# EXOGENOUS PRODUCT DIFFERENTIATION AND THE STABILITY OF COLLUSION

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*Abstract*

*The stability of collusion in quantities in a differentiated duopoly is analised, and the result is compared to that emerging in the case of price-setting behaviour. It turns out that quantity collusion is generally better sustained than price collusion, unless products are almost perfect substitutes. Under both quantity and price competition, the social damage associated with collusion is increasing in the degree of substitutability.*

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## 1. Introduction

The issue of cartel stability has received wide attention in the recent oligopoly theory. In this short note, I want to focus on the influence exerted by product differentiation on firms' ability to collude. Within the address approach, this question has been tackled in several papers (Chang, 1991; Jehiel, 1992; Ross, 1992 and Häckner, 1994). Within the non-address approach, the main contributions are those of Deneckere (1983), Majerus (1988), Ross (1992) and finally Rothschild (1992). Ross confines his attention to the interplay between product differentiation and price collusion, showing that the critical discount rate is initially decreasing and then increasing in the degree of substitutability between products. Deneckere, as well as Rothschild, compares cartel stability under price and quantity competition, though resorting to different models. The first author claims that collusion is better sustained in price setting games when substitutability is high, while the reverse holds when substitutability is low; on the contrary, the second author finds that a high degree of substitutability fosters collusion in quantity, and *viceversa*. Finally, Majerus has shown that the latter result does not hold as the number of firms increases. In the following pages, I am going to investigate collusive behaviour in a differentiated quantity-setting duopoly adopting the same modelization as in Ross (1992). On the basis of my own and Ross' results it is possible to claim that quantity collusion is generally more stable than price collusion, except when goods are very close substitutes. This conclusion is in sharp contrast with Rothschild (1992) while it is partially in line with Deneckere (1983).

## 2. The model

The model I adopt relies on a quadratic utility function for the representative consumer, as in Dixit (1979), Singh and Vives (1984) and Ross (1992). Individuals derive utility from the consumption of two substitute goods, $x_1$ and $x_2$, and a numeraire $y$, according to

$$U = ax_1 + ax_2 - \frac{1}{2}(bx_1^2 + 2dx_1x_2 + bx_2^2) + y \qquad (1$$

that, given $p_y$=1, yields the following inverse demand function for good $i$:

$$p_i = a - bx_i - dx_j, \quad i, j = 1, 2, \quad i \neq j. \tag{2}$$

The ratio $r = d/b \in [0,1]$ measures the degree of exogenous differentiation between the two goods. If $r$=0 they are completely independent, while if $r$=1 they are perfect substitutes. Both firms produce at the same constant marginal cost $c$. Thus, the profit function of firm $i$ is the following:

$$\pi_i = (p_i - c)x_i, \tag{3}$$

from which firm $i$'s reaction function in the quantity space can be derived:

$$\frac{\partial \pi_i}{\partial x_i} = a - c - 2bx_i - dx_j = 0. \tag{4}$$

Solving (4), we obtain $x_i^N = (a-c)/(2b+d)$ and $\pi_i^N = b(a-c)^2/(2b+d)^2$ as the optimal quantity and equilibrium profit for firm $i$ in the simultaneous noncooperative duopoly game.

Let us now focus on cartel behaviour. The objective of the cartel is to maximize joint profits $\Pi^C = \pi_i^C + \pi_j^C$. The generic first order condition is:

$$\frac{\partial \Pi^C}{\partial x_i} = a - c - 2bx_i - 2dx_j = 0, \tag{5}$$

from which we can easily obtain firm $i$'s optimal collusive output $x_i^C = (a-c)/2(b+d)$ and then equilibrium profits $\pi_i^C = (a-c)^2/4(b+d)$.

We are finally able to investigate firm $i$'s cheating behaviour, provided firm $j$ sticks to the collusive agreement. By substituting $x_j^C = (a-c)/2(b+d)$ into (4) and solving for $x_i$, we get $x_i^D = (a-c)(2b+d)/4b(b+d)$ as firm $i$'s deviation output, while deviation profits are

$\pi_i^D = [(a-c)(2b+d)]^2/16b(b+d)^2$.

By now we are well acquainted with the notion of collusion stability in an infinitely repeated game, so that I can confine myself to recall that the critical discount rate

$$\rho^* = \frac{\pi^C - \pi^N}{\pi^D - \pi^C} \tag{6}$$

provides a direct measure of cartel stability. Provided that the model is symmetric, index *i* can be dropped. On the basis of the above calculations, the critical discount rate in the Cournot case is the following:

$$\rho_C^* = \frac{4(r+1)}{4(r+1)+r^2}, \tag{7}$$

which is equal to 1 when goods are completely independent ($r$=0), while it is equal to 8/9 when there is perfect substitutability ($r$=1). Furthermore, it is easily checked that in the relevant range of parameters both the first and the second derivatives of $\rho_C^*$ respect to $r$ are non-positive, so that the critical discount factor is decreasing and concave in the degree of substitutability.

The critical discount rate under Bertrand competition (Ross, 1992, pp.7-8) is defined as follows:

$$\rho_B^* = \frac{4(1+r)}{(2-r)^2} \quad \forall r \in [0, 0.73[; \quad \rho_B^* = \frac{r^4}{(2-r)^2(r^2+r-1)} \quad \forall r \in [0.73, 1]. \tag{8}$$

The discontinuity in $\rho_B^*$ is due to the fact that for $r$>0.73 the model ovestates the gain from cheating and violates the constraint that all quantities must be positive. When this constraint is duly accounted for, the second expression in (8) obtains. Thus, under Bertrand behaviour cartel stability initially decreases and then increases as product differentiation shrinks. A comparison between (7) and (8) shows that quantity collusion is generally more stable than price collusion.

Since $\rho_C^* = \rho_B^*$ when $r$=0.96155, the reverse is true only for $r \in ]0.96155, 1]$, i.e., when products are very close substitutes.

As far as social welfare is concerned, Ross (1992, p.8-9) shows that under Bertrand competition the ratio $SW^C/SW^N$ decreases as $r$, i.e., the degree of substitutability, increases. Thus, the larger is product differentiation the less harmful collusion will be. It can be quickly checked that the same holds in the Cournot setting as well. Social welfare is defined as the sum of consumer surplus and industry profits:

$$SW^J = \sum_i \pi_i^J + CS^J, \quad i = 1, 2; \quad J = N, C, \tag{9}$$

which in the two cases under consideration yields:

$$SW^N = \frac{(a-c)^2 (3+r)}{(2+r)^2}; \quad SW^C = \frac{3(a-c)^2}{4(1+r)}. \tag{10}$$

It is straightforward to verify that both $SW^N$ and $SW^C$ are decreasing and convex in $r$. Since the same also holds for the ratio $SW^C/SW^N$, the above claim is thus proved.

## 3. Conclusions

I have investigated the issue of cartel stability in a quantity-setting duopoly where products are characterized by an exogenous degree of substitutability. The comparison between my results and those derived by Ross (1992) for a price-setting duopoly shows that collusion in quantities is more easily sustained than collusion in prices for a wide range of parameters, while the opposite holds only when the degree of substitutability between products is almost complete. This conclusion differs completely with that reached by Rothschild (1992) while it confirms to a certain extent that of Deneckere (1983). Finally, the level of social welfare associated with the cartel increases as product differentiation increases, independently of whether firms set prices or quantities.

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