

# Excess Capacity in Oligopoly with Sequential Entry<sup>1</sup>

Luca Lambertini - Gianpaolo Rossini

Department of Economics

University of Bologna

Strada Maggiore 45

I-40125 Bologna, Italy

fax 0039-051-2092664

e-mail [lamberti@spbo.unibo.it](mailto:lamberti@spbo.unibo.it)

e-mail [rossini@spbo.unibo.it](mailto:rossini@spbo.unibo.it)

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## Abstract

We analyse sequential entry in a quantity-setting oligopoly model. Firms have the option to adopt either a productive capacity which is optimal at the time of entry or a smaller one. This capacity may be suitable either for the steady state or just some time after entry. In the latter case firms never carry idle capacity, while in the former they keep spare capacity in the steady state. In the Cournot-Nash setting, a subgame perfect equilibrium may result in firms investing in capacity that will turn out to be idle later, depending on the size of the market and the rental price of capital. Older firms have larger spare capacity than later entrants and we can tell the age of a firm from its unused capacity. If market size is large enough, excess capacity turns out to be socially optimal.

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# 1 Introduction

Casual observation points to industries made up of firms of different size. The existing literature explains this stylized fact either through R&D races or through the description of the dynamic evolution of an industry. The former view relies upon either cost or quality differentials across firms, generated by R&D activities in process or product innovation (see Reinganum, 1989; Shaked and Sutton, 1983; Lehmann-Grube, 1997, inter alia). The latter view integrates demand and supply factors, with and without uncertainty (see Lucas, 1978; Jovanovic, 1982; Ericson and Pakes, 1995; Fishman and Rob, 1995, 1999). Our contribution nests into this strand of literature, by relating the evolution of the industry to the issue of whether firms have the incentive to hold excess capacity in the long-run equilibrium. This question has received a considerable amount of attention in modelling entry barriers in static multi-stage models. However, to our knowledge, it hasn't yet been investigated in a model where entry takes place over an arbitrarily long time span.

In this vein, strategic investment in productive capacity remains quite an open question with reference to a framework of sequential entry in oligopoly. The two main issues at stake are a) the social efficiency of the entry process, and b) the incentive for firms to hold excess capacity. When there are set-up costs the inefficiency of entry is mainly due to accommodation by incumbents refraining from price competition. As a result the reduction of the output level of incumbent firms (the "business stealing effect") makes entry more desirable to new entrants than to society (von Weizsäcker, 1980; Perry, 1984; Mankiw and Whinston, 1986; Nachbar, Petersen and Hwang, 1998). A second strand of literature is devoted to the use of idle capacity as a strategic device by incumbent firms. The early contribution on this topic (Spence, 1977) claims that incumbents may install excess capacity to prevent entry.

Subsequently, Dixit (1979, 1980) shows that this procedure is not consistent, since investing in idle capacity cannot be a credible threat, i.e., Spence's equilibrium is not subgame perfect. A later development in this direction rescues Spence's contribution by relating the incentive to hold excess physical capital to the slope of the best reply functions of firms in the market subgame (Bulow, Geanakoplos and Klemperer, 1985). When reaction functions are positively sloped, i.e. there is strategic complementarity among products, we observe redundant capital commitment in a subgame perfect equilibrium. Dixit's conclusion holds for strategic substitutability.

A more recent strand of literature has dealt with entry deterrence in an uncertain environment. Kulatilaka and Perotti (1992) find that higher volatility in an uncertain market leads firms to invest earlier and to commit to higher capacity. Hopenhayn (1992) develops a stochastic model of entry and exit, where uncertainty is technological and firm-specific and firms' turnover takes place also in the steady state. Gabszewicz and Poddar (1997) again point to excessive capacity when demand is not certain. A similar conclusion can be found in Maskin (1999) who finds that excessive capacity occurs as an entry deterrence strategy under either technological or market uncertainty in a Cournot setting. On the contrary, Somma (1999) shows that a lower commitment is preferred when there is a high probability that a more efficient technology may appear in the second period.

Our purpose is to investigate a dynamic entry process. We assess the incentive for firms, selling a homogeneous good, to invest in excess capacity in a model where entry takes place over time. In this respect we depart from a large literature where entry is analysed in timeless models without discounting (Prescott and Visscher, 1977; Boyer and Moreaux, 1986; Eaton and Ware, 1987; Vives, 1988; Anderson and Engers, 1994).

We consider sequential entry, with a single firm entering the market at each period, in continuous time. When the role of real time is properly taken

into account, firms have the option to install a capacity which ranges between the capacity that is optimal at the time of entry and the capacity they foresee will be optimal in the steady state. If they adopt a capacity that is strictly larger than the one of the steady state they will be capacity constrained from the time of entry to the time at which their capacity will be optimal. From then on they operate with unused capital.

We evaluate the capacity decisions under the solution concept of the standard Cournot-Nash equilibrium. Each period incumbents and entrants set quantities simultaneously in a market characterised by strategic substitutability. We establish that firms adopt a capacity which depends upon the cost of capital, the size of the market and the past history of entry. No firm enters with a capacity which is redundant at the time of entry. Idle capacity surfaces later as they come closer to the steady state. Unlike what happens in Spence (1977), the emergence of idle capacity in the long run equilibrium is due to the incentive to exploit the temporary rent which dissipates as we approach the steady state. Therefore, if in the long run equilibrium firms adopt excess capacity, then the size of a firm's installed capital is inversely related to the date of entry, revealing thus the firm's age.

Consumers' welfare maximization is not always against excess capacity. Especially, when market size is relatively large excess capacity is equivalent to accelerating the path to the steady state where profits will be zero and consumers shall get the most out of the entry process.

The remainder of the paper is organised as follows. In section 2 we provide the basic setup for sequential entry models. In section 3 we analyse Cournot-Nash behaviour. In section 4 we go through second best welfare analysis. The results are summarised in section 5.

## 2 The set up

Consider a quantity-setting oligopoly, over continuous time  $t \in [0; 1)$ : Entry takes place sequentially over continuous time  $t \in [0; \zeta]$ ; where  $\zeta$  is the time at which the market reaches the steady state where the last entrant just breaks even. That is, the steady state is reached at time  $\zeta$  when the ...rm  $\zeta + 1$ 's discounted flow of operative profits just covers the cost of capital acquired at time  $\zeta$ ; defined as  $k_\zeta$ : In each period  $t$  a single ...rm  $t + 1 \in [1; \zeta + 1]$  enters the market with a capacity  $k_{t+h}$  which is at least as large as the steady state capacity,  $k_\zeta$ ; and weakly lower than the optimal capacity at the time of entry,  $k_t^*$ , for all  $t < \zeta$ : Without loss of generality, we assume that capital does not depreciate over time. Our framework is one of perfect information and certainty.

We assume that ...rms produce a homogeneous good at a constant unit cost  $c$ ; as long as individual production does not exceed capacity. Otherwise, we suppose, for the sake of simplicity, that the marginal cost becomes infinitely large. At any time  $t$  the inverse market demand is:

$$p_t = \max \{0; a - Q_n\} \quad (1)$$

where  $n = t + 1$  is the number of ...rms in the market at time  $t$ : Each ...rm has the option to choose its capital endowment  $k_{t+h} \in [k_\zeta; k_t^*]$ : For any ...rm entering at  $t \in [0; \zeta]$ ; we have that  $k_\zeta < k_t^*$ : From  $t$  to  $t + h$ ; ...rm  $i = t + 1$  that adopts  $k_{t+h}$  is capacity constrained. Over  $t \in [t + h; 1)$ ; the ...rm plays her best reply against the overall quantity produced by rivals:

$$q_{iz} = \begin{cases} k_{t+h} & \text{if } Q_{-i}(z) \geq k_{t+h} \\ q_{iz}^*(Q_{-i}(z)) & \text{if } Q_{-i}(z) < k_{t+h} \end{cases} \quad (2)$$

Notice that, at time  $t + h$ ;  $q_{iz}^*(Q_{-i}(z)) = k_{t+h}$ : Moreover, from  $\zeta$  onwards,  $q_{iz}^*(Q_{-i}(z)) = q_{iz}^*(\zeta q_\zeta^*) = q_\zeta^*$ : For the last ...rm entering the market at time  $\zeta$ ; the optimal choice is obviously to set up the steady state capacity  $k_\zeta$ :

Define:

<sup>2</sup> The instantaneous operative profit over  $z \in [t; t + h]$  accruing to firm  $i$  entering at time  $t \in [0; \zeta]$  as

$\frac{1}{2} \int_z (p_z - c) q_{iz} = (a_i - k_{t+h} - Q_{i;z}) k_{t+h}$ : The population of earlier entrants in general is composed partly by capacity constrained firms and partly by other firms which can play their best replies, i.e.:

$$Q_{i;z} = \sum_{j=1}^n q_j + \sum_{l=m+1}^z k_l ; j \in i : \quad (3)$$

For later reference, define  $K_l = \sum_{l=m+1}^z k_l$ :

<sup>2</sup> The instantaneous operative profit over  $v \in (t + h; \zeta]$  accruing to firm  $i$  entered at time  $t \in [0; \zeta]$  as

$\frac{1}{2} \int_v (p_v - c) q_{iv} = (a_i - q_{iv} - Q_{i;v}) q_{iv}$ : The population of later entrants in general is composed partly by capacity constrained firms and partly by other firms which can play their best replies, i.e.:

$$Q_{i;v} = \sum_{j=1}^n q_j + \sum_{w=u+1}^v k_w ; j \in i : \quad (4)$$

For later reference, define  $K_w = \sum_{w=u+1}^v k_w$ :

<sup>2</sup>  $N = \zeta + 1$  as the number of firms in the steady state, hence  $i \in [1; N]$ :

<sup>2</sup>  $\frac{1}{2} s_{ss} = (a_i - N k_i - c) k_i$  as the steady state operative profit of a single firm, over  $t \in (\zeta; 1)$ :

<sup>2</sup>  $\frac{1}{2}$  as the discount rate, equal across firms and constant over time. The same discounting belongs to the social planner. The rental price of capital is also equal to  $\frac{1}{2}$ :

<sup>2</sup>  $s = a_i - c$  as the net size of the market (defined by Dixit (1979) as net absolute advantage when referred to a single firm).

<sup>2</sup> The number of firms entered up to  $\zeta$  is  $\zeta + 1$ : Therefore, steady state capacity is  $k_\zeta = s/(\zeta + 2)$ :

### 3 Firms' behaviour

The discounted flow of profits accruing to firm  $i = t + 1$ ; entering at  $t \in [0; \zeta]$ ; over the period  $[t; t + 1)$  is given by:

$$b_{i,t+1;t}(k) = \int_t^{t+h} \frac{1}{4} \mu_{iz} \zeta e^{i \frac{1}{2} z} dz + \int_{t+h}^{\zeta} \frac{1}{4} \mu_{iv} \zeta e^{i \frac{1}{2} v} dv + \int_{\zeta}^{t+1} \frac{1}{4} \mu_{ss} \zeta e^{i \frac{1}{2} r} dr - \frac{1}{2} k_{t+h}; \quad (5)$$

where  $k_{t+h} \in [k_\zeta; k_t(K_I)]$ :

Notice that, over  $t \in (\zeta; t + 1)$ ; all firms play  $k_\zeta$ : Therefore, the choice of  $k_{t+h}$  is unaffected by the discounted flow of profits from steady state onwards,  $\int_{\zeta}^{t+1} \frac{1}{4} \mu_{ss} \zeta e^{i \frac{1}{2} r} dr$ ; which we disregard in the remainder. As a result, the firm's choice of capital installment is defined as follows:

$$k_{t+h}^a = \arg \max_{k_{t+h}} b_{i,t+1;t}(k) = \int_t^{t+h} \frac{1}{4} \mu_{iz} \zeta e^{i \frac{1}{2} z} dz + \int_{t+h}^{\zeta} \frac{1}{4} \mu_{iv} \zeta e^{i \frac{1}{2} v} dv - \frac{1}{2} k_{t+h} \quad (6)$$

where  $b_{i,t+1;t}(k) = b_{i,t+1;t}(k) - \int_{\zeta}^{t+1} \frac{1}{4} \mu_{ss} \zeta e^{i \frac{1}{2} t} dt$ : We prove the following:

**Lemma 1** Firm  $i$ 's profits are:

$$\int_t^{t+h} \frac{1}{4} \mu_{iz} \zeta e^{i \frac{1}{2} z} dz = \frac{\mu_{s_i} K_I \eta}{m + 1} k_{t+h} \int_t^{t+h} e^{i \frac{1}{2} z} dz; \quad (7)$$

over  $z \in [t; t + h]$ ; and

$$\int_{t+h}^{\zeta} \frac{1}{4} \mu_{iv} \zeta e^{i \frac{1}{2} v} dv = \frac{\mu_{s_i} K_w \eta}{u + 1} k_{t+h} \int_{t+h}^{\zeta} e^{i \frac{1}{2} v} dv; \quad (8)$$

over  $v \in (t + h; \zeta]$ :

**Proof.** Consider the first part of the Lemma. A firm which, at any  $z \in [t; t + h]$ ; is not capacity constrained, produces the output  $q_{jz}$  given by the solution of the following first order condition (FOC):

$$\frac{\partial \mu_{jz}}{\partial q_{jz}} = s_i - 2q_{jz} \prod_{j=1}^n q_{jz}^{-1} - \bar{K} = 0; \quad (9)$$

i.e.,  $q_{jz}^a(\bar{K}) = (s_i \bar{K}) = (m + 1)$ ; where  $\bar{K} = K_l + k_{t+h}$  is the overall capacity of the subpopulation of farms which are capacity constrained at time  $z$ : Plugging  $q_{jz}^a(\bar{K})$  into  $\int_t^{t+h} \frac{1}{2} q_{iz}^a e^{i \frac{1}{2} z} dz$  and simplifying, proves the first statement in the Lemma.

Now consider the second statement. Over  $v \in (t + h; \infty]$ ; farm  $i$  (entered at  $t$ ) is no longer constrained, and, at any  $v$ , maximises instantaneous profits

$$\frac{1}{2} q_{iv}^a = \frac{2(s_i - K_w)}{u + 1} \quad (10)$$

by playing the best reply  $q_{iz}^a(K_w) = (s_i - K_w) = (u + 1)$ : This yields optimal instantaneous profits  $\frac{1}{2} q_{iv}^a [q_{iz}^a(K_w)] = (s_i - K_w)^2 = (u + 1)^2$ : This completes the proof. ■

On the basis of Lemma 1, we can write  $b_{t+1;t}(k)$  as follows:

$$b_{t+1;t}(k) = \frac{(s_i - K_l - k_{t+h}) e^{\frac{1}{2}(t+h)} + e^{\frac{1}{2}t} k_{t+h}}{\frac{1}{2}(m + 1) e^{\frac{1}{2}(2t+h)}} + \frac{e^{\frac{1}{2}t} (s_i - K_w)^2}{\frac{1}{2}(u + 1)^2 e^{\frac{1}{2}(t+h+\infty)}} \quad (11)$$

which can be differentiated w.r.t.  $k_{t+h}$  to obtain the following FOC:

$$\frac{\partial b_{t+1;t}(k)}{\partial k_{t+h}} = \frac{(s_i - K_l - 2k_{t+h}) e^{\frac{1}{2}(t+h)} + e^{\frac{1}{2}t} k_{t+h}}{\frac{1}{2}(m + 1) e^{\frac{1}{2}(2t+h)}} = 0 \quad (12)$$

whose solution is:<sup>1</sup>

$$k_{t+h}^a = \frac{(s_i - K_l)}{2} + \frac{\frac{1}{2} e^{\frac{1}{2}(t+h)} (m + 1)}{2(e^{\frac{1}{2}h} - 1)} \quad (13)$$

<sup>1</sup>The second order condition for a maximum:

$$\frac{\partial^2 b_{t+1;t}(k)}{\partial k_{t+h}^2} = - \frac{2 e^{\frac{1}{2}(t+h)} + e^{\frac{1}{2}t}}{\frac{1}{2}(m + 1) e^{\frac{1}{2}(2t+h)}}$$

is always met.

From (13), it appears that for the first firm, entering at  $t = 0$ ; capacity is determined exclusively by the size of the market and intertemporal discounting. This produces a viability condition for the entry process to start:

Lemma 2 The necessary condition for the entry process to start with firm 1 choosing  $k_h^a$  is  $s > s^0 = \frac{\frac{1}{2}e^{\frac{1}{2}h}}{e^{\frac{1}{2}h} - 1}$  :

Proof. For the first firm,  $K_1$  is necessarily nil. Moreover,  $t = 0$ : Plugging these values in (13), we obtain the expression for the capacity of firm 1,  $k_h^a$ : It is then immediate to verify that this capital level is positive if  $s > \frac{\frac{1}{2}e^{\frac{1}{2}h}}{e^{\frac{1}{2}h} - 1}$  : ■

Before proceeding to establish optimum conditions for the choice of capacity, observe that:

Lemma 3 Choosing  $k_i$  at the time of entry is admissible for all  $s > 0$ :

Proof. To prove this claim, it suffices to check that

$$k_i = \frac{s}{i + 2} \cdot s \text{ for all } i + 1 \geq 0 \quad (14)$$

which is always true. ■

Sufficient conditions for the optimal choice of capacity by the generic firm  $t + 1$  are stated in the following:

Proposition 1 For any  $\frac{1}{2} > 0$ ; there exists a threshold value of the market size  $s > s^0 > 0$ ; such that:

2 for all  $s > s$ ; maximum profits obtain at  $k_{t+h}^a > k_i$  ;

2 for all  $s \in [0; s]$ ; maximum profits obtain at  $k_i$  :

For all positive values of  $\frac{1}{2}$  and  $s$ ;  $k_{t+h}^a < k_t(K_1)$  ; where  $k_t(K_1)$  is the capacity that firm  $t + 1$  would choose as a best reply against  $K_1$  :

Proof. Compare (13) with  $k_{\zeta} = s(\zeta + 2)$ : This yields:

$$k_{t+h}^a = k_{\zeta} \text{ for all } s = \frac{K_1 e^{\frac{1}{2}h} (\zeta + 1) + \frac{1}{2} e^{\frac{1}{2}(t+h)} (m+1) (\zeta + 2)}{\zeta (e^{\frac{1}{2}h} (\zeta + 1))} \quad \mathfrak{s} : (15)$$

The inequality  $\mathfrak{s} > s^0$  can be checked by plugging  $K_1 = 0$  and  $m = 0$  into  $\mathfrak{s}$  and comparing it against  $s^0$  as from Lemma 2. This proves the ...rst part of the Proposition.

To prove the second statement we compare  $k_{t+h}^a$  with

$$k_t(K_1) = \frac{s \zeta K_1}{2} = \frac{s \zeta \sum_{l=m+1}^{\infty} K_l}{2} \quad (16)$$

to obtain:

$$k_t(K_1) - k_{t+h}^a = \frac{\frac{1}{2} e^{\frac{1}{2}(t+h)} (m+1)}{2 (e^{\frac{1}{2}h} (\zeta + 1))} > 0 : \quad (17)$$

Notice that, when  $s \in [0; \mathfrak{s}]$ ; ...rms choose  $k_{\zeta}$  irrespective of whether  $s$  is larger or smaller than  $s^0$ : For all  $s \in [0; s^0)$ ; ...rms never choose  $k_{t+h}^a$  as it is both suboptimal and too large w.r.t. the size of the market; for all  $s \in [s^0; \mathfrak{s}]$ ;  $k_{t+h}^a$  is admissible but suboptimal. ■

Proposition 1 produces a few relevant corollaries. The ...rst is the following:

**Corollary 1** Optimal capacity  $k_{t+h}^a$  only depends upon the past history of the entry process.

To prove it, just observe expression (13), which depends on  $K_1$  but not on  $K_w$ ; i.e., the generic ...rm's capital commitment at date  $t$  is determined by the overall capacity accumulated by earlier entrants.

Now we assess the behaviour of  $k_{t+h}^a$  as  $t$  increases towards  $\zeta$ ; in order to characterise the time pattern of excess capacity as the market approaches the steady state. This is summarised in the following:

**Proposition 2** For all  $s > \bar{s}$ ; optimal capacity  $k_{t+h}^a$  is everywhere decreasing and concave in  $t$ :

**Proof.** To prove the above statement, just calculate first and second derivatives of  $k_{t+h}^a$  w.r.t.  $t$ :

$$\frac{\partial k_{t+h}^a}{\partial t} = i \frac{\frac{1}{2}^3 (m+1) e^{\frac{1}{2}(t+h)}}{2 (e^{\frac{1}{2}h} i - 1)} < 0 ; \quad (18)$$

$$\frac{\partial^2 k_{t+h}^a}{\partial t^2} = i \frac{\frac{1}{2}^4 (m+1) e^{\frac{1}{2}(t+h)}}{2 (e^{\frac{1}{2}h} i - 1)^2} < 0 ; \quad (19)$$

■

The intuition attached to Proposition 2 is that a casual observer looking at the market in steady state is able to tell older firms from younger firms simply by looking at their respective installed capacities.

## 4 Second best welfare analysis

Here we assess the behaviour of a planner w.r.t. capital commitment  $k_{t+h}$ ; given firms' output decisions at the market stage, as given by (2). To this aim, we calculate the social welfare levels over the periods  $[t; t+h]$  and  $[t+h; \infty]$ :

$$SW(t; t+h) = \frac{(K_l + k_{t+h}) (2s i - K_l i - k_{t+h}) + ms^2(m+2)}{2(m+1)^2} ; \quad (20)$$

$$SW(t+h; \infty) = \frac{K_w (2s i - K_w) + us^2(u+2)}{2(u+1)^2} ; \quad (21)$$

The planner would choose  $k_{t+h}$  so as to maximise social welfare over the whole time horizon up to the steady state, i.e.,  $SW(t; \infty) = SW(t; t+h) + SW(t+h; \infty)$ : The FOC is:

$$\frac{\partial SW(t; \infty)}{\partial k_{t+h}} = \frac{(s i - K_l i - k_{t+h}) e^{\frac{1}{2}h} i - 1}{\frac{1}{2}(m+1)^2 e^{\frac{1}{2}(t+h)}} i - \frac{1}{2} = 0 ; \quad (22)$$

yielding<sup>2</sup>

$$k_{t+h}^{sb} = \frac{(s_i - K_i) e^{\frac{1}{2}h} i - 1 - \frac{1}{2}(m+1)^2 e^{\frac{1}{2}(t+h)}}{e^{\frac{1}{2}h} i - 1} \quad (23)$$

It is easily verified that

$$\frac{\partial k_{t+h}^{sb}}{\partial t} < 0 \text{ and } \frac{\partial^2 k_{t+h}^{sb}}{\partial t^2} < 0 \quad (24)$$

so that  $k_{t+h}^{sb}$  is everywhere decreasing and concave in  $t$ :

It remains to assess whether the per-form capital endowment (23) in the second best equilibrium is larger than the privately optimal capital (13). This is done in the following:

**Proposition 3** For any  $\frac{1}{2} > 0$ ; there exists a threshold value of the market size  $\mathfrak{b} > 0$ ; such that:

<sup>2</sup> for all  $s > \mathfrak{b}$ ; we have  $k_{t+h}^{sb} > k_{t+h}^a$ ;

<sup>2</sup> for all  $s \in (0; \mathfrak{b})$ ; we have  $k_{t+h}^{sb} < k_{t+h}^a$ ;

**Proof.** Compare (23) with (13). This yields:

$$k_{t+h}^{sb} - k_{t+h}^a = \frac{(s_i - K_i) e^{\frac{1}{2}h} i - 1 - \frac{1}{2}(2m+1)(m+1)e^{\frac{1}{2}(t+h)}}{2(e^{\frac{1}{2}h} i - 1)} \quad (25)$$

which is positive if

$$s > \frac{K_i e^{\frac{1}{2}h} i - 1 + \frac{1}{2}(2m+1)(m+1)e^{\frac{1}{2}(t+h)}}{(e^{\frac{1}{2}h} i - 1)} \equiv \mathfrak{b}; \quad (26)$$

and conversely if  $s \in (0; \mathfrak{b})$ : This concludes the proof. ■

<sup>2</sup>The SOC

$$\frac{\partial^2 SW(t; \zeta)}{\partial k_{t+h}^2} = i \frac{e^{\frac{1}{2}h} i - 1}{\frac{1}{2}(m+1)^2 e^{\frac{1}{2}(t+h)}} < 0$$

is always met.

Observe that the above result can be reformulated in terms of residual market demand, by noting that

$$k_{t+h}^{sb} > k_{t+h}^a \text{ if } s_j \leq K_l > \frac{\frac{1}{2}^2(2m+1)(m+1)e^{\frac{1}{2}(t+h)}}{(e^{\frac{1}{2}h} - 1)}; \quad (27)$$

where  $s_j \leq K_l$  is the size of the residual market at time  $t$ ; when capacity-constrained firms have installed an overall capacity  $K_l$ :

Moreover, we wish to investigate the parameter regions where second best social welfare is maximised alternatively at  $k_{t+h}^{sb}$  or  $k_{\zeta}$ . This establishes the following

**Proposition 4** For any  $\frac{1}{2} > 0$ ; there exists a threshold value of the market size,  $\bar{s} > 0$ ; such that:

$$\geq \text{ for all } s > \bar{s}; \max^n SW^{sb} \text{ obtains at } k_{t+h}^{sb};$$

$$\geq \text{ for all } s \leq (0; \bar{s}); \max^n SW^{sb} \text{ obtains at } k_{\zeta};$$

**Proof.** To prove the above statement compare (23) with  $k_{\zeta}$  to obtain the following

$$k_{t+h}^{sb} = k_{\zeta} \text{ for all } s = \frac{K_l e^{\frac{1}{2}h} - 1 + \frac{1}{2}^2 e^{\frac{1}{2}(t+h)} (m+1)^2 (\zeta+2)}{(\zeta+1)(e^{\frac{1}{2}h} - 1)} - \bar{s}; \quad (28)$$

■

We now wish to establish a ranking over  $\bar{s}$ ;  $\underline{s}$ ;  $\underline{b}$ :

**Proposition 5** For all  $K_l > K_l^0$ ; we have  $\bar{s} > \underline{s} > \underline{b}$ ; for all  $K_l \leq (0; K_l^0)$ ; we have  $\bar{s} < \underline{s} < \underline{b}$ :

Proof. Simply observe that the critical level of  $K_1$ ; at which  $\bar{s} = \underline{s} = \underline{\theta}$ ; is:

$$K_1^0 = \frac{\frac{1}{2} e^{\frac{1}{2}(t+h)} (m+1)(m_{\ell} - 1)}{e^{\frac{1}{2}h} i - 1} \quad (29)$$

■

We are now in a position to give a comparative picture of the planner's preferences over the entry process, vis à vis the ...rms' behaviour.

Theorem 1 Suppose  $K_1 > K_1^0$ : Then we have

A]  $s > \bar{s}$ ,  $k_{t+h}^{sb} > k_{t+h}^a > k_{\ell}$ : The planner chooses  $k_{t+h}^{sb}$ ; ...rms choose  $k_{t+h}^a$ :

B]  $s \in (\underline{s}; \bar{s})$ ; the planner chooses  $k_{\ell}$  while ...rms adopt  $k_{t+h}^a$ :

C]  $s \in (0; \underline{s})$ ; both the planner and ...rms choose  $k_{\ell}$ :

Suppose  $K_1 \in (0; K_1^0)$ : Then we have

D]  $s > \underline{\theta}$ ,  $k_{t+h}^{sb} > k_{t+h}^a > k_{\ell}$ : The planner chooses  $k_{t+h}^{sb}$ ; ...rms choose  $k_{t+h}^a$ :

E]  $s \in (\underline{s}; \underline{\theta})$ ;  $k_{t+h}^a > k_{t+h}^{sb} > k_{\ell}$ : The planner chooses  $k_{t+h}^{sb}$ ; ...rms choose  $k_{t+h}^a$ :

F]  $s \in (\bar{s}; \underline{s})$ ; the planner chooses  $k_{t+h}^{sb}$ ; ...rms choose  $k_{\ell}$ :

G]  $s \in (0; \bar{s})$ ; both the planner and ...rms choose  $k_{\ell}$ :

Proof. The Theorem is a direct consequence of Propositions 1, 3 and 4. As an illustration, we concentrate our attention to points [A, B, C]. Consider the case  $K_1 > K_1^0$ : Suppose  $s > \bar{s}$ : If so, Propositions 1, 3 and 4 establish that both the planner and the ...rms choose excess capacity, with social incentives towards excess capacity being larger than private incentives. This proves [A].

Now take  $s \in (\underline{s}; \bar{s})$ : In this range, Proposition 4 tells that the planner would like ...rms to adopt steady state capacity. However, Proposition 1 leads ...rms to choose  $k_{t+h}^a > k_{\ell}$ : This proves [B].

Finally, consider  $s \in (0; \bar{s})$ : In this range, Propositions 1 and 4 entail that it is both socially and privately optimal to choose  $k_j$ : This proves [C]. ■

Theorem 1 can be interpreted as a description of the tradeoff between the cost of capacity on one side and the effect of larger capacity on market price, outputs and surplus on the other side. As an illustration, consider points [A, B, C]. If the market is very small, then both private and social incentives point to the adoption of the steady state capacity. Since surplus is quadratic in market size, in such a range it is more desirable to save on installment costs. The opposite holds if the market is sufficiently large. If so, then excess capacity is appealing also to the planner, as the temporary gain in welfare more than offsets the cost of idle capital in the ensuing story of the industry. Finally, notice that, when the incentive to adopt excess capacity exists for the planner, then it is higher than for firms, due to the fact that the planner takes into account the sum of industry profits and consumer surplus.

## 5 Concluding remarks

In a static quantity-setting framework, the only reasonable solution concept is the Nash equilibrium, in that there is no reason to expect that any firm may have an unchallenged ability to move first. This is the major result proved by d'Aspremont and Gérard-Varet (1980) and Hamilton and Slutsky (1990). Therefore, when an entry process is described in a single period model, it is not rational for firms to operate with excess capacity. This is the basic objection to Spence's (1977) conclusion raised by Dixit (1980).

In the light of the foregoing analysis, this conclusion may change when entry takes place sequentially in continuous time.

Firms install a capacity which ranges between the one that is optimal at the time of entry and the one that is best suited for the steady state when

a zero profit condition dictates operative scale of production. The choice of capacity depends upon the cost of capital, the net size of the market (reservation price minus marginal cost) and the past history of entry in the market.

We are able to find threshold values of the market size beyond which firms enter with a productive capacity that will be partly idle after a while and in the steady state. For a constant cost of capital and a given net size of market, we are able to tell the age of firms from their capacity, since older firms are more likely to carry excess capacity in the steady state, due to their incentive to extract as much surplus as possible in their very first stay in the market. Younger firms enter with a capacity that will be much closer to the one they will use in the steady state where they will carry less idle capacity.

Second best welfare analysis provides a thorough assessment of the entry process. In particular, if market size is large enough, then both private and social incentives point to the adoption of excess capacity. Moreover, the socially preferred result is for firms to enter with larger capacity than it would be privately optimal.

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