

# Monopoly, Quality, and Network Externalities<sup>1</sup>

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## Abstract

We describe the behaviour of a monopolist supplying a vertically differentiated good with network externalities. Assuming a fixed cost of quality improvements, we show that the presence of network externalities enhances the incentive to expand output associated with scale economies. Although the quality distortion operated by the monopolist increases with network externalities, the output expansion effect is dominant, so that the welfare loss due to monopoly power shrinks as the role of network externalities in determining consumers' satisfaction becomes more relevant.

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# 1 Introduction

The case for or against regulating a monopolist supplying goods whose production involves a large amount of fixed costs and a negligible unit variable cost, has been long debated, and it is now being re-assessed concerning markets where consumer utility is characterised by network externalities, i.e., it is positively related to the number of consumers who purchase the same good.<sup>1</sup> However, to our knowledge, the interplay between network externalities and the monopolist's choices concerning product quality and the output level, has not been evaluated so far.

Our aim is to provide a theoretical framework apt to address this issues. We shape a monopoly model where quality improvements hinge upon fixed costs, which can be thought of as the R&D effort. In the existing literature on vertical differentiation, the case of variable convex costs has been widely investigated. The main question is whether a monopolist supplies the socially optimal quality, or distorts it so as to induce self-selection on the part of consumers. The earliest contributions (Spence, 1975; Sheshinski, 1976) deal with a single-product monopolist. Their main conclusions are that (i) for a given output level, quality is over or undersupplied by the monopolist as compared to social planning, depending on whether the marginal valuation of quality is above or below the average valuation of quality (if they coincide, the monopolist supplies the same quality as the social planner); and (ii) the monopolist undersupplies quality if his output is close to the socially optimal one.

Several other contributions investigate a continuous model where the monopolist supplies a range of qualities, with a technology analogous to that assumed in Spence (Mussa and Rosen, 1978; Itoh, 1983; Maskin and Riley, 1984; Besanko, Donnenfeld and White, 1987). This literature highlights that, in order to discriminate among buyers with different characteristics, the monopolist increases the slope of the price-quality gradient compared to the social optimum. This is achieved by offering a quality range broader than the one that would be available under social planning or perfect competition. This points to the adoption of Minimum Quality Standards to correct quality distortion (Besanko, Donnenfeld and White, 1987).

To our knowledge, the role of fixed costs in shaping the behaviour of a monopolist, has received scanty attention, a relevant exception being Gabszewicz et al. (1986).<sup>2</sup> In their paper, however, fixed costs are exogenously given, and therefore do not affect the optimal choice of quality.

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<sup>1</sup>Seminal contributions in the theory of network externalities are Katz and Shapiro (1985; 1986); Farrell and Saloner (1985; 1986). For an overview, see Katz and Shapiro (1994) and the special issue of the International Journal of Industrial Organization, edited by Economides and Encaoua (1996).

<sup>2</sup>The opposite holds in the field of oligopoly competition. See Ronnen (1991), Motta (1993), Lambertini (1997), Lehmann-Grube (1997) and Scarpa (1998), inter alia.

We adopt a model where a single-product monopolist supplies a good whose production entails a fixed cost convex in the quality level, and consumers' utility function contains a network externality component. We evaluate the monopolist's performance against the social optimum in a general setting where the monopolist, in response to the presence of network effects, may over or under supply product quality as compared to the social optimum.

Resorting to a specification of the model where the distribution of consumers is uniform and the cost function is quadratic, it is possible to ascertain that the monopolist always undersupplies product quality and, as long as the market is only partially covered, such distortion is increasing in the extent of network externalities. The latter finding seemingly points to the need for quality regulation. However, this does not imply that we either could or should aim at increasing the intensity of competition in such a market, for several reasons. First of all, it is well known that, when production involves fixed costs, a competitive market structure cannot obtain (Shaked and Sutton, 1982; 1983). Second, the presence of brand-specific network externalities disrupts the conventional monotonic relationship between the number of firms on one side and consumer surplus and social welfare on the other. As a consequence, the appropriate comparison has to be carried out between a profit-seeking monopolist and a public firm maximising social surplus. We show that (i) the monopoly output and social welfare are increasing in the extent of network externalities; and (ii) the social planner serves all consumers independently of network externalities. These facts lead to a relevant conclusion, namely, that when the level of network externalities is non-negligible, the welfare loss due to monopoly power decreases as network externalities increase. To the extent that our assumptions are acceptable, our analysis implies that, in industries where the utility each individual derives from purchase is strongly related to the number of consumers patronising the same good or brand, the case for regulation is much weaker than what we usually think according to conventional wisdom.

The paper is organized as follows. Section 2 presents the general model. A specific formulation is introduced in section 3, where we derive both the monopoly equilibrium and the social optimum, which are then comparatively evaluated in section 4. Section 5 concludes.

## 2 The model

Consider a monopoly market for a good whose utility depends both on intrinsic characteristics, which are represented by quality  $q$ , and by the amount of market demand  $x$ . Consumers are characterised by parameter  $\mu$ , which represents the individual marginal willingness to pay for quality:<sup>3</sup> they are distributed with

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<sup>3</sup>As emphasised by Tirole (1988, ch. 2),  $\mu$  may also be interpreted as the reciprocal of the marginal utility of money. This implies that  $\mu$  increases as income increases, and conversely.

density  $f(\mu)$  over the interval  $[\underline{\mu}; 1]$ , with  $\underline{\mu} < 1$ : The number of individuals is normalised to 1. Each consumer buys at most one unit of the good, the resulting net surplus being:

$$U = \max_{\mu} f(\mu) + \beta x_i - p; 0 \quad (1)$$

where  $p$  is the price charged by the monopolist, while  $\beta$  (the same for all the agents) is a positive coefficient representing the weight of the network externality in the utility function. Let  $\beta^o(\beta; p; q; f(\mu))$  define the marginal willingness to pay of the consumer who is indifferent between buying and not buying:

$$\beta^o(\beta; p; q; f(\mu)) = \mu : \mu q + \beta \int_{\underline{\mu}}^{\mu} f(z) dz - p = 0 \quad (2)$$

Then, market demand is

$$x = \int_{\underline{\mu}}^{\beta^o} f(\mu) d\mu \quad \text{where } \underline{\mu} = \max\{\underline{\mu}; 1; \beta^o\} \quad (3)$$

When  $\max\{\underline{\mu}; 1; \beta^o\} = \beta^o$ ; partial market coverage obtains; when  $\max\{\underline{\mu}; 1; \beta^o\} = \underline{\mu}$ ; full market coverage obtains, i.e.,  $x = 1$ . Consumer surplus is

$$CS = \int_{\underline{\mu}}^{\beta^o} U(\mu) f(\mu) d\mu \quad (4)$$

On the supply side, production involves a fixed cost  $C = C(q)$ ; with  $C^0; C^{00} > 0$ : Variable costs are assumed away. This amounts to saying that quality is the result of R&D efforts, whose cost is increasing in the quality level, while it is unrelated to the scale of production. The profit function is then

$$\pi_M = p \int_{\underline{\mu}}^{\beta^o} f(\mu) d\mu - C(q) \quad (5)$$

Obviously, social welfare is  $SW = \pi_M + CS$ : Following Spence (1975), we evaluate the social incentive to modify product quality, in correspondence of the monopoly optimum. We prove the following:

**Proposition 1** Given  $f(\mu)$ ; in the monopoly optimum where  $\frac{\partial \pi_M}{\partial q} = 0$ ; the derivatives  $\frac{\partial CS}{\partial q}$  and  $\frac{\partial SW}{\partial q}$  may have either sign under partial market coverage.

**Proof.** See Appendix A.1.

The above proposition states that, in response to the presence of network effects, the monopolist may over or under supply product quality as compared to the social optimum.

### 3 A model with quadratic costs and uniform distribution

In order to characterise in detail the influence of network externalities on the provision of quality and welfare, we investigate a version of the model which is widely adopted in the existing literature. In particular, we assume that (i)  $C(q) = q^2$ ; and (ii) the population of consumers is uniformly distributed over  $[\hat{\mu}_i - 1; \hat{\mu}_i]$ : Recall that the marginal consumer is characterised by a willingness to pay  $\hat{\mu} = (p_i - \hat{p}_i) = (q_i - \hat{q}_i)$ , so that under partial market coverage (pmc) and full market coverage (fmc); respectively, market demand is

$$x = \hat{\mu}_i \frac{p_i - \hat{p}_i}{q_i - \hat{q}_i} = \frac{q_i \hat{\mu}_i - p_i}{q_i - \hat{q}_i} \text{ for all } (p_i; q_i) \text{ such that } \hat{\mu} \geq 2 (\hat{\mu}_i - 1; \hat{\mu}_i): \text{ (pmc)} \quad (6)$$

$$x = 1 \text{ for all } (p_i; q_i) \text{ such that } \hat{\mu} > \hat{\mu}_i - 1: \text{ (fmc)} \quad (7)$$

In either case, the monopoly profit function is  $\pi_M = px_i - q^2$ :

#### 3.1 Profit maximization

In this section, we first treat separately the alternative settings of partial and full market coverage. Then, we proceed to establish the parameter ranges where the monopolist adopts, alternatively, one regime or the other.

##### 3.1.1 Partial market coverage

Suppose  $\hat{\mu} \geq 2 (\hat{\mu}_i - 1; \hat{\mu}_i]$ : Then, partial market coverage obtains and the monopolist's profits are given by:

$$\pi_M^{pmc} = p \frac{q_i \hat{\mu}_i - p_i}{q_i - \hat{q}_i} - q^2 \quad (8)$$

This expression has to be maximized with respect to the two choice variables: price and quality.<sup>4</sup>

The first order condition with respect to price is:

$$\frac{\partial \pi_M^{pmc}}{\partial p} = \hat{\mu}_i + \frac{q_i \hat{\mu}_i - 2p_i}{q_i - \hat{q}_i} = 0 \quad (9)$$

which leads to  $p = \bar{p}q = 2$ . Plugging this expression for  $p$  in (8) and taking the derivative with respect to  $q$  yields:

$$\frac{\partial \pi_M^{pmc}}{\partial q} = \frac{\hat{\mu}_i^2 q}{2(q_i - \hat{q}_i)} - \frac{\hat{\mu}_i^2 q^2}{4(q_i - \hat{q}_i)^2} - 2q = 0 \quad (10)$$

<sup>4</sup>The same results obviously follow from the maximisation of profits with respect to quantity and quality.

The quality level provided by the monopolist is therefore:<sup>5</sup>

$$q_M^{pmc} = \bar{q} + \frac{\mu^2 + \mu k}{16} \quad (11)$$

where  $k = \mu^2 - 32\bar{q}$ : Clearly, the admissible range for  $\bar{q}$  is  $[0; \mu^2/32]$ : We will show below that the upper bound of this interval is never binding. The following result can be established:

**Lemma 1** The optimal monopoly quality and price under partial market coverage are everywhere decreasing in  $\bar{q}$ :

**Proof.** Taking the derivative of (11) and simplifying, we get

$$\text{sign} \frac{\partial q_M^{pmc}}{\partial \bar{q}} = \text{sign} \left( -\frac{\mu}{8} \right) = \text{sign} (-) < 0 \quad (12)$$

which is negative for all  $\bar{q} \in [0; \mu^2/32]$ : To prove that  $p_M^{pmc}$  is also everywhere decreasing in  $\bar{q}$ ; it suffices to observe that  $p_M^{pmc} = \bar{p} - \mu q_M^{pmc} = 2 - \mu \bar{q} - \frac{\mu^3 + \mu^2 k}{16}$ . ■

The equilibrium price simplifies to  $p_M^{pmc} = 2 - \mu \bar{q} - \frac{\mu^3 + \mu^2 k}{16}$ ; and market demand is  $x_M^{pmc} = \mu + 8\bar{q} - (\mu + k)$ : The resulting equilibrium profit is  $\pi_M^{pmc} = (3\mu - k)(16\bar{q} + \mu^2 + \mu k)^2 / (256(\mu + k))$ : Obviously, in the limit, as  $\bar{q}$  tends to zero, these results coincide with those derived in the standard model without network externalities,  $q_M^a = \mu^2/8$ ;  $p_M^a = \mu^3/16$  and  $x_M^a = \mu/2$  (see Lambertini, 1997). For any strictly positive  $\bar{q}$ , it is easy to see that  $p_M^{pmc}$  is always lower than  $p_M^a$ , while  $x_M^{pmc}$  is greater than  $x_M^a$ : In particular,

**Lemma 2** The monopoly output under partial market coverage is everywhere increasing in  $\bar{q}$ :

**Proof.** Taking the derivative of  $x_M^{pmc}$ ; we get

$$\frac{\partial x_M^{pmc}}{\partial \bar{q}} = \frac{8(\mu - k)}{k(\mu + k)^2} > 0; \quad (13)$$

being  $\mu - k > 0$  for all acceptable  $\bar{q}$ : ■

Notice that, unlike what happens in the variable cost case without network externalities (Spence, 1975), as long as the monopolist does not serve the whole market, a distortion is observed both in quality and in quantity. Positive network

<sup>5</sup>Second order conditions are met throughout the calculations performed in the paper, although they are omitted for the sake of brevity.

externalities increase the quality distortion made by the monopolist, who supplies a lower quality at a lower price, so as to expand the output level in order to serve lower income consumers. The welfare implications of these distortions can be traced out calculating the level of consumer surplus (CS) and the level of social welfare (SW):

$$CS_M^{pmc} = \int_{\hat{\mu}}^{\bar{\mu}} (\mu q_M^{pmc} + \theta x_M^{pmc} - p_M^{pmc}) d\mu = \frac{(16\theta + \hat{\mu}^2 + \hat{\mu}k)^3}{128(\hat{\mu} + k)^2} \quad (14)$$

$$SW_M^{pmc} = \frac{(16\theta + \hat{\mu}^2 + \hat{\mu}k)^2 [(3\bar{\mu} - k)(\bar{\mu} + k) + 2(16\theta + \hat{\mu}^2 + \hat{\mu}k)]}{256(\hat{\mu} + k)^2} \quad (15)$$

In the admissible range of  $\theta$ , it can be shown that

$$\frac{\partial q_M^{pmc}}{\partial \theta} > 0; \quad \frac{\partial CS_M^{pmc}}{\partial \theta} > 0; \quad \frac{\partial SW_M^{pmc}}{\partial \theta} > 0: \quad (16)$$

The above inequalities can be given the following interpretation. Profits increase with the weight of network externalities, because the positive effects of output expansion and quality reduction outweigh the negative effect due to the reduction of price. On the other hand, consumer surplus becomes larger as network externalities increase, because the beneficial effects of price reduction and output expansion more than offset the loss due to a lower product quality. Furthermore, observe that  $\partial^2 SW_M^{pmc} / \partial \theta^2 > 0$  and  $\partial^2 q_M^{pmc} / \partial \theta^2 < 0$ ; considered together, these derivatives imply  $\partial^2 CS_M^{pmc} / \partial \theta^2 > 0$ :

### 3.1.2 Full market coverage

In this case, profits write simply  $\pi_M^{fmc} = p - q^2$ . Since profits are always increasing in price, the monopolist always chooses the highest price compatible with full market coverage, given by:

$$p(q) = (\hat{\mu} - 1)q + \theta \quad (17)$$

Therefore, the monopolist chooses the quality which maximizes  $\pi_M^{fmc} = (\hat{\mu} - 1)q + \theta - q^2$ : The first order condition is:

$$\frac{\partial \pi_M^{fmc}}{\partial q} = \hat{\mu} - 1 - 2q = 0; \quad (18)$$

yielding optimal quality  $q_M^{fmc} = (\hat{\mu} - 1)/2$ : The price set by the monopolist is  $p_M^{fmc} = p(q_M) = (\hat{\mu} - 1)^2/2 + \theta$ : Therefore, we can state

**Lemma 3** The optimal monopoly price under fmc is everywhere increasing in  $\theta$ :

Social welfare at equilibrium amounts to:

$$SW_M^{fmc} = \theta + \frac{(\hat{\mu} - 1)^3}{4}; \quad (19)$$

### 3.1.3 Partial vs full market coverage

The monopolist's profits under partial market coverage are the following:

$$q_M^{pmc} = \frac{(3\bar{\mu} - k)(16\bar{c} + \bar{\mu}^2 + \bar{\mu}k)^2}{256(\bar{\mu} + k)} \quad (20)$$

while, under full market coverage, profits are:

$$q_M^{fmc} = \bar{c} + \frac{(\bar{\mu} - 1)^2}{4} \quad (21)$$

The monopolist chooses to serve all the market only if  $q_M^{fmc} > q_M^{pmc}$ , and the conditions under which this inequality holds involve the values of  $\bar{c}$  and  $\bar{\mu}$ : The following holds:

**Proposition 2**  $q_M^{fmc} > q_M^{pmc}$  and fmc obtains, in the following parameter regions:

$$\bar{\mu} \geq 2 \text{ [; } 4=3] \text{ and } \bar{c} \geq \frac{\bar{\mu}k}{\bar{\mu}(4 + \bar{\mu})^3} ; \bar{\mu}^2 ; 6\bar{\mu} ; 4 =8:$$

$$\bar{\mu} > 4=3 \text{ and } \bar{c} \geq \max \{0; (3\bar{\mu} ; \bar{\mu}^2 ; 2)=4\} .$$

In the remainder of the space  $\bar{\mu}; \bar{c}$ ; the opposite holds and pmc obtains.

Observe that the boundaries  $\frac{\bar{\mu}k}{\bar{\mu}(4 + \bar{\mu})^3} ; \bar{\mu}^2 ; 6\bar{\mu} ; 4 =8$  and  $(3\bar{\mu}; \bar{\mu}^2; 2)=4$  are both below  $\bar{\mu}^2=32$  for all  $\bar{\mu}$ : This entails that the condition for the reality of  $q_M^{pmc}$  is never binding.

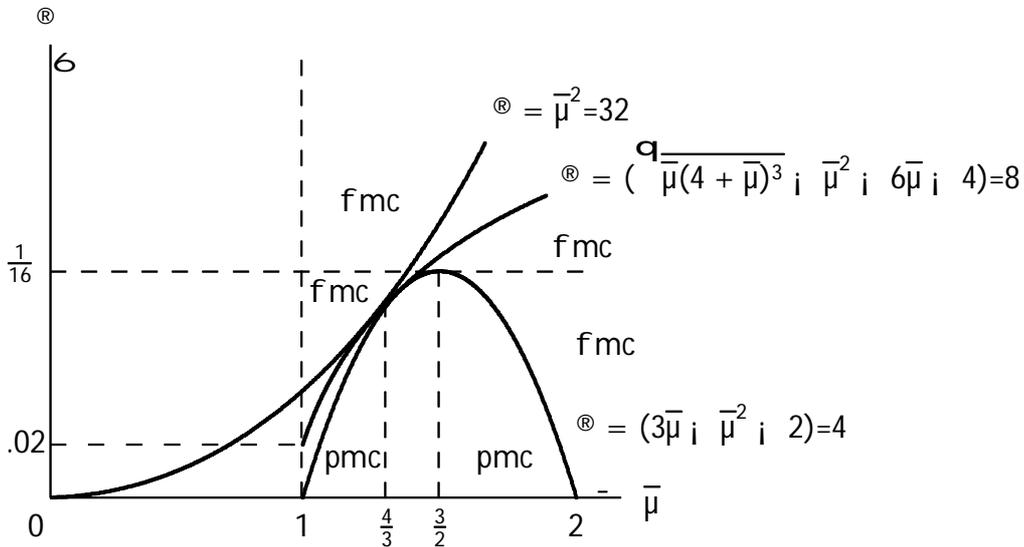
The above proposition has the following relevant corollaries

**Corollary 1** For all  $\bar{c} \geq 2 \text{ [(} \frac{16}{5} ; 11)=8; 1=16]$ , the monopolist's optimal output is non-monotone in  $\bar{\mu}$ :

The proof follows immediately from the observation that the monopolist may be induced to serve all consumers even if the marginal willingness to pay for quality is relatively low, provided that the network effect is sufficiently large to compensate for a low valuation of quality.

The market coverage policy chosen by the monopolist in the space  $\bar{\mu}; \bar{c}$  is described in Figure 1, where the domain of  $\bar{\mu}$  is to the right of the dashed line at  $\bar{\mu} = 1$ .

Figure 1 : Full vs partial market coverage under monopoly



The effects of both  $\mu$  and  $\bar{\mu}$  on the optimal quality can now be described. The following holds:

**Proposition 3** For all  $\bar{\mu} \in [1; 4/3]$ ; optimal monopoly quality, price and output are discontinuous in  $\mu$ ; along  $\mu = \frac{\bar{\mu}(4 + \bar{\mu})^3 - \bar{\mu}^2 - 6\bar{\mu} - 4}{8}$ ; For all  $\bar{\mu} \in [4/3; 2]$ ; optimal monopoly quality, price and output are continuous in  $\mu$ ; along  $\mu = (3\bar{\mu} - \bar{\mu}^2 - 2)/4$ . Optimal quality is monotonically increasing in  $\bar{\mu}$  and non-increasing in  $\mu$ :

**Proof.** See Appendix A.2.

### 3.2 Welfare maximization

A benevolent social planner maximizes social welfare with respect to price and quality. As is well known, network externalities being absent, the planner would price at marginal cost, serving all consumers. Consequently, any positive  $\mu$  can be expected not to affect the planner's output decision. Indeed, given (1), the standard solution of the first order conditions relative to welfare maximization leads to the following value of  $\hat{\mu}$ :

$$\hat{\mu} = \frac{\mu(2\theta(8\theta + \mu^2 + \mu k))}{\mu(3\theta\mu + \frac{\mu^3}{4} + \theta k + \frac{\mu^2 k}{4})} : \quad (22)$$

It can be easily shown that this value is negative for every  $\theta > 0$ : This means that the social planner would choose a price and a quality such that the market is more than totally served, which of course cannot be the case. Therefore, we impose full market coverage ( $x = \mu$ ;  $\hat{\mu} = 1$ ) from the outset. In such a case, social welfare is

$$SW = \mu q + \theta \int \frac{q}{2} \mu q^2 : \quad (23)$$

The first order condition is:

$$\frac{\partial SW}{\partial q} = \mu \int 2q \mu \frac{1}{2} = 0 : \quad (24)$$

Therefore,  $q_{SP} = (2\bar{\mu} - 1) = 4$ : Social welfare in equilibrium is  $SW_{SP} = \theta + [\mu(\mu - 1)] = 4 + 1 = 16$ , which is obviously larger than  $SW_M^{fmc}$ ; the difference amounting to  $1 = 16$ . In order to guarantee full market coverage, the price cannot be higher than (17). Substituting  $q_{SP}$  into (17) yields  $p(q_{SP}) = (2\mu^2 - 3\mu + 1) = 4 + \theta$ : Since social welfare does not depend on the price level, the social planner can choose any  $p \in [0; p(q_{SP})]$ : The difference  $p(q_{SP}) - p$  simply implies a transfer in favour of consumers. This result also implies that, as it usually happens when some kind of externality is involved, welfare maximization does not require marginal cost pricing.

## 4 Monopoly vs social planning

We are now in a position to compare the choices of the social planner with those of the monopolist under both market coverage regimes, within the model investigated in the previous section. We can state the following:

**Proposition 4** The optimal monopoly quality is always lower than  $q_{SP}$ : Moreover, the difference  $q_{SP} - q_M^{pmc}$  is increasing in  $\theta$ ; for all  $\theta$  such that the monopolist chooses partial market coverage.

**Proof.** Consider first the case where the monopolist covers the market entirely. We have that  $q_{SP} - q_M^{fmc} = 1 = 4$ ; i.e., the difference between the two quality levels is positive and independent of both  $\theta$  and  $\bar{\mu}$ : Second, in the case where the monopolist covers the market only partially, observe that, if  $\theta = 0$ ;  $q_M^{pmc} = q_M^a = \bar{\mu}^2 = 8 < q_{SP}$  for all  $\bar{\mu} \in [1; 2]$ : Then, from lemma 1, we know that  $\partial q_M^{pmc} / \partial \theta < 0$  in the admissible range for  $\theta$  and  $\bar{\mu}$ : This suffices to prove the proposition. ■

We can conclude that, also in the case of full market coverage, the monopolist undersupplies quality, setting a price which can be lower than the price the social

planner would choose: providing a higher quality, the social planner can set a price which is  $(\bar{\mu} + 1)\frac{1}{4}$  higher than  $p_M$ , being still able to satisfy the full market coverage condition (17).

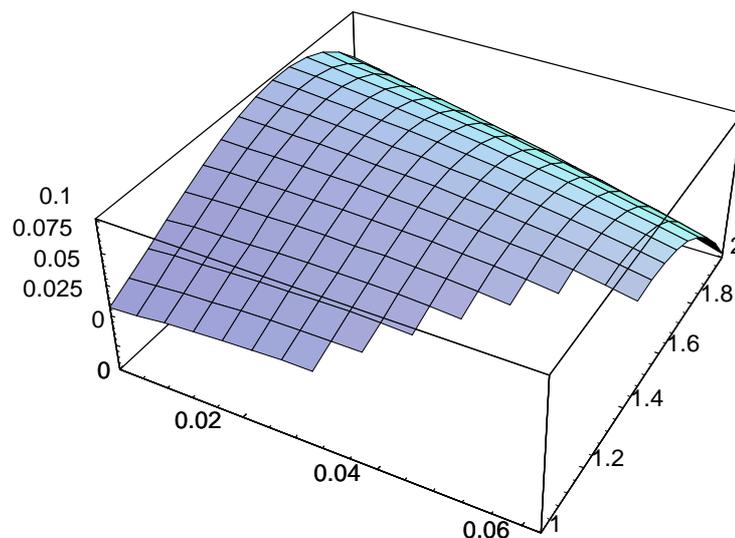
The comparative evaluation of social welfare in the two cases is summarised in the following proposition:

**Proposition 5** Independently of the extent of market coverage,  $SW_{SP} > SW_M$  for all admissible  $\theta$  and  $\bar{\mu}$ : However,

- <sup>2</sup> Whenever the monopolist serves all consumers,  $SW_{SP} \geq SW_M^{fmc} = 1/16$ :
- <sup>2</sup> Whenever the monopolist covers the market only partially, then  $SW_{SP} \geq SW_M^{pmc}$  is non-monotone both in  $\theta$  and in  $\bar{\mu}$ :

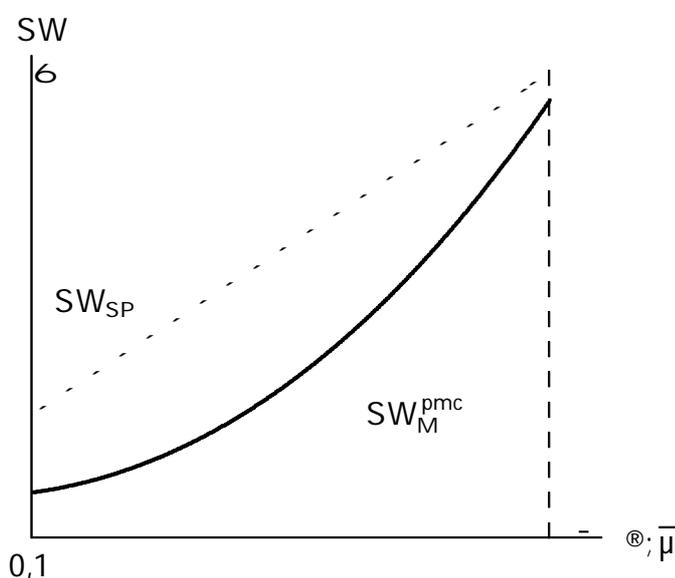
**Proof.** The result that  $SW_{SP} > SW_M$  for all admissible  $\theta$  and  $\bar{\mu}$  is fairly intuitive, because, for any pair  $f^\theta; \bar{\mu}$ ; the social planner can at least replicate the monopolist's performance in terms of social welfare. Comparing  $SW_{SP}$  with  $SW_M^{pmc}$  over the parameter space  $\theta \in [0; 1/16]$ ;  $\bar{\mu} \in [1; 2]$ ; the surface  $SW_{SP} \geq SW_M^{pmc}$  appears as in Figure 2, which conveys the following information: (i) for any admissible  $\theta$ ; the difference  $SW_{SP} \geq SW_M^{pmc}$  is non-monotone in  $\bar{\mu}$ ; (ii) when  $\theta$  is close to 0 and  $\bar{\mu}$  is close to 1, the difference  $SW_{SP} \geq SW_M^{pmc}$  is increasing in  $\theta$ ; while the opposite happens in any other region of the parameter space  $f^\theta; \bar{\mu}$ :

Figure 2 : Welfare difference, social planning vs monopoly ( $\theta \in [0; 1/16]$ ;  $\bar{\mu} \in [1; 2]$ )



The non-monotonicity observed in both cases is to be traced back to the behaviour of social welfare under monopoly. To ascertain this, notice that the derivative of  $SW_{SP}$  w.r.t. either  $\theta$  or  $\bar{\mu}$  is linear, while the derivative of  $SW_M^{pmc}$  w.r.t. the same parameters is a convex curve. They cross at a value of the relevant parameter which belongs to its admissible interval. Suppose that  $\theta$  and  $\bar{\mu}$  are alternatively fixed at an appropriate value. This allows us to plot the two social welfare levels in two dimensions, obtaining in both cases a picture like figure 3.

Figure 3 : Comparative welfare assessment



As we already know, this picture obtains either (i) if we let  $\bar{\mu}$  vary, given any arbitrary  $\theta \in [0; 1=16]$ , or (ii) if we let  $\theta$  vary, provided that  $\bar{\mu}$  is close to 1. ■

The above discussion has some relevant implications as to the scope for regulation when consumer preferences are characterised by network externalities. First, from figure 2 it clearly appears that (i)  $SW_{SP}$  i  $SW_M^{pmc}$  is single-peaked in  $\bar{\mu}$  for any given  $\theta$ ; with a global maximum at  $(\bar{\mu} = 1:675; \theta = 0)$ ; and (ii) excluding the region where the network externality and the marginal willingness to pay are both close to the lower bounds of their respective admissible intervals, any increase in  $\theta$  reduces the welfare loss imputed to the monopolist. The

straightforward corollary to this result is that, as long as the monopolist does not serve all consumers, the argument for regulation generally becomes weaker as the extent of network externalities increases.

## 5 Discussion and concluding remarks

According to the current antitrust legislation, both in Europe and in the US, the case for the intervention of the legal authorities arises whenever competition is threatened or already eliminated, regardless of any welfare considerations. The foregoing analysis sheds some new light on the amount of welfare loss and the resulting need for public intervention in a monopoly market for a product whose network externalities are a relevant component of consumer's utility. A discussion on these issues is currently taking place regarding the market for mobile telephones (in Europe) and the software industry (in the US). The US Department of Justice's case against Microsoft<sup>6</sup> focusses upon the behaviour of Microsoft to preserve and extend its monopoly power in the software industry. Specific allegations concern

- (1) monopolization of the market for PCs operating systems;
- (2) anti-competitive bundling of the Microsoft Internet browser with the Windows OS;
- (3) anti-competitive contractual arrangements with various vendors of related goods.

The foregoing analysis sheds some light on point (1) above, under two respects. First, the existence of network externalities may entail that monopoly is not so evil as one usually thinks. When production technology involves high development costs, the presence of network externalities enhances the incentive towards output expansion associated with decreasing average cost. On the other hand, the quality distortion operated by the monopolist increases as the weight of network externalities increases. However, on the aggregate, the output expansion effect dominates the quality distortion, yielding as a result that the welfare loss due to monopoly power shrinks as the role of network externalities in determining consumers' satisfaction becomes more relevant. Second, the externality entails that the market be completely served in the social optimum. This produces a relevant implication, namely, that marginal cost pricing is not a necessary condition for welfare maximization. Hence, marginal cost should not be taken as a benchmark to evaluate the pricing policy of Microsoft.

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<sup>6</sup>For an exhaustive account of the related debate, see the web page <http://raven.stern.nyu.edu/networks>, by Nicholas Economides.

Ever since the Sherman Act (1890) was passed in the US, according to the established wisdom on anti-competitive behaviour, any attempt at monopolising a market should be prosecuted, in that it evidently involves a welfare loss. Although the correct interpretation of the Sherman Act is still a matter of debate, a widely accepted view is that its main economic goal was to minimise the deadweight loss due to monopoly power (Bork, 1966).<sup>7</sup> Basically, this position provides the background for the legal action against Microsoft.<sup>8</sup> In summary, the existing antitrust laws prompts for the intervention of legal authorities whenever there is evidence of abuse of a dominant position, under the presumption that monopoly power necessarily implies a significant deadweight loss. However, what appears as an abuse of dominant position might well be the outcome of endogenous mechanisms characterising markets with large network externalities, which may jeopardise the existence of a competitive or at least oligopolistic equilibrium (see Lambertini and Orsini, 1998).

Obviously, the above considerations are not fully conclusive. In the model we have considered here, the output expansion effect jointly exerted by scale economies and network externalities clearly goes in the direction of a welfare increase, but it must be taken into account that a different preference structure might alter the results significantly. In particular, preference for variety may play a decisive role. This is emphasised by Church and Gandal (1992), who investigate how the provision decision by software firms determines whether multiple hardware technologies can coexist in the market, or whether standardization obtains, with only one hardware technology supplied with software in equilibrium. They show that when consumers place a high value on software variety there is a suboptimal amount of standardization by the market.

Another relevant issue is that, in several markets, consumers generate benefits from the purchase of at least two components (again, an important example is the software-hardware industry, or bundling of different softwares). In such cases, the intertemporal distribution of purchases becomes crucial. If a single firm is the unique supplier of all components, two issues must be tackled, namely,

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<sup>7</sup>This is reinforced in Section 3 of the Federal Trade Commission Act (1914), which prohibits those practices that have the effect of binding a consumer to a particular supplier, "where the effect" of such practices "may be substantially to lessen competition", i.e., the Congress of the US meant to prohibit any behaviour that could interfere with the opportunity for equally efficient rivals to compete (Fox, 1981).

<sup>8</sup>The route taken by the European Community to shape its competition policy is somewhat the same as in the US, but the Treaty of Rome contains a paragraph which finds no correspondence in the US antitrust law. Article 85, paragraph 2, establishes that a business strategy should be neither prohibited nor prosecuted if it "contributes to improving the production or distribution of goods or to promoting technical or economic progress, while allowing consumers a fair share of the resulting benefit, and which does not ... afford ... the possibility of eliminating competition in respect of a substantial part of the products in question". This entails that the European Community competition policy is not aimed at maximising social welfare, defined as the sum of consumers' and producers' surplus.

multiproduct pricing and intertemporal pricing. If more than one firm operates in the market, then switching costs become relevant as well (see Klemperer, 1992, 1995; Beggs and Klemperer, 1992; Farrell et al., 1998, *inter alia*).

Finally, goods characterised by network externalities may well be durable goods, and there arises a need for modelling the intertemporal choices of both producers and consumers (see Cabral, Salant and Woroch, 1998; Fudenberg and Tirole, 1998).

For these reasons, future research should produce a deeper understanding of these phenomena in order to design appropriate policy interventions in industries where network externalities are a relevant feature.

# Appendix

A.1. Proof of Proposition 1. Consider first the monopoly optimum under partial market coverage, i.e., when  $\mu < 1$ ;  $\beta = \beta^0$ ;  $\mu = \mu_q + \int_{\beta^0}^1 f(z) dz$ ;  $p = 0$ . The relevant first order condition is:

$$\frac{\partial \pi_M}{\partial q} = \int_{\beta^0}^1 p \left( f(\beta) - \beta_q^0 \right) + p_q^0 \int_{\beta^0}^1 f(\mu) d\mu - C^0 = 0; \quad (a1)$$

where  $\beta_q^0 = \frac{\partial \beta^0}{\partial q}$ ;  $f(\mu) = \int_{\beta^0}^1 f(z) dz$ ; and  $p_q^0 = \frac{\partial p}{\partial q}$ : As quality increases, the location of the marginal consumer shifts in a way determined by:

$$\beta_q^0 = \frac{p_q^0 \int_{\beta^0}^1 C^0}{p f(\beta^0)}; \quad (a2)$$

where

$$\text{sign } \beta_q^0 = \text{sign } p_q^0 \int_{\beta^0}^1 C^0; \quad (a3)$$

Differentiating social welfare w.r.t. quality yields

$$\frac{\partial SW}{\partial q} = \frac{\partial \pi_M}{\partial q} + \frac{\partial CS}{\partial q} = \frac{\partial \pi_M}{\partial q} + \int_{\beta^0}^1 \mu \left( f(\beta) - \beta_q^0 \right) p_q^0 f(\mu) d\mu + \int_{\beta^0}^1 \beta_q^0 \int_{\beta^0}^1 f(\mu) d\mu \left( f(\beta) - \beta_q^0 \right); \quad (a4)$$

In the monopoly optimum,  $\frac{\partial \pi_M}{\partial q} = 0$ , and the above expression simplifies to:

$$\frac{\partial SW}{\partial q} = \frac{\partial CS}{\partial q} = \int_{\beta^0}^1 \mu \left( f(\beta) - \beta_q^0 \right) p_q^0 f(\mu) d\mu + \int_{\beta^0}^1 \beta_q^0 \int_{\beta^0}^1 f(\mu) d\mu \left( f(\beta) - \beta_q^0 \right); \quad (a4')$$

which can be rearranged as:

$$\frac{\partial CS}{\partial q} = \int_{\beta^0}^1 \mu p_q^0 f(\mu) d\mu - \beta_q^0 \int_{\beta^0}^1 \mu \left( f(\beta) - \beta_q^0 \right) f(\mu) d\mu + 2 \beta_q^0 \int_{\beta^0}^1 f(\mu) d\mu \left( f(\beta) - \beta_q^0 \right); \quad (a5)$$

Observe that, when  $\theta = 0$ ; expression (a5) coincides with the well known condition in Spence (1975). The presence of network externalities adds the last term where  $\theta$  appears explicitly, and modifies the other terms as well. ■

A.2. Proof of Proposition 3. Consider the first statement in proposition 3. Observe that, for  $\mu \in [1/4, 3/4]$ ;  $x_M^{pmc} < 1$  along  $\theta = \frac{\mu}{\mu(4+\mu)^3} - \mu^2 - 6\mu - 4 = 0$ : This goes along with an analogous discontinuity in quality and price. To prove the second statement in proposition 3, it suffices to check that  $x_M^{pmc} = 1$ ;  $p_M^{pmc} = p_M^{fmc}$

and  $q_M^{\text{pmc}} = q_M^{\text{fmc}}$  along  $\bar{\mu} = (3\bar{\mu}^2 - \bar{\mu}^2 - 2) = 4$ ; which is the relevant boundary between pmc and fmc for all  $\bar{\mu} \in [4; 32]$ . To prove the third statement, consider first  $q_M^{\text{fmc}}$ : This is increasing in  $\bar{\mu}$  and invariant in  $\theta$ : As to  $q_M^{\text{pmc}}$ ; we know from lemma 1 that  $\frac{\partial q_M^{\text{pmc}}}{\partial \theta} > 0$  for all  $\theta \in [0; \bar{\mu}^2 - 32]$ : Moreover, we have

$$\frac{\frac{\partial q_M^{\text{pmc}}}{\partial \bar{\mu}}}{\frac{\partial q_M^{\text{pmc}}}{\partial \theta}} = \frac{\frac{\mu}{\bar{\mu} + \bar{\mu}^2 - 32} - \frac{1}{2}}{16 - \bar{\mu}^2 - 32\theta} ; \quad (\text{a6})$$

which is positive for all  $\theta \in [0; \bar{\mu}^2 - 32]$ : Notice that such interval contains the region where pmc obtains. This concludes the proof. ■

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