COURNOT VS STACKELBERG EQUILIBRIA

WITH ENTREPRENEURIAL AND LABOUR MANAGED FIRMS

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Abstract
The issue of equilibrium selection in a duopoly game between a profit maximizing and a labour managed firm is addressed under either price or quantity competition with product differentiation. If firms can choose the timing of moves before competing in the relevant market variable, the Bertrand game yields multiple equilibria, while the Cournot game has a unique subgame perfect equilibrium with the profit maximizing firm in the leader's role and the labour managed firm in the follower's role. Due to a lower total output, the Cournot-Stackelberg equilibrium yields a lower level of social welfare as compared to the simultaneous equilibrium. This reduces the incentive to transform an LM duopoly into a mixed one.

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1. Introduction

A large body of literature deals with the issue of choosing roles in sequential duopoly games. In the context of duopolistic competition between profit maximizing (PM) firms, Gal-Or (1985) and Dowrick (1986) show that, provided firms are symmetric, the slope of their respective reaction functions in the relevant strategic variable, i.e., either price or quantity, determines whether they prefer to act as a leader or a follower. Specifically, both firms would prefer to be the leader (follower) in quantity (price) setting games if reaction functions are downward (upward) sloping, due to the presence of strategic substitutability (complementarity) between goods (see Bulow et al., 1985). The results reached by the above contributions are extended to the case of differentiated products by Gal-Or (1985) and Boyer and Moreaux (1987).

In a recent paper, Okuguchi (1993b) investigates the preferences of labour managed (LM) firms as for the distribution of roles under both Bertrand and Cournot competition and product differentiation, finding out that, in sharp contrast to what happens when only entrepreneurial firms are involved, in the case of a pure LM duopoly, reaction functions are upward sloping regardless of the kind of competition, be that in prices or quantities. Hence, both LM duopolists would prefer to act as a follower, independently of the strategic variable being set.

Even though the comparison between the payoffs accruing to duopolists in simultaneous and sequential games, as well as the conclusions drawn from it, is relevant in itself, it does not provide any answer to the main question, namely, whether firms' preferences would allow for any of the sequential or simultaneous equilibria to endogenously emerge as *the* equilibrium of the underlying game one could envisage, i.e., a game where firms are first required to announce the timing of their respective moves and then proceed to set the relevant variable in order to maximize their own objective function in the basic market game. This issue has been tackled in a very influential paper by Hamilton and Slutsky (1990). They embed simultaneous and sequential play into an extended game with observable delay where players must set both the strategic variable of the basic game and the time to set that variable. The latter process is actually a logical preplay stage which is not observed. If players decide to move at the same time, a simultaneous equilibrium is observed, and *viceversa*. It is noteworthy that the decision to play early rather than at a later stage is not sufficient to yield Stackelberg leadership, since an analogous decision by the rival determines the emergence of a simultaneous Nash equilibrium. Thus, a Stackelberg equilibrium (or sequential play) with one player moving first and the rival second will be the only subgame perfect equilibrium of the extended game if only one of the two possible sequential play outcomes Pareto-dominates the simultaneous play outcome (Hamilton and Slutsky, 1990, Theorem IV, p.37). Otherwise, when both players share the same preferences over the sequence of moves and the follower's payoff dominates that associated with simultaneous play, then both sequential equilibria (as well as a mixed strategy one) are subgame perfect equilibria of the extended game, so that in principle it is impossible to know which of them will be actually observed (Hamilton and Slutsky, 1990, Theorem III, p.36).

Applying the tools provided by Hamilton and Slustky (1990), I want to address a question which so far, to the best of my knowledge, has remained neglected, i.e., which preferences characterize a mixed duopoly game between a profit maximizing and a labour managed firm, and consequently which kind of equilibrium one can expect to obtain in such a game if firms can decide the timing of moves before proceeding to compete in prices or quantities.

The behaviour of LM firms in mixed oligopolies has been described by several authors (see, *inter alia*, Cremer and Crémer, 1992; Delbono and Rossini, 1992; Rossini and Scarpa, 1993; Okuguchi, 1993a). They have highlighted the peculiar behaviour of LM firms under quantity competition, yielding an upward sloping reaction function[1] instead of the usual downward sloping one characterizing the PM firm. Nevertheless, all these contributions investigate to various aims simultaneous play under either quantity or price setting behaviour. I will show that, when a preplay stage in the sense of Hamilton and Slutsky (1990) is introduced,

1. However, the reaction function of an LM firm is not necessarily upward sloping. See Miyamoto (1982, p.13).

(i) simultaneous play is not to be expected under neither form of competition; (ii) Cournot behaviour yields as the unique subgame perfect equilibrium of the extended game the Stackelberg equilibrium with the PM firm moving first, and (iii) Bertrand behaviour leads to multiple equilibria in which both firms would prefer to move late or play in mixed strategies.

These results have some interesting implications as for the issue of reforming Eastern European economies. Delbono and Rossini (1992) evaluate the feasibility of alternative reforms of LM markets consisting in the passage to a mixed oligopoly or a horizontal merger where the resulting firm maximizes an objective function in which a positive weight is assigned to either entrepreneurial profit or social welfare. In analysing the case of a mixed duopoly, they only consider simultaneous Nash equilibria. In the present paper, it is shown that only Stackelberg equilibria should be taken into account. Hence, it turns out that a reform based on either the privatization or the nationalization of a labour managed firm implies a smaller social gain than it could be expected on the basis of previous literature.

The remainder of the paper is structured as follows. Betrand competition is described in Section 2. Section 3 is devoted to Cournot competition. Policy implications are discussed in Section 4. Finally, Section 5 contains concluding comments.

2. Bertrand competition

In order to safeguard the comparability of what follows with at least a part of the existing literature, I basically adopt the same symbology and assumptions as in Okuguchi (1993b). The magnitudes related to the PM and LM firms are identified as *P* and *C*, respectively.

Both firms produce through the following technology:

$$l_i = h_i(x_i), \quad i = C, P \tag{1}$$

where l_i is the amount of labour employed by firm i and x_i is the quantity produced by the same firm. The technology is fully characterized by the following derivatives:

$$h_i^{'} > 0, \quad h_i^{''} > 0, \tag{2}$$

i.e., the marginal productivity of labour is decreasing. Firms operate in a market for differentiated goods, whose demand is

$$x_i = g^i(p_i, p_j), \quad i, j = C, P, \quad i \neq j, \tag{3}$$

where (see Okuguchi, 1993b, pp.2-3):[2]

$$\partial g^i/\partial p_i \equiv g_i^i < 0, \quad \partial g^i/\partial p_j \equiv g_j^i > 0, \quad -g_i^i > g_j^i; \tag{4.1}$$

$$\frac{\partial^2 g_i}{\partial p_i \partial p_j} \equiv g_{ij}^i \leq 0, \quad g_j^i + p_i g_{ij}^i > 0. \tag{4.2}$$

The inequalities in (4.1) state that (i) an increase in firm *i*'s price induces a decrease in the demand for her own product, (ii) the two goods are substitutes, and (iii) the own price effect is larger than the cross price effect. The inequalities in (4.2) are needed for the reaction function of the LM firm to be positively sloped.

Since under the above assumptions Okuguchi (1993a,b) has shown that in a Bertrand

2. These assumptions, as well as those introduced in the remainder of the paper, hold for instance when linear demand functions are considered.

setting the reaction function of an LM firm is positively sloped irrespectively of the nature of the rival, I can confine myself to investigate the characteristics of the entrepreneurial firm's reaction function. I am going to prove the following:

LEMMA 1. Under Bertrand competition, the reaction function of the profit maximizing firm is upward sloping.

PROOF. The objective function of the PM firm is the following:

$$\pi_P^B = p_P g^P(p_C, p_P) - h_P(x_P) - k_P \tag{5}$$

where k_P defines the entrepreneurial firm's fixed cost. The first order condition for profit maximization w.r.t. price is:

$$\frac{\partial \pi_P^B}{\partial p_P} = g^P(p_C, p_P) + p_P g_P^P - h_P^{'} g_P^P = 0. \tag{6}$$

Assume the second order condition is satisfied. It is known (see Bulow et al., 1985) that the slope of the reaction function has the same sign as the derivative of (6) w.r.t p_C:

$$sign\,\frac{\partial p_P}{\partial p_C} = sign\,\frac{\partial^2 \pi_P^B}{\partial p_P \partial p_C} \tag{7}$$

Accordingly, it is sufficient to determine the sign of

$$\frac{\partial^2 \pi_P^B}{\partial p_P \partial p_C} = g_C^P + p_P g_{PC}^P - h_P^{'} g_{PC}^P - h_P^{''} g_P^P g_C^P; \tag{8}$$

on the basis of the above assumptions, it is quickly established that the sign of (8) is positive. Hence, the reaction function of the PM firm in the price space is upward sloping. *Q.E.D.*

Provided that the reaction function of the PM firm is positively sloped, as claimed in Lemma 1, and the reaction function of the LM firm is also increasing, as shown by Okuguchi (1993b), I am going to show what is stated in the following:

PROPOSITION 1. The extended Bertrand game between a profit maximizing firm and a labour managed firm has multiple equilibria. None of them is simultaneous.

PROOF. Since both reaction functions are positively sloped, this setting is a special case of the general situation depicted by Hamilton and Slutsky (1990, pp.36-41) in their Theorems III, V(Aii) and VI. According to these theorems, when both reaction functions are increasing the extended game with observable delay, where players first choose the timing of moves and then proceed to play, has multiple equilibria. Namely, both sequential play are subgame perfect equilibria; moreover, there exists a mixed strategy equilibrium in which firms randomize over the strategies "moving first" and "moving second". This is due to the fact that both reaction functions intersect the Pareto superior set, i.e., the set of all pair of prices yielding payoffs that dominate those associated with the simultaneous equilibrium. *Q.E.D.*

3. Cournot competition

In this Section, optimization w.r.t. quantity is analised. If the domain of the demand function (3) is a rectangular region, it can be inverted to obtain:

$$p_i = f^i(x_i, x_j), \quad i, j = C, P, \quad i \neq j, \tag{9}$$

with

$$\partial f^i / \partial x_j \equiv f^i_j < 0; \tag{10.1}$$

$$\frac{\partial^2 f^i}{\partial x_i \partial x_j} \equiv f^i_{ij} \quad \in \quad [0, -\frac{f^i_j}{x_i}[. \tag{10.2}$$

Assumption (10.1) is borrowed from Okuguchi (1993b, p.4). Assumption (10.2), which is a bit tighter than the corresponding condition in Okuguchi (1993b, p.4), and is borrowed from Okuguchi (1993a, p.29), implies that firm *i*'s marginal revenue decreases as her rival's output increases. Provided firm *i* acts as a profit maximizer, this condition also implies that her own reaction function is negatively sloped (see Novshek, 1985; Dixit, 1986; Okuguchi, 1993a, *inter alia*).[3] Provided that the reaction function of the LM firm is upward sloping (Okuguchi, 1993a,b), the following holds:

PROPOSITION 2. The Stackelberg equilibrium with the profit maximizing firm moving first and the labour managed firm moving second is the only subgame perfect equilibrium of the extended Cournot game.

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3. Cournot behaviour may induce a profit maximizing firm to consider her rivals as strategic complements. This happens when a large dominant firm competes against a population of smaller rivals. See Bulow et al. (1985, p.500).

PROOF. Since reaction functions exhibit opposite slopes, this is a special case of the general setting defined by Hamilton and Slutsky (1990, pp.36-41) in their Theorems IV and V(B). If there exists a preplay stage in which firms may choose the timing of moves, both decide to avoid playing simultaneously and, having opposite preferences over the distribution of roles in sequential play, since the PM firm prefers to lead while the LM firm prefers to follow, they agree on appointing the leader's role to the entrepreneurial firm.[4] *Q.E.D.*

4. Policy implications

The results obtained in the previous sections have some interesting implications as far as industrial policy is concerned. In the existing literature, the issue of restructuring Eastern European economies has been usually tackled through the comparison of the welfare levels associated with LM and mixed oligopolies under the assumption that firms adopt Cournot behaviour and move simultaneously (Cremer and Cremèr, 1992; Delbono and Rossini, 1992; Kahana, 1994). In particular, Delbono and Rossini (1992) evaluate the viability of alternative reforms where the initial LM monopoly is turned into either a mixed duopoly or a firm with a mixed objective function. In the former case an LM firm competes against either a private or a public firm, while in the latter the monopolist maximizes an objective function which takes into account either entrepreneurial profits or social welfare, together with value added per worker. The authors conclude that, whatever reform is adopted, it leads to a welfare improvement with respect to the *status quo*. In a recent note, Kahana (1994) shows that it wouldn't be rational for an LM monopolist to operate with several plants, so that as the relevant *status quo* one should rather consider an LM oligopoly. However, this does not modify significantly the results reached by Delbono and Rossini.

4. To the best of my knowledge, the only other case yielding unanimous preferences over the distribution of roles has been provided by Singh and Vives (1984) analysing a duopoly game between entrepreneurial firms where one firm optimizes w.r.t. price and the other w.r.t. quantity. This yields reaction functions of opposite slopes.

Thus, on the basis of the above remarks, it appears that the relevant comparison involves the welfare levels associated with sequential play in both the LM and the mixed duopoly. For obvious reasons, I adopt the same setting as in Delbono and Rossini (1992, p.228). Firms supply a homogeneous product and behave *à la* Cournot. The inverse market demand function is linear:

$$p = a - x_i - x_j. \tag{11}$$

Firms are characterized by the following technology:

$$x_i = \sqrt{l_i}, \tag{12}$$

denoting a decreasing marginal productivity of labour. Firm *i*'s total costs are:

$$C_i(x_i) = x_i^2 + k \tag{13}$$

where k denotes setup costs, which are assumed to be equal for both firms. Money wage is normalised to 1. The objective function of an LM firm is defined as follows:

$$v_i^C = \frac{p x_i - k}{x_i^2} \tag{14}$$

while that of a PM firm is:

$$\pi_j^C = p x_j - x_j^2 - k. \tag{15}$$

Finally, social welfare (gross of fixed costs) corresponds to:

$$SW = aX - \frac{X^2}{2} - \sum_i x_i^2 \tag{16}$$

where $X = x_i + x_j$ is total production.

4.1. Sequential play in the LM duopoly

Consider first the setting where both firms maximize income per worker, under the assumption that firm *i* takes the lead. Thus, firm *i* must choose her quantity in order to maximize the objective function (14) under the constraint given by the reaction function of firm *j*, which is the following:

$$x_j = \frac{2k}{a - x_i} \tag{17}$$

The total quantity produced in such a setting, which can be labelled as X_{Sdc}, is lower than the quantity supplied in correspondence of the simultaneous Nash equilibrium, X_{dc} (see the Appendix). As a consequence, the social welfare levels associated with the two equilibria can be ranked as follows:

$$SW_{dc} > SW_{Sdc} \quad \forall a^2 > 8k, \tag{18}$$

where the constraint on market size appearing in (18) warrants that both firms are active at the simultaneous Nash equilibrium (and thus also at the Stackelberg equilibrium, where they produce less).

4.2. Sequential play in the mixed duopoly

Assume now that a PM firm competes against an LM firm, in such a way that the former leads while the latter follows. The entrepreneurial firm sets her own output level so as to solve the following problem:

$$\max_{x_P^C} \quad \pi_P = p x_P - x_P^2 - k \tag{19}$$

$$s.t.: \quad x_C = \frac{2k}{a - x_P} \tag{20}$$

The total output supplied at equilibrium, labelled X_{Sdcp}, is smaller than the total output associated with the simultaneous Nash equilibrium, X_{dcp} (see the Appendix). Hence, the following inequality can be quickly established:

$$SW_{dcp} > SW_{Sdcp} \quad \forall a^2 > \frac{128}{9}k. \tag{21}$$

The relative size of a and k must satisfy the inequality in (21) in order for both firms to operate on the market at the simultaneous Nash equilibrium (see Delbono and Rossini, 1992, p.231).

4.3. Welfare comparison

I am now in a position to compare the welfare levels associated with the equilibria considered in Delbono and Rossini (1992) with those generated by the Stackelberg equilibria above. To this aim, I can resort to the following magnitudes:

$$\Delta SW_N = \frac{SW_{dcp} - SW_{dc}}{SW_{dc}} \tag{22}$$

$$\Delta SW_S = \frac{SW_{Sdcp} - SW_{Sdc}}{SW_{Sdc}} \tag{23}$$

The index in (22) measures the rate of increase in welfare due to the transformation of an LM duopoly into a mixed duopoly, under the assumption of simultaneous play, while (23) yields the same information under sequential play. In the viable range of parameters ($a^2>128k/9$), it can be established that, given the size of fixed costs, (i) both ΔSW_N and ΔSW_S increase as a increases, and (ii) $\Delta SW_N > \Delta SW_S$, i.e., the increase in social welfare due to the envisaged reform is smaller when sequential rather than simultaneous play is considered. In the light of the perspective described by Delbono and Rossini (1992), who take also into account the possibility of a horizontal merger between the two firms, the fact that $SW_{Sdcp} < SW_{dcp}$ makes merger appear socially preferable to a mixed duopoly for a wider range of parameter values. As a last remark, one can notice that, provided that with a linear market demand a public firm's reaction function is downward sloping,[5] the analysis of a mixed duopoly with a public and an LM firm would lead to conclusions largely analogous to those I have just outlined.

5. Conclusions

I have analysed the nature of the equilibria arising in a mixed duopoly setting where a profit maximizing and a labour managed firm compete either in prices or in quantities. In the light of the contribution by Hamilton and Slutsky (1990), some of the several possible equilibria can be selected as candidate subgame perfect equilibria of the extended game where firms first declare their respective preferences over the timing of moves and then proceed to optimize

5. Delbono and Scarpa (1995) investigate the conditions under which a public firm's Cournot reaction function is upward sloping.

respect to the relevant market variable.

Under Bertrand competition, due to the fact that reaction functions are both upward sloping, the extended game exhibits multiple equilibria, namely both the Stackelberg equilibria and one in mixed strategies where firms randomize over moving first or second. Under Cournot competition, the entrepreneurial firm's reaction function is downward sloping while the labour managed firm's one is upward sloping, so that the extended game has a unique subgame perfect equilibrium, i.e., the Stackelberg equilibrium where the profit maximizing firm is appointed the leader's role. Hence, the simultaneous Nash equilibrium in pure strategies does not emerge as a subgame equilibrium from either the Bertrand or the Cournot extended game.

The analysis of Cournot competition has provided new insights on the relative feasibility of alternative reform plans in Eastern European countries. Cournot-Stackelberg equilibria yield lower total production as compared to simultaneous equilibria in both LM and mixed duopoly. This amounts to saying that the social welfare level associated with sequential play is lower than the one yielded by simultaneous play under both market regimes. Hence, the incentive to turn an LM market into a mixed one appears now lower than it seemed to be on the basis of previous contributions, while merger appears as a more advantageous alternative.

Appendix

A.1. The leader's output in the LM duopoly

The solution to the leader's problem in the pure LM duopoly is given by:

$$x_i = \frac{2}{3}a - \frac{2k}{3a} + (\eta + \phi)^{1/3} - \frac{a(6k + 3a^2 - (2F/a - 2a)^2)/3}{[27a^3(\eta + \phi)]^{1/3}} \qquad (a.1)$$

where

$$\eta = \frac{-8k^3 + 42a^2k^2 - 6a^4k - a^6}{27a^3} \qquad \phi = \frac{1}{3a}\sqrt{\frac{96a^2k^3 - 20k^4 - 24a^4k^2 + 2a^6k}{3}} \qquad (a.2)$$

The output of the follower, firm j, can be obtained through her reaction function (17). Since it must be that $a > x_i + x_j$, the following constraint is to be satisfied:

$$a^2 > k(3 + 2\sqrt{2}). \qquad (a.3)$$

A.2. The leader's output in the mixed duopoly

When the PM firm plays the leader's role in the mixed duopoly game, her production amounts to:

$$x_P = \frac{a}{2}\left(\frac{3}{2} + \frac{1}{\psi^{1/3}} + \frac{\psi^{1/3}}{4}\right), \qquad (a.4)$$

where

$$\psi = -\frac{(16k + a^2)}{8} + a\sqrt{\frac{8k^2 + a^2 k}{2}}. \qquad (a.5)$$

Again, the quantity produced by the follower, in this case the LM firm, can be computed resorting to her own reaction function (20). Finally, the condition that must be met in order for market price to be positive at equilibrium is the following:

$$a^2 > 2k. \qquad (a.6)$$

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