The basic framework is the same as before, with $t \in]0, 1[$, $a = 1$, $c < 1$, and so on.

		firm j	
		0	TC
firm i	0	$\pi_i^A = \pi_j^A$	$\pi_i^B \quad \pi_j^B$
	TC	$\pi_i^D \quad \pi_j^D$	$\pi_i^E = \pi_j^E$

Matrix 2 - the reduced form of the 2-strategy game

From the reduced form of the game I obtain the following result:

Proposition 2 *In a Cournot duopoly game, firms decide to invest in transport and communication technology according to the cost of the R&D investment required. Depending on the level of k, therefore, it is possible to find different subgame perfect Nash equilibria in pure strategies.*

When the cost of R&D k is low, the game shows a NE in dominant strategies where both firms invest in TC. For very low levels of k such an equilibrium is also Pareto efficient, while as k increases the game shows a Prisoner's Dilemma, because the aggregate payoff of firms would be maximized by not investing at all.

For intermediate values of k, the game becomes a Chicken Game and there exist two asymmetric equilibria in which only one firm invests in TC.

Finally, for high values of k, the game shows a NE in dominant strategies where both firms do not invest in TC. Such an equilibrium turns out to be Pareto-efficient from the firms' standpoint.

Proof. I prove the above preposition by comparing the equilibrium profits appearing in Matrix 2. Before proceeding with such a comparison, it is important to notice that I have to take into account a different binding threshold value of t to ensure the sustainability of the game. Remember that $c \leq 1$ (case E) , $c \leq t$ (case A) and $t \geq \frac{2c}{1+c}$ (cases B and D). It is easy to prove that $\frac{2c}{1+c} > c$. As a consequence, the threshold value becomes $t > \bar{t} = \frac{2c}{1+c}$, which is weaker than the one found for Matrix 1.

Comparing equilibrium profits along the principal diagonal, from 24 I get $\pi_{i,j}^A < \pi_{i,j}^E$ if $k < k_1$. Furthermore, from 30 and 37, $\pi_i^A(\pi_j^A) < \pi_i^D\ (\pi_j^B)$ if $k < k_3$ and $\pi_i^B(\pi_j^D) < \pi_i^E\ (\pi_j^E)$ if $k < k_5$. I have then to compare the threshold values of k appearing in this context. However, from 52 I have that, in the acceptable region of parameters, $k_1 < k_5 < k_3$. It is easy to demonstrate that such a result holds in this case too.

Depending on the value taken by k[22], it is possible to find three different types of Nash equilibria:

$$\begin{array}{c} 0 < k < k_5 \Longrightarrow \text{ one NE, } \textit{(TC, TC)} \\ k_5 < k < k_3 \Longrightarrow \text{two NE, } \textit{(TC, 0)} \text{ and } \textit{(0, TC)}\ . \\ k_3 < k \Longrightarrow \text{ one NE, } \textit{(0,0)} \end{array} \tag{66}$$

In the first interval considered, in fact, for firm i $\pi_i^A < \pi_i^D$ and $\pi_i^B < \pi_i^E$, while for firm j $\pi_j^A < \pi_i^B$ and $\pi_j^D < \pi_j^E$. As a consequence, both firms invest in TC, but they maximize their aggregate profit only if $0 < k < k_1$, being $\pi_{i,j}^A < \pi_{i,j}^E$. On the other hand, for $k_1 < k < k_5$, the game becomes a *Prisoner's Dilemma,* given that $\pi_{i,j}^A > \pi_{i,j}^E$. For $k_5 < k < k_3$, I obtain that $\pi_i^A < \pi_i^D$ $(\pi_i^A < \pi_i^B)$ but $\pi_i^B > \pi_i^E$ $(\pi_j^D > \pi_j^E)$ and then the game shows two asymmetric

[22] All the values of k are reported in Appendix 1.

equilibria. Finally, for $k > k_3$, $\pi_i^A > \pi_i^D$ ($\pi_i^A > \pi_i^B$) and $\pi_i^B > \pi_i^E$ ($\pi_j^D > \pi_j^E$), thus leading to an equilibrium in dominant strategies where firms do not invest in TC R&D. This equilibrium is Pareto-efficient, because it is also satisfied $k > k_1$.

It can be easily checked that the sequence of payoffs presented is invariant as the value of the parameter c varies within the admissable range. ■

This second model confirms then what I found in the previuos section. There is a region of k ($k_5 < k < k_3$) where the game allows for two asymmetric NE, where only one firm invests in TC, while the other decides not to invest. Remind that by investing in TC R&D a firm can expand her market by serving more consumers. But for certain levels of k the market sustains only the presence of one firm undertaking TC R&D. The commitment taken by one firm to invest in TC is then sufficient to deter the rival from pursuing the same R&D strategy. It is interesting to notice that such a commitment turns out to be credible because of the R&D expenditure k which it requires. Suppose that firm i invests in TC. Given such a choice, firm j would reach an higher profit by not investing in TC. If firm j invested in TC as well, in fact, the quantity arriving on the market would be so large to drop the price down to an unprofitable level.

6 Conclusions

The Information Revolution which has recently taken place deserves a particular attention, especially for the effects that it can have on industrial competition. The discussion has been focused more than in the past on firms' strategic decisions in the allocation of R&D resources among different projects. It is commonly observed that firms conduct R&D either along process innovation or along product innovation. Notwithstanding the relevance of this argument, however, the issue of product innovation vs. process innovation has been surprisingly neglected in the literature and only recently some contributions have been addressed to this topic. This work constitutes then an attempt to analyse the factors which direct R&D expenditure toward product innovation or toward process innovation. In my analysis, I employed a particular kind of process innovation, i.e. an investment in transport and communications technologies. I considered in fact a Cournot duopoly setting where firms simultaneously select whether to invest in product innovation or in transport and communication.

The two forms of R&D under consideration have different effects on firm's profits and I tried to analyse the strategic behaviour animating firms' competition in the market. I therefore used a two-stage three strategy game, where firms first decide among three options (invest in PI, invest in TC or not invest at all) and then they compete on the market by setting quantities. The analysis revealed the presence of various subgame perfection Nash equilibria, depending on the relative efficiency of the R&D effort. Such an efficiency has been measured by combining the values taken by the parameter t, which indicates the initial percentage of the product which arrives at destination, with different levels of k, the cost of doing R&D. As a result, I obtained both symmetric and asymmetric equilibria. Starting from the symmetric ones, both firms invest in TC when such a strategy is very effective, i.e when a relatively low expenditure k leads to a massive increase in the fraction which arrives at destination. As the initial t increases, on the other hand, firms prefer to undertake PI, introducing in this way a certain degree of product differentiation. When the cost k becomes prohibitive, firms decide not to invest. As for the asymmetric equilibria, only one firm either invests in TC (for low values of t) or in PI (for higher values of t), and this occurs for intermediate values of k. In section 5 I also analysed a two strategy game and this gave me the opportunity to understand the nature of the TC investment, which is something more than a kind of process innovation.

It is important to remind that the results obtained in my model are strongly influenced by the assumptions introduced. In fact, two simplifications were very useful but their impact on the results has to be taken into account. Firstly, I assumed that an investment of a fixed k in TC is sufficient to completely eliminate the waste of product during the freight phase, thus leading to $t = 1$. A first extension of my analysis should consider an increase in the percentage t as a function of the R&D expenditure. Secondly, I fixed the values of the parameter γ measuring the product differentiation. In this case too, it should be interesting to model such a parameter as a function of the R&D expenditure.

Furthermore, this kind of analysis can be also applied to the study of oligopolistic markets with trade and R&D investments. For example, it would be interesting to expand my model by considering the role of R&D investment in transport and communication *vis à vis* product innovation in a duopoly with trade.

Even if the extensions suggested above would definitely add more insight into my analysis, I think that the results appearing in this paper are very interesting, especially because they constitute an attempt to analyse a topic which has not been widely explored.

7 Appendix 1

In Appendix 1 I will demonstrate the validity of many results appearing in section 4. In particular, I will divide my analysis in three parts, corresponding to the three subsections 4.1.1, 4.1.2 and 4.1.3. Calculations and simulations will be computed by using the program *Mathematica* (Wolfram, 1991). A very useful command is the plot in three dimensions of the difference between the profits considered. The numerical value of the difference considered appears in the vertical axis, while in the horizontal axes are reported the values of $c \in (0, 1)$ and $t \in (0, 1)$. This device will turn out to be particularly useful in evaluating profits in cases where both firms invest in R&D and the parameter k does not have discriminatory power.

7.1 Appendix 1.A

Let me start with the specification of the results appearing in section 4.1.1. It can be easily verified that:

$$\pi^A_{i,j} < \pi^E_{i,j} \;\; if \;\; k < k_1 = \frac{c\,(t-1)\,(c-2t+ct)}{9t^2}$$

and

$$\pi^A_{i,j} < \pi^I_{i,j} \;\; if \;\; k < k_2 \cong 0.049 + \frac{0.049}{t^2}c^2 - \frac{0.098}{t}c.$$

A little more difficult is the comparison between $\pi^E_{i,j}$ and $\pi^I_{i,j}$, given that I cannot use the parameter k for discriminatory purposes. I will then use the parameter t to measure the effectiveness of the two different R&D investments, thus ranking the above profits. To begin with, by solving the difference $\pi^E_{i,j} - \pi^I_{i,j} = 0$ w.r.t the parameter t, I find that $\pi^E_{i,j} > \pi^I_{i,j}$ for $t_{1a} < t < t_{1b}$ and $\pi^E_{i,j} < \pi^I_{i,j}$ for $t < t_{1a}$ and for $t > t_{1b}$, where:

$$t_{1a} \cong \frac{\frac{1}{2}c\,[60\,(c-1)-72]}{25c^2-50c-11};\; t_{1b} \cong \frac{\frac{1}{2}c\,[60\,(1-c)-72]}{25c^2-50c-11}.$$

However, I have to take into account the feasibility restraint $t > \bar{t} = \frac{2c}{5/4+3/4c}$ and simple algebra allows me to show that $t_{1a} < \bar{t} < t_{1b}$. As a consequence, given that I am interested only in the region where $\bar{t} < t < 1$, $\pi^E_{i,j} > \pi^I_{i,j}$ for $\bar{t} < t < t_{1b}$ and $\pi^E_{i,j} < \pi^I_{i,j}$ for $t_{1b} < t < 1$, thus verifying the results of formula 26, where I use $t_1 = t_{1b}$ to simplify the notation. The same threshold value of t allows for the comparison between k_1 and k_2 (see 27), which are the values of k which discriminate between $\pi^A_{i,j}$ and $\pi^E_{i,j}$ and between $\pi^A_{i,j}$ and $\pi^I_{i,j}$ respectively.

7.2 Appendix 1.B

- Profits for i (j) when j (i) does not invest at all:

By simple calculations I get:

$$\pi^A_i(\pi^A_j) < \pi^D_i\;(\pi^B_j)\;if\;\; k < k_3 = \frac{4c\;(t-1)\,(c-1)}{9t},$$

$$\pi^A_i(\pi^A_j) < \pi^G_i\;(\pi^C_j)\;\; if \;\; k < k_4 \cong 0.037 + \frac{0.037}{t^2}c^2 - \frac{0.074}{t}c$$

As for the comparison between $\pi_i^D(\pi_j^B)$ with $\pi_i^G(\pi_j^C)$, by solving the difference $\pi_i^D - \pi_i^G = 0$ ($\pi_j^B - \pi_j^C = 0$) w.r.t the parameter t, I get that $\pi_i^D > \pi_i^G$ $\left(\pi_j^B > \pi_j^C\right)$ for $t_{2a} < t < t_{2b}$ and viceversa for $t < t_{2a}$ and for $t > t_{2b}$, where:

$$t_{2a} \cong \frac{\frac{1}{2}c\left[12c - 14 - 13.86\,(c-1)\right]}{12c^2 - 12c - 11};\; t_{2b} \cong \frac{\frac{1}{2}c\left[12c - 14 + 13.86\,(c-1)\right]}{12c^2 - 12c - 11}$$

However, by considering the restraint $t > \overline{t} = \frac{2c}{5/4 + 3/4c}$, it is easy to find that $t_{2a} < \overline{t} < t_{2b}$. As a consequence, by focusing on the area in which $\overline{t} < t < 1$ and given that $t_1 < t_2$ (see 34), the results of 32 hold and I can simplify the notation by $t_2 = t_{2b}$. As before, the same threshold value of t allows for the comparison between k_3 and k_4.

- Profits for i (j) when j (i) invests in TC:
  as for $\pi_i^B(\pi_j^D)$ vs. π_i^E (π_j^E) and $\pi_i^B(\pi_j^D)$ vs. π_i^H (π_j^F), it is easy to find that:

$$\pi_i^B(\pi_j^D) < \pi_i^E\ (\pi_j^E)\ if\ \ k < k_5 = \frac{4c\,(1-t)\,(t-c)}{9t^2},$$

$$\pi_i^B(\pi_j^D) < \pi_i^H\ (\pi_j^F)\ \ if\ \ k < k_6 \cong 0.02\left(1 + \frac{c}{t}\right) - 0.06c\,(1+c) - \frac{(0.1 + 0.2t)}{t^2}c^2.$$

As for the comparison between $\pi_i^E(\pi_j^E)$ with $\pi_i^H(\pi_j^F)$, by solving the difference $\pi_i^E - \pi_i^H = 0$ ($\pi_j^E - \pi_j^F = 0$) w.r.t the parameter t, one obtains that $\pi_i^E > \pi_{i,j}^H$ $\left(\pi_j^E > \pi_j^F\right)$ for $t_{3a} < t < t_{3b}$ and viceversa for $t < t_{3a}$ and for $t > t_{3b}$, where:

$$\begin{aligned} t_{3a} &\cong \frac{\frac{1}{2}c\left[-11520 - 6912c + 10560\,(c-1)\right]}{1729c^2 - 10370c - 575}, \\ t_{3b} &\cong \frac{\frac{1}{2}c\left[-11520 - 6912c - 10560\,(c-1)\right]}{1729c^2 - 10370c - 575}. \end{aligned}$$

Taking into account the feasibility condition $t > \overline{t} = \frac{2c}{5/4 + 3/4c}$, it is easy to find that $t_{3a} < \overline{t} < t_{3b}$. It is then proved the result of 39, where I use $t_3 = t_{3b}$. The same threshold value of t allows for the comparison between k_5 and k_6.

- Profits for i (j) when j (i) invests in PI:
  by comparing $\pi_i^C(\pi_j^G)$ vs. π_i^F (π_j^H) and $\pi_i^C(\pi_j^G)$ vs. π_i^I (π_j^I) I get:

$$\pi_i^C(\pi_j^G) < \pi_i^F\ (\pi_j^H)\ if\ \ k < k_7 \cong 0.42c\left(\frac{1-t}{t}\right) + 0.34c^2 - \frac{(0.08+0.25t)}{t^2}c^2$$

$$\pi_i^C(\pi_j^G) < \pi_i^I\ (\pi_j^I)\ \ if\ \ k < k_8 \cong 0.027\left(1+\frac{c^2}{t^2}\right) - \frac{0.056}{t}c.$$

As for $\pi_i^F(\pi_j^H)$ vs. $\pi_i^I(\pi_j^I)$, by adopting the same method as before I obtain that $\pi_i^F > \pi_i^I\ \left(\pi_j^H > \pi_j^I\right)$ for $t_{4a} < t < t_{4b}$, while the opposite holds for $t < t_{4a}$ and $t > t_{4b}$, where:

$$\begin{aligned} t_{4a} &\cong \frac{\frac{1}{2}c\left[192c - 362 + 352\,(c-1)\right]}{256c^2 - 320c - 21}, \\ t_{4b} &\cong \frac{\frac{1}{2}c\left[192c - 362 - 352\,(c-1)\right]}{256c^2 - 320c - 21} \end{aligned}$$

Taking into account the feasibility condition $t > \bar{t} = \frac{2c}{5/4+3/4c}$, it is easy to find that $t_{4a} < \bar{t} < t_{4b}$ and this proves formula 46, in which $t_4 = t_{4b}$. The same threshold value of t allows to compare k_7 vs. k_8, as it appears in 47.

7.3 Appendix 1.C

In this appendix I will prove the main result appearing in section 4.1.3 by following the same order in which the intervals of t are divided. To simplify the notations I will not report all the exact values of the parameters considered, which are nonetheless available in *Mathematica* files.

- $\bar{t} < t < t_1$:

in this interval I have to compare k_1, k_3, k_5 and k_7. It is easy to show that $k_3 > k_1$, $k_3 > k_5$ and $k_3 > k_7$ for $c \in (0,1)$ and $t \in (0,1)$. Moreover, $k_5 > k_1$ for $t > t_9 = \frac{3c}{2+c}$ but such a result can be extended to the all interval of validity, given that $\bar{t} > t_9$[23]. In the admissable region of parameters, then, it is always confirmed that:

$$k_3 > k_1,\ k_3 > k_5,\ k_3 > k_7\ \ and\ \ k_5 > k_1 \tag{67}$$

As a consequence, I get that $k_1 < k_5 < k_3$ and $k_7 > k_3$, thus verifying the result appearing in 52 and 53.

[23] I am in fact only interested in values of t grater than $\bar{t}$ to ensure the existence of the matrix under consideration.

Lastly, $k_5 > k_7$ for $t_{7a} < t < t_{7b}$, while the opposite holds outside the interval. Feasibility restraints limit my analysis to values for which $t > \bar{t}$ and, being $t_{7a} > \bar{t}$, I have that:

$$k_5 > k_7 \;\; if \;\; t_{7a} < \; t < t_{7b}; \; k_7 > k_5 \;\; if \;\; \bar{t} < \; t < t_{7a} \; and \, t_{7b} < t < 1. \tag{68}$$

Furthermore, $t_{7a} > t_1$ and this implies that in the interval of t under consideration it is always verified that $k_7 > k_5$. In other words,

$$k_7 > k_5 \;\; in \, \bar{t} < t < t_1 \tag{69}$$

The result appearing in 54 is then easily verified by combining 67 with 69.

- $t_1 < t < t_2$:

in this interval I have to compare k_5, k_7, k_3 and k_2. From 67 I know that $k_3 > k_5$ and $k_3 > k_7$. Moreover, combining 68 with the fact that $t_{7a} > t_2$,

$$k_7 > k_5 \;\; in \, t_1 < t < t_2. \tag{70}$$

As for k_2, it is always verified that, in the admissable region of parameters,

$$k_2 > k_5 \tag{71}$$

Let me consider now k_2 vs. k_7: simple algebra shows that $k_2 > k_7$ for $t < t_{5a}$ and $t > t_{5b}$. The opposite holds for $t_{5a} < t < t_{5b}$. However, by considering the feasibility condition $t > \bar{t} = \frac{2c}{5/4+3/4c}$, I get $t_{5a} < \bar{t} < t_{5b}$ and this implies that $k_2 > k_7$ for $t > t_5$ and $k_2 < k_7$ for $\bar{t} < t < t_5$, where $t_5 = t_{5b}$. As for the position of t_5, I found that $t_1 < t_5 < t_2$ and then:

$$k_2 < k_7 \; if \, t_1 < t < t_5 \;\; and \;\; k_2 > k_7 \;\; if \, t_5 < t < t_2 \tag{72}$$

I have now to compare k_2 with k_3: the first step is to find the threshold values of t. I obtain that $k_2 > k_3$ for $t < t_{6a}$ and $t > t_{6b}$, while the opposite holds for $t_{6a} < t < t_{6b}$. Furthermore, $t_{6a} < \bar{t} < t_{6b}$ and this implies that $k_2 > k_3$ for $t > t_6$ and $k_2 < k_3$ for $\bar{t} < t < t_6$, where $t_6 = t_{6b}$. By considering the interval under study, it is also easy to find that $t_1 < t_6 < t_2$ and then:

$$k_2 < k_3 \; if \, t_1 < t < t_6 \;\; and \;\; k_2 > k_3 \;\; if \, t_6 < t < t_2 \tag{73}$$

Finally, $t_5 < t_6$ and I can combine 67, 71, 72, 73 to obtain the result appearing in 56.

- $t_2 < t < t_4$:

in this interval the threshold values of k which enter in the analysis are k_2, k_4, k_5 and k_7. From 71 I know that $k_2 > k_5$; moreover, in the admissable region of parameters, it is always verified that $k_2 > k_4$ and $k_4 > k_5$. This yields to the following result:

$$k_5 < k_4 < k_2 \tag{74}$$

As for k_2 vs. k_7, given that $k_2 > k_7$ for $t > t_5$, such a result is valid within the interval considered, because the values of t are always greater then t_5[24]. In other words:

$$k_2 > k_7 \text{ in } t_2 < t < t_4. \tag{75}$$

The most difficult part is represented by the evaluation of k_5 vs. k_7 and k_4 vs. k_7. As far as k_5 vs. k_7 is concerned, from 68 I know that $k_5 > k_7$ for $t_{7a} < t < t_{7b}$, while the opposite holds outside the interval found. Furthermore, $\bar{t} < t_2 < t_{7a} < t_4 < t_{7b}$ and this implies that:

$$k_5 < k_7 \ \mathit{in}\ t_2 < t < t_7 \ \mathit{and}\ k_7 < k_5 \ \mathit{in}\ t_7 < t < t_4, \tag{76}$$

where $t_7 = t_{7a}$. Let me now consider k_4 vs. k_7: simple algebra shows that $k_4 > k_7$ for $t < t_{10a}$ and $t > t_{10b}$, while $k_4 < k_7$ for $t_{10a} < t < t_{10b}$. By taking into account the feasibility condition I get $t_{10a} < \bar{t} < t_{10b}$ and then $k_4 > k_7$ only for $t > t_{10b}$. However, by comparing the values of t appearing in this part I obtain that $t_{10b} < t_2 < t_4$, and then in the interval under consideration $t > t_{10b}$, leading to:

$$k_4 > k_7 \ \mathit{in}\ t_2 < t < t_4. \tag{77}$$

By combining the results appearing in 74, 75,76 and 77 it is immediate to demostrate the validity of 58.

- $t_2 < t < t_4$:

I have to analyse the relation among k_2, k_4, k_5 and k_8. In 74 I found that $k_5 < k_4 < k_2$. I need only to find the relative position of k_8. In the admissable region of parameters it is possible to show that:

$$k_8 < k_2 \ \mathit{and}\ k_8 < k_4. \tag{78}$$

[24] In fact, $k_2 < k_7$ for $\bar{t} < t < t_5$, as it was demostrated above, but I am studying the interval $t_2 < t < t_4$, with $t_2 > t_5$ and then $k_2 > k_7$.

The only difficulty is given by the evaluation of k_5 vs. k_8. By using the same method as before, I find that $k_8 > k_5$ for $t < t_{8a}$ and $t > t_{8b}$, while $k_5 > k_8$ for $t_{8a} < t < t_{8b}$. The feasibility conditions lead to $t_{8a} < \bar{t} < t_{8b}$ and then $k_8 > k_5$ only for $t > t_{8b}$, while $k_5 > k_8$ for $\bar{t} < t < t_{8b}$. Furthermore, the comparison of the threshold values of t shows that $t_4 < t_8 < t_3$, with $t_8 = t_{8b}$ to simplify the notation. As a consequence:

$$k_5 > k_8 \ \ in\ t_4 < t < t_8 \ \ and\ \ k_8 > k_5 \ \ in\ t_8 < t < t_3. \tag{79}$$

By combining the results appearing in 74, 78 and 79 it is then possible to easily verify 61.

- $t_3 < t < 1$:

in this last interval I compare k_2, k_4, k_6 and k_8. From 74 and 78 it is immediate to get $k_8 < k_4 < k_2$. I need only to find the relative position of k_6. In the admissable region of parameters it is possible to show that:

$$k_6 < k_2 \ and\ k_6 < k_4. \tag{80}$$

As for k_8 vs. k_6, it is easy to obtain that $k_8 > k_6$ for $t < t_{11a}$ and $t > t_{11b}$ and viceversa. However, $\bar{t} < t_{11a} < t_{11b}$, which implies $k_8 > k_6$ only for $\bar{t} < t < t_{11a}$ and for $t > t_{11b}$. The opposite holds in the interval $t_{11a} < t < t_{11b}$, where $k_6 > k_8$. Furthermore, the comparison of the threshold values of t shows that $t_{11a} < t_{11b} < t_3$, and given that I am studying an interval where $t > t_3$, then $t > t_{11b}$. As a consequence:

$$k_8 > k_6 \ \ in\ t_3 < t < 1 \tag{81}$$

The result appearing in 64 is then confirmed by combining 74, 78, 80 and 81.

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