DO FIRMS COMPETE WHEN DEMAND IS LOW?
A Model of Spatial Differentiation

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Abstract

In a spatial competition model, changes in firms' competitive behaviour may occur when the hypothesis that individual gross surplus is positive in equilibrium is relaxed. We prove that there exists a region of the relevant parameter where firms' behaviour mimics collusion, while in another range they find it optimal to isolate from each other and behave monopolistically.

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1. Introduction

The horizontal differentiation model introduced by Hotelling (1929) has produced a wide stream of literature focusing on how firms startegically exploit the possibility of choosing their respective locations in the product space in order to soften price competition.

Two major points are worth stressing. First, the attention paid to product differentiation and its bearings on equilibrium profits has left many questions on the nature of price competition in such settings virtually unanswered. Second, horizontal differentiation has generally been dealt with under the assumptions of inelastic demand and full market coverage.

This paper is devoted to the investigation of short-run price behaviour when an exogenous shock affecting nominal magnitudes brings about a contraction in market demand, which in turn may induce firms to noncooperatively adopt price rules of monopolistic or quasi-cooperative flavour. Thus, the present analysis gives a temptative but suggestive answer in the positive to the question whether the intensity of competition may be inversely related to the level of market demand, as it has been informally raised by Stiglitz (1984).

2. The model

Consider a duopoly in which firms 1 and 2 sell a physically homogeneous good along a segment of unit length. Unit production costs are assumed to be constant and can be normalised to zero without any loss of generality. Consumers are uniformly distributed over the segment, with total density 1. Each consumer buys at most one unit of the good, drawing from consumption a gross surplus s - invariant across consumers - and paying a full price p_i+d^2, where p_i is the mill price charged by firm i and d is the distance between the consumer and the patronized firm. Total demand is equal to 1, i.e., all consumers are served, if the indirect utility function

$$U = s - p_i - d^2 \qquad (1$$

is non-negative for all consumers. In such a case, as shown by D'Aspremont et al. (1979), the Nash symmetric equilibrium prices turn out to be $p_i = 1$, while the equilibrium locations are the endpoints of the segment, 0 and 1. However, this solution clearly requires $s \geq 5/4$. Provided that location can be regarded as a long run choice, one might argue how firms' pricing behaviour may be affected by an exogenous shock reducing s below the above threshold level. Given that

s can be interpreted as the reciprocal of the marginal utility of income (or money), this amounts to investigating, within the relevant range of parameters, the bearings of a shock affecting nominal income on the results yielded by this class of models (at least in this simplified version).[1] We shall proceed as follows. We shall study firms' optimal behaviour for $s \in]0, 5/4[$. Two basic settings emerge. For very low values of s, each firm's demand is independent of the other's, so that the market does not allow for any strategic interaction, and monopolistic pricing is necessarily observed. For higher values of s, demands overlap, so that some scope arises for strategic interaction - although this is not necessarily exploited - and firms' behaviour must be studied by deriving their reaction functions in the price space.

2.1. Isolated markets

Since for the consumer located in 1/2 transportation costs amount to 1/4, it is clear that for s<1/4 the profit accruing to each firm is independent of the rival's behaviour. Thus, both firms behave monopolistically, maximizing a profit function defined as follows:

$$\pi_i^M = p_i\sqrt{s - p_i}\,; \quad i = 1, 2, \tag{2}$$

where $\sqrt{s - p_i}$ defines firm i's demand. This yields:

$$p_i^* = \frac{2}{3}s; \quad \pi_i^* = \frac{2}{3}s\sqrt{\frac{s}{3}}. \tag{3}$$

2.2. Overlapping demands

If $s \in [1/4, 5/4[$, firms may potentially compete for the consumers located in an area at the center of the linear city, which widens as s increases. To derive the firms' reaction functions, we have to proceed in two steps, defining (a) the profit function of firm i for any price charged by firm j, $\pi(p_i \mid p_j)$; (b) the optimal pricing rule for firm i given the price charged by firm j.

(a) Derivation of the profit function. First, notice that $\pi(p_i \mid p_j)$ is actually independent of p_j for $p_j > \hat{p}_j$, where $\hat{p}_j = 2\sqrt{s} - 1$ for s<1, while $\hat{p}_j = s$ for $s \geq 1$. Thus, for any s, $\pi_i = \pi_i^M = p_i\sqrt{s - p_i}$,

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1. Provided that locations are either exogenous or fixed in the short run, the present analysis could be easily extended to both the case of n firms and that of the circular city described by Salop (1979).

if $p_j \geq \hat{p}_j$.

If instead $p_j < \hat{p}_j$, then there exists a price $\tilde{p}_i(p_j)$ such that the net surplus of the consumer who is indifferent between the two firms is nil:

$$\tilde{p}_i = p_j + 2\sqrt{s - p_j} - 1. \tag{4}$$

If firm i sets a price $p_i \geq \tilde{p}_i(p_j)$, firm i gains monopoly profits as defined by (2); otherwise, if $p_i < \tilde{p}_i(p_j)$, she obtains the following duopolistic profits:

$$\pi_i^D = \frac{p_i}{2}(p_j - p_i + 1), \tag{5}$$

with π_i^D intersecting π_i^M at $\tilde{p}_i(p_j)$ from below.[2] Therefore, we have established the following:

PROPOSITION 1: assume $s \geq 1/4$; then, for any $p_j < \hat{p}_j$, profits of firm i are defined as $\pi_i(p_i \mid p_j) = \min(\pi_i^M; \pi_i^D)$.

(b) Derivation of the reaction functions. The reaction function of firm i is defined as the optimal choice of p_i given p_j. Taking into account the profit function $\pi_i(p_i \mid p_j)$ referred to in Proposition 1, three possibilities arise. Denoting with p_i^M and p_i^D the prices maximizing, respectively, π_i^M and π_i^D, which are concave and single-peaked, then the optimal price is: (i) $p_i^M \quad if \quad \tilde{p}_i(p_j) \leq p_i^M$; (ii) $p_i^D \quad if \quad \tilde{p}_i(p_j) > p_i^M \quad \& \quad p_i^D < \tilde{p}_i(p_j)$; (iii) $\tilde{p}_i(p_j) \quad if \quad p_i^D > \tilde{p}_i(p_j) > p_i^M$.

Consider case (i). We have $\tilde{p}_i(p_j) \leq p_i^M$ if

$$p_j + 2\sqrt{s - p_j} - 1 \leq \frac{2}{3}s, \tag{6}$$

implying

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2. Notice that $\tilde{p}_i > 0$ for $p_j < 2\sqrt{s} - 1$, which coincide with $\hat{p}_j$ when s<1. If $\hat{p}_j = s$, as for $s \geq 1$, then for all $p_j \in [2\sqrt{s} - 1, s]$, $\tilde{p}_i < 0$, and for all $p_i > 0$ firm i's profits are given by π_i^M.

$$p_j \geq \overline{p}_j = \frac{2}{3}s - 1 + 2\sqrt{\frac{s}{3}}. \qquad (7$$

Then, for all $p_j \geq \overline{p}_j$, firm i's reaction function is

$$p_i^*(p_j) = \frac{2}{3}s. \qquad (8$$

For all $p_j < \overline{p}_j$, we have to check whether (ii) or (iii) holds. Clearly, (ii) holds for all values of $p_j < \overline{p}_j$ satisfying

$$p_i^D(p_j) < \tilde{p}_i(p_j) \quad \Leftrightarrow \quad \frac{p_j + 1}{2} < p_j + 2\sqrt{s - p_j} - 1, \qquad (9$$

while for all other values of $p_j < \overline{p}_j$, case (iii) holds. Solving (9), we get

$$p_j < p_j^0 = -5 + 4\sqrt{s+1}; \qquad (10$$

notice that $p_j^0 > 0 \quad \forall \quad s > 9/16$.

For finite values of s, we may now sum up firms' pricing behaviour in table 1.

INSERT TABLE 1 HERE

Notice that the strategic complementarity in prices usually observed in product differentiation models (see Beath and Katsoulacos, 1991, p.22), arises here only for s>9/16. However, strategic interaction does not necessarily yields a Nash equilibrium in prices where the latter are strategic complements. Indeed, the relation between equilibrium prices and gross surplus is described by the following:

PROPOSITION 2: (i) for $s \leq 3/4$, p*=2s/3; (ii) for $s \in]3/4, 5/4[$, p*=s-1/4; (iii) for $s \geq 5/4$, p*=1.

PROOF. In order to prove the above Proposition, it suffices to notice that $\overline{p}_j \leq 2s/3$ if $s \leq 3/4$; and $p_j^0 \geq 1$ if $s \geq 5/4$. *Q.E.D.*

A few remarks are now in order. For $s \in]1/4, 3/4]$, although demands overlap and thus in principle competition is possible, firms find it optimal not to compete; they behave monopolistically and the market is not fully covered. Furthermore, for $s \in [3/4, 5/4[$, a symmetric Nash equilibrium emerges in an area where reaction functions are downward sloping and thus strategic substitutability is observed. In such an equilibrium, the net surplus of the indifferent consumer located at the middle of the segment is nil. Finally, it is most noteworthy that in this case strategic interaction leads firms to adopt a pricing behaviour which mimics collusion.

3. Conclusions

The main finding of this paper is that firms' propension to compete is inversely related to the level of demand, here approximated by the individual gross surplus from purchase. There are demand configurations which might support competition with positive profits, and yet firms find it optimal to set prices in a monopolistic or at least quasi-cooperative way. This is in line with that large body of the literature aimed at showing a procyclical pattern of competitiveness and thus a countercyclical pattern of the real price, e.g. Rotemberg and Woodford (1991). Two extensions, namely, the endogenisation of costs and the explicit modelling of individual demand price responsiveness, as suggested by Stiglitz (1984), are topics left to future research.

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$s<1/4$	$p_i = \frac{2}{3}s$
$s \in [1/4, 9/16[$	$p_i = p_j + 2\sqrt{s - p_j} - 1 \quad if \quad p_j < \overline{p}_j$
	$p_i = \frac{2}{3}s \quad if \quad p_j \geq \overline{p}_j$
$s \geq 9/16$	$p_i = \frac{p_j + 1}{2} \quad if \quad p_j < p_j^0$
	$p_i = \tilde{p}_i(p_j) = p_j + 2\sqrt{s - p_j} - 1$ $if \quad p_j \in [p_j^0, \overline{p}_j[$
	$p_i = \frac{2}{3}s \quad if \quad p_j \geq \overline{p}_j$

Table 1. Optimal pricing behaviour