VCO Behavioral Model Based on the Nonlinear-Discrete Convolution Approach

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Abstract — A purely mathematical, behavioral model for Voltage Controlled Oscillators (VCO) is derived. It is based on a Non-Linear Discrete Convolution approach [1] which allows a time-domain formulation compatible with commercial system-level simulators. The approach enables the VCO nonlinear dynamics to be described with great accuracy and computational efficiency, and its influence on the performance of an entire subsystem to be investigated. Model validation, implementation in commercial system-level simulation tools as well as application examples are provided in the paper.

I. INTRODUCTION

The analysis of complex systems (e.g., an RF front-end) is nowadays very important to evaluate the final performance of an entire communication system, in terms, for instance, of the antenna transmitting power or the bit error rate (BER). In this context, however, transistor-level simulation requires a large CPU effort and long simulation time, due to the large number of active and passive components which must be accounted for. To overcome this problem, computationally efficient models are required. Accurate behavioral models of the main building blocks included in the communication system, (i.e., voltage controlled oscillators, power amplifiers, mixers, etc.) play a very important role in this context, since they represent the starting step for reliable system performance prediction with a reasonable amount of computational resources. Behavioral models may also result in a higher compatibility with the digital simulation engines used for the mixed-mode analog-digital co-simulation.

Recently the Nonlinear Integral Approach [3,4] has been successfully adopted by different authors for the system-level, behavioral modeling of different circuits and components [5-9]. This approach, under mild and well justified hypotheses of “short-term-memory” system behavior, takes into account simultaneously the nonlinear and the dynamic effects. In particular, a behavioral model for VCOs has been proposed and validated in [8,9]. However, the formulation adopted in those papers is oriented to Harmonic Balance (HB) steady-state circuit analysis and makes the model not useful for system-level simulation tools based on time domain numerical techniques.

A new type of VCO behavioral model based on the Nonlinear Discrete Convolution approach [1,2] is proposed in this paper. The time domain formulation of the model allows its direct implementation in system-level simulators. In particular, the implementation in the

II. THE NONLINEAR DISCRETE CONVOLUTION VCO MODEL

The difficulties related to the accurate modeling of a VCO behavior, are mainly due to the simultaneous presence of nonlinear and dynamic phenomena. In particular, by considering only the instantaneous frequency, the VCO response to an input control voltage can be generically expressed [4,9] as:

\[ f_{osc}(t) = \psi|v(t - \tau)|^{\frac{1}{\tau}} \quad (1) \]

where \( \psi \) is a nonlinear functional which formally represents the nonlinear dependence of the instantaneous oscillator frequency at the generic instant \( t \) on the present and past values of the control voltage signal over a finite memory time interval \( T_M \). Relation (1) can be conveniently simplified in order to allow a feasible model identification. In fact, the functional description can be linearised with respect to suitably defined “dynamic deviations” \( e(t, \tau) = v(t - \tau) - v(t) \), since these keep small even in the presence of large voltage amplitudes under the justified hypothesis of short memory conditions (i.e., small \( T_M \)) or, in other words, if the duration of the nonlinear memory effects of the VCO are short enough with respect to the time dynamic of the controlling voltage. This implies [4] that \( T_M \ll 1/\omega_{max} \), where \( \omega_{max} \) is the maximum angular frequency where spectral components of the controlling voltage are present with an amplitude large enough to stimulate nonlinear behavior in the circuit. Under these hypotheses, equation (1) can be expressed by means of a modified Volterra approach that leads to the following Nonlinear Integral Model (NIM) [3,4] expression:

\[ f_{osc}(t) = f_{QS}(v(t)) + \int_0^{T_M} s_j(v(t), \tau)(v(t - \tau) - v(t)) \, d\tau \quad (2) \]

In (2) \( f_{QS} \) describes the quasi static characteristic while the second term represents a purely-dynamic, single-fold
convolution integral between the voltage dynamic deviations \( e(t, \tau) = v(t-\tau) - v(\tau) \) and the pulse response function \( s_f(v(t), \tau) \) which is non-linearly controlled by the instantaneous input voltage \( v(t) \).

By considering discrete-spectrum signals, model expression (2) has been rewritten in [8,9] in a HB formulation oriented to steady-state simulation and successfully implemented in HB-based CAD tools. However, in system-level simulation, time domain analysis is also adopted. To implement the NIM model in such a kind of environment, equation (2) can be rewritten [1,2] in a time-domain, system-simulation-oriented expression by dividing the memory time \( [0,T_M] \) of the VCO into a suitable number \( N_D \) of intervals of width \( \Delta \tau \) (so that \( T_M = N_D \Delta \tau \)). This leads to a new formulation of the NIM model (2), the Nonlinear Discrete Convolution (NDC) approach [1,2], where the dynamic deviation is a function of a finite number of points in the \( \tau \)-domain, while the voltage controlled dynamic pulse response is expressed by means of a \( p \) index which "discretises" the convolution integral by a finite summation:

\[
 f_{osc}(t) = f_{qs}(v(t)) + \sum_{p=1}^{N_D} s_{fp}(v(t)) \cdot (v(t-p\Delta \tau) - v(t))
 \]

The NDC formulation (3) allows an easy implementation of the model by directly using basic tools available in commercial CAD environments [10].

III. MODEL IDENTIFICATION AND VALIDATION

The model extraction procedure simply starts from the identification of the function \( f_{qs} \) by performing oscillator steady-state HB analyses as a function of the quiescent control voltage. Once the static response of the VCO has been identified, the weights \( s_{fp}(v(t)) \) of the voltage dynamic deviations in (3) can be computed for any value of the controlling voltage \( v(t)=V_B \) by minimizing the mean square discrepancy between the small-signal complex frequency sensitivity, simulated at transistor level for the original VCO circuit, and the corresponding value predicted by the behavioral model (3). More precisely, linearising and then Fourier transforming (3), the model "fitting" to the transistor-level complex frequency sensitivity \( (F/V) \) can be expressed in the form:

\[
 \frac{F_k}{V_k}_{V_B,\omega_k} = \left. \frac{\partial f_{qs}}{\partial v} \right|_{V_B} + \sum_{p=1}^{N_A} s_{fp}(V_B) e^{-j\omega_k p \Delta \tau} - 1
 \]

The first term in (4) represents the static sensitivity, obtained by simply differentiating the static characteristic \( f_{qs} \) in various bias points. When a suitably wide set of different bias points \( V_{bl} (l = 1,...,N_B) \) and frequencies \( \omega_k \) \((k = 1,...,N_d)\) has been chosen, expression (4) provides a set of \( N_B \times N_d \) complex equations, which impose the congruence between the model small-signal response and the complex frequency sensitivity \( F/V \) computed through transistor-level simulations of the VCO. More precisely, this requires to simulate the circuit response by applying to the voltage controlled oscillator, for every angular frequency, a small-signal sinusoidal voltage represented by the phasor \( V_k \).

It should be noted that equations (4) are linear with respect to the \( N_B \times N_d \) unknowns \( s_{fp}(V_B) \), which are the characteristic parameters of the purely dynamic part of the NDC model. Therefore, the model extraction procedure is based on simple and reliable algorithms for the mean-square solution of an over-determined systems of linear equations.

The behavioral model implemented in this paper corresponds to the MOS differential VCO circuit shown in Fig.1.
As an example of the accuracy of the new behavioral model in predicting the VCO nonlinear dynamics, Fig. 2 shows the mean value of the frequency deviation with respect to the oscillation frequency plotted as a function of the controlling voltage amplitude (a sinusoidal voltage excitation at the frequency of 500kHz was superimposed on a quiescent control voltage of 1.8V).

The agreement between the transistor-level simulation and the NDC nonlinear dynamic behavioral model is very good.

IV. IMPLEMENTATION OF THE NEW MODEL IN AWR VSS

The new VCO model has been implemented in the AWR Visual System Simulator which enables model implementation to be carried out in different ways: by using primitive models and VSS support of hierarchic designs, by incorporating MATLAB models or by creating C/C++ code for the models. In this work the NDC model has been preliminary implemented by means of VSS primitive models. In particular, the static characteristic \( f_{QS} \) and the \( N_D \) “weights” \( s_f \) were simply implemented through look up tables as functions of the controlling voltage. The delays \( p\Delta \tau \) were defined by means of ad hoc operators available in VSS. Moreover, sum and multiplication operators were also used as can be seen in Fig.3, where the sub-schematic used to implement the \( \tilde{p} \)th term of the summation in (3) is shown. In particular, it is possible to see the block used to read the \( s_f \) function from a corresponding look-up-table file controlled by the input control voltage. This function is weighed by the signal difference between the instantaneous input voltage and the same signal delayed of \( \tilde{p}\Delta \tau \) seconds, accordingly to equation (3). A similar structure is used for each term in (3).

For the identification and implementation of the behavioral model for the VCO in Fig.1, a memory time \( T_M = 10 \) nsec and a number \( N_D = 3 \) of time delays have been adopted. The memory time clearly depends on the specific circuit considered and is, approximately, some order of magnitude lower than the reciprocal of the tuning bandwidth. As far as the number \( N_D \) of time delays is concerned, a higher number of discretisation steps (that implies small \( \Delta \tau \)) clearly improves the approximation of the integral in (2), but, at the same time, increases the model complexity and, consequently, the CPU time required for the evaluation of the model response. A value of \( N_D = 3 \) was found to be a good trade-off between model accuracy and complexity.

V APPLICATION EXAMPLES

To evaluate how the nonlinear dynamic effects of a VCO may influence the performance of a more complex system as a PLL, the VCO behavioral model has been inserted in a frequency synthesizer system which consists of a classical scheme (see Fig.4) composed of a charge-pump phase frequency detector, a loop filter, a frequency divider and a reference oscillator. All these components where described through standard models available in the VSS library.

Fig.4: Classical scheme of a PLL described through behavioral models provided in the VSS environment.

The transient response of the PLL system was simulated by considering both a purely static (although nonlinear) VCO model and the new VCO nonlinear dynamic model. The two different settling behaviors are plotted in Fig.5, where the PLL response to a step on the frequency divider ratio after a 1\textmu s initial delay is shown.

Fig.5: Different system-level simulations of PLLs transient responses. The adoption of the nonlinear dynamic VCO model correctly puts in evidence the oscillating PLL settling response which is hidden when a purely static nonlinear VCO model is used.
The response of the PLL including the VCO nonlinear dynamic effects reaches the desired frequency with an oscillating behavior which is not put in evidence using a purely static VCO model. In the latter case, dynamic effects are present only in the loop filter. The “oscillating” behavior is evident especially for “fast” PLLs. The term “fast” relates to the time constants of the loop filter: fast PLL have smaller time constants and faster response. In such a condition the dynamic effects of the VCO are usually important in the simulation of the frequency synthesizer dynamics. These results are coherent with those provided in other papers [11].

The oscillator behavioral model has also been utilized to evaluate how the dynamic effects of a VCO could influence the constellation of a digitally modulated signal and, consequently, the BER performance of a telecommunication system. As an example a direct frequency modulation scheme has been considered where the controlling, modulating voltage of the VCO is a 16-QAM signal. After an FM demodulation of the output signal, the complex envelope has been computed and plotted in a complex plane obtaining the constellation traced in Fig.6. This figure shows the behavior of the system in the presence of a VCO with important dynamic effects: the 16 symbols in the IQ plane are quite scattered around the 16 ideal points corresponding to the case of a VCO having an ideal instantaneous frequency/voltage relationship.

VI. CONCLUSION

A Nonlinear Discrete Convolution behavioral model for VCOs has been proposed. The model, which takes into account both nonlinear and dynamic effects, can be identified through a systematic procedure on the basis of static and dynamic VCO analyses and provides accurate performance prediction under large signal operation. The time-domain formulation of the model makes its implementation in commercial CAD System Simulators very simple. Validation results and application examples based on the model implementation in the AWR VSS simulator have been provided.

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