Multiproduct Firms, Product Differentiation, and Market Structure[1]

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Abstract

This paper provides a survey of the literature on multiproduct firms under either endogenous or exogenous product differentiation. The main aim is to identify the reasons why firms supply product lines rather than single goods. Factors pertaining to the demand side, such as brand loyalty and switching costs, are treated here. Moreover, the incentive towards product proliferation is investigated in connection with the issue of the persistence of monopoly. The consequences of product proliferation on social welfare are also discussed.

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1 Introduction

The choice on the part of firms to supply product lines rather than single goods is commonly observed in several markets. However, industrial economists started investigating this aspect of firms' behaviour relatively late. Neither the theory of monopolistic competition, nor the Sylos Labini-Bain-Modigliani paradigm, offered a satisfactory description of market interaction among multiproduct firms (for a discussion, see Robinson, 1953). The early studies in this direction justified the supply of product lines on the grounds of production costs: within the theory of contestable markets (Baumol, Panzar and Willig, 1982), the existence of multiproduct firms seemed to be a consequence of economies of scope. A subadditive cost function,[1] combined with the absence of sunk costs, would make it optimal for firms to operate on several markets, supplying several goods.

However, the theory of contestable markets devotes a limited amount of attention to demand-side incentives towards product proliferation. This is the viewpoint adopted by Brander and Eaton (1984), who focus on the interplay between consumer's demand for differentiated goods on one side, and the strategic and technological effects affecting firms' behaviour, on the other side. Relying on a theoretical model where the analysis confines to Cournot competition, Brander and Eaton verify that firms' strategic decisions as to product range and output level may lead to market equilibria where firms supply product ranges characterised by a high degree of substitutability. This result is derived under the assumption that each firm's product range consists in a given number of varieties, and is therefore subject to a fairly natural critique, namely, that firms may endogenously alter the span of their product range for strategic reasons.[2] This is the route taken by several subsequent contributions (Wernerfelt, 1986; De Fraja, 1992).

We intend to supply a summary of the advances achieved by the literature on multiproduct firms, under the alternative assumptions of exogenous and endogenous product differentiation, emphasising in particular the incentives towards product proliferation on the demand side. This implies that we will not proceed to a detailed illustration of (i) supply-side incentives[3] (although they will pop up occasionally in the remainder of the survey); and (ii) the foundations of the

[1] Let x_i define the output level of good i. A cost function $C(x_i)$ is subadditive if $C(x_i+x_j) \leq C(x_i) + C(x_j)$, for all x_i and x_j.

[2] This idea is closely related to the literature on endogenous differentiation (Hay, 1976; Prescott and Visscher, 1977; Eaton and Lipsey, 1978; Lane, 1980; Judd, 1985; Bonanno, 1987), where firms may adopt product proliferation as a foreclosure strategy. This view has found some empirical support (Schmalensee, 1978).

[3] We refer the reader to the seminal contributions due to Panzar and Willig (1981); Baumol, Panzar and Willig (1982); or to the exhaustive surveys by Brock (1983) and Spence (1983).

theory of multiproduct firms, guaranteeing the existence of equilibrium.[4]

The theory of multiproduct firms has evolved along four main lines of research. The first is marked by a clear heritage from the theory of contestable markets: firms choose the span of the product line on the basis of the tradeoff between economies of scope and scale, but they are unable to choose the characteristics of products. The second considers endogenous differentiation, introducing thus a tradeoff between specialisation and the ability to discriminate among consumers with different incomes and preferences. The third focuses on the issue of the persistence of monopoly in settings where the entry of new firms is conditional upon the acquisition of production rights over new varieties. The fourth introduces the idea that consumers may bear switching costs, either real or perceived as such, related to the purchase of product lines. Consequently, consumers' brand loyalty can be so high that they purchase goods from one firm only. The switching costs approach allows to nest the former three approaches into a unique picture.

The remainder is structured as follows. Sections 2 and 3 contain a review of Brander and Eaton (1984), where product substitutability is an exogenous parameter, and no brand loyalty effect is operating. Endogenous differentiation is dealt with in sections 4-6, considering both models of horizontal differentiation (Bonanno, 1987; Martinez-Giralt and Neven, 1988) and models of vertical differentiation (Mussa and Rosen, 1978; Champsaur and Rochet, 1989, *inter alia*). The strategic use of product proliferation, with or without an explicit description of R&D competition, is analysed in section 7 (Gilbert and Newbery, 1982; Shaked and Sutton, 1990). Section 8 reviews the literature on switching costs (Klemperer, 1992; 1995). Concluding remarks are in section 9.

2 Demand-side vs scale economies and the optimal product line

The contribution by Brander and Eaton (1984) is closely related to the contestability theory, but for the fact that they abandon the assumption of scope economies in production, and introduce an analogous assumption concerning demand, in order to investigate the interaction between this and economies of scale in determining the firms' optimal choices of their respective product lines.

Consider a duopoly where each firm produces two goods. The available goods are indexed 1, 2, 3 and 4. Suppose each good in the pairs $(1,2)$ and $(3,4)$ are close substitutes for the other good in the same pair, while pairs $(1,3)$, $(1,4)$, $(2,3)$ and $(2,4)$ are formed by weakly substitute goods. To determine the extent of substitutability, examine the inverse demand functions. Define p_i the market

[4] As to the theorems on concavity of the profit function and the existence of equilibrium in both perfectly and imperfectly competitive markets with multiproduct firms, see MacDonald and Slivinsky (1987); Okuguchi and Szidarovsky (1990); Anderson and de Palma (1992); De Fraja (1994).

price of variety i, and x_i the demand for the same variety. The demand vector is thus $X = (x_1, x_2, x_3, x_4)$. The inverse demand function for good i writes $p_i = p_i(x_1, x_2, x_3, x_4) = p_i(X)$. Now define:

$$p_{ij} = \frac{\partial p_i}{\partial x_j};\ p_{ik} = \frac{\partial p_i}{\partial x_k};\ p_{ii} = \frac{\partial p_i}{\partial x_i}\ . \tag{1}$$

If we say, e.g., that goods 1 and 2 are reciprocally closer substitutes than goods 1 and 3, we mean that the reaction of p_1 to a variation in x_2 is, in absolute value, larger than the reaction of p_1 to a change in x_3. Hence, in general, we can state that $|p_{ij}| > |p_{ik}|$ if i and j are close substitutes while i and k are weak substitutes.

As to production technology, each firm bears a sunk cost k for each variety. The unit variable cost is assumed to be constant, and the total cost of producing x_i units of variety i is:

$$C_i = cx_i + k. \tag{2}$$

From (2), it follows that average production cost is decreasing:

$$AC = \frac{C_i}{x_i} = c + \frac{k}{x_i}\ . \tag{3}$$

Notice that this cost function is not subadditive. Hence, product line decisions depend solely upon the interaction between demand incentives and economies of scale. As an illustration, consider first the behaviour of a monopolist.

2.1 The monopolist's optimal product line

First of all, it can be established that, if the monopolist decides to offer two products, she will find it profitable to supply imperfect substitutes. To grasp the intuition at the basis of this result, suppose the monopolist produces goods 1 and 2, which are close substitutes, and let $x_1 = x_2$. Now, for a given x_1, imagine that the monopolist switches from good 2 to good 3, with $x_3 = x_2 = x_1$. Define the two product vectors as $X' = (x_1, x_2, 0, 0)$ and $X" = (x_1, 0, x_3, 0)$. The effect of the variation in the product vector on the price of good 1 is

$$\Delta p_1 = p_1(X") - p_1^{'}(X'). \tag{4}$$

The mean value theorem (see Rosenlicht, 1968) allows us to rewrite (4) for a vector $X^* \in [X', X"]$, as follows:

$$\Delta p_1 = [p_{13}(X^*) - p_{12}(X^*)]\, x_i\ ,\ i = 1, 2, 3. \tag{5}$$

Since $p_{ij} < p_{ik} < 0$, the price variation observed for good 1 must be positive. By symmetry, this must hold for variety 2 as well. Consequently, for a given pair of output levels, both prices and profits must increase. Hence, the two-product monopolist, in the absence of entry threats, finds it optimal to supply two varieties characterised by low substitutability. Doing otherwise would increase price competition with her own product range.

To verify this result, consider the following demand functions (Singh and Vives, 1984):

$$\begin{aligned} p_1 &= a - x_1 - dx_2 - g(x_3 + x_4) \\ p_2 &= a - x_2 - dx_1 - g(x_3 + x_4) \\ p_3 &= a - x_3 - dx_4 - g(x_1 + x_2) \\ p_4 &= a - x_4 - dx_3 - g(x_1 + x_2) \ , \end{aligned} \tag{6}$$

where parameters $0 < g < d < 1$ measure the degree of substitutability between products. In particular, parameter d measures the substitutability between goods 1 and 2 or goodss 3 and 4, while parameter g measures the substitutability within pairs $(1,3)$, $(1,4)$, $(2,3)$ and $(2,4)$. Suppose the monopolist supplies a pair of close substitutes, e.g., products 1 and 2. In this case, $x_3 = x_4 = 0$ and the profit function is $\pi^M = (p_1 - c)x_1 + (p_2 - c)x_2 - 2k$. The solution of first order conditions (FOCs)

$$\frac{\partial \pi^M}{\partial x_i} = a - c - 2x_i - 2dx_j = 0, \quad i,j = 1,2;\ i \neq j \tag{7}$$

is $x_1 = x_2 = (a-c)/[2(1+d)]$, and the resulting profits are $\pi^M = (a-c)^2/[2(1+d)] - 2k$. The same obtains if the monopolist supplies goods 3 and 4. Replacing d with g in output levels and profits gives the equilibrium magnitudes when the monopolist produces any pair of the set $\{(1,3),(1,4),(2,3),(2,4)\}$. As $g < d$, observing that monopoly profits are inversely related to the degree of substitutability suffices to confirm the above conclusions.

2.2 The duopolists' optimal product lines

Duopoly competition takes place in three stages. In the first, each firm decides how many products to supply; this choice affects the exploitation of scale economies. In the second, each firms chooses which products to supply, on the basis of their reciprocal substitutability. In the third, market competition takes place, either in quantities or in prices. In principle, one can imagine that these decisions may be taken either sequentially or simultaneously. However, firms' behaviour in the real world suggests that decisions concerning product line should precede those related to the marketing stage.[5] The solution concept of the whole game is the subgame perfect equilibrium by backward induction (Selten, 1975).

For the sake of simplicity, let's bypass the first stage and focus on the more relevant case, where each firm supplies two products. Label firms A and B,

[5] We strictly follow the authors' procedure (Brander and Eaton, p. 325). Moreover, a sequential solution by backward induction ensures the existence of an equilibrium in pure strategies.

and suppose they compete in quantities. If firm A produces the pair $(1, 2)$ and firm B produces the pair $(3, 4)$, we have what Brander and Eaton call *market segmentation*. Alternatively, each firm might supply a pair of weaker substitutes, e.g., $(1, 3)$ and $(2, 4)$. If so, product lines are *interlaced*. Consider firm A's profits in the two settings, starting with the segmentation case:

$$\pi_A^S = p_1 x_1 + p_2 x_2 - c(x_1 + x_2) - 2k. \tag{8}$$

Superscript S stands for *segmentation*. The FOC for profit maximisation with respect to product 1 is

$$\frac{\partial \pi_A^S}{\partial x_1} = MR_1 + p_{21} x_2 - c = 0, \tag{9}$$

where $MR_1 = p_{11} x_1 + p_1$ represents marginal revenue on product 1. Second order conditions (SOCs) are

$$\frac{\partial^2 \pi_A^S}{\partial x_1^2} \leq 0; \tag{10}$$

$$\left(\frac{\partial^2 \pi_A^S}{\partial x_1^2} \cdot \frac{\partial^2 \pi_A^S}{\partial x_2^2} \right) - \left(\frac{\partial^2 \pi_A^S}{\partial x_1 \partial x_2} \cdot \frac{\partial^2 \pi_A^S}{\partial x_2 \partial x_1} \right) > 0. \tag{11}$$

The FOC and SOCs for good 2 are obviously analogous to (9-10), with the appropriate subscripts. By symmetry, we are also able to state that firm B's FOC and SOCs are analogous to (9) and (10-11), respectively. Solving these conditions yields the Cournot-Nash equilibrium pertaining to the third stage. As the model is symmetric, all quantities and prices must coincide. Brander and Eaton assume further that $MR_{ij} < 0$, i.e., marginal revenue of variety i is decreasing in the output level of variety j, for all i and j.

Consider now the case where firm A supplies goods $(1, 3)$. The FOC w.r.t. good 1 becomes

$$\frac{\partial \pi_A^I}{\partial x_1} = MR_1 + p_{31} x_3 - c = 0, \tag{12}$$

where superscript I stands for *interlaced*. *Mutatis mutandis*, SOCs are as in (10-11).

On these grounds, it can be shown that market regime S yields higher profits than market regime I. To see this, observe that in regime S all goods must be sold at the same price p^S, and likewise in regime I all of them are sold at price p^I. Then, we have three possible situations: either $p^I = p^S$; or $p^I > p^S$; or, finally, $p^I < p^S$. Both the first and the second case lead to a contradiction. Consider the first: if the equilibrium prices were the same in regimes I and S, also output levels and consequently marginal revenues would coincide. However, we have assumed that $p_{21} < p_{31} < 0$; therefore, the conditions for optima cannot be satisfied by $p^I = p^S$. Consider then the second case, i.e., $p^I > p^S$. This would imply $x^I < x^S$

and, as MR_{ij} must be negative, it would also entail that the marginal revenue characterising good 1 is higher in regime I than in regime S. Again, under the assumption that $p_{21} < p_{31} < 0$, conditions (9) and (12) cannot be satisfied. Thus, the only plausible case is $p^I < p^S$.

Examine now industry profits $\Pi = \sum_{i=1}^4 p_i x_i - c\sum_{i=1}^4 x_i - 4k$. Their variation following a change in x_1 is

$$\frac{\partial \Pi}{\partial x_1} = p_1 + \sum_{i=1}^{4} p_{i1} x_i - c. \tag{13}$$

In regime S, we have

$$\frac{\partial \Pi^S}{\partial x_1} = p_{31} x_3 + p_{41} x_4 < 0, \tag{14}$$

as, by (9), $MR_1 + p_{21}x_2 - c = 0$. Likewise, $\partial\Pi/\partial x_1 < 0$ also in regime I, where (12) holds. It appears that industry profits decrease as any x_i increases. It follows that, given the concavity of profits w.r.t. output levels, industry profits (as well as individual profits) must decrease in switching from S to I, as outputs increase.

It is possible to check the validity of these results by using demand functions (6) introduced in the previous subsection. To this aim, we compare setting S where firm A produces the pair $(1, 2)$ and firm B produces the pair $(3, 4)$, with setting I where firm A produces the pair $(1, 3)$ and firm B produces the pair $(2, 4)$. Calculations are quite straightforward and are left to the curiosity of the reader. Since the model is completely symmetric, we confine our attention to the incentive for firm i towards a segmented market structure, measured by

$$\text{sign } \pi_i^S - \pi_i^I = \text{ sign } d^2 + d^3 + 2dg + 2d^2g - 3g^2 + dg^2 - 4g^3 \,. \tag{15}$$

The difference $\pi_i^S - \pi_i^I = difsi$ is plotted against d and g in figure 1. Since $g < d$, figure 1 reveals that structure S is preferred to structure I in the whole admissible range of parameters. Hence, by the symmetry of the model, both firms prefer S over I. The validity of the results concerning industry profits obviously follows from this observation.

We are now in a position to investigate product line decisions. In this stage, firms take the number of products as given and choose the composition of the product range to be supplied, anticipating Cournot competition at the market stage. Examine first the case where these choices are simultaneous. A firm's best reply to the choice of two particular varieties by the rival consists simply in offering the remaining two, and any 2×2 partition of the four-product set is a Nash equilibrium of this stage, under both regimes. Otherwise, if one firm has a first-mover advantage, she will surely choose to produce two close substitutes, inducing the rival to take an analogous decision. Hence, segmentation univocally arise when the solution concept is the Stackelberg equilibrium. As to the selection between Nash and Stackelberg equilibria, the fact that the Stackelberg

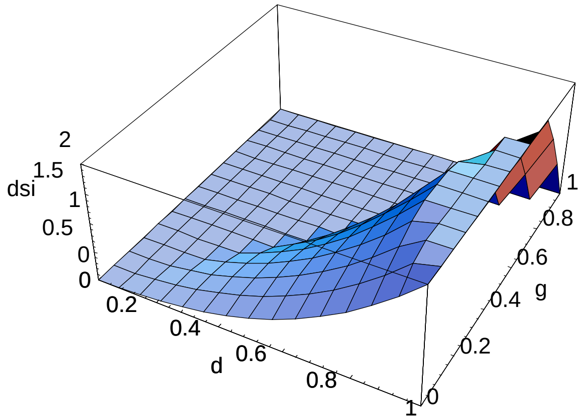


Figure 1:

equilibrium Pareto-dominates all possible Nash equilibria makes it appear as the natural solution of this stage, in that both leader and follower find it preferable to a Nash equilibrium.[6]

The decision pertaining to the width of the product line can be now briefly characterised. In the most unfortunate circumstances, market demand would be so low that, if a firm finds it convenient to offer just one product, the rival does not enter the market at all. In the opposite situation, market demand might be so high that both firms would find it optimal to offer four products each. Notice that this is made possible by the assumption of Cournot competition. Under price-setting behaviour, this would entail zero profits. Above we focused on the equilibria arising when best replies at this stage are symmetric and implies the supply of a product line consisting of two varieties by each firm. This situation can be expected to obtain when demand is neither so low to prevent firms from entering, nor so high to induce them to compete head-to-head over the whole product range.

In summary, Brander and Eaton provide a set of fairly general conditions under which segmentation obtains at equilibrium. Relaxing the assumption that each firm supplies two differentiated varieties, Wernerfelt (1986) considers the possibility for each firm to offer a single standardised good satisfying (imperfectly) all consumers. Wernerfelt finds that, if consumer tastes are homogeneous and the level of per product sunk costs is above a critical threshold, the optimal choice is

[6]This approach to equilibrium selection reflects the state of the art characterising game theory in the early 1980s. The concept of Stackelberg-solvable game, i.e., that a game can be solved *à la* Stackelberg if there is at least one such equilibrium dominating the Nash ones, is due to d'Aspremont and Gérard-Varet (1980). The endogenisation of timing has been further formalised by Hamilton and Slutsky (1990).

indeed to offer a single variety. Otherwise product proliferation is more attractive (see section 6).

We have focused on the interplay between scale economies, product substitutability and demand-side incentives towards product proliferation, overlooking the role of scope economies in production. De Fraja (1992) tackles this problem, in a duopoly model where each firm can supply at most two products, under either quantity or price competition. He proves that, in general, the equilibrium number of products is larger than the socially optimal one, which would allow full exploitation of scale economies. This is due to the incentive for firms to exploit scope economies, whenever available, which have no significant bearing on consumer surplus.

In the remainder, we deal with endogenous differentiation models. We start, in the next section, by considering the behaviour of a horizontally differentiated monopoly.

3 Spatial monopoly

The behaviour of a spatially differentiated monopolist is studied by Bonanno (1987) in the quadratic transportation cost version (d'Aspremont, Gabszewicz and Thisse, 1979) of Hotelling's (1929) linear city. To illustrate his contribution, we proceed by induction, starting with the case where the monopolist supplies a single variety.

Unit production cost is assumed to be constant and is normalised to zero. Consumers are uniformly distributed, with density equal to one, along a segment of unit length which can alternatively be interpreted as a geographical space or the consumers' preference space. Each individual purchase a unit of the good, so that the market is fully served. The indirect utility function is

$$V = u - t(x - m)^2 - p; \quad t > 0, \tag{16}$$

where u is gross surplus, t is the unit transportation cost rate, x is the location chosen by the monopolist, m is the location of a generic consumer and p is the mill price. Given full market coverage, profit and price coincide, and one needs to find the profit-maximising price when $x = 1/2$. By substitution, we get $V = s - t(1/2 - m)^2 - p$, from which, taking into account that the optimal price must drive to zero the net surplus of marginal consumers in 0 and 1, we obtain $p^M(1) = u - t/4 = \pi^M(1)$.

Consider now the case where the monopolist offers two varieties. Clearly, optimal locations are $x_1 = 1/4$ and $x_2 = 3/4$, i.e., the monopolist segments the market in two sub-markets of the same size, with marginal consumers at 0, 1/2 and 1, the consumer at 1/2 being indifferent between the two varieties.[7] The

[7] Note that 1/4 and 3/4 are also the socially optimal locations, i.e., those which maximise social welfare (and minimise total transportation costs). Hence, a peculiar characteristic of

monopolist will then set the maximum price compatible with the hypothesis of full market coverage, implying that consumers at 0, 1/2 and 1 enjoy zero surplus in equilibrium. Hence, in both sub-markets, $p^M(2) = u - t/16$. Increasing the number of varieties, it is quickly verified that optimal price is $p^M(n) = u - t/(2n)^2$.

This model can be used to analyse strategic product proliferation by a monopolist, as a barrier to entry by outside competitors. Suppose the introduction of each variety entails a sunk cost k, independent of location. However, once the latter is chosen, the sunk cost prevents the incumbent from modifying location to react to entry. In the simplest setting, monopoly profits become $\pi^M(1) = u - t/4 - k$. If an outside firm decides to enter, she obviously locates either in 0 or in 1. Then, given locations, suppose firms play a Nash equilibrium in prices. Their profits are $\pi^I = 49t/144 - k$ and $\pi^E = 25t/144 - k$, with superscripts I and E standing for *incumbent* and *entrant*, respectively. It appears that the incumbent remains a monopolist if $k \in (25t/144, 49t/144]$, in that for all values of k within such interval the entrant's profits are negative. Otherwise, if $k \in (0, 25t/144]$, the outsider enters at 0 or 1 with non-negative profits. To avoid this, the incumbent could enlarge her product line, lowering the profitability of any market niche available to the outsider (see Schmalensee (1978) for a case study supporting this view).

4 Monopoly and product quality

The behaviour of the vertically differentiated multiproduct monopolist has received wide attention in the theoretical literature. The basic issues raised in this field are two. The first concerns the choice by a profit-maximising monopolist between quality and quantity distortions, for a given number of varieties. The second regards the difference between the optimal monopoly behaviour and the socially efficient one. A third issue is that of describing the conditions under which we expect to observe a monopoly rather than a competitive market structure (Gabszewicz, Shaked, Sutton and Thisse, 1986). This topic nests into the debate on natural oligopoly and the so-called *finiteness property*, establishing that if quality improvements mainly affect fixed (R&D) costs while marginal cost is flat, a market for vertically differentiated products may accomodate a finite number of firms gaining strictly positive profits at equilibrium (Shaked and Sutton, 1983). Here, we focus on the two issues mentioned above. We will briefly dwell upon the third in section 6.

The first results on quality and quantity distortion in monopoly trace back to Spence (1975) and Sheshinski (1976). They establish that a monopolist can,

the Hotelling model consists in yielding the same equilibrium under both monopoly and social planning. Under the first regime, the monopolist minimise the maximum distance between the generic consumer and the generic variety in order to raise the price and maximise profits, while under the second regime the social planner does the same in order to minimise the overall disutility from transportation costs, $TC = t\sum_i (x_i - m)^2$.

alternatively, restrict the output level for a given quality, or distort quality for a given output. This entails that the quality supplied at the monopoly optimum is lower that supplied at the social optimum if the marginal quality evaluation characterising the average consumer is higher than the marginal evaluation of quality characterising the marginal consumer, and conversely. This is due to the fact that, in setting quality, the monopolist considers the marginal consumer while the social planner considers the average consumer. This conclusion can be reached in a single-good setting. The possibility that the monopolist expands the product line in order to induce a self-selection mechanism on the part of consumers is considered in several later contributions (Mussa and Rosen, 1978; Itoh, 1983; Maskin and Riley, 1984; Besanko, Donnenfeld and White, 1987, 1988; Lambertini, 1997). In this stream of literature, unit production cost is constant w.r.t. quantity but is increasing in the quality level, so that this model cannot give rise to a natural oligopoly (or monopoly). The basic idea underlying all these contributions is that the monopolist expands the product line so as to discriminate between consumers with different incomes. In the next subsections, we follow Lambertini (1997).

4.1 Full market coverage

Suppose consumers are uniformly distributed, with unit density, over $[\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} = \overline{\theta} - 1$ and $\overline{\theta} \geq 1$.[8] Parameter θ denotes marginal willingness to pay for quality, which can be thought of as the reciprocal of marginal utility of money (see Tirole, 1988, chapter 2). As income increases, consumer's willingness to pay for a higher quality increases as well. Under full market coverage, each consumer buys one unit of the differentiated good, provided that the net surplus from purchase is non-negative:

$$V = \theta q_i - p_i \geq 0 \,, \quad i = 1, 2, ...n \,, \tag{17}$$

where q_i and p_i are the quality level and price of variety i. On the supply side, total production costs for variety i are $C_i = tq_i^2 x_i$, where x_i is the output level and t is a positive parameter affecting marginal cost.

In what follows, we compare the behaviour of the monopolist with the behaviour of a social planner (or perfect competition), for a given number of products. Before doing that, it is useful to characterise briefly the interval of socially preferred qualities (Cremer and Thisse, 1994). When price equals marginal cost, i.e., $p_i = tq_i^2$, the utility-maximising quality level for a consumer indexed by θ is $q_i^* = \theta/(2t)$. Hence, the interval of socially preferred qualities is $[\underline{\theta}/(2t), \overline{\theta}/(2t)]$.

Under full market coverage, (17) holds as an equality for the marginal consumer at $\underline{\theta} = \overline{\theta} - 1$, while it holds as a strict inequality for all other consumers. For the sake of simplicity, consider first the case where a single variety is supplied.

[8] The assumption of a uniform distribution is not restrictive. Spence (1975) shows that most of the results we are about to derive hold for a wide class of distributions.

The objective function of the planner is social welfare, defined as follows:

$$SW(1) = \int_{\underline{\theta}}^{\overline{\theta}} (\theta q - tq^2) d\theta \,. \tag{18}$$

It is evident that price is irrelevant, in that, under full coverage, it only affects the distribution of surplus between producer and consumers, leaving unchanged its overall size. From the FOC w.r.t. quality we obtain $q^{SP} = (\overline{\theta} - 1)/(4t) = \left(\underline{\theta}/(2t) + \overline{\theta}/(2t)\right)/2$, i.e., the social planner produces the variety preferred by the average consumer.

The monopolist's objective is to maximise

$$\pi^M(1) = p - tq^2 \,. \tag{19}$$

The derivative of (19) w.r.t. price is everywhere positive. This implies that monopoly price is the highest price which is compatible with the assumption that the consumer at $\underline{\theta}$ be able to buy, i.e., $p^M = \underline{\theta} q$. Plugging it into (19), we obtain $\pi^M(1) = q(\underline{\theta} - tq)$, which is maximised at $q^M = \underline{\theta}/(2t)$, the quality preferred by the marginal consumer. Hence, the monopolist undersupplies quality as compared to the social optimum.

Now suppose two qualities are produced, $q_H > q_L > 0$. Demand functions are $x_H = \overline{\theta} - (p_H - p_L)/(q_H - q_L)$ and $x_L = (p_H - p_L)/(q_H - q_L) - (\overline{\theta} - 1)$. Objective functions become

$$\pi^M(2) = (p_H - tq_H^2)x_H + (p_L - tq_L^2)x_L \,; \tag{20}$$

$$SW(2) = \int_{\underline{\theta}}^{h} (\theta q_L - tq_L^2) d\theta + \int_{h}^{\overline{\theta}} (\theta q_H - tq_H^2) d\theta \,, \tag{21}$$

where $h = (p_H - p_L)/(q_H - q_L)$ is the marginal willingness to pay of the consumer who is indifferent between q_H and q_L. The monopolist sets $p_L = \underline{\theta} q_L$ and, plugging it into $\partial \pi^M(2)/\partial p_H = 0$, she obtains the optimal price for the superior variety. Equilibrium qualities can be obtained by solving the corresponding first and second order conditions, yielding $q_H^M = (2\overline{\theta} - 1)/(4t)$ and $q_L^M = (2\overline{\theta} - 3)/(4t)$. The solution to the planner's problem is given by $q_H^{SP} = (4\overline{\theta} - 1)/(8t)$ and $q_L^{SP} = (4\overline{\theta} - 3)/(8t)$. A quick comparison reveals that

$$q_H^{SP} - q_H^M = \frac{1}{8t} \,; \; q_L^{SP} - q_L^M = \frac{3}{8t} \,, \tag{22}$$

that is, quality distortion increases as we move down along the quality spectrum. This phenomenon arises because of the monopolist's attempt at extracting more surplus from sales in the higher segment of the market: the distortion of the low quality is meant at making the switching from the high to the low-quality good unattractive for rich consumers. As the number of varieties increase, the monopolist's behaviour can be characterised likewise. For the sake of brevity, we confine to the case of n goods. The upper and lower limits of the quality ranges supplied by the monopolist and the social planner are, respectively:

$$q_H^M = \frac{n\overline{\theta} - 1}{2nt} \; ; \; q_L^M = \frac{n\overline{\theta} - 2n + 1}{2nt} \; ; \tag{23}$$

$$q_H^{SP} = \frac{2n\overline{\theta} - 1}{4nt} \; ; \; q_L^M = \frac{2n\overline{\theta} - 2n + 1}{4nt} \; . \tag{24}$$

As a result, with n varieties, the two regimes supply the following degrees of differentiation:

$$q_H^M - q_L^M = \Delta q^M = \frac{n-1}{nt} \; ; \; q_H^{SP} - q_L^{SP} = \Delta q^{SP} = \frac{n-1}{2nt} \; , \tag{25}$$

from which $\Delta q^M / \Delta q^{SP} = 2$, i.e., the monopolist's product range is twice as large as the social planner's. Moreover, $\lim_{n\to\infty} \Delta q^M = 1/t$, $\lim_{n\to\infty} \Delta q^{SP} = 1/(2t)$, implying that, as the quality spectrum becomes continuous, the social planner provides each consumer with his own preferred quality, while the monopoly range is twice as large, for all n. As stressed by Dupuit as early as 1849, this is not aimed at deteriorating the position of low-income consumers, but rather at forcing high-income consumers to pay the prices at which superior qualities are sold, in that lower qualities are not attractive.

4.2 Partial market coverage

Under partial coverage of the market, some low-income consumers are not served, and condition (17) is violated in the right neighbourhood of $\underline{\theta}$. In this situation, given n, the monopolist and the social planner produce the same qualities, although the monopolist restricts the output to half the amount supplied by the planner (or under perfect competition). Again, we consider initially the case of a single variety. The marginal consumer locates at p/q, so that market demand is $x = \overline{\theta} - p/q$ and the profit function is $\pi^M(1) = (p - tq^2)x$. FOCs are

$$\frac{\partial \pi^M(1)}{\partial q} = tp + \frac{p^2}{q^2} - 2\overline{\theta} qt = 0; \tag{26}$$

$$\frac{\partial \pi^M(1)}{\partial p} = \overline{\theta} - \frac{2p}{q} + qt = 0, \tag{27}$$

from which we get $p^M = 2\overline{\theta}^2/(9t)$; $q^M = \overline{\theta}/(3t)$. Equilibrium output is $x^M = \overline{\theta}/3$, and profits amount to $\pi^M(1) = \overline{\theta}^3/(27t)$. Notice that partial market coverage obtains if and only if $\overline{\theta} \leq 3$; when $\overline{\theta} > 3$, the analysis carried out under full market coverage holds. As to the social planner, his aim is the maximisation of

$$SW(1) = \int_{p/q}^{\overline{\theta}} (\theta q - tq^2) d\theta. \tag{28}$$

Solving the social optimum problem entails producing the same quality supplied by the profit-maximising monopolist, although sold at marginal cost, i.e., $p^{SP} = \overline{\theta}^2/(9t)$. Moreover, the output level under social planning is $x^{SP} = 2\overline{\theta}/3$.

Consider now the case of two varieties, whose demand functions are:

$$x_H = \overline{\theta} - \frac{(p_H - p_L)}{q_H - q_L} \; ; \; x_L = \frac{(p_H - p_L)}{q_H - q_L} - \frac{p_L}{q_L} \, . \tag{29}$$

Solving the monopolist's and the social planner's optimum problems yields:

$$q_H^M = q_H^{SP} = \frac{2\overline{\theta}}{5t} \; ; \; q_L^M = q_L^{SP} = \frac{\overline{\theta}}{5t} \; ; \tag{30}$$

$$X^M = \frac{2\overline{\theta}}{5} = \frac{X^{SP}}{2} \; ; \; x_i^M = \frac{\overline{\theta}}{5} = \frac{x_i^{SP}}{2} \; , \; i = H, L. \tag{31}$$

Equilibrium qualities and quantities (30-31) reveal that the behaviour of the monopolist coincide with that of the social planner on the quality side, while a distortion is observed on the output side. The extension to the case of n varieties is now rather straightforward:

$$q_i^M = q_i^{SP} = \frac{i\overline{\theta}}{t(2n+1)} \; , \; i = 1, 2, ...n; \tag{32}$$

$$X^M = \frac{n\overline{\theta}}{2n+1} = \frac{X^{SP}}{2} \; ; \; x_i^M = \frac{X^M}{n} = \frac{x_i^{SP}}{2} \; , \; i = 1, 2, ...n, \tag{33}$$

from which one immediately obtains $\lim_{n\to\infty} X^M = 1/2$ and $\lim_{n\to\infty} X^{SP} = 1$. This implies that, as the number of varieties becomes infinitely high, the profit-maximising monopolist serves only the richer half of the market, while a social planner would serve all consumers.

We are now in a position to evaluate the choice of the monopolist between full market coverage (with quality distortion) and partial market coverage (with output distortion), in the parameter range where both are admissible. To this aim, it suffices to consider the case of a single good. The monopolist prefers to distort quantity rather than quality if $\overline{\theta}^3/(27t) > (\overline{\theta} - 1)^2/(4t)$. This condition is met for all $\overline{\theta} < 3$; hence, in this range the monopolist will choose to supply the same quality as the social planner (or perfect competition), but restrict the output level by fifty per cent. If $\overline{\theta} = 3$, the monopolist is indifferent between output and quality restrictions, while for all $\overline{\theta} > 3$ full market coverage with quality distortion is the only admissible regime. These considerations carry over to the case of n varieties, for the appropriate values of $\overline{\theta}$.

5 Multiproduct firms in oligopoly markets with endogenous differentiation

Within the literature on product differentiation, the behaviour of multiproduct firms operating under oligopolistic competition has received rather scanty attention. The main contributions in this field are due to Martinez-Giralt and Neven (1988) and Champsaur and Rochet (1989), who deal with horizontal and vertical differentiation, respectively.

5.1 Horizontal differentiation

Martinez-Giralt and Neven (1988) examine two version of the spatial differentiation model. In the first, they assume that consumers are distributed along a linear city, as in Hotelling (1929) and d'Aspremont, Gabszewicz and Thisse (1979). In the second, they assume instead that consumers are distributed along a circle, as in Salop (1979). In both cases, the main issue addressed by the authors is whether at least one firm has any incentive to supply more than one product, and they get to the conclusion that such an incentive does not exist. This result can be intuitively explained in the following terms. For the sake of simplicity, consider a duopoly. The choices open to a firm that may consider to proliferate her product range are described by figures 2 and 3. In figure 2, firm 1 is single-product, while firm 2 supplies two goods, both located to the right of firm 1, at L_{21} and L_{22}. In figure 3, firm 1 is again a single-product unit, but her good is located between the two varieties offered by firm 2. Martinez-Giralt and Neven show that it is convenient for firm 2 to adopt the strategy represented in figure 2, in that it allows her to isolate one product from the competition exerted by the good supplied by firm 1. In other words, the market configuration depicted in figure 2 is less competitive than the configuration represented by figure 3, where both firm 2's varieties compete directly with the single one produced by firm 1.

Figure 2

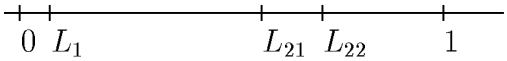
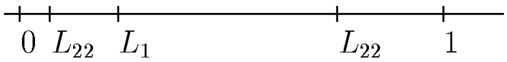


Figure 3



Nevertheless, the bunching of firm 2's products on the same side of firm 1 entails a high degree of competition between themselves, leading firm 2 to supply one product only.[9] Seemingly, this implies that we should not observe multiproduct firms under horizontal differentiation. However, Martinez-Giralt and Neven are aware that this result depends on the absence of scale and scope economies, as well as entry threats by outsiders. Another element which is overlooked here is brand loyalty. We will come back to it in section 8.

5.2 Vertical differentiation

Champsaur and Rochet (1989) consider price competition between multiproduct firms in the vertical differentiation model with variable costs of quality improvements, originated by Mussa and Rosen (1978). As this model is a generalisation of the spatial differentiation model with convex transportation costs (Cremer and Thisse, 1991), the results obtained by Champsaur and Rochet largely replicates those by Martinez-Giralt and Neven.

Suppose consumers are uniformly distributed over the interval $[\underline{\theta}, \overline{\theta}]$, with $\underline{\theta} > 0$ and $\overline{\theta} = \underline{\theta} + \alpha$. Full market coverage is assumed. Every consumer purchases one unit, his indirect utility function being $U = u(\theta, q_i) - p_i(q_i) \geq 0$, where q_i is the quality level and p_i is the price set by firm i. Two firms, H and L, operate in

[9] Or, alternatively, two undifferentiated varieties. As long as we do not consider fixed costs, producing two identical varieties or a single good is the same. With fixed costs attached to every variety the incentive for firm 2 to minimise differentiation between L_{21} and L_{22} would certainly lead her to drop one product.

the market, each of them supplying a continuous range of qualities, $q_H \in [q_H^-, q_H^+]$ and $q_L \in [q_L^-, q_L^+]$. Unit production costs are convex in quality and constant in quantity, with total costs being $C_i = c_i(q_i, x_i)$; $\partial C_i/\partial q_i > 0$; $\partial^2 C_i/\partial q_i^2 > 0$; $\partial C_i/\partial x_i > 0$; and $\partial^2 C_i/\partial x_i^2 = 0$.

Consider first the case where quality intervals are disjoint, i.e., $q_L^+ < q_H^-$. In this setting, there exists a pure-strategy Nash equilibrium at the price stage, for all conceivable quality intervals such that this inequality is satisfied. All varieties' demands, in equilibrium, are strictly positive.[10] Consider then the case of partial overlapping between the quality intervals, where $q_L^+ \geq q_H^-$, i.e., there exists at least one product which is offered by both firms. Also in this situation there always exists a pure-strategy equilibrium in prices. However, a necessary and sufficient condition for both firms to gain strictly positive profits in equilibrium, is that both q_L^+ and q_H^- belong to the interval of consumers' preferred qualities.[11] If this condition is met, then the price equilibrium is also unique.

We are now in a position to examine what happens in the quality stage, confining our attention to the setting where both firms are active. First of all, it is possible to show that, when $u(\theta, q_i) = \theta q_i$ and $C_i = tq_i^2 x_i$, there exists an equilibrium in pure strategies at the quality stage, where both firms' profits are positive, and firms are single-product. This is a straightforward extension of a result contained in Cremer and Thisse (1994) and Lambertini (1996). Hence, the same incentive towards shrinking the product range into a single variety emerges here, as in the horizontal model examined in the previous subsection. Moreover, provided each firm supplies an interval of qualities, her decisions are summarised by the behaviour of the lower and upper bounds of such interval, q_i^- and q_i^+. These values affect the profits of firm i in very different ways, respectively. Firms' profits, when quality intervals are disjoint, i.e., when $q_L^+ < q_H^-$, can be written as follows:

$$\Pi_H = \pi_H(q_L^+, q_H^-) + \pi_H(q_H^+, \infty);\ \Pi_L = \pi_L(q_L^+, q_H^-) + \pi_L(q_H^-, -\infty). \qquad (34)$$

The first term on the right-hand side of both expressions in (34) represents the profit for firm i in the quality range where she competes with the rival, that is, the central part of the market. Such a profit can be defined as a *pure differentiation profit*. This is the same profit each firm would get if she produced a single variety. The second term is independent of the behaviour of the rival and can be labelled as a *pure segmentation profit*. When instead $q_L^+ \geq q_H^-$, at least partial overlapping obtains, and both firms' profits reduce to the pure segmentation profits in the ranges where there is no overlapping, while in the quality interval where both firms operate with the same varieties, profits are driven to zero by a standard

[10] A simple proof of these claims, in the case where product intervals collapse into a single variety, can be found in Cremer and Thisse (1994) and Lambertini (1996).

[11] When, e.g., $c_i(q_i, x_i) = tq_i^2 x_i$, the interval of socially preferred qualities is $[\underline{\theta}/(2t), \overline{\theta}/(2t)]$. See previous section.

Bertrand argument. Hence, in the corresponding Nash equilibrium in qualities, if neither quality interval degenerates into a single product and both firms' profits are positive, it must necessarily be that $q_L^+ < q_H^-$, i.e., quality intervals must be disjoint in that this strategy ensures larger profits as compared to the case of partial overlapping.

Finally, it can be given an intuitive argument revealing that, once a no-overlapping equilibrium configuration has been chosen, firms find it optimal to produce quality ranges of infinite extensions, with $q_H \in [q_H^-, \infty)$ and $q_L \in (-\infty, q_L^+]$. This is due to the fact that, even if qualities located very far from the socially preferred ones will not be purchased by any consumer, each firm prefers to foreclose any market niche adjacent to hers.[12] Obviously, the production of an infinite number of varieties is admissible only on purely theoretical grounds. The incentive towards product innovation and proliferation as a barrier to entry is dealt with in the next section.

6 Product innovation and the persistence of monopoly

The issue of the incentives towards both process and product innovation under monopoly *vis à vis* oligopoly or perfect competition is a *vexata quaestio* in the literature on industrial organization, dating back to the seminal contributions by Schumpeter (1942) and Arrow (1962). In the recent literature, the main contributions on the issue of the persistence of monopoly are due to Gilbert and Newbery (1982) and Reinganum (1983). These authors explicitly deal with process innovation, that is, with an R&D activity aimed at reducing the marginal cost associated with the production of an existing good, but their analysis can be easily reformulated in terms of a product innovation. Imagine an auction by an independent lab holding the rights over the innovation. This lab is willing to sell the patent of infinite duration over the innovation to the highest bidder between an incumbent and an outsider that, in case she wins, can enter the market which will thus become a duopoly made up by single-product firms. Otherwise, if the incumbents bids more and gets the patent, monopoly will persist with a two-product firm. Label these two firms as I (*incumbent*) and E (*entrant*). The current profit of firm I as a single-good monopolist is $\pi_I^M(1)$, where superscript M, as usual, stands for *monopoly*. The profit she would obtain by winning the auction is $\pi_I^M(2)$. Finally, the firms' duopoly profits, when the outsider obtains the patent, are $\pi_I^D(1)$ and $\pi_E^D(1)$, where superscript D stands for *duopoly*. The incentive to innovate exists for the incumbent whenever $\pi_I^M(2) \geq \pi_I^M(1)$, and the

[12] In a recent paper, De Fraja (1996) extends the vertical differentiation model originated by Gabszewicz and Thisse (1979) to show that firms may leave some gaps in their product lines, this choice being independent of fixed costs, while related to the structure of variable costs and consumer preferences.

incumbents wins the auction if

$$\pi_I^M(2) \geq \pi_I^D(1) + \pi_E^D(1), \tag{35}$$

or, equivalently, if the price that I is willing to pay for the rights over the innovation is larger than the maximum price that E is willing to pay. Since a two-product monopolist must necessarily be able to gain at least the same profits as two non-colluding firms operating with one product each, condition (35) is certainly met and monopoly persists. This condition describes what is known as the *efficiency effect* (Fudenberg and Tirole, 1986; Tirole, 1988) or the *incentive to preempt* (Katz and Shapiro, 1987), summarising the tradeoff between static and dynamic efficiency characterising monopoly. This result, derived by Gilbert and Newbery (1982), holds under perfect certainty. When the innovation race is uncertain, another effect enters the stage, namely the so-called *replacement effect*, which shows that the innovation might be attained by the outsider because the incumbent "rests on her laurels" (Reinganum, 1983). Overall, the net effect, and the outcome of the innovation race or auction, is ambiguous.

A similar approach is adopted by Shaked and Sutton (1990). They drop the auction metaphor, and their answer to the question of whether monopoly persists or not relies exclusively on factors operating on the demand side, economies of scope being assumed away. Shaked and Sutton examine price competition in a model where each product requires the same amount of fixed costs, and they identify two elements: (i) as to the monopolist, the incentive to introduce a new good depends on the market demand for it, net of the demand loss borne by the existing good because of product proliferation, with part of the demand initially satisfied by the "old" good being relocated to the new one. This *expansion effect* is measured by the increase in monopoly profits as a result of innovation; (ii) as to the potential entrant, the incentive to introduce a new product depends only on the absolute demand level expressed by the market for this good, when it is offered under duopoly, without any externality on the other good. This incentive is weakened by the fact that entry generates a duopoly with price competition. Accordingly, it can be labelled as *competitive effect*. Market structure in equilibrium is then determined by the relative size of these two effects.

We illustrate the problem through a simple example. Assume there are only two firms and two goods, each of them being produced at most by one firm.[13] The (sunk) fixed cost attached to each good is F. Define $\pi(j,k)$ the operative profit (i.e., gross of fixed costs) of a firm, when she produces j goods and the rival produces k goods, with $j,k \in \{0,1,2\}$. Hence, single-product operative monopoly profit is $\pi(1,0)$; two-product monopoly profit is $\pi(2,0)$; and, finally, individual

[13] Notice that this assumption makes Shaked and Sutton's model similar to those dealing with the persistence of monopoly, and, at the same time, quite different from the view adopted by Brander and Eaton (1984) and De Fraja (1992).

duopoly profit with single-product firms is $\pi(1,1)$. As a last assumption, suppose $\pi(2,0) > \pi(1,0)$ and $\pi(2,0) > 2\pi(1,1)$. The expansion effect is measured by:

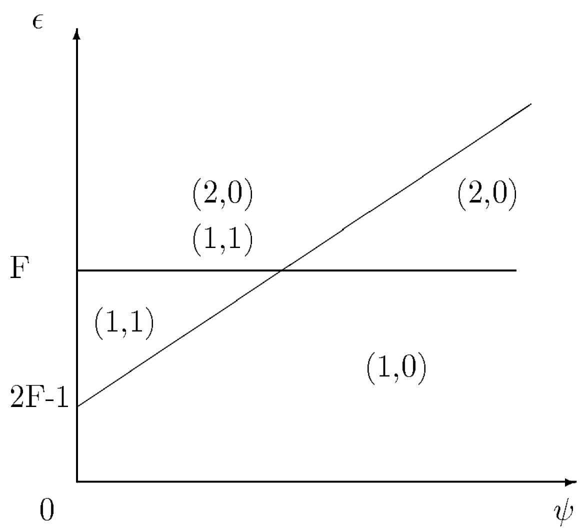
$$\epsilon = \frac{\pi(2,0) - \pi(1,0)}{\pi(1,0)} > 0, \tag{36}$$

i.e., by the rate of increase in the monopolist's profits when she expands the product range. The competitive effect is measured by:

$$\psi = \frac{\pi(2,0) - 2\pi(1,1)}{2\pi(1,1)} > 0. \tag{37}$$

That is, ψ describes the incentive towards monopolising the market. Notice that ψ increases as price competition becomes tougher, and conversely. In the limit, if products are perfect substitutes, Bertrand competition implies that $\pi(1,1) = 0$. Therefore, in such a situation ψ tends to infinity, while ϵ is nil. Conversely, when the two goods are completely independent, $\epsilon = 1$ and $\psi = 0$. For the sake of simplicity, normalise $\pi(1,0) = 1$ and consequently $F \in (0,1)$. Simple calculations show that $\pi(1,1) = F$ if $\epsilon = 2F\psi + 2F - 1$, and $\pi(2,0) - \pi(1,0) = F$ if $\epsilon = F$. This allows to identify the equilibrium market structure in the space $\{\psi, \epsilon\}$, i.e., as a function the relative size of the two effects. By using the relationships $\epsilon = 2F\psi + 2F - 1$ and $\epsilon = F$, we can investigate the equilibrium through figure 4, where we consider $F \in (1/2, 1)$.

Figure 4



The pairs in brackets indicates the number of products supplied by firms. An interpretation can be given in the following terms. Consider an increase in ϵ, for a given competitive effect ψ along the horizontal axis. If the expansion effect is sufficiently high, both $(1,1)$ and $(2,0)$ are Nash equilibria. Consider then the opposite perspective where, for a given expansion effect ϵ, the competitive effect increases. Observe that, if the value of ψ is sufficiently high, the unique equilibrium structure is monopoly, be that either single- or two-product: intuitively, as the intensity of price competition increases, the dominant position initially held by the incumbent becomes safer. Likewise, for a given level of ψ, the incentive for the monopolist to enlarge her product range is increasing in the size of the expansion effect.

These results, derived under a fairly general set of assumptions, can obviously be expected to hold in many specific settings as well. A natural example that immediately springs to mind is endogenous differentiation, either horizontal or vertical. As noted by Shaked and Sutton, in such a setting both the effects considered above become completely endogenous, and will depend upon the extent of market coverage, in particular the expansione effect (i.e., the incumbent's incentive to monopolise the market through product proliferation) may drastically shrink under full market coverage, as assumed, e.g., by Bonanno (1987) in examining entry deterrence in the Hotelling model.

7 Switching costs and multiproduct firms

The casual observation of consumption choices in many markets suggests that those individuals who have already bought a certain product from a given firm must bear a cost (whether real or just perceived is relatively irrelevant) if they decide to purchase a good offered by another firm, even if these two goods are completely or at least largely equivalent under many respects. This real or perceived *switching cost* gives firms some degree of monopoly power in a repeated purchase setting, or in the affine setting where consumers purchase a set of complement goods. An example is given by hi-fi equipments: it is frequently observed that a customer buys all components of the same brand, even if there is no need to do so, in that all products supplied by different firms are reciprocally compatible in terms of electrical specifications.[14] In this case, consumers perceive a cost (or a disutility) associated with pure "brand" heterogeneity. A different case is that of computers, where indeed the existence of different standards (Mac vs IBM) generates a tangible switching cost.[15] Be that as it may, what matters is that

[14] There exists an exception to this rule, namely, the fact that MC (moving coil) and MM (moving magnet) phono cartridges need completely different pre-amplification stages due to their different output voltages.

[15] The issue of supplying product lines in the presence of different standards is examined in the literature on compatibility and network externalities. For an introduction and overview, see Shy (1995, chapter 10).

the influence exerted by such costs on consumer behaviour confronts firms with a tradeoff, at any point in time, between attracting new customers by setting low prices currently and obtain a return in the future volume of demand, and exploiting the established market position by setting high prices which, in turn, my jeopardise any future increase in demand.

Although relatively recent, the literature on switching costs is wide (see Klemperer, 1995). Here, we confine to the analysis of its main features, by exposing a simplified version of a model by Klemperer (1992). It is convenient to consider first a duopoly with single-product firms. Define as p_i the price set by firm i, q_i the output level, π_i the profit function and $s_i = q_i/(q_i + q_j)$ the market share of firm i. Production entails a constant marginal cost c. Firms cannot price-discriminate. The number of consumers is N, and each consumer, characterised by a reservation price equal to R, purchases one unit either from firm i or from firm j. Assume there is only one marketing period, with an initial condition (inherited possibly from the past history of the market) according to which in the previous period a fraction s_i of consumers patronised firm i, and obviously a fraction $s_j = 1 - s_i$ patronised firm j. Each consumer has the same switching cost σ. If this is sufficiently high, the unique noncooperative equilibrium of the market game is observationally equivalent to a collusive equilibrium, in that equilibrium profits are indeed the same that firms would attain by colluding. The reason for this is that firm i cannot steal customers from firm j, unless she decreases her price at least to $p_i = p_j - \sigma$.[16] As price discrimination is ruled out by assumption, such a price reduction entails a uniform profit loss on all customers initially served by firm i, i.e., her market share s_i. It is easily verified that this loss is larger than the increase in profits generated from stealing the rival's demand. Therefore, the optimal choice for both firms consists in setting the price equal to the reservation price R.

Consider now the case where each firm can choose which product to supply out of a pair of goods, labelled 1 and 2, that may be perfect or imperfect substitutes. Each consumer purchases, overall, one unit, combining the two available varieties in proportions f and $(1 - f)$, with $f \in (0, 1)$. As a consequence, if $f = 1/2$, maximum differentiation is observed in the consumption basket. The utility accruing to a generic consumer, gross of price(s) and switching costs, if any, is

$$U = R + \nu - \mu \left(\frac{1}{2} - f \right)^2 > R \quad \forall \quad f \in (0, 1). \tag{38}$$

Parameter ν measures consumer's evaluation of differentiation in consumption, while parameter μ measures the disutility incurred by the consumer if he buys the two goods in different proportions. To simplify further the exposition, assume

[16] The same result obtains if firm i expands output so as to reduce p_i below such a threshold. Hence, intuitively the equilibrium outcome mimics collusion independently of the strategic variable considered.

$s_i = s_j = 1/2$ and $\nu > 2\sigma > R - c > \mu > 0$.[17] Under these conditions, it can be shown that firms get larger profits when they supply perfect substitutes, that is, when both firms sell either good 1 or good 2. To verify this result, observe that when they do so, their individual profits are $\pi_i = \pi_j = N(R-c)/2$, as in the previous example. When instead they produce imperfect substitutes, a consumer previously served by firm i obtains $R - p_i$ if he continues to buy from firm i, while he gets $R - p_j - \sigma$ if he switch to firm j; and he obtains

$$R + \nu - \mu \left(\frac{1}{2} - f \right)^2 - f p_i - (1-f) p_j - \sigma \tag{39}$$

if he combines the two products, in which case his utility is maximised at $f = 1/2 + (p_j - p_i)/(2\mu)$. If ν is sufficiently large, all consumers choose to combine the two goods, and therefore profits are defined as follows:

$$\pi_i = (p_i - c) \left[\frac{1}{2} + \frac{p_j - p_i}{2\mu} \right] N. \tag{40}$$

It is now easy to check that, if $\nu > \sigma + \mu/4$, the market subgame obtaining when firms have chosen to supply imperfect substitutes has a unique equilibrium in which $p_i = p_j = c + \mu$ and $\pi_i = \pi_j = N\mu/2$. This proves that, in the presence of switching costs, differentiation is not profitable. By extension, it can be shown that this result carry over to the case where firms indeed offer product lines rather than single goods.

This implies that switching costs play a crucial role in explaining firms' behaviour in differentiated markets, where firms may use product proliferation strategically. The reason for this is intuitive: if preferences induce consumers to purchase more than one good, then a single-product firm is worse off when faced with the competition of a multiproduct firm, because she forces her own consumers to choose between bearing a switching cost or giving up differentiation, i.e., the consumption of a product line.[18] This fact induces each firm to offer the same number of products as the rival firms. However, the softer price competition due to the switching costs reduces firms' incentives towards differentiation in the product space. If product lines were differentiated, consumers might indeed purchase a combination of lines notwithstanding the switching cost, provided that the beneficial effect of differentiation were sufficiently large. Being aware of this, firms prefer to offer product lines which are perfect substitutes for each other

[17] In this example, utility from consumption is discontinuous at $f = 0$ and $f = 1$, which have been appropriately excluded from the admissible interval. However, the results we are about to derive hold for utility functions $U(f)$ where $U(0) = U(1) = R$; $U(f) = R + \nu - \mu(1/2 - f)^2$, with $f \in [\xi, 1-\xi]$; and, for any other f, $U(f) \leq U(\xi)$, where ξ is a positive constant appropriately chosen.

[18] Notice the analogy between this model and that of Brander and Eaton (1984), summarised in section 2. Of course, as switching costs tend to zero, Brander and Eaton's results obtain.

and exploit fully the anti-competitive effect of switching costs. This is what leads Klemperer to state that "competing head-to-head may be less competitive". The existence of switching costs implies that prices are higher when products are close (in the limit, perfect) substitutes, than in the opposite case, and this contrasts with the standard literature on product differentiation where the opposite happens. Another relevant implication of this model is that the search for market niches still not preempted by rivals might drastically loose relevance, with imitation becoming more relevant than innovation.

The latter issue, however, is controversial. Indeed, as far as social welfare is concerned, switching costs imply the possibility for firms to offer excess product variety as compared to social optimum (Klemperer and Padilla, 1997). The intuition behind this is that, if consumers prefer to patronise a single producer, the firm offering a wider range of products might attract all consumers. Therefore, innovation within a line, and thus product proliferation, would have the well known effect of strengthening the position of a firm in the market. Yet, expanding product lines may be inefficient from the firms' viewpoint as well. Gilbert and Matutes (1993) show that the choice of offering a product line by all firms operating in a certain market can be the consequence of a prisoner's dilemma. Each firm is aware that it would be preferable to produce a single good, if this strategy were associated to a credible commitment on the part of every firm. In the absence of such a commitment, product proliferation is a dominant strategy and the outcome is inefficient for all firms within the industry.

8 Concluding remarks

The theory of product differentiation allows us to understand how firms may acquire some market power by decreasing substitutability between products, and how firms can deter entry by widening their product ranges (Schmalensee, 1978; Brander and Eaton, 1984). In doing so, they face a tradeoff between economies of scale in a single product and economies of scope over a product line (De Fraja, 1992). This is particularly true in the case of endogenous differentiation, where pure profit incentives, in the absence of entry threats, drive firms to reduce the width of their product lines (Bonanno, 1987; Martinez-Giralt and Neven, 1988; Champsaur and Rochet, 1989).

This holds as long as we take the producer's viewpoint. As soon as the consequences on consumer surplus and social welfare are accounted for, it emerges that firms' choices are obviously socially inefficient, with very few exceptions. The degree of differentiation which is optimal for firms is usually excessive from a social standpoint. Moreover, the need to create barriers to entry leads firms to underexploit scale economies and favour instead scope economies disproportionately. This also implies that firms can be expected to be inefficient in producing each single good, in that they will tend not to minimise average costs. This fact is driven by market forces, and it is but another epiphany of the well known tradeoff between static and dynamic efficiency stressed initially by Schumpeter (1942).

To some extent, the introduction of switching costs (Klemperer, 1992) into the picture restores consumers sovereignty, in that firms' decisions over their respective product lines are conditional upon these costs, which are equivalent, on the demand side, to the sunk costs that may operate on the supply side. Consumer sovereignty ends, though, as soon as the presence of switching costs produces a quasi-collusive market outcome. These considerations open an interesting perspective on the possibility for firms to exploit the success of a certain brand, i.e., brand loyalty on the part of consumers, in extending her activity in other markets. The strategic use of brand loyalty through *umbrella brands* allows firms to enter new markets by bearing the informational costs otherwise borne by consumers, who, in turn, may condition the global success of such firms. In this light, Klemperer's contribution recasts in modern terms a debate opened by Chamberlin (1933).

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