Advertising as a Signal of Quality, A New Explanation

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Abstract
The present article provides a unified explanation for several phenomena related to advertising by firms. (i) Advertising without repeat purchase of the product, (ii) advertising from established brands, or post-introductory, (iii) simultaneous advertising from low and high quality firms, (iv) its persistence and pro-cyclicality. The explanation is original because it rests upon oligopolistic interaction. The analysis hinges upon two fundamental results. The first is that advertising allows separation when a signal *via* prices only does not. The second is that purely dissipative advertising can be used to strategically deter entry. Hence, a link is established between entry deterrence and signaling.

Keywords: Advertising, signaling, entry deterrence, imperfect information, oligopoly, vertical differentiation.

1. Introduction

The[1] explanation of advertising by Nelson [1974] is well known. In a context where consumers repeatedly purchase an experience good—namely one of a quality that can be learned only after purchase—apparently wasteful advertising campaigns induce rational consumers to realize that the advertised brands sell a superior product. Consumers would not re-purchase the good after a first trial if it were not of good quality, and the introductory advertising costs could not be recovered in subsequent periods.

Nelson arguments—that apply to any form of wasteful expenses by firms and not just to advertising—rest on *repeat purchase*. Along those lines several authors have developed rigorous models to scrutinize the validity of Nelsonian theories (see among others Milgrom and Roberts [1986], Schmalensee [1982]; see also Martin [1994] for a recent survey). The limitation in the Nelsonian theory is that it can account only for introductory advertising which is terminated after the product becomes known. Moreover, most of the literature considers the strategic interaction among *one firm* and the population of consumers. Kihlstrom and Riordan [1984] represent an exception since they consider a perfectly competitive industry with free entry, where firms are price-takers. They obtain the result that advertising equilibria can arise also when the interaction among firms and consumers is not repeated, provided the high quality firms have lower variable costs than their low quality rivals.

There seems to lack a unified theory which can explain several phenomena related to advertising.

First, advertising or other wasteful and observable selling expenses are common in markets where consumption has a transient nature. Think for instance to the owners of restaurants that crowd some touristic areas (this example is also in Tirole [1989]), who invest sometimes considerable sums in luxurious settings, silver cutlery, number of waiters, and the like, although they do not expect to receive the same customers repeatedly. One may observe that some restaurants will spend money on these items and others will not. However, a theory based on repeat purchase cannot assign an informative role to these expenditures.

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Second, as a cursory look at television campaigns may confirm, firms with brands of long-established reputation also spend considerable sums on advertising. This phenomenon cannot be explained by Nelsonian theories, unless one admits a constant inflow of new consumers at each period who are unaware of the quality of the brand being advertised and who enter a repeat purchase process. Another way out would be to assume that consumers do not have perfect recall—or that the characteristics of the products on the market change fast while purchases happen infrequently[2]. It seems difficult to accept the idea however that the magnitude of this inflow be large enough to justify the large stream over time of advertising expenditures that one observes. Similarly, this type of advertising is not encompassed by the explanation in Kihlstrom and Riordan since, although they do not assume repeat purchase, firms need to advertise only in the introductory phase.

Third, the empirical literature (as well as some non-signal theories of advertising like Dorfman and Steiner[1954]) had long since noted the relative constancy of the advertising-sales ratio of firms over time (see for instance Schmalensee [1972]). This means in particular that advertising varies pro-cyclically.

Fourth, in various markets where firms of higher quality compete against rivals of lower quality, advertising by low quality firms is conspicuous and sometimes it compares to that of the high quality firms.

The explanation provided in the present paper encompasses all these four types of advertising and it therefore enlarges the quota of advertising expenditures that can be attributed to a signaling effort by firms. Furthermore, quite surprisingly, the explanation we provide confirms an old idea, namely that high advertising expenditures can be used as a strategic entry barrier. It is worth emphasizing that the entry preventing attribute of advertising is derived in the present paper without recurring to *ad hoc* assumptions about the influence of advertising on consumers' tastes.

The model below abandons the *repeat purchase* (which however could be reintroduced at no cost) and the *one-firm* assumptions. It explicitly considers the problem of finding separating instruments for the case where two firms compete on the same market. None of these two firms has an established reputation but it can be of high or of low quality; therefore the consumers are confronted with the problem of interpreting the price-advertising strategies of both firms at the same time. Interesting situations may arise that do not appear in the monopoly case. For instance, a firm may try and exploit the consumers imperfect information by copying the strategy adopted by its rival. This type of mimicking behavior is rather different from the one to which the signaling literature in economics is used: in our case, in fact, it is quite obvious that if two firms send the same signal (whether along or outside the equilibrium path) then the consumers must attribute them the same probability of being of a given type. In other words

[2] We thank Guido Candela for drawing our attention to this case.

each firm is aware that the strategy it plays is liable to affect the beliefs that the uninformed party, here the consumers, holds about the *other* firm's type as well as of its own type.

This feature of beliefs does not depend on the number of informed parties—it is evident that in the job market game of Spence [1973] there are many workers but the beliefs of the employer on the type of worker Smith do not take into account what worker Jones has spent on education[3]. It depends rather on the property that the payoff (here the profits) of an informed player depends upon what the other player does (e.g. its pricing behavior), and on the fact that this reciprocal dependence is common knowledge to all players.

The paper proceeds as follows. First, in Section 2, the problem with one firm only is introduced and the notation established. The main reason for Section 2 rests not upon its novelty but on the need to briefly clarify why in the absence of repeat purchase the monopoly case is insufficient to explain any form of advertising, and to pave the way for the remaining Sections.

Section 3, then, constitutes the core of the paper. Two firms compete, each being of one among two possible types, each firm knows its type and that of the competitor while consumers do not observe the realized state of Nature but only the strategies played by the firms. On the basis of this observation consumers form their beliefs. The purpose of the Section is that of deriving a characterization of separating equilibria that does *not* depend on the belief system adopted by consumers. As a method, the approach consists of writing down two necessary conditions that the strategies played by the two firms must satisfy in the states of Nature when their qualities differ: these simply say that neither firm must find it profitable to mimic exactly the behavior of the rival when their types differ. Only using these two conditions it is shown that there are cases in which a separating equilibrium, if it exists, *must* involve the use of advertising by the high quality firm if the quality types differ. The first task of the present research is therefore accomplished in Section 3, since an explanation is provided for advertising that does not impinge on repeat purchase.

The issue of existence of separating equilibria is solved in Appendix 2, which, however, does not develop a complete typology of possible belief systems since this is irrelevant for our results and it would lead us astray from the non-technical issues that motivate the analysis.

In Section 4, the idea that entry may be deterred through advertising is examined using the results of Section 3. The Section presents an extension of the basic model obtained introducing the possibility that the duopoly market structure be contestable by a firm which can also be of two types: low and "very low", and which can mimic the behavior of a low quality. A true low-

[3]Similarly, if the industry is perfectly competitive a firm cannot affect with its behavior the beliefs that consumers hold about the types of the other firms. Khilstrom and Riordan [1984] model therefore does not share this feature.

quality firm must then also protect its position, and it can do so *only* through advertising. Interestingly, this type of advertising does not serve to separate from existing firms, but from potential entrants, and therefore where observed it may be erroneously ascribed to other reasons not depending on imperfect information about quality. This type of advertising persists over time as long as the threat by lower quality firms persists, and it varies in the same direction as demand varies, namely it varies pro-cyclically.

Our explanation of the entry barrier attribute of advertising is not based upon the effects on consumers' tastes, as advertising is here purely dissipative. It is interesting to note that in an article on limit pricing and advertising Bagwell and Ramey [1988] used price-advertising couples as informative strategies by an incumbent to signal its cost of production to an entrant. They found that purely dissipative advertising could not be used as an entry barrier.

Other models (of repeat-purchase) that are related to the study of experience goods are the reputation models like Klein and Leffler [1981] and Shapiro [1983], based on the importance of quality premia for the inducement to a monopolist to introduce a high quality instead of a low one (see also Riordan [1986]). In this respect our analysis also has some consequences, in spite of the assumption we make that the types of the firms are chosen by Nature. Where firms cannot rely on repeat purchase, in fact, a high quality can be introduced only if it can separate from the low qualities.

2. Monopoly in the Absence of Repeat-Purchase

The main implications of the analysis can be derived *via* a simple example, already used for the monopoly case by Bagwell and Riordan [1991], and here rearranged, where needed, to introduce a vertically differentiated duopoly[4].

Consider a market where a monopolist sells a good that can be of quality H or L. Unit production costs are c for the high quality and 0 for the low quality. There are no fixed costs of production, although the monopolist can voluntarily add a discretionary amount of fixed cost, a, in the form of wasteful advertising campaigns. Each consumer buys either one unit or none. If the good is of quality H the consumer's utility after purchase is

$$u_H = v + m - p_H \tag{2.1}$$

[4]Although several examples exist (see Gabszewicz and Thisse[1979] seminal paper, and Shaked and Sutton [1981]) we do not have yet a general theory of vertical differentiation in oligopoly under *complete* information. The authors have worked out a set of necessary conditions for a duopoly to be defined as vertically differentiated, and have checked that the main character of the analysis can be preserved in that more general set-up. The exposition, however, then becomes mingled to conceptual issues pertaining to the full information generalization of the vertical differentiation models that do not add any useful insights while the difference between our explanation of advertising and the traditional one becomes unclear.

where v is a positive constant, p_H is the price paid to the seller, and m is a taste parameter, ranging in the unit interval $[0,1]$, uniformly distributed over this range so that the consumers population has unit mass. If a consumer does not buy her utility is zero. The consumer that is indifferent between buying or not a high quality product has taste parameter $m = p_H - v$. The demand to the monopolist of high quality under perfect information is then

$$D_H(p_H) = \min[1 + v - p_H, 1]$$

if $1 + v \geq p_H$, and $D_H = 0$ if $1 + v \leq p_H$.

If the good is of low quality, then utility is assumed to be invariant with m and to be given by

$$u_L = v - p_L. \tag{2.2}$$

Therefore demand to the monopolist of low quality under perfect information is $D_L = 1$ if $v \geq p_L$, and $D_L = 0$ if $v < p_H$.

The perfect information prices are $p_H^m = \max\{(1 + v + c)/2, v\}$ and $p_L^m = v$ for the high and the low quality respectively. The case where $(1 + v + c)/2 \leq v$ is of no interest since then the monopolist has no incentive to separate when her quality is H. Eliminating this case, then, the full information profits are respectively given by $\pi_H^m = [(1 + v - c)/2]^2$ and $\pi_L^m = v$.

Under imperfect information, assume that consumers know that the monopolist may be of one of the two qualities, so that there are only two possible states of Nature, denoted by *(H)* and *(L)*. Let σ denote a couple (p, a) of price and advertising used by the monopolist. For any observation of σ the consumers form a belief about the product quality. More precisely a belief is defined as a function $b(\sigma)$ indicating the probability that the product is of high quality. Then, given a particular price and advertising combination chosen by the firm, the consumer's expected utility is $u(\sigma) = b(\sigma)(v + m - p) + (1 - b(\sigma))(v - p)$, with $u(\sigma) = u_H$ if $b(\sigma) = 1$ and $u(\sigma) = u_L$ if $b(\sigma) = 0$.

A separating equilibrium, loosely speaking, is constituted by strategies and beliefs with the property that σ, say, is played if the state is H, and σ_L, say, is played if the state is L, and the beliefs are such that no deviation from these strategy prescription is profitable. It is well known that application of the elimination of dominated strategies implies that at a separating equilibrium $\sigma_L = (p_L^m, 0)$, and we shall not repeat the argument here (see Milgrom and Roberts 1986). This implies that there are two necessary conditions for separation, they are respectively:

$$(i) \qquad (1 + v - p)p - a \leq v, \tag{2.3}$$

and

$$(ii) \qquad (1 + v - p)(p - c) - a \geq v - c. \tag{2.4}$$

Condition (i) states that a strategy σ can be part of a separating equilibrium only if the monopolist has no incentive to play σ if its quality is low. Condition

(ii) states that a strategy σ can be part of a separating equilibrium only if the monopolist has no incentive to deviate from the play of σ to the play of $(v, 0)$ if the quality is high. Note that, as Milgrom and Roberts[1986] show, although many other deviations are possible from the equilibrium prescription, the two deviations here stated are generally regarded as crucial. In fact it is generally the case that one can find a system of beliefs that supports the play of σ in state *(H)* and of $(v, 0)$ in state *(L)* if σ satisfies the two conditions (i) and (ii).

The characterization of σ is usually the focus of the analysis, since it characterizes the type of signal used. In general σ is not uniquely identified by the constraints, while these rather identify a set of regions in the space of couples (p, a) to which σ must belong. In this respect the following result indicates that the use of advertising is not essential for separation.

Proposition 1: *Under monopoly if there exists a strategy $\sigma = (p, a)$ with $a > 0$, that satisfies constraints (i) and (ii), then there also exists a strategy $\sigma' = (p', 0)$ satisfying the same constraints.*

Proof: See the Appendix.

Note that Proposition-1 can be interpreted as follows: the existence of a separating equilibrium, under monopoly and in a one-shot game, does not impinge on the possibility of advertising; in other terms, the possibility to advertise does not enlarge the spectrum of circumstances under which separation may occur. This however does not mean that advertising can be excluded on some game theoretic ground because to do that one needs use of an equilibrium selection criterion. This is done in Milgrom and Roberts[1986] who show that advertising resists the application of standard refinements criteria only if repeat purchase is assumed, and we do not restate the argument here.

There is an economic reason, however, why advertising is not necessary: it is more costly to signal through advertising than through price. The proof of this statement is simple: define $a(p) = (1 + v - p)p - v$. For any given price this function gives the minimal amount of advertising that is necessary to separate. Then, maximize the profit of the high quality monopolist with respect to price and advertising under the constraint that $a \geq \max[0, a(p)]$. This is equivalent to maximize with respect to p the function $\Pi(p) = (1 + v - p)p - a(p)$ over the interval p in $[0, p^a]$ where p^a is the highest root of $0 = (1 + v - p)p - v$. Since $a(p)$ is monotone decreasing in p, and since $\Pi'(p) > 0$ the profit so written is increasing in price, so that a maximum obtains at p^a with $a(p^a) = 0$.

Saying it differently, consider again the separation constraint (i) above. Since advertising adds to the costs of a high quality monopoly as much as it adds to those of a low quality it is relatively easier to imitate than a policy of high price: the latter is more costly in terms of lost profit (unit margin times demand) for a monopoly of low quality, who produces at a low marginal cost, than for one of high quality, who produces at a high marginal cost.

3. Duopoly

3.1. The structure of competition under full information

Before the study of the signaling game described in the Introduction above, it is necessary to briefly present the full information results of competition in duopoly. Assume that the cost conditions are the same as for the monopoly case, again without loss of generality, assume that $c_L = 0$. Let the buyer's utility be described by the expressions (2.1) and (2.2) above. The consumer that is indifferent between buying quality H or L has taste parameter $m(p_H, p_L) = p_H - p_L$, if $0 < p_H - p_L < 1$. Then it is easy to show that the demand to the high quality firm is the continuous function $d_H(p_H, p_L)$ described by

$$d_H(p_H, p_L) = \min\left\{1 - p_H + \min[p_L, v], 1\right\}, \quad if \quad p_H < 1 + \min[p_L, v] \tag{3.1}$$

$d_H(p_H, p_L) = 0$ otherwise.

Note here that for any value of p_L, $d_H(p_H, p_L) = 0$ for $p_H \geq 2$.

Similarly, demand to firm of type L is given by

$$d_L(p_H, p_L) = \min\left\{p_H - p_L, 1\right\}, \quad if \quad p_L < p_H, \tag{3.2}$$

and $d_L(p_H, p_L) = 0$ otherwise.

The two demand functions are depicted in Figure 1.

The reaction function for firm H is described by the function

$$\hat{p}_H = (1/2)(1 + p_L + c), \quad if \quad \max[c-1, 0] \leq p_L \leq v, \tag{3.3}$$

if by contrast $p > v$, then firm L has zero demand and firm H reaction function consists of its monopoly price, that is

$$\hat{p}_H = (1/2)(1 + v + c), \quad if \quad p_L > v.$$

Finally, if firm L quotes a price lower than $c - 1$ then firm H cannot quote a price equal to $(1/2)(1 + p_L + c)$ as this is lower than its marginal cost, therefore we set $\hat{p}_H = c$ for $c - 1 \leq p_L$.

Similarly, the reaction function for firm L is

$$\hat{p}_L = \frac{p_H}{2}, \quad if \quad \frac{p_H}{2} < v, \tag{3.4}$$

and it is $\hat{p}_L = v$ otherwise. A Nash equilibrium in prices under full information, with both firms enjoying a positive market share[5], and positive profits occurs if the two reaction functions cross at the point (p_L^*, p_H^*) given by

$$(p_L^*, p_H^*) = \left(\frac{2(1+c)}{3}, \frac{(1+c)}{3}\right). \tag{3.5}$$

When the crossing of the two reaction functions occurs at this point both firms enjoy a positive market share (see Figure 2 for an example). This is possible if and only if c and v are in the set $\mathcal{S}$ defined by $\mathcal{S} = \{(c,v) \mid c \in [0,2], v \geq (1+c)/3\}$ [6]. It is henceforth assumed:

Assumption 1 : $(c,v) \in \mathcal{S}$.

3.2. Necessary Conditions

To analyze the duopoly case we shall use a methodology that parallels as close as possible the one traditionally used for the monopoly. First the necessary conditions for separation shall be determined, then a characterization of the separating equilibria is derived, and finally (in an Appendix) the existence of separating equilibria is demonstrated.

Unlike in the monopoly case the consumers observe two couples of price-advertising strategies, one for each firm. There are four states of Nature: *(L,L)* designs the state where both firm are of type L, *(L,H)* and *(H,L)* design the states where one firm is H and the other is L, *(H,H)* designs the state where both are of type H. Let ω denote the set of states that are admissible, here let therefore be $\omega = \{(L,L),(L,H),(H,L),(H,H)\}$. At the first stage of the game Nature chooses a state and only the two firms observe Nature's move, they then simultaneously choose, at the second stage, their strategies (σ_i, σ_j) . Finally, at the third stage, consumers form beliefs and purchase from one or the other firm.

There are at least two equivalent ways of representing beliefs. The first assigns to any strategy profile (σ_i, σ_j) played by the two firms, i and j, a four dimensional vector of probabilities, one for each state. The second assigns to each strategy profile an ordered couple $(b_i(\sigma_i, \sigma_j), b_j(\sigma_j, \sigma_i))$ that represents the probabilities

[5]We are not interested to equilibria where only one firm survives, for this reason also we have not introduced fixed costs in the cost functions. Note that in the example here treated the low quality firm at equilibrium always enjoys a positive market share, while the high quality firm could be out of the market if its cost is too high. It is this situation that we rule out.

[6]The price p_L^* is an equilibrium price if and only if $p_L^* \equiv \frac{(1+c)}{3} \leq v$. The price of firm H is higher than c if $c < \frac{2(1+c)}{3}$, i.e. if $c < 2$. Furthermore, since the market demand to firm H at prices (p_L^*, p_H^*) is equal to $(1 - ((1+c)/3))$, the condition $c < 2$ also guarantees that this be positive. Whence the definition of the set S in the text.

attributed to the event that firms i and j be the high quality firm. The equivalence of the two representations is self evident. For our convenience we adopt the second.

At first sight the task of defining beliefs as a function from couples of strategies (σ_i, σ_j) (four-dimensional vectors) to couples of probabilities $(b_i(\sigma_i, \sigma_j), b_j(\sigma_i, \sigma_j))$ is formidable. Luckily it is possible to obtain results that are belief-independent, in the sense that they apply to any system of beliefs that sustain a separating equilibrium. This exempts us from the task of justifying the choice of a particular structure of beliefs for the results that shall be obtained in the present Section.

Let us transform the notation slightly so that $b_i(\sigma_i \mid \sigma_j)$ denotes the probability assigned to the event that firm i is H when this firm plays σ_i against the play of σ_j by firm j. The notation makes it clear that firms are symmetric in the sense that for any σ, one has $b_i(\sigma \mid \sigma) = b_j(\sigma \mid \sigma)$, i.e. that firms are assigned the same belief when they play identical strategies[7]. Even if this is an obvious property we state it formally for convenience:

Property 1*: If the firms adopt identical strategies then the consumers beliefs assign to either firm an identical probability of being of high quality.*

A (fully) separating equilibrium is one where: (i) beliefs assign a unique vector $(b_i(\sigma_i \mid \sigma_j), b_j(\sigma_i \mid \sigma_j))$ to each strategy profile; (ii) the firms play pure strategies that lead to a different action profile in each state; (iii) beliefs are consistent with the firms' strategies[8]. Let us proceed to the analysis of the strategies (σ_H, σ_L), that must be played *at a separating equilibrium* when the realized state is *(H,L)* or *(L,H)*—the strategies that are played at the other states of Nature do not affect the characteristics of (σ_H, σ_L) that are discussed here.

(A) First, it must be a property of any separating equilibrium of the game that when the state is *(L,H)* or *(H,L)* the firm of type L must not find it convenient to deviate, from the action σ_L that the equilibrium prescribes, to the use of the action σ_H that is prescribed for the type-H firm.

(B) Second, it must be a property of any separating equilibrium that when the state is *(L,H)* or *(H,L)* the firm of type H has no incentive to deviate from the equilibrium prescription σ_H and use the strategy σ_L that is prescribed to firm L.

[7] A more stringent symmetry requirement is that $b_i(\sigma' \mid \sigma'') = b_j(\sigma' \mid \sigma'')$. Although this restriction is quite natural it is not necessary for the analysis here performed, ad it is therefore not used.

[8] Note that point (i) in the definition excludes partially separating equilibria, i.e. equilibria where some but not all states are pooled. We are not interested in partial separation here.

Note that since the deviation in A induces symmetric beliefs $b(\sigma_H \mid \sigma_H)$, it "jams" the signal sent by firm H. But it does *not* necessarily imply that L is perceived as H by consumers: whether on or out the equilibrium path, in fact, observing σ_H at both firms may not lead to the belief that with probability one the state is (H, H).

These two properties lead, taken in turn, to two necessary conditions on the equilibrium strategies.

For $k, j \in \{H, L\}$, let $\pi_j(\sigma_j, \sigma_k; b(\sigma_j \mid \sigma_k))$ denote the payoff to firm of type j when (i) playing strategy σ_j against strategy σ_k played by its rival of type k, and (ii) given the induced belief on its type $b(\sigma_j \mid \sigma_k)$.

Necessary condition A, which may be called *"no-jamming condition"*[9] writes as:

$$\text{(A)} \qquad \pi_L(\sigma_H, \sigma_H, b(\sigma_H \mid \sigma_H)) \leq \pi_L(\sigma_L \mid \sigma_H; 0).$$

Necessary condition (B) writes as

$$\text{(B)} \qquad \pi_H(\sigma_L, \sigma_L, b(\sigma_L \mid \sigma_L) \leq \pi_H(\sigma_H \mid \sigma_L; 1).$$

Recall again that the analysis is, for the time being, concerned with the properties of the strategies played when the state is *(L,H)* or *(H,L)* in a separating equilibrium. These two conditions are necessary, for *any* given belief system implying particular values of $b(\sigma_H \mid \sigma_H)$ and $b(\sigma_L \mid \sigma_L)$. If it is possible to show that the choice of these two values is irrelevant for the following results then the last influence of beliefs on the analysis of necessary conditions (A) and (B) is removed[10].

3.3. Necessary conditions and signaling

The plan of the argument in this subsection is the following:

1) We show that condition (A) is most favorable to price-alone strategies when $b(\sigma_H \mid \sigma_H) = 0$.
2) We assume $b(\sigma_H \mid \sigma_H) = 0$ and show that with price-alone strategies separation cannot always obtain.

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[9]The term signal jamming is used by Fudenberg and Tirole 1986, in a context where an informed player may prevent another uninformed player from observing a signal .

[10]Note that since σ_H and σ_L are by definition part of a separating equilibrium they reveal the state *(H,L)* or *(L,H)* and there is no choice but 0 or 1, as it applies, for the R.H.S in the inequalities in (A) and (B).

3) We analyze condition (B) and show that when separation fails through price-alone strategies it can be obtained with a positive level of advertising expenditures.

To begin with, let define a strategy for a firm as a function that describes its choice of action for each given state of nature—firms move simultaneously after the state is chosen by Nature. An action is a couple (p, a) constituted by a price and a level of fixed cost advertising, a, with $a \geq 0$. First, it is useful to state the following result.

Lemma 1: Independently of the consumers' system of beliefs, at a separating equilibrium of the game firm L in the state (H,L) plays $\sigma_L = (\hat{p}_L, 0)$.

Proof: see Appendix 1.

Obviously this result greatly simplifies the analysis. The strategy played by firm L in state *(L,H)* is, in the duopoly example here considered, $\hat{p}_L = p_H/2$ and zero advertising, or, in our notation, $\sigma_L = \left(\frac{p_H}{2}, 0\right)$.

Let us analyze the implications of condition (A) for our duopoly example.

If firm L imitates the price and advertising σ used by firm H then consumers will form beliefs $b(\sigma_H \mid \sigma_H)$. It is then crucial to determine the role of $b(\sigma_H \mid \sigma_H)$ for the characterization of the separating strategies and to choose the value for it that does not interfere with the results.

First, given $b(\sigma_H \mid \sigma_H)$, then the consumer's utility from buying when both firms use the same strategy is given by $b(\sigma_H \mid \sigma_H)u_H + (1 - b(\sigma_H \mid \sigma_H))u_L$; that is, shortening the notation, $b(v + m - p) + (1 - b)(v - p)$. This implies that the consumer indifferent between buying and not buying is $m^b = (p - v)/b$. So that total demand is $D_b = 1 + v/b - p/b$ if $p > v$ and it is $D_b = 1$ if $p < v$[11].

Second, going back to condition A (no-jamming), for a couple of price and advertising (p_H, a), used by firm H , if $p_H < v$, then all consumers buy one unit, and demand to each firm is 1/2, so that if it plays σ_H firm L has profit $\pi_L(\sigma_H, \sigma_H, b(\sigma_H \mid \sigma_H)) = \frac{p_H}{2} - a$. If it plays $\hat{p}_L\ (p_H) = \frac{p_H}{2}$ (and zero advertising) it makes profit $\pi_L(\sigma_L \mid \sigma_H; 0) = \left(\frac{p_H}{2}\right)^2$. In the case $p_H < v$, then the value of p_H and of a that satisfy condition A must satisfy

$$\frac{p_H}{2} - a \leq \left(\frac{p_H}{2}\right)^2, \tag{3.6}$$

Note that for $p_H > v$ condition A does not write as in (3.6) above, because demand is not inelastic to price as not all consumers necessarily buy. The L.H.S.

[11] Note that the demand D_b corresponds to demand D_H of Section 2, when $b = 1$, and to D_L when $b = 0$. While its elastic part rotates downward as b is decreased form 1 to zero.

term is then equal to $D_b \cdot \frac{p_H}{2} - a$, where b represents the beliefs held by consumers. Then for $p_H > v$ the condition writes as

$$(1 + \frac{v - p_H}{b})\frac{p_H}{2} - a \leq \left(\frac{p_H}{2}\right)^2 \tag{3.7}$$

Conditions (3.6) and (3.7) when combined define a continuous function $a(p_H)$, which is concave shaped and it has a kink in correspondence of $p_H = v$, as depicted in Figure 3. The graph of the function to the right of v is shifted, with its pivot in the point with coordinates $(v, a(v))$, in the right direction as b increases from 0 to 1. For $b = 0$ it coincides with the horizontal axis[12], in fact, $p^b = v$ for $b = 0$ and $p^b > v$ for $b > 0$.

The function $a(p_H)$ delimits the lower bound for a in a separating strategy σ_H for any given level of p_H.

Third, and finally, note that the range of prices for which a high price and no advertising strategy can separate is the interval of prices at the right of the intersection of the curve $a(p_H)$ with the price axis. Since this intersection moves rightward as $b(\sigma_H \mid \sigma_H)$ is increased, the range of prices that can separate without advertising is larger the lower is the value of $b(\sigma_H \mid \sigma_H)$. To get the desired result, therefore we shall set $b(\sigma_H \mid \sigma_H) = 0$ so as to "consider the less favorable case for the necessity of advertising".

Assumption 2: $b(\sigma_H \mid \sigma_H) = 0$.

Note that under this assumption the function $a(p_H)$ depends on the value of $b(\sigma_H \mid \sigma_H)$ only if $v < 2$, in which case it look as in the right graph depicted in Figure 3, in the opposite case in fact there is no kink and $a(p_H) = (p_H/2) - (p_H/2)^2$, as depicted in Figure 4 below.

A *first* condition for a price-alone strategy to work as a signal is that the price is higher than v (and in particular higher than p^b if violating Assumption 2 one had $b > 0$). Notice incidentally that such a price may be higher than the full information price, in particular it must be higher than that price if $v > p_H^* = 2(1+c)/3$.

Simply looking at Figure 3 it is clear that if v is larger than 2 then a price-alone strategy implies $p_H > 2$ and therefore it yields zero demand to firm H. Then we can state

[12]Formally, let $a_s(p_H) = \frac{p_H}{2} - \left(\frac{p_H}{2}\right)^2$, for $p_H < v$, and let $a_b(p_H) = p_H\left(\frac{b+v}{2b}\right) + p_H^2\left(\frac{2+b}{4b}\right)$ for $p_H \in [v, p^b]$, where $p^b = (2b+2v)/(2+b)$ is the highest root of $a_b(p_H)$. Then, recalling that $d_H = 0$ for $p_H \geq 2$,

$$a(p_H) = a_s(p_H) \qquad if \quad p_H < v,$$

$$a(p_H) = a_b(p_H), \qquad if \quad p_H \in [v, p^b],$$

and $a(p_H) = 0$ if $p > p^b$.

Proposition 2: If $v > 2$ then no strategy with zero advertising can achieve separation.

In fact if $v > 2$ the only prices that can signal firm H in the state *(H,L)* are higher than 2, but then this firm receives zero demand as stated in Subsection 3.1 above, and the equilibrium. The fact that a high value of v destroys the equilibria where price-alone strategies are used can be interpreted economically. We leave this discussion however for the concluding Section.

It is now possible to ask whether with $v > 2$ advertising can separate where price-alone strategies cannot. Let us turn now for this purpose to the implications of condition (B).

Note that when firm L plays according to its best reply, $\hat{p}_L = p_H/2$, then the demand to firm H under full information is given by

$$d_H(p_H, \hat{p}_L) = 1 - p_H/2. \tag{3.8}$$

so that

$$\pi_H(\sigma_H \mid \sigma_L; 1) = \ (1 - p_H/2)p_H.$$

Note to start that the demand to firm H (and to firm L) in state *(H,L)* if H plays the same strategy as firm L, should in general be determined by the beliefs held by consumers when the couple (σ_L, σ_L) is observed. However, in the case here analyzed the matter is simplified by Lemma 1: since L plays according to its full information best response to p_H, it is never the case that $p_L > v$. If both firms play p_L, therefore, all consumers buy even if they think that the state is *(L,L)*, so that the market is entirely covered for any belief $b(\sigma_L \mid \sigma_L)$. Therefore, again, beliefs are irrelevant here, and the tie is broken as it is customary by assuming that demand is split in equal parts[13].

Condition (B) describes a function, $A(p_H)$, that defines the maximal amounts of advertising consistent with separation. This function looks generically as a quasi-concave function with a kink at the point where $p_H = 2c$, (see discussion

[13] In a model where $D_L(p)$ has no vertical sections, but it is downward sloping wherever it is positive valued, then the assumption that $b(\sigma_L, \sigma_L) = 0$ would play the role that the assumption $b(\sigma_H, \sigma_H) = 0$ plays for condition (A). The calculations would only become less straightforward while the main conclusions would not be altered.

of Figure 4 below) and it is defined by[14]

$$A(p_H) = (p_H - c)\left(1 - \frac{p_H}{2}\right) = a_y(p_H), \tag{3.10}$$

for $p_H < 2c$, and by

$$A(p_H) = (p_H - c)\left(1 - \frac{p_H}{2}\right) - \frac{1}{2}\left(\frac{p_H}{2} - c\right) = a_z(p_H), \tag{3.11}$$

for $p_H > 2c$.

Note that $a_z = a_y$ at $p_H = 2c$ so that $A(p_H)$ is continuous[15]. Furthermore $2c$ is also a maximum of $A(p_H)$ if $p_H > 0.5$. While for $c < 0.5$ the maximum is at the right of $p_H = 2c$ and it coincides with the maximum of $a_z(p_H)$.

The interplay of the curves $a(p_H)$ and $A(p_H)$ determines the regions in the space of (p_H, a) couples to which the separating strategy σ_H, in state *(H,L)* or *(L,H)*, must necessarily belong if a separating equilibrium of the game exists.

Note, to start, that if $v < 2$ strategies involving a zero amount of advertising and that satisfy the constraint derived by condition (A) always exist. If and only if $A(p_H)$ intersects the horizontal axis to the right of v[16], however, there are price-alone strategies that also satisfy constraint (B).

For the purposes of the present Section—namely to demonstrate that the possibility of advertising enlarges the set of parameter ranges for which separation may occur over and above the range for which separation may obtain through prices only—it is sufficient to show that, when $v > \min\{z(c), 2\}$, there exist ranges for the cost parameter c for which a strategy σ_H with strictly positive advertising satisfies necessary conditions (A) and (B).

Proposition 3. For $c < 1$ and $v > \min\{z(c), 2\}$ the only non-trivial separating equilibria that may exist involve a strictly positive level of advertising in the states (L,H) and (H,L). Furthermore, for c<1/2 the high quality firm has a strictly positive profit, while for c≥1/2 its profits are null but its market share positive.

[14]The profit to firm H if the equilibrium strategies are played is

$$(p - c)d_H(p_H, p_H/2) - a = (p - c)(1 - p_H/2) - a.$$

Then condition B writes as

$$(p_H - c)\left(1 - \frac{p_H}{2}\right) - a \geq \max\left\{\frac{1}{2}\left(\frac{p_H}{2} - c\right), 0\right\}. \tag{3.9}$$

The right hand term is zero if $p_H < 2c$, that is when firm H would make negative profits if it deviated to the price of its rival, but just needs to make non-negative profits.

[15]The function A coincides with the $Min[a_z, a_y]$ over the relevant range and so it can be more easily retained.

[16]The condition is that the largest root of $a_z(p_H)$, denoted $z(c) = \frac{1}{4}(3 + 2c + \sqrt{9 - 4c + c^2})$ is larger than v.(Note that for all $c > 0$, $z(c) > c$ and that the roots of $a_y(c)$ are c and 2).

As an informal proof for the case $c < 0.5$ consider the graph represented in the left part of Figure 4, which has been drawn for a value of $v > 2$, and of $c = 0.25$: the curve $a(p)$ is recognized as starting from the origin, and $A(p)$ starts from the point c, where $a_y(p_H) = 0$. The two constraints there cross at the point where $p_H = 2c$, and at the point where $p_H = 1$. It can be shown that for $c < 1/2$ these crossings occur in the same way as depicted in that graph[17], i.e. one has that for p_H in the range $[2c, 1]$ the inequality $A(p) > a$ holds, while outside that range the opposite inequality holds (the right hand graph in Figure 4 has been drawn for a value of $c = 0$, the valid region shrinks in size as c is increased form zero to 0.5). It follows that in general, for $c < 1/2$, the region to which a separating strategy σ_H must belong is formed by (i) a non-empty region where advertising is strictly positive; (ii) a (possibly empty) region consisting of all points on the abscissa lying in the interval $[v, z(c)]$. This proves the first part of the Proposition.
(To prove the part of the proposition regarding the case $c > 1/2$ consider Figure 5, which has been drawn under the hypothesis that $c = 0.7$. It is apparent from Figure 5 that if $v > 2$, then a separating equilibrium if it exists is given by the point of tangency of $A(p_H)$ and $a(p_H)$. The same situation arises for all values of $c \geq 0.5$, because for these values one can easily check that $A(p_H)$ has a unique maximum at the point where $p_H = 2c$, and that at $p_H = 2c$ one has $a(p_H) = A(p_H)$, so that the equilibrium σ_H is unique in these cases[18]). $\square$

We will finally note that a price couple equal to the full information prices for the high and low quality may be used at a separating equilibrium if the high quality firm accompanies the full information price by an adequate expenditure in advertising. For instance, for $c = 0.25$ one has that $p_H^* = 5/6$ and $a = a_s(5/6)$ may constitute a separating strategy σ_H for H.

The important feature of the results so far obtained is that they, in the same line as for the monopoly case, establish a *characterization* of equilibria, preceding a complete specification of beliefs.

Existence is discussed in Appendix 2.

3.4. Persistence

As it is clear, the plausibility that advertising must be used in a one-shot set-up, implies that it can be used also in situations where repeat purchase is not present. For instance in the case of the restaurants quoted in the Introduction. The luxurious setting of one of two restaurants can be interpreted as an ostentation

[17]In particular, $p_H = 2c$ is the point where a_s, a_y, a_z, all cross; $p_H = 1$ is always one of the roots of the equation $a_s = a_z$, and it is the largest root for $c < 1/2$.

[18]When $c = 1$ then $2c = 2$ and the point of tangency of the two constraints implies that $p_H = 2$. For $c > 1$ the two constraints do not touch any more since $A(p_H) < 0$ for $c > 1$, so that the separating equilibria with advertising do not exist for these values of c.

of "waste" that persists over time as it must signal to new consumers at each period. Similarly, the repetition of promotion campaigns on television by some brands may have the same function. Furthermore, as it shall be shown below persistence can occur when renown brands may be induced to advertise in order to deter the entry of other firms.

4. Advertising as an Entry Barrier

Assume there is a third firm in addition to the two previously considered. This firm is a producer of lowest quality, in the sense that it can be one of two types: L or 0, but not of type H. If a buyer purchases the good 0 at price p_0 she gets a utility $u_0 = -p_0$. To simplify matters assume that a flop can never be mistaken for a high quality product, but it is indistinguishable from a low quality product, an assumption that could be relaxed in a more complete version of this game.

To keep with the approach of short term interaction, we assume that the entry, price, and advertising decisions are all simultaneously taken. If we had assumed repeat purchase, and a multi-period framework, it would be possible to modify the example here in order to consider the behavior of incumbents against potential entrants.

In this game it is common knowledge that there can be such a producer, called a "flyer-by-the-night", in the sense that everybody knows that if three firms are on the market one could be a flop and must leave the market after having deceived consumers once. We shall concentrate on the asymmetric states *(H,L,0)* and those obtained permuting the order of the entries. Since quality H cannot by hypothesis be imitated by the flop, in these states the flop must imitate the low quality if it wants to enter. Then if the state *(H,L,0)* realizes firm L has to prevent the flop from entering[19].

If the type 0 firm enters and replicates the strategy of firm L then consumers cannot distinguish which firm is L and which is the flop. Firm H, by contrast may separate from firm L using a strategy that can be described analogously to what has been done above so as to prevent firm L from imitating—for brevity we do not re-state the conditions (A) and (B) for this game.

Note that if the flop enters and imitates firm L, then this firm and the flop are both believed to be flops with probability 0.5 and will share the demand addressed to the corresponding expected quality as it shall be briefly shown. The consumers know that there is at most one flop, hence their expected utility from buying from one of the two firms that uses the strategy $\sigma_L = (p_L, a_L)$ is

[19] Under the assumption that a high quality cannot be mistaken with a flop, when the state is *(H,H,0)* the flyer–by-the-night cannot enter. If the state is *(L,L,0)*, under revelation one would observe advertising from both low quality firms, as it is briefly explained in the following footnote. Finally, if the state is *(L,L,L)* all firms play the Bertrand equilibrium with a price equal to marginal cost (here zero) and no advertising.

$u_f = 0.5[v - p_L]$. The consumer indifferent from either firm playing σ_L and firm H is $m^* = p_H - p_L - (0.5)v$. The total demand addressed to the two firms L and *0* is therefore equal to m^*.

Assume then that at state *(L,H,0)* firm L must deter the flop with a (separating) strategy $\sigma_L = (p_L, a_L)$. A necessary condition for this strategy to deter the flyer-by-the-night is that if this enters and plays exactly σ_L it makes a loss.

The no-jamming constraint preventing a flop from entering and imitate firm L is:

$$\frac{p_L}{2}(p_H - p_L - \frac{v}{2}) - a_L \leq 0. \tag{4.1}$$

The R.H.S. in the inequality above is justified by the fact that the full information price and profit of a flop is zero. The L.H.S. is simply the profit of a flop that *enters and imitates* a firm of type L that is playing (p_L, a_L) against a firm H that is playing (p_H, a_H).

If $a_L = 0$ the flyer-by-the-night always finds it profitable to enter so that advertising is necessary for firm L to keep the flop out[20]. Obviously, a necessary condition for the low quality firm to separate from the flop through advertising is that by so doing it makes a profit higher than if it accommodates the entry of the flyer-by-the-night. The firm L profit after accommodation is $\frac{p_L}{2}(p_H - p_L - \frac{v}{2})$. And the price p_L that maximizes this profit is equal to $p^\ell \equiv (1/2)(p_H - v/2)$. A deviation from the entry preventing price p_L that belongs to $\sigma_L = (p_L, a_L)$, to p^ℓ entails a profit equal to $(1/4)(p_H - v/2)^2$. Therefore a condition that $\sigma_L = (p_L, a_L)$ must satisfy is that

$$p_L(p_H - p_L) - a_L > (1/4)(p_H - v/2)^2. \tag{4.2}$$

Finally, the behavior of firm H must assure that firm L does not want to imitate firm H in a duopoly, i.e. that $p_L(p_H - p_L) - a_H > p_L(p_H - p_L) - a_L$(this again is the no-jamming condition).

The lesson from this second example is that the theory here exposed can account for (i) low quality firms advertising together with high quality firms, (2) persistence of both firms advertising over time.

Note that the existence region for high-price and no advertising for firm H shrinks further if the low quality firm advertises. In fact the L.H.S. of all conditions (A) and (B) is lowered by the fixed amount a_L, so that the incentive constraint $a_z(p)$ is shifted upward, and the range of v-values that are larger than the root z_c shrinks accordingly as a_L is increased.

[20] When the state is *(L,L,0)* the two firms of type L can deter the entry of the third firm by using a price-advertising couple (p_{LL}, a_{LL}) satisfying the no-jamming condition $p_{LL}\frac{1}{3} - a_{LL} \leq 0$, which ensures that entry is not profitable. The two type L firms make nonnegative profits provided $p_{LL}\frac{1}{2} - a_{LL} \geq 0$. Clearly, there are many such couples (p_{LL}, a_{LL}), one example is $(p_{LL}, a_{LL}) = (v, \frac{v}{3})$, at which both firms make zero profits.

Finally, assume that the market size increases or decreases over time as in a cyclical behavior of demand. For instance assume that the mass of consumers is M_t where the subscript t denotes a date. If advertising is used as an entry barrier, then it persists; however it must also vary according to demand conditions: the no-jamming constraint preventing a flop to enter and imitate firm L becomes:

$$\frac{p_L}{2}(p_H - p_L - \frac{v}{2})M_t \leq a_L. \tag{4.3}$$

As M_t varies along the cycle so varies the minimum amount of advertising of firm L.

As a conclusion we get that *advertising varies pro-cyclically.*

5. Concluding remarks on advertising, signaling, and price policy

When advertising is used as an entry barrier, since in that case it is clear that a price-alone strategy can always be mimicked by an entrant of lower quality, it arises as a natural equilibrium behavior. When it is just used by a high quality to separate from a low quality its relative merits over a price-alone strategy should be intuitively explained as they seem to apply for the duopoly and not for the monopoly case.

Note first that to obtain separation when one firm is of high quality and the other is of low quality the former must discourage imitation by visibly wasting some resources. This can be done by an abnormal increase (for the case of a decrease in price see comments below) in price, or by adding the fixed costs of an advertising campaign. A high price, however is painful in terms of lost customers that switch to the rival supplier. This effect is larger the more elastic is the firm's demand to the price difference. For high enough prices if quality does not command a high premium the high quality is out of the market and cannot separate. The quality premium for firm H in our model is higher (in percentage terms) the lower is the parameter v. A high v induces a low percentage difference in utility $(u_H - u_L)/u_L = (m - p_H + p_L)/(v - p_L)$. This accounts in part for the difficulty to obtain separation through a high price and no advertising when v is high. By using advertising and an appropriate price together, the high quality firm may succeed in discouraging imitation without raising its price too much above that of the rival.

Second, the price of the low quality firm at a separating equilibrium is a decreasing function of the price of the high quality firm (in monopoly this is not so). This means that the lower the price that is used by the latter for separation, the lower the margins that it can enjoy by imitating the low quality firm. This makes it easier for the high quality to be discouraged from mimicking the low quality. Also in this respect the use of advertising, since it allows a reduction in the signaling price of the high quality firm renders the separation easier.

More generally, the importance of a low price in a competitive framework is quite obvious, while it is not so in the context of a monopoly. By focussing on the single-firm problem some authors have stressed the importance of low introductory prices as a signaling policy (see Schmalensee[1978]), while others have attributed more importance to high-price policies as signaling devices (see Bagwell and Riordan[1991]). The explanation we use here does not conflict with either those views of low (or of high) introductory prices as signals, although we stress the importance of advertising components of a signaling strategy that *allow* a price policy to be a signal[21]. In our example a price higher than the full information price—when revelation is spontaneous the full information price itself—may signal without advertising. But there are situations where advertising is *necessary* for prices higher than, equal to, or lower than the full information price to separate.

The old idea that the entry of new firms may be deterred through advertising finds its counterpart in our model in the idea that signaling through advertising allows a lower price than signaling through price alone and reduces the market share for the rivals. The low quality firm by advertising can use a price low enough to discourage entry of a firm with a still lower quality, while without advertising the same price cannot deter the entrants.

This type of advertising is most likely to continue over time and to vary in a pro-cyclical fashion, as it is directed not so much to separate one incumbent from another incumbent but to discourage, through time, the mimicking from potential entrants.

[21] The signaling role of prices is also the focus of Wolinski [1983] and Cooper and Ross [1984]. In these works, however, some consumers are informed about product qualities or can acquire information at a cost (i.e. the product is a search good). The question there studied is then the extent to which equilibrium prices transmit information from informed to uninformed agents. It is high prices that have received the most attention in those works.

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6. Appendix 1

Proof of Proposition 1

Proof: Suppose that (p, a) *satisfies both (i) and (ii). Suppose further that* $(p, 0)$ *does not satisfy either one or both (i) and (ii). Since* (p, a) *satisfies (ii)* $(1 + v - p)(p - c) - a - v + c \geq 0$ *then, a fortiori* $(1 + v - p)(p - c) - v + c \geq 0$. *The lowest value of* a *that can be associated to* p *is therefore given by (i) and is* $(1 + v - p)p - v \equiv a^+$. *Then, assume* $\sigma = (p, a^+)$. *By continuity of the function* $(1 + v - p)p$ *there exists a price* $p' > p$ *such that* $(1 + v - p')p' = v$. *Hence assume the strategy* $\sigma' = (p', 0)$, *which verifies constraint (i), is used instead of* σ. *Constraint (ii) for* σ' *writes as* $(1 + v - p')(p' - c) - v + c \geq 0$. *Since* σ *satisfies (ii), for this inequality to be verified, it is sufficient that* $(1 + v - p')(p' - c) - v + c \geq (1 + v - p)(p - c) - v + c - a^+$. *But this writes as* $(1+v-p')p' - c(1+v-p') - v \geq -(1+v-p)c$ *or* $(1+v-p') \leq (1+v-p)$, *which is true since* $p' > p$ *and since both expressions in parentheses are non-negative. The argument applies also for all strategies (p,a) with* $a > v^+$ *and it is therefore complete.* $\square$

Proof of Lemma 1:

> *Proof: Suppose that at an equilibrium firm L plays an action* σ' *different from* σ_L *when the state is (H,L). By definition of a Separating Equilibrium firm L is perceived as L and its opponent as H. However, by deviating to strategy* σ_L, *if it is still perceived as L with probability one it will play according to its best reply against* σ_H, *and if it is perceived as L with probability less than one, it will receive a higher demand than if it is with probability one, and its profit will increase, therefore, in either case there is no strategy available to firm L that dominates* σ_L. $\square$

7. Appendix 2: Existence

The necessary conditions for the existence of a separating equilibrium are sufficient to characterize the nature of the signal used, but are not sufficient to guarantee existence in general. Recall furthermore that the analysis has been so far concerned only with the actions prescribed in states *(H,L)* and *(L,H)*, and the other actions have been left unspecified.

The present Section fills this gap and shows that existence is guaranteed, in one of the simplest belief systems that can be envisaged, if only an additional condition is imposed upon σ_H. Of course the equilibria here found satisfy (as all equilibria must) the characterization in Section 3 above based upon conditions (A) and (B). To eliminate the influence of the value taken by $b(\sigma_H \mid \sigma_H)$ upon the result that advertising may be necessary, we assume that $b(\sigma_H \mid \sigma_H) = 0$, so as to render as large as possible the set of situations in which a price alone strategy of the type $\sigma_H = (p, 0)$ can work as a signal.

Assume that consumers beliefs have the following structure. (i) If they observe that no firm uses advertising and that prices are equal to c at both firms consumers believe that the state is *(H,H)*. (ii) If only one firm plays σ_H and if this satisfies (A) and (B) above, then they believe that this firm is H and the other is L, *unless* the other firm also plays either σ_H, or $(0,0)$ in which case they believe that state is *(L,L)*. (iii) Any other strategy couple different from those contemplated by (i) and (ii) induces the belief that the state is *(L,L)*.

Formally:

(i) $(b_i(\sigma_i \mid \sigma_j), b_j(\sigma_i \mid \sigma_j)) = (1,1)$ if $(\sigma_i, \sigma_j) = ((c,0),(c,0))$;

(ii) $(b_i(\sigma_i \mid \sigma_j), b_j(\sigma_i \mid \sigma_j)) = (1,0)$ if $(\sigma_i, \sigma_j) = (\sigma_H, x)$ with $x \notin \{\sigma_H, (0,0)\}$ and with σ_H satisfying conditions (A) and (B);

(iii) $(b_i(\sigma_i \mid \sigma_j), b_j(\sigma_i \mid \sigma_j)) = (0,0)$ otherwise.

Note that (iii) encompasses the symmetric couple (σ_H, σ_H) so that these beliefs imply that condition (A) is written with $b(\sigma_H, \sigma_H) = 0$.

It must be shown now that the equilibrium strategy prescribe that: if the state is *(L,L)* then σ_i and σ_j are $(0,0)$ at each firm; If the state is *(H,L)* they are $\sigma_H = (p_H, a)$ by firm H and $\sigma_L = (p_H/2, 0)$ by firm L, with σ_H satisfying condition 3.2 , 3.3 and a condition that shall be defined below; finally, if the state is *(H,H)* they are $(c,0)$ by each firm.

First, if the state is *(H,L)* or *(L,H)* the strategies played must be σ_H, and $(p_H/2, 0)$ respectively. The best deviations from equilibrium under the assumed system of beiefs are σ_H for firm L and $(p_H/2, 0)$ for firm H. But these deviations cannot be profitable since σ_H satisfies conditions (A) and (B) above. This couple of actions is therefore a couple of mutual best replies.

Second, if the state is *(L,L)* then the play of a null price and of no advertising by both firms must constitute a couple of best replies: if a firm deviates to any other strategy with a positive price it shall still be perceived as L, therefore it cannot increase its profit above zero. Therefore the couple $((0,0),(0,0))$ is a couple of mutual best replies.

It remains to be verified whether under the state *(H,H)* the candidate that the beliefs propose, namely the couple $((c,0),(c,0))$ is a pair of mutual best replies. It is immediate to see that the only possible deviation left to either firm by the beliefs is the play of strategy σ_H instead of $(c,0)$. That deviation generates an observation of the kind listed in (ii) in the belief system, and therefore affords the deviating firm with a demand and a profit pertaining to a high quality firm that confronts a low quality opponent quoting price c. This profit is $(p_H - c)(1 + c - p_H) - a$. Therefore since the play of the equilibrium recommendation implies a null profit to both firms, the condition for no deviation to σ_H writes as

$$(p_H - c)(1 + c - p_H) - a \leq 0. \tag{7.1}$$

It follows that a separating equilibrium exists if the three conditions (A), (B) and (7.1) are met by σ_H.

Note for further reference that the crossing of (7.1) and of $a(p)$ occurs exactly for a value of $p_H = 2(1 + c)/3 \equiv p_H^*$, the full information price for firm H. Therefore the full information price may be used as a signal if coupled with an advertising campaign costing $a(p_H^*)$. In fact also some slightly lower prices may belong to the admissible region for σ_H. Recalling form Proposition 2 that when $c < 1/2$ the region for which signaling with a positive advertising exists is non-empty and it implies a price in $[2c, 1]$, one can easily check that condition (7.1) is satisfied for some couples (p_H, a) when $p_H^* \in [2c, 1]$, namely when $c < 2$.

The issue of existence is therefore resolved.

One may note, to complete the discussion, that there can exist many (in general an infinite number of) equilibria when the regions as described in the preceding paragraph are non-empty. While we do not address here the issue of selecting among equilibria, it is interesting to note that solely based on the characterization of equilibria, one can conclude that in some cases equilibria without advertising do not exist.

Of course, like in the case of a monopoly, one can construct belief systems that sustain only pooling equilibria, but since the focus here is on separation and not on the issue of pooling versus separation we do not pursue a detailed analysis here.

The system of beliefs in (i) - (iii) shares a feature that is most general in games with two signal senders, namely that beliefs depend upon what both players do, or, in short, beliefs are "correlated".

It is possible to generate separating equilibria of this game also by using systems with "uncorrelated" beliefs. For instance, beliefs of this type assign probability one of being H to any firm playing a strategy that satisfies some conditions (in particular (A) and (B)), whatever the other player does. Obviously this implies that the observation of a strategy couple like (σ_H, σ_H) implies belief $b(\sigma_H \mid \sigma_H) = 1$. This is valid only if ω comprises the state *(H,H)*. It is possible to show that there exist systems of non correlated beliefs sustaining separating equilibria.

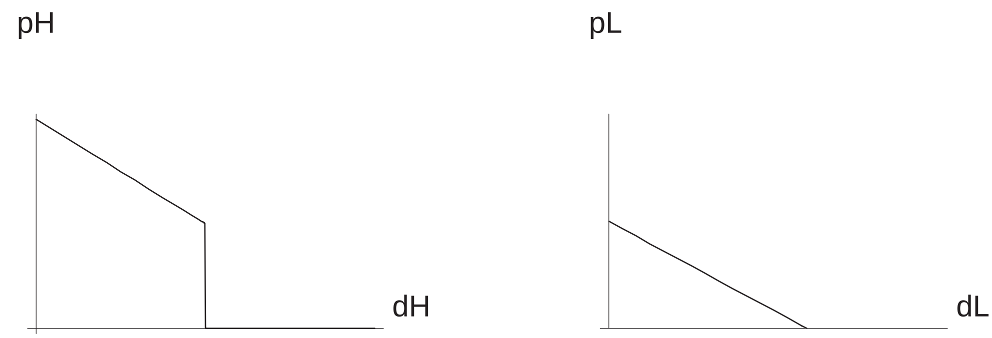

Figure 1

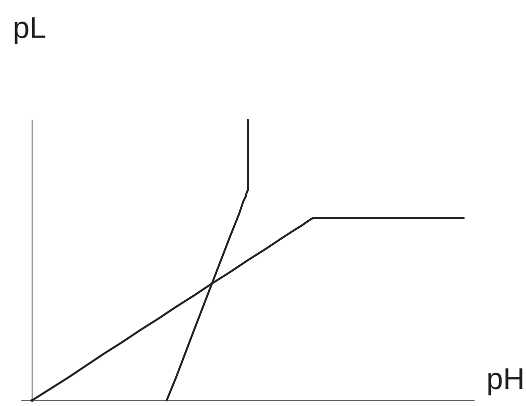

Figure 2

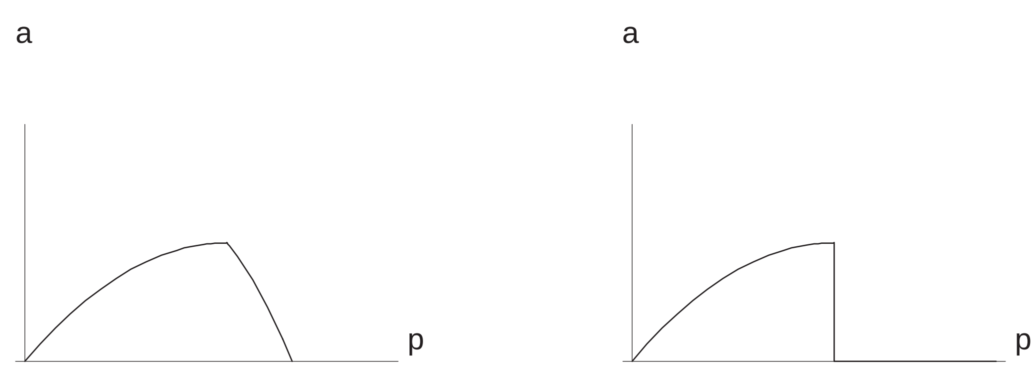

Figure 3

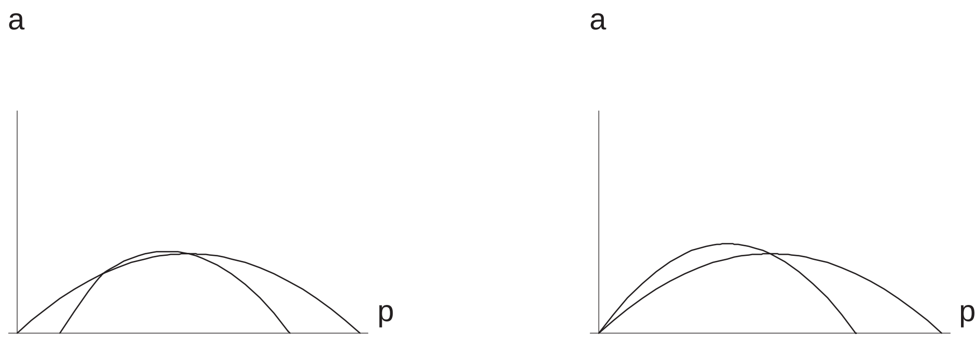

Figure 4

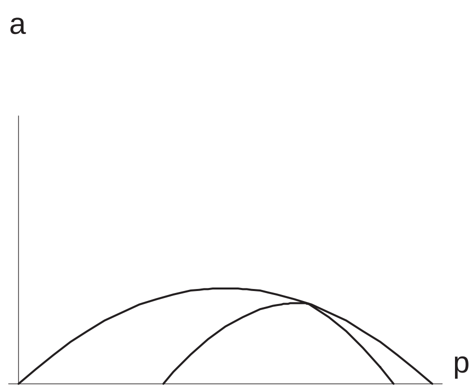

Figure 5