Aggregate uncertainty, political instability and redistribution

Giorgio Bellettini*

Università’ di Bologna

and

University of Pennsylvania

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Abstract

This paper associates political instability to real shocks affecting the income of the median voter, in a two-period model where two political parties set redistribution in order to defend the interests of well-defined constituencies. Implemented policies affect future voting outcomes and an intertemporal trade-off arises for the parties since their optimal one-period strategy does not maximize the probability of being reelected. The higher the volatility of the real shock, the more likely that parties deviate from the optimal one-period strategy by choosing a conservative strategy, which increases their chances of reelection and the expected lifetime utility of their constituencies. (JEL D72, E62)

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1. Introduction

What does political instability mean? How does it affect fiscal policies and governments’ behaviour? Most economists seem to share the common belief that political instability is a synonymous for political conflicts, turmoil and violence, leading to uncertainty about future property rights which deters productive activities and, in particular, investment decisions. This idea has found support in a recent empirical literature (see Alesina et al. [3], Alesina and Perotti [4] and Barro [8]).

However obvious it may seem, this argument needs some further specifications. In fact, the statement depends crucially on the definition of political instability and on its causes. For instance, in representative democracies political instability can be related to the probability that the current government is defeated in the next elections; if we assume that individuals have perfect information about the type of policies that will be implemented by the successors, an increase in this measure of instability does not necessarily lead to higher uncertainty about future policies.¹

The recent theoretical literature about politics and economics introduces political instability in order to provide an explanation for the observed cross-country differences in fiscal policies and for their suboptimality, especially with regard to debt policies. In this literature, political instability occurs because voter participation or the composition of the electorate may change and it entails a change in the identity of the median voter. Examples of this approach can be found in Alesina and Tabellini [6] [7], Cukierman, Edwards and Tabellini [11], Persson and Svensson [20] and Tabellini and Alesina [23].

¹The meaning and use of political instability has been the subject of a voluminous political-science literature. See, for example, Ake [2] and Hurwitz [14] and the references therein cited.
This paper aims at studying the relationship between political instability and fiscal policies (in particular, redistributive policies) starting from a different interpretation of political instability. In particular, I associate political instability with real uncertainty, developing the idea that, similarly to private economic decisions, political outcomes cannot be exactly predicted because voters face real random shocks which affect their opportunity sets. If rational and selfish agents choose policies in order to maximize their own indirect utility function, and the arguments of this function have some stochastic components, electoral results become uncertain and future policy cannot be perfectly anticipated.

A natural consequence of this approach is to relate the degree of political instability to the volatility of the real shock. Thus, in the framework of this paper, the higher is the volatility of the shock, the higher is the degree of political instability and uncertainty.

In order to formalize these ideas, I develop a two-period model where:

1. there is only one policy issue (the amount of redistribution);

2. there are only two parties, Left and Right, with no free-entry;

3. parties are perfect agents of different income groups;

4. the income of the middle class in the second period has a stochastic component.

In our model, parties are not interested in being elected per se, as in the Down-sian approach to political competition, but insofar as they have the opportunity to implement the policies which are the most preferred by their constituencies.\(^2\)

\(^2\)See Downs [12].

\(^3\)The importance of party’s ideology in explaining macroeconomic policies has found ample evidence in the political science literature. See, for example, Hibbs [13] and Tufte [24].
Equivalently, parties can be seen as citizen candidates who implement their preferred policy, if elected.\footnote{For examples of the citizen candidates approach to democratic policy choice, see Besley and Coate [10] and Osborne and Slivinski [17].} Since the main purpose of the paper is to propose a definition of political instability related to real uncertainty, and study its implications on the choice of fiscal policies, I kept the political system as simple as possible. In particular, I take the number of parties as exogenous.

With regard to the economic environment, I will present a modified version of Perotti [18] model about education and growth, where the author studies the long-debated relation between income distribution and economic growth. Individuals live for two periods and they are characterized by their human capital endowment. Investment in education has a fixed cost and yields a fixed return. Due to the lack of a credit market, an income at least equal to the fixed cost is required to invest in education.

Revenues from a proportional income tax are redistributed as per capita lump-sum subsidies. Since investment decisions are undertaken in the first period after fiscal policy is implemented, the amount of redistribution affects the level of investment by altering the level of after-tax income accruing to each class. After investment, some agents (in particular, the middle class) are hit by an income shock: at the beginning of the second period, a fraction of them will belong to the rich class; the rest will become poor.

Elections are held at the beginning of each period. Citizens perfectly foresee the policy that would be chosen by each party and cast their vote for the party whose preferred policy yield them the highest level of utility. The party that gets the largest number of votes is elected and implements its optimal policy.

A crucial feature of this model is that expectations about future electoral results are conditional on the current policy, which determines the future distri-
bution of income together with the random shock. In particular, the left party is most likely to be elected in the second period when the level of redistribution in the first period is so low that only the rich invest, and the distribution of income becomes right skewed. On the contrary, the probability that the right party is elected is maximized when both the rich and the middle class invest.

Two are the main findings of this paper. First, when parties choose their optimal fiscal policy, they always face an intertemporal trade-off between maximizing current utility of their constituencies and maximizing probability of future reelection. Therefore, they may find intertemporally optimal not to reach their constituencies' highest level of current consumption, if the gain in terms of probabilities of future electoral success and future consumption is sufficiently high. Second, this gain is higher, the higher is the variance of the real shock. Thus, when the variability of the shock is large and the degree of political instability is high, both parties are more likely to adopt a conservative policy whereby the lower current welfare of their constituencies is traded against the opportunity of capturing a larger percentage of future votes.

The last section illustrates these results through some numerical examples where I assume that the random fraction of middle income agents who become poor follows a uniform distribution with constant mean. By altering the support of the distribution, I am able to evaluate the effects on the politico-economic equilibrium due to changes in the volatility of the shock. These examples show that the volatility of the shock can substantially affect the optimal policy of the two parties and the results of the elections.

As I discussed earlier, this work is closely related to the recent literature about endogenous fiscal policy which attempts to study the relationship between political instability and fiscal policies. However, in this literature political instability is identified with factors such as random voting turnout or changes in the com-
position of the eligible voting population. Thus, electoral results turn out to be exogenous and current policies cannot affect the probability of electoral success.

This paper is also linked to the literature that investigates the relationship between income distribution, political equilibrium and economic growth, even if the role of political instability is not taken into account (see, for example, Alesina and Rodrik [5], Krussel and Rios-Rull [15] [16], Perotti [18], [19] and Persson and Tabellini [21]).

The paper is organized as follows. Section 2 sets out the model. Sections 3 and 4 analyze the political equilibrium which arises in the one-period and in the lifetime case. Section 5 studies the relation between the volatility of the shock and the equilibrium fiscal policies. Section 6 presents some numerical examples and Section 7 concludes.

2. The model

I use Perotti [18] human capital accumulation model with some alterations to characterize the relationship between real uncertainty, political instability and fiscal policy.

The economy consists of an infinite number of agents who live for two periods and who are described in their role as consumers, workers and voters.

Agent $i$ belongs to one of three types and is indexed by his endowment of efficiency units of labor in the first period $n_i^1$, where $n_1^1 < n_2^1 < n_3^1$. Each type's fraction is given by $\mu_i$, where $\mu_i < 0.5 \forall i$ and $\sum_{i=1}^{3} \mu_i = 1$, so that none of them can dominate the elections, and the decisive voter always belongs to the middle class. Furthermore, I assume that $n_3^1 < N_1$, where $N_1$ denotes the average income in period 1.

The utility function takes the form
\[ u^i(c^i_1, c^i_2) = c^i_1 + c^i_2 \] (2.1)

where \( c^i_t \) is the consumption of an agent \( i \) at time \( t \).

The consumption good is produced using a linear production function:

\[ y^i_t = n^i_t \] (2.2)

where \( n^i_t \) represents the efficiency units of labor of an agent \( i \) at time \( t \).

Investment technology is such that a fixed amount of consumption good \( \gamma \) in period 1 yields a return of \( \rho \) units of good in period 2. The investment technology is person specific and there are no other intertemporal markets.\(^5\)

In both periods there is lump-sum redistribution financed through a linear income tax and the budget is always balanced.

On the political side, there are two parties, L and R, whose objective function is to maximize the welfare of types 1 and 3, respectively.\(^6\) In each period, the party in power chooses the level of taxation, \( \tau_t \). I assume that given \( \rho > \gamma \), there exists an upper bound \( \tau_2 \equiv \frac{\rho - \gamma}{\rho} \) such that the tax rate in period 2 cannot exceed this level. This assumption ensures that investing in education always yields a higher utility than not investing.\(^7\)

\(^5\)This fixed-cost, fixed-return specification of the investment technology and the lack of credit markets seem plausible when applied to investment in education.

\(^6\)This political system captures the idea that "...Democrats are best serving the interests of downscale groups and the Republicans are best accommodating the interests of upscale groups" and that "the middle class is the battleground of electoral competition between the parties" (Hibbs [13], p.214, 216). Examples of economic models based upon this two-party system can be found in Aghion and Bolton [1] and Roemer [22].

\(^7\)In this model, we are not interested in the traditional time-consistency problems associated with dynamic taxation. Assuming the existence of an upper bound
Parties cannot commit to future policies during the campaign; elections are held at the beginning of each period, and the party which gets the largest number of votes is appointed. Citizens vote sincerely for the party which maximizes their expected utility.

Two points are worthwhile underlining here. First, agents' preferences over tax policy depend exclusively on their income. Second, our assumption about \( \mu^i \) implies that in each period the decisive voter belongs to the middle class. Thus, in order to formalize our concept of political uncertainty, I will introduce a stochastic element which affects only the middle class income in the second period.

The law of motion of the individual endowment of the efficiency units \( n^i \)'s is given by:

\[
n_2^i = n_1^i + \rho e^i + \varepsilon^i
\]

(2.3)

where \( e^i \) is an indicator function which is equal to one if type \( i \) has invested and zero if type \( i \) has not invested. By assumption, the random shock \( \varepsilon^i \) is equal to 0 for types 1 and 3, whereas \( \varepsilon^2 \) is a discrete random variable which takes two values, \( \varphi \) and \( \theta \). More specifically, I assume that in the second period the "lucky" middle agents have an income equal to the initial endowment of the rich class plus the return from investment, if they invested. Similarly, the "unlucky" agents will end up with the same endowment as the poor plus \( \rho \), if they invested. Thus:

\[
n_2^2 = \begin{cases} 
    n_1^1 + \rho e^3 & \text{for the "unlucky"} \\
    n_1^3 + \rho e^2 & \text{for the "lucky"}
\end{cases}
\]

Note that in the second period the only source of difference between the middle class and the other classes is represented by the investment decision, \( e^2 \). For the tax rate in the last period rules out the possibility of full taxation, which would be the time-consistent solution for the left-wing party.
example, if all classes invest, \( c^i = 1 \ \forall i \), and in the second period the middle class disappears.

We can regard this stochastic element as an additive shock to the middle-class’ income in the second period. An alternative formulation would relate the stochastic term to variability in the return to investment, but I chose this specification because I want uncertainty to play a role even in the case where people do not invest.

The distribution of the random variable \( \varepsilon^2 \) depends upon an aggregate shock \( z \); in particular:

\[
P(\varepsilon|z) = \alpha, \quad P(\bar{\varepsilon}|z) = 1 - \alpha
\]

where \( 0 \leq \alpha \leq 1 \). By the law of large numbers, these ex-ante probabilities are also the actual fractions of population which are either "lucky" or "unlucky". The higher is \( \alpha \), the "worst" will be the state of the world in the second period.

I will henceforth refer to the aggregate shock by means of \( \alpha \) instead of \( z \), since the two formulations are completely equivalent. The aggregate shock \( \alpha \), where \( \alpha \) represents the fraction of middle income people who will become poor, is assumed to lie on the closed interval \( \Omega = [\underline{\alpha}, \bar{\alpha}] \subseteq [0, 1] \) and to be distributed according to a density function \( f(\alpha) \), assumed to be symmetric with mean \( \hat{\alpha} \) and variance \( \sigma^2 \).

Finally, the lack of credit markets implies the following individual budget constraints:

\[
c_1^i \leq (1 - \tau_1) n_1^i + T_1 - \gamma c^i \tag{2.4}
\]

\[
c_2^i \leq (1 - \tau_2) n_2^i + T_2 \tag{2.5}
\]

where \( T_i = \tau_i N_i \) is the amount of redistribution in period \( t \).

The timing of the model is as follows:
In this framework, the only economic decision that agents have to take is whether to invest in education or not. Therefore we can write:

The consumer’s problem. In period 1 agent \( i \) solves the following problem:

\[
V^i(n^i_1, N, \tau_1) = \max_{\tilde{\epsilon} \in \{0,1\}} \{ c^i_1 + E(c^i_2) \}
\]

subject to:

\[
c^i_1 \leq (1 - \tau_1) n^i_1 + T_1 - \gamma \tilde{\epsilon}^i
\]

\[
c^i_2 \leq (1 - \tau_2) n^i_2 + T_2
\]

where \( E(\cdot) \) is the expectation operator.

It can be shown (see Perotti [18]) that the solution to this problem is given by:

\[
\tilde{\epsilon}^i = \begin{cases} 
1 & \forall i \text{ s.t. } n^i_1 \geq \tilde{n} \\
0 & \forall i \text{ s.t. } n^i_1 < \tilde{n}
\end{cases}
\]

where \( \tilde{n}(\tau_1) \) is implicitly defined by:

\[
(1 - \tau_1) \tilde{n} + T_1 - \gamma = 0
\]

Given the tax rate \( \tau_1 \), people whose initial endowment is higher than the threshold level \( \tilde{n} \) invest in education; on the contrary, people whose endowment is below this threshold are liquidity constrained and therefore they cannot invest.

In this context, the government can use fiscal policy to affect the number of people who invest, recognizing that changes of the tax rate affect the threshold
level $\tilde{n}$. The way the level of taxation affects this threshold and the level of investment depends on whether the average income $N_t$ is greater or smaller than the investment cost $\gamma$. In the remaining of the paper I will assume that $N_t > \gamma$ and $n_t^1 < \gamma$.\footnote{These two assumptions together imply that the rich invest regardless of the level of taxation, whereas the middle class and the poor invest if and only if the amount of redistribution is large enough to relax their liquidity constraint.} We can see from (2.7) that in this case $\frac{\partial \tilde{n}}{\partial \gamma} < 0$, i.e. the higher the degree of redistribution, the lower the initial endowment which is required to invest.

3. The one-period problem

As I already pointed out, in each period parties choose the level of taxation in order to maximize the welfare of their constituencies. Furthermore, before that, in each period agents vote to decide which party will be in office in that period.

The political equilibrium in the second period can be characterized fairly easily. Parties observe the level of income of their constituencies and they set the level of taxation accordingly.

Thus, the problem of the left-wing party in the second period can be written as:

$$\varphi_2^L (n_2^1, N_2) = \arg \max_{\varphi_2 \in (0, \Phi_2)} \left\{ (1 - \varphi_2) n_2^1 + \varphi_2 N_2 \right\} \quad (3.1)$$

Similarly, the problem of the right-wing party is:

$$\varphi_2^R (n_2^3, N_2) = \arg \max_{\varphi_2 \in (0, \Phi_2)} \left\{ (1 - \varphi_2) n_2^3 + \varphi_2 N_2 \right\} \quad (3.2)$$

Given that $n_2^1 < N_2$ and $n_2^3 > N_2$, the solutions to (3.1) and (3.2) are $\varphi_2^L = \Phi_2$ and $\varphi_2^R = 0$, respectively. Thus, we have shown the following:
Proposition 1. The optimal one-period strategy for the left-wing party is to maximize the level of taxation, whereas minimum taxation is the optimal one-period strategy for the right-wing party.

Let us now turn to the voting problem in the second period. As we explained in the previous section, in this model agents cast their vote by comparing the indirect utility associated with $\varphi^L_2$ and $\varphi^R_2$.

As we already know, in each period the decisive voter belongs to the middle class. Note that, depending on the aggregate shock, the decisive voter in period 2 can be either a "lucky" or an "unlucky" one. Thus, if we denote by $\psi^k_2$ the party chosen in period 2 by an individual $k$ belonging to the middle class, we can write:

$$\psi^k_2(\tau_1, \alpha, \varepsilon^k) = \arg \max_{j \in \{L, R\}} \left\{ (1 - \varphi^j_2) n^k_2(\tau_1, \varepsilon^k) + \varphi^j_2 N_2(\tau_1, \alpha) \right\} \quad (3.3)$$

Next, we need to order people by their preferred party, to find out who will win the election in period 2. In other words, we are looking for $m$ such that:

$$\mu^1 + \sum_{k \leq m} \theta^k(\alpha) = \frac{1}{2} \quad (3.4)$$

where $\theta$ denotes fraction. Finally, the equilibrium condition requires that the most preferred policy by $m$ be the policy which is actually implemented, i.e. $\varphi^m_2(\tau_1, \alpha, \varepsilon^m) = \tau_2(\tau_1, \alpha)$.

Let us now define $\alpha^*$ as the level of the shock such that $\mu_1 + \alpha^* \mu_2 = \frac{1}{2}$. It is easily verified that, if $\alpha > \alpha^*$, the income of the decisive voter in the second period is given by: $n_1^3 + \rho c^2$ and that if $\alpha < \alpha^*$ the income of the decisive voter is given by: $n_1^3 + \rho c^2$. Obviously, if the decisive voter is poorer (richer) than the average, the left (right) party will be elected.
In the remainder of this section, I will show how the electoral results in the second period can be influenced by the party in power in the first period. In order to do so, we need some additional definitions.

Let $\tilde{\gamma}_i^1$ denote the lowest level of taxation such that agent $i$ can invest. In other words, $\tau_1 = \tilde{\gamma}_i^1$ implies $\tilde{n}(\tau_1) = n_i^1$, i.e. the threshold level is exactly equal to agent $i$ income. Using (2.7), we can write:

$$\tilde{\gamma}_i^1 = \frac{\gamma - n_i^1}{N_1 - n_i^1} \quad (3.5)$$

Recall, also that the liquidity constraint is never binding for the rich because $n_1^3 > \tilde{n}(\tau_1) \forall \tau_1 \in [0, 1]$. Therefore, the lowest tax rate such that the rich can invest is equal to $\tilde{\gamma}_1^3 = 0$.

As it will become immediately clear, the level of taxation in the first period determines the subset of aggregate shocks such that, if the realized shock falls in that subset, the left party will be elected in the second period, whereas if the shock falls in the complementary set, the right party will be elected. Let $\Omega_L(\tau_1)$ represent a subset of $\Omega$ such that when the shock takes value $\alpha \in \Omega_L(\tau_1)$, the left party is elected in the second period.

Consider now the case where $\tau_1 \geq \tilde{\gamma}_1^1$. In this case, the amount of redistribution in period 1 is so high that all agents have the opportunity to invest. Once the shock hits the economy, a fraction $\alpha$ of the middle class becomes poor and a fraction $1 - \alpha$ becomes rich. Thus, in period 2 there will be only two classes: a poor class with income $n_1^1 + \rho$ and mass $\mu^1 + \alpha \mu^2$ and a rich class with income $n_1^3 + \rho$ and mass $\mu^3 + (1 - \alpha) \mu^2$. In this case, we can conclude that $\Omega_L(\tau_1 \mid \tau_1 \geq \tilde{\gamma}_1^1) = [\alpha^*, \bar{\alpha}] \equiv \Omega_L^1$.

Second, consider a decrease in the level of redistribution such that only the rich and the middle class are allowed to invest, i.e. $\tilde{\gamma}_1^2 \leq \tau_1 < \tilde{\gamma}_1^1$. In this case, at the beginning of the second period we will have three income classes, $n_1^1$, $n_1^1 + \rho$ and $n_1^3 + \rho$, with fractions $\mu^1$, $\alpha \mu^2$, and $\mu^3 + (1 - \alpha) \mu^2$ respectively. It is clear that
the left-wing party will be elected in the second period if and only if $\mu^1 + \alpha \mu^2 \geq \frac{1}{2}$ (that is, $\alpha \geq \alpha^*$) and $n^1_1 + \rho < N_2(\tilde{\tau}_1^2, \alpha)$.

If we now define $\delta \in R$ such that for any $\alpha \leq \delta$, $n^1_1 + \rho \leq N_2(\tilde{\tau}_1^2, \alpha)$, we can conclude that the left party will be elected if and only if the aggregate shock $\alpha$ will lie between $\alpha^*$ and $\delta$, i.e.

$$\Omega_L^2 \left( \tau_1 \mid \tilde{\tau}_1^2 \leq \tau_1 < \tilde{\tau}_1^1 \right) = \Omega_L^2 = \begin{cases} [\alpha^*, \delta] & \text{if } \delta > \alpha^* \\ \emptyset & \text{if } \delta \leq \alpha^* \end{cases}$$

Finally, consider the case where $0 \leq \tau_1 < \tilde{\tau}_1^2$. Here there is low redistribution and only the rich can invest. Therefore at the beginning of period 2 we have three classes, $n^1_1$, $n^2_1$ and $n^3_1 + \rho$, with fractions $\mu^1 + \alpha \mu^2$, $(1 - \alpha) \mu^2$, and $\mu^3$ respectively.

If $\eta \in R$ is implicitly defined by:

$$n^3_1 = N_2(0, \eta)$$

then applying the same reasoning as before:

$$\Omega_L \left( \tau_1 \mid \tau_1 < \tilde{\tau}_1^2 \right) = \Omega_L^3 = \begin{cases} [\alpha^*, \eta] \cup [\alpha^*, \infty] & \text{if } \eta < \alpha^* \\ \Omega & \text{if } \eta \geq \alpha^* \end{cases}$$

Summarizing, we have shown the following results:

**Proposition 2.** For any $\tau_1 \in [0, 1]$, there exists a set $\Omega_L(\tau_1) \subseteq \Omega$, such that $\forall \alpha \in \Omega_L$, the left party is elected in period 2, and $\forall \alpha \notin \Omega_L$, the right party is elected in period 2.

**Corollary 1.** For any $\tau_1 \in [0, 1]$, $\Omega_L(\tau_1) \in \{\Omega_L^1, \Omega_L^2, \Omega_L^3\}$. More specifically, $\Omega_L(\tau_1 \mid \tau_1 \geq \tilde{\tau}_1^1) = \Omega_L^1$, $\Omega_L(\tau_1 \mid \tilde{\tau}_1^2 \leq \tau_1 < \tilde{\tau}_1^1) = \Omega_L^2$, $\Omega_L(\tau_1 \mid \tau_1 < \tilde{\tau}_1^2) = \Omega_L^3$, where $\Omega_L^1 \supseteq \Omega_L^2 \supseteq \Omega_L^3$. 

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The intuition behind Corollary 1 is the following. On the one hand, when only the rich invest, the distribution of income tends to be skewed to the right. If the shock is so 'good' that the average income in the second period is high enough, even the lucky middle agents will fall below the average and only the initially rich will vote for the right party. On the other hand, when everybody invests and there are only two classes, the rich together with the lucky middle class will vote for the right party. Finally, when both the rich and the middle class invest, the distribution of income becomes skewed to the left and, for bad shocks, it may happen that the whole middle class will vote for the right party.

It is worthwhile anticipating here that, as we will extensively see in the next section, the second part of Corollary 1 highlights the intertemporal trade-off faced by the parties between maximizing their short-run utility and maximizing their chances of reelection. The next section is entirely based upon this result.

4. The lifetime problem

In this section I will characterize the optimal tax rate which would be chosen by each party if it was elected in the first period.

In order to highlight the relationship between aggregate uncertainty and fiscal policies, let us first look at the politico-economic equilibrium which would arise if there was no income shock and therefore no uncertainty at all. In this case, it is easy to show the following:

**Proposition 3.** When there is no aggregate uncertainty, the left-wing party maximizes the level of redistribution in both periods and it always wins the elections.

**Proof.** Consider the policy vector \( \tau = (1, \tau_2) \), where the left (right) entry represents the tax rate in the first (second) period. When \( \tau_1 = 1 \), all three classes
invest and the position of each of them relative to the average will be the same in every period. Therefore, the tax vector \( \tau \) maximizes overall consumption for any agent who is initially below the average. Clearly, given \( n_1^2 < N_1 \), this policy proposal cannot be defeated by any other proposal. ■

It should be clear that the last proposition depends crucially on the assumption that, initially, the income of the middle class is below the average. In the opposite case where the median voter is initially above the average, the result of the proposition should be reversed, and the right party would always win the elections by minimizing the degree of redistribution in every periods.

When we introduce real uncertainty, the maximization problem of the parties changes. In fact, in the first period, they set the level of taxation in order to maximize the expected value of their constituency’s consumption, conditional on the probability distribution \( f(\alpha) \).

Thus, each of them will solve the following problem:

\[
\varphi_1 \left( n_1^1, N_1 \right) = \arg \max_{\tau_1 \in [0, 1]} \left\{ c_1^1 + E(c_2^1) \right\}
\]

subject to:

\[
c_1^1 = (1 - \varphi_1) n_1^1 + \varphi_1 N_1 - \gamma c_1^1 (\tau_1)
\]

\[
c_2^1 = (1 - \tau_2 (\varphi_1, \alpha) n_2^1 (\varphi_1) + \tau_2 (\varphi_1, \alpha) N_2 (\varphi_1, \alpha)
\]

where \( j \in \{1, 3\} \).

As we know from the previous section, the level of \( \tau_2 \) will be either maximum \( (\tau_2 = \pi_2) \) or minimum \( (\tau_2 = 0) \) depending on which of the two parties is elected; therefore, the expected value of consumption of agent \( j \) in period 2 can be written as follows:

\[
E(c_2^1) = n_2^1 (\varphi_1) + \pi (\varphi_1) \pi_2 \left[ \bar{N}_2 (\varphi_1) - n_2^1 (\varphi_1) \right] + \text{cov} (\tau_2, N_2)
\]

(4.2)
where $N_2$ denotes the expected average income in the second period, obtained by
letting $\alpha = \hat{\alpha}$ and $\pi(\varphi_1)$ is the probability that the left-wing party is elected when
$\tau_1 = \varphi_1$. Using the results of the previous section, it is immediate to derive the
probability $\pi$ that the left party will be elected in the second period, given $\varphi_1$. In
fact, by integration of the probability function $f(\alpha)$ over the sets $\Omega^L_1$, we obtain
$\pi^L = \int_{\Omega^L_1} f(\alpha) d(\alpha)$. Note that $0 \leq \pi^L \leq \pi^1 \leq \pi^3 \leq 1$.

Let us now try to characterize the solutions to problem (4.1) for both parties. Beginning with
the left-wing party, let us first restrict the set of tax rates which can be candidate for an optimum. Clearly, the one-period optimal solution $\tau_1 = 1$
is a candidate also for the overall problem. Consider now a decrease in the level of
taxation. Lower redistribution reduces consumption of the poor in the first period
and, if the poor becomes liquidity constrained, it decreases their consumption also in
the second one. However, if the tax rate is lowered to a level where only the
rich invest, the probability that the left party is elected in the second period is
maximized and the benefit in terms of expected future consumption may outweigh
the loss in current consumption.

If we denote by $\tau^3_1$ the highest tax rate such that only the rich invest, we can
conclude that $\tau^3_1$ is the only other candidate as a solution to the overall problem
for the left party.

Thus, using (4.1) and (4.2), the problem of the left party reduces to:

$$
\varphi^L_1 \left( n_1^1, N_1 \right) = \arg \max_{\varphi_1 \in \{ \tau_1, \varphi_1 \}} \left\{ (1 - \varphi_1) n_1^1 + \varphi_1 N_1 - \gamma e^1 (\varphi_1) + n_2^1 (\varphi_1) + \pi (\varphi_1) \tau_2 \left[ N_2 (\varphi_1) - n_2^1 (\varphi_1) \right] + \text{cov} (\tau_2, N_2) \right\} \tag{4.3}
$$

If we compare the levels of utility which can reached by the left party when
$\varphi_1 = 1$ and $\varphi_1 = \tau^3_1$, respectively, it is easy to show that, after some algebra, their
difference becomes:
\[ G^L(1, \tau_1^3) = \left(1 - \tau_1^3\right) \left( N_1 - n_1 \right) + \rho - \gamma - \tau_2 \mu^3 \rho - \tau_2 \left( \pi^3 - \pi^1 \right) \left[ \bar{N}_2 \left( \tau_1^3 \right) - n_1^1 \right] \]

If the left party decreases the degree of redistribution to \( \tau_1^3 \), it incurs a cost in the first period equal to the amount of resources which are not distributed to the poor, that is: \( (1 - \tau_1^3) (N_1 - n_1^1) \). It also incurs a cost in the second period equal to \( \rho - \gamma \), because at that level of taxation the poor are liquidity constrained and cannot invest.

On the other hand, there are two types of benefits. First, should the left party be elected in the second period, a higher amount of resources will be redistributed. Second, the probability that the left part is elected increase. These benefits sum up to:

\[ \tau_2 \mu^3 \rho + \tau_2 \left( \pi^3 - \pi^1 \right) \left[ \bar{N}_2 \left( \tau_1^3 \right) - n_1^1 \right] \]  

Thus, we can summarize this discussion in the following:

**Proposition 4.** In the presence of real uncertainty, \( \varphi_1^L = 1 \) if and only if \( G^L > 0 \). Otherwise, \( \varphi_1^L = \tau_1^3 \).

Let us turn now to the right-wing party. The right party has two different incentives to deviate from the optimal one-period strategy \( (\tau_1 = 0) \). First, as we already know, the right-wing party increases its chances of reelection by increasing the level of taxation. Second, the increased redistribution in the first period will increase the size of the "pie" to be shared in the second one and it will decrease the amount of income that the rich will loose should the left party be elected.
This latter benefit is highest when the amount of resources to be redistributed in the second period is maximized, that is when all three classes invest.\(^9\)

Summarizing, there are three candidates as the optimal tax rates for the right party: \(\tau_1 = 0\), \(\tau_1 = \tilde{\tau}_1^2\) which maximizes its probability of electoral success, and \(\tau_1 = \tilde{\tau}_1^1\) which minimizes the consumption loss of the rich in case of electoral defeat of the right party.

Thus, the problem for the right party can be rewritten as follows:

\[
\varphi_1^R(n_1^3, N_1) = \arg\max_{\varphi_1 \in \{0, \tilde{\tau}_1^2, \tilde{\tau}_1^1\}} \left\{ (1 - \varphi_1) n_1^3 + \varphi_1 N_1 - \gamma c^3(\varphi_1) + n_2^3(\varphi_1) + \pi(\varphi_1) \bar{\pi}_2 \left[ \bar{N}_2(\varphi_1) - n_2^3(\varphi_1) \right] + \text{cov}(\bar{\pi}_2, \bar{N}_2) \right\}
\]

(4.6)

Looking at (4.6), consider an increase in the level of taxation from \(\varphi_1\) to \(\varphi_1^i\). The first period cost for the right party is given by: \(A(\varphi_1, \varphi_1^i) \equiv (\varphi_1^i - \varphi_1)(N_1 - n_1^3)\). The change in the second period expected consumption is the sum of two terms. The first term is equal to \(\pi(\varphi_1^i)[\bar{N}_2(\varphi_1^i) - \bar{N}_2(\varphi_1)]\) and represents the increased amount of per capita transfers in case the left party is in power in the second period. On the other hand, the second term is given by \(\pi(\varphi_1^i)[\bar{N}_2(\varphi_1) - n_2^3 - \rho]\) and it is equal to the different amount of ”stolen” resources induced by the change in the expected electoral results.

Let \(B(\varphi_1, \varphi_1^i)\) and \(C(\varphi_1, \varphi_1^i)\) denote these two terms, respectively. If we now define \(G^R(\varphi_1, \varphi_1^i) \equiv A + B + C\), we can write the following:

**Proposition 5.** In the presence of real uncertainty, \(\tau_1^R \in \{0, \tilde{\tau}_1^2, \tilde{\tau}_1^1\}\). The optimal tax rate \(\tau_1^R\) is such that \(G^R(\tau_1^R, \tau_1) > 0\) \(\forall \tau_1 \neq \tau_1^R\).

\(^9\)The incentive to deviate from short-run optimal policies in order to increase the total amount of resources to be redistributed plays a role in Perotti [19]. Note, however, that in our model this incentive is relevant if and only if there is uncertainty about future electoral results.
To conclude this section, we need to analyze the voting problem in the first period. Given the solutions to (4.3) and (4.6), the winning party will be the one whose policies maximize the expected value of lifetime utility of the middle class (i.e. the median voter). Similarly to (3.3), if we denote with \( \psi_1^2 \) the party chosen in the first period by the middle class, we can write:

\[
\psi_1^2 (n_1^2, N_1) = \arg \max_{j \in \{L, R\}} \{ c_1^2 + E(c_2^2) \}
\]

subject to:

\[
c_1^2 = \left( 1 - \tilde{\varphi}_1^2 \right) n_1^2 + \varphi_1^2 N_1 - \gamma e^2 (\varphi_1^2)
\]

\[
E(c_2^2) = \hat{n}_2^2 + \pi(\varphi_1^2) \pi_2 \left( \hat{N}_2(\varphi_1^2) - \hat{n}_2^2 \right) + \text{cov}(\tau_2, N_2) + \text{cov}(\tau_2, n_2^2)
\]

where \( \tilde{n}_2 \equiv \tilde{\alpha} n_1^2 + (1 - \tilde{\alpha}) n_1^3 + \rho e^2 (\varphi_1^2) \) denotes the expected income of a middle class agent in the second period. Once again, the equilibrium condition requires that \( \varphi_1^2 = \tau_1 \), that is, the level of taxation in the first period must be equal to the most preferred level by the middle class.

Notice that, when the expected income of the middle class is high enough, that is \( \hat{N}_2(\tau_1) - \hat{n}_2^2 < 0 \), the middle class anticipates to be hurt by future redistribution and it may decide to support the right-wing party already in the first period (see example 2 in Section 6). Indeed, this may happen if and only if \( \varphi_1^2 > 0 \).

5. Uncertainty and fiscal policy

How does a change in the variance of the shock \( \sigma^2 \) affect the electoral results and the optimal choices of the party?

In order to answer these type of questions, I will henceforth assume that the mean of the distribution of the shock, \( \tilde{\alpha} \), is equal to \( \alpha^* \). In other words, the expected value of the shock is such that, in the second period, the sum of the
fractions of the poor and the unlucky middle class is exactly equal to one half, which is obviously equal to the sum of the fractions of the rich and the lucky middle class. Furthermore, I will restrict the attention to the cases where neither of the two parties will be elected in the second period with probability one, no matter what policy is implemented in the first period.\textsuperscript{10}

Having introduced these restrictions, we can state the following result:

**Proposition 6.** The higher is $\sigma^2$, the higher is the gain in terms of probability of future electoral success if parties deviate from their optimal one-period strategies.

**Proof.** For any $\alpha \leq (>) \hat{\alpha}$, since the probability function is symmetric, $\frac{dF(\alpha)}{d\sigma^2} \geq (<=) 0$. As we know from previous definitions, $\pi^3 \pi^1 = F(\eta)$; given $\alpha < \eta < \hat{\alpha}$ we get $\frac{d}{d\sigma^2}(\pi^3 - \pi^1) = \frac{dF(\eta)}{d\sigma^2} > 0$. Similarly, note that $(1 - \pi^2) - (1 - \pi^3) = \pi^3 - \pi^2 = F(\eta) + (1 - F(\delta))$; again, given $\alpha < \eta < \hat{\alpha}$ and $\hat{\alpha} < \delta < \overline{\alpha}$, we have $\frac{d}{d\sigma^2}(\pi^3 - \pi^2) > 0$. \hfill \blacksquare

Proposition 6 follows from the fact that whenever parties move away from their radical short-run strategies, they can capture the additional probability of success in case the shocks at the tails of the distribution occur. The more uncertain is the distribution of the shock, the higher the gain from this move. For instance, consider the left party. As we know, the optimal short-run strategy is to set $\tau_1 = 1$, whereas the strategy that maximizes the probability of future electoral success is to set $\tau_1 < \overline{\tau}_1^2$. Thus, the difference between its highest probability of success and the probability associated to the optimal short-run strategy is given by $\pi^3 - \pi^1 = $\textsuperscript{10}Notice that these assumptions imply that we are limiting the analysis to the case where $\alpha < \eta < \hat{\alpha} < \delta < \overline{\alpha}$. If we dropped these assumptions, the direction of the response of the electoral results and the political equilibrium to changes in the variance of the shock may become a priori ambiguous.
\[ f_{\alpha} f(\alpha) d(\alpha). \] If \( \eta < \bar{\alpha} \), the value of this integral increases when the variance of the shock increases. A similar argument holds for the right-party, where the difference between its highest probability of success and the probability associated to the optimal short-run strategy is given by \( \pi^3 - \pi^2 = \int f(\alpha) d(\alpha) + \int f(\alpha) d(\alpha) \) which is increasing with the variance if \( \eta < \bar{\alpha} \) and \( \epsilon > \bar{\alpha} \).

Once we have established the former result, we can look at the implications of this result on the actual fiscal policy which are chosen by the parties. In particular, we can show the following:

**Corollary 2.** The higher is \( \sigma^2 \), the more likely that both parties will deviate from their optimal one period strategy.

**Proof.** First of all, recall that \( \frac{d\pi^1}{d\sigma^2} = 0, \frac{d\pi^2}{d\sigma^2} < 0, \frac{d\pi^3}{d\sigma^2} > 0 \). Let us consider the problem of the left-wing party. From (4.4), taking into account the derivatives above, we easily obtain that \( \frac{dG^L}{d\sigma^2} < 0 \).

In the case of the right-wing party, looking at (4.6), we can similarly conclude that \( \frac{dG^R (0, \bar{z}_1)}{d\sigma^2} < 0 \) and \( \frac{dG^R (0, \bar{z}_1)}{d\sigma^2} < 0 \). [\( \blacksquare \)]

Corollary 2 shows that the larger is the variance of the shock, the more likely that the parties will become moderate. Consider, for example, the left-wing party. When the variance of the shock increases, the probability that a large fraction of middle agents will be hit by the good shock increases. If the left-wing party decreases the level of redistribution so that these agents are not allowed to invest, their future income may be lower that the future average income and they may end up voting for the left party. Similarly, a higher volatility of the shock induces the right-wing party to move away from its radical one-period optimal strategy and to become moderate.
In the next section, I will construct some numerical examples where, for high enough values of $\sigma^2$, the right-wing party optimal strategy will be given by $\tau_1 = \tau_1^2$. It will be also shown that this "moderate" strategy may cause the right party to win the elections in the first period, although the median voter in that period is poorer than the average (see example 2 in the next section).

6. Some examples

This section presents two examples, where I assume that the random variable $\alpha \in [\underline{\alpha}, \bar{\alpha}]$ follows a uniform distribution with mean $\bar{\alpha}$ and variance $\sigma^2$. In these exercises, I will keep the mean constant and allow the variance to vary by changing the support of the distribution.

The solutions to the lifetime maximization problems of the two parties are shown in tables 1 to 6. Entries in the tables represent the expected value of the lifetime utility of each party's constituencies (that is, the poor and the rich), for given $\tau_1$ (rows) and $\sigma^2$ (columns). For instance, let us consider the case of the left party. The first row in table 1 shows the values of the expected utility for the poor when $\tau_1 = 1$, whereas the second row shows these values when $\tau_1 = \tau_1^2$ (see (4.3)). Similarly, in the case of the right party, the three rows in table 2 show the value of the expected utility of the rich when $\tau_1 = 0, \tau_1 = \tau_1^2, \tau_1 = \tau_1^1$, respectively (see eq.(4.6)). Clearly, for each value of the variance (column), parties will choose the level of taxation which corresponds to the highest entry.

Tables 3 shows the electoral results in the first period. According to (4.7), in order to decide which party will win the elections in the first period, we have to compare the indirect utility of the middle class for the different policies chosen by the two parties.

The first example, which will be treated as the baseline case, assumes that the
parameters of the models take the following values:

<table>
<thead>
<tr>
<th>The Baseline Case</th>
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<tbody>
<tr>
<td>$\mu_1$</td>
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<td>0.35</td>
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In this case, we will look at the political equilibria for three different values of the variance of the shock:

$$\left(\sigma^2_s\right) = \begin{pmatrix} 0.006 & 0.018 & 0.037 \end{pmatrix}$$

where $s = 1, 2, 3$. For each value of $\sigma^2$, we calculated the probabilities of success of the left party $\pi^1, \pi^2, \pi^3$ using the procedure described in section 3. The probabilities turned out to be equal to:

$$\Pi = (\pi_{is}) = \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0.489 & 0.28 & 0.196 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}$$

where $\pi_{is} = \pi^i$ when $\sigma^2 = \sigma^2_s$ with $i, s \in \{1, 2, 3\}$.

For this example, the political equilibrium is described by tables 1, 2, and 3. First of all, note that all entries in the first row of table 1 are higher than those in the second row. Thus, the left party always chooses $\tau_1 = 1$ for any level of the variance.\(^{11}\) On the other hand, as table 2 shows, the right party always sets $\tau_1 = 0$ when $\sigma^2 = 0.006$, but it moves to $\tau_1 = \tilde{\tau}_1^2 = 0.286$ when $\sigma^2 = 0.018$ and $\sigma^2 = 0.037$, in order to capture a large increase in the probability of reelection.\(^{12}\) However, from table 3 we can conclude that the median voter always votes in favor of the left-wing party, even when the right party chooses the higher tax rate.

\(^{11}\)In these examples, $\pi^3$ is always equal to $\pi^1$. Therefore, there is no reason for the left party to decrease the level of taxation.

\(^{12}\)Recall that $\tilde{\tau}_1^2$ is calculated using (3.5) where $i = 2$. 

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Let us now construct another example. In particular, example 2 alters the fractions of the poor and the rich agents so that $\mu_1 = 0.39$ and $\mu_3 = 0.18$. In this case, we study the political equilibrium for the following values of the variance of the shock:

$$\left(\sigma_s^2\right) = \begin{pmatrix} 0.006 & 0.013 & 0.02 \end{pmatrix}$$

The corresponding probabilities are:

$$\Pi' = \left(\pi'_{is}\right) = \begin{pmatrix} 0.5 & 0.5 & 0.5 \\ 0.068 & 0.047 & 0.038 \\ 0.5 & 0.5 & 0.5 \end{pmatrix}$$

The new political equilibrium is now described by tables 4, 5 and 6. It is immediate to notice that, in this example, when the variance of the shock is equal to 0.02, not only the right-wing party chooses a positive level of taxation $\tau_1^2 = 0.388$, but it also wins the election in the first period and it is almost sure to be reelected in the second one, given $\pi'_{23} = 0.038$.

7. Conclusion

This paper links political instability to aggregate uncertainty in a two-period model where two political parties choose the level of redistribution in order to maximize the welfare of their constituencies. The electoral results in the second period depend on the median voter’s income which, in turn, depends on both the realization of an aggregate shock and the policy which was implemented in the first period.

In the absence of the random shock, or if parties were dictators, they would replicate the same strategy over time: a left-wing dictator would always maximize the amount of redistribution, whereas a right-wing dictator would always
minimizes it. However, when the real aggregate shock is introduced, other politico-economic equilibria may arise and this work analyzes how do they depend on the volatility of the shock.

The main results of the paper are the following. First, we show that the optimal radical policies, which would be chosen by the parties in the dictatorship case, induce a future distribution of income which is not the most favorable for their future reelection. Therefore, an intertemporal trade-off arises for the parties between maximizing current utility and maximizing the probability of electoral success. Second, the higher the volatility of the shock, the higher the increase in the probability of electoral success if parties move from their short-run optimal policies to a more conservative policy.

Section 6 presents two numerical exercises that are performed in order to illustrate these results. The aggregate shock is assumed to follow a uniform distribution with constant mean; instead, the variance is allowed to change through variations of the support of the distribution. These examples show how changes of the variance of the shock may substantially affect the politico-economic equilibrium. In fact, in both examples, when the volatility is high enough, the optimal level of redistribution for the right party changes from zero to a positive level; moreover, in the second example, the electoral results change as a consequence of the increased volatility.

An interesting extension of this work would be to incorporate the analysis in a more dynamic setting, in order to study the relationship between real uncertainty, political uncertainty, and economic growth. This line of research is missing in most of the recent literature about endogenous fiscal policy and growth, where the distribution of income is stable over time and there cannot be any shift in the winning majority (see, for example, Alesina and Rodrik [5], Bertola [9], Persson and Tabellini [21]).
In a overlapping generations structure, conflicts of interests between agents of different ages belonging to the same class may arise. It would therefore be interesting to analyze a politico-economic model with both inter and intra-classes conflicts, even though computing dynamic political equilibria with agents' heterogeneity is usually a complicated task (see Krusell and Rios-Rull [15]).
References


Example 1

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<tr>
<th>Table 1: The Left-Wing Party</th>
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<td>2.033</td>
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Example 2

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<th>Table 6: The Electoral Results</th>
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<td>Left</td>
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