Delegation and Product Differentiation

Luca Lambertini*

Dipartimento di Scienze Economiche
Università degli Studi di Bologna

September 1993

JEL classification numbers: L13, L22

Abstract

The effects of the delegation of control to managers are investigated in a duopolistic market for differentiated goods. It appears that delegation is profitable to shareholders under Cournot competition, provided that the rival firm maximizes profit.

* *Acknowledgements*

I wish to thank Flavio Delbono, Paolo Garella and Gianpaolo Rossini for helpful suggestions. The usual disclaimer applies.

1. Introduction

In his influential paper, Vickers (1985) shows, *inter alia*, that the separation between ownership and control, and, consequently, the presence of sales in the objective function of a firm, can be advantageous from the shareholders' viewpoint, if the rival firms adopt a profit-maximizing behavior. This result is derived under the assumption of Cournot competition with a homogeneous good.[1]

In this paper, we investigate the same perspective in a market for differentiated goods, under both quantity and price competition. It is shown that Vicker's (1985) result is obtained in the limit, as the substitutability between products increses. Moreover, under price competition, attaching a positive weight to sales turns out to be detrimental to firm's profit, since any increase in supply due to managerial preferences squeeze price and thus profits.

2. Cournot competition

This section is devoted to the analysis of the equilibrium arising from quantity competition between a profit-maximizing firm and a firm whose shareholders delegate the control over their assets to managers. We assume that information is complete and symmetric, in order to avoid any incentive problem between owners and managers and concentrate exclusively on the effects brought about by delegation.

Following Dixit (1979) and Singh and Vives (1984), we assume the representative consumer maximizes the following quadratic utility function:

$$U = ax_1 + ax_2 - \frac{1}{2}(bx_1^2 + 2dx_1x_2 + bx_2^2) + y, \quad a,b,d \geq 0, \tag{1}$$

where x_i, i=1,2, defines the quantity of good i being consumed, while y is a numeraire good. Given p_y=1, the utility function described by (1) yields the following linear inverse demand functions:

$$p_i = a - bx_i - dx_j, \quad i,j = 1,2, \quad i \neq j, \tag{2}$$

1. In two related papers, Fertshman and Judd (1987) and Sklivas (1987) show that firms' owners can distort managers' objective from strict profit maximization for strategic reasons, and the separation between ownership and control gives the owners the opportunity to take advantage of this because of the effects on rival managers' behavior.

in the region in which both quantities are positive. Moreover, we assume $b \geq d$, which means that the ratio $r = d/b \in [0, 1]$ is a measure of the degree of differentiation. If r=1, the two goods are perfect substitutes, while if r=0 the respective demands are completely independent.

As for firms, assume firm 1 attaches a weight θ to sales while firm 2 is strictly a profit maximizer. Firms adopt the same technology, which shows constant returns to scale:

$$C_i = cx_i, \quad i = 1,2, \quad c \leq a. \tag{3}$$

Firms noncooperatrively maximize w.r.t. quantity the following objective functions, respectively:

$$M_1 = (p_1 - c)x_1 + \theta x_1, \tag{4}$$

$$\pi_2 = (p_2 - c)x_2. \tag{5}$$

The first order conditions are:

$$\frac{\partial M_1}{\partial x_1} = a - c - 2bx_1 - dx_2 + \theta = 0, \tag{6}$$

$$\frac{\partial \pi_2}{\partial x_2} = a - c - 2bx_2 - dx_1 = 0, \tag{7}$$

and the equilibrium quantities are:

$$x_1^* = \frac{2ab - 2bc - ad + cd + 2b\theta}{(2b - d)(2b + d)}; \tag{8}$$

$$x_2^* = \frac{2ab - 2bc - ad + cd - d\theta}{(2b - d)(2b + d)}. \tag{9}$$

Substituting (8-9) into (4), we obtain the equilibrium profit of firm 1 as a function of θ:

$$\pi_1^* = \frac{(2ab - 2bc - ad + cd + 2b\theta)(2ab^2 - 2b^2c - abd + bcd - 2b^2\theta - d^2\theta)}{(2b - d)^2 (2b + d)^2}. \tag{10}$$

Consequently, shareholders can properly choose the value of theta, in order to maximize (10):

$$\frac{\partial \pi_1^*}{\partial \theta} = \frac{2abd^2 - 2bcd^2 - ad^3 - cd^3 - 8b^3\theta + 4bd^2\theta}{(2b-d)^2(2b+d)^2} = 0, \tag{11}$$

yielding:

$$\theta^* = \frac{d^2(a-c)(2b-d)}{4b(2b^2-d^2)}, \tag{12}$$

which is always positive.[2] Substituting (12) into (11), we get

$$\pi_1^* = \frac{(a-c)(2b-d)^2}{8b(2b^2-d^2)}. \tag{13}$$

Let $\pi_1^*/\pi_2^* = R^*(\theta^*)$. It is easy to check that, given $b \geq d$,

$$R^*(\theta^*) = \frac{2(2b-d)^2(2b^2-d^2)}{(4b^2-2bd-d^2)} \geq 1 \tag{14}$$

and

$$\lim_{d \to 0} R^*(\theta^*) = 1; \quad \lim_{d \to b} R^*(\theta^*) = 2. \tag{15}$$

This means that if goods are perfect substitutes, i.e., $d=b$, Cournot competition with homogeneous products yields the same results obtained by Vickers (1985, p.142).

Alternatively, we can investigate the problem described by the following:

$$\max_{\theta} \quad R^*(\theta). \tag{16}$$

---

2. The second order conditions for a maximum are also met.

Define $\theta' = argmaxR^*(\theta)$. We have that:

$$\theta' = \frac{d(a-c)(2b-d)}{4b^2-2bd-d^2}, \tag{17}$$

which, in the viable range of the parameters, is always positive, and yields:

$$R^*(\theta') = \frac{(2b-d)^2}{4b(b-d)}, \tag{18}$$

which is greater than 1. It remains to be established whether $\theta' > \theta^*$. After some simple, albeit tedious calculations, we have:

$$\theta' > \theta^* \quad iff \quad r^3 - 2r^2 - 4r + 8 > 0, \quad r = \frac{d}{b}. \tag{19}$$

The roots of (19) are r_1=-2, r_2=r_3=2, which lie outside the interval [0, 1]. Accordingly, $\theta' > \theta^*$, i.e., the value of θ that maximizes the ratio between π_1^* and π_2^* is larger than the value of θ maximizing π_1^*. This implies that the weight attached by the managers of firm 1 to sales shouldn't be too large, in order to not squeezing profits.

3. Bertrand competition

Assume now firms compete in prices, with the same technology described by (3). The direct demand system is:

$$x_i = \frac{a}{b+d} - \frac{bp_i}{b^2-d^2} + \frac{dp_j}{b^2-d^2}, \quad i,j = 1,2, \quad i \neq j. \tag{20}$$

The first order conditions are, respectively:

$$\frac{\partial M_1}{\partial p_1}=\frac{a(b-d)+dp_2-d(2p_1-c-\theta)}{b^2-d^2}=0; \qquad (21)$$

$$\frac{\partial \pi_2}{\partial p_2}=\frac{a(b-d)+dp_1-d(2p_2-c)}{b^2-d^2}=0. \qquad (22)$$

Solving the system (21-22), we obtain:[3]

$$p_1^*=\frac{a(b-d)+bc}{2b-d}-\frac{2b^2\theta}{4b^2-d^2}, \qquad (23)$$

$$p_2^*=\frac{ab(b-d)+b^2c}{2b-d}-\frac{b^2d\theta}{4b^2-d^2}, \qquad (24)$$

yielding

$$\pi_1^*=b(2ab^2-2b^2c-abd+bcd-ad^2+cd^2-2b^2\theta)(2ab^2-2b^2c$$
$$-abd+bcd-ad^2+cd^2+2b^2\theta-d^2\theta)/(b^2-d^2)(2b-d)^2(2b+d)^2, \qquad (25)$$

$$\pi_2^*=\frac{b(2ab^2-2b^2c-abd+bcd-ad^2+cd^2-bd\theta)^2}{(b^2-d^2)(2b-d)^2(2b+d)^2}. \qquad (26)$$

Maximizing π_1^* w.r.t. θ, we obtain:

$$\theta^*=\frac{d^2(a-c)(2b^2-bd-d^2)}{4b^2(d^2-2b^2)}, \qquad (27)$$

which in the viable range of the parameters is always negative.[4] The interpretation of this result is straightforward: under Bertrand competition, the presence of sales in the objective function of the managerial firm implies too large a supply and a lower profit margin. Accordingly, in

3. The second order conditions are also satisfied.

4. See Lyons (1986) for the derivation of an analogous result with homogeneous goods.

order to maximize profits, shareholders attribute a negative weight to sales, or, alternatively, managers must be work-averse.[5]

The corresponding equilibrium profits are, respectively:

$$\pi_1^* = \frac{(a-c)^2(b-d)(2b+d)^2}{8b(b+d)(2b^2-d^2)}; \tag{28}$$

$$\pi_2^* = \frac{(a-c)^2(b-d)(4b^2+2bd-d^2)^2}{16b(b+d)(2b^2-d^2)^2}. \tag{29}$$

Evaluating $R^*(\theta^*)$, we have:

$$R^*(\theta^*) > 1 \quad iff \quad b < -\frac{3}{4}d, \tag{30}$$

which means that, under Bertrand competition, attaching a weight to sales, i.e., delegating control to managers is detrimental to the performance of the firm, whose profit turns out to be lower than those accruing to a strictly profit-seeking firm.

Furthermore, exploring the following perspective:

$$\max_{\theta} \quad R^*(\theta), \tag{31}$$

we get:

$$\theta' = \frac{d(a-c)(b-d)(2b+d)}{b(4b^2+2bd-d^2)}, \tag{32}$$

which is always positive. As in the Cournot framework, we have thus $\theta' > \theta^*$, but for the fact that under Bertrand competition, profit maximization requires $\theta^* < 0$.

5. See Dixon and Manning (1986) for an example.

4. A comparative evaluation

In order to assess the role of sales in the two frameworks, we have to evaluate the following ratio:

$$\frac{\pi^*_{1C}}{\pi^*_{1B}} = \frac{(2b-d)^2(b+d)}{(a-c)(b-d)(2b+d)^2}, \tag{33}$$

where subscripts B and C stand for Bertrand and Cournot competition, respectively. The above ratio is greater than 1 if $a>b>d>c$.

Moreover, it is easy to show that, in the viable range of the parameters:[6]

$$\lim_{d\to 0} \theta^*_C = 0, \quad \lim_{d\to b} \theta^*_C = \frac{a-c}{4}; \tag{34}$$

$$\lim_{d\to 0} \theta^*_B = \lim_{d\to b} \theta^*_B = 0. \tag{35}$$

A last remark is now in order. As pointed out by Vickers (1985, pp.141-2), attaching a weight to sales allows the managerial firm to obtain the same profit as in the sequential game in which she acts as a Stackelberg leader in the relevant strategic variable. As a result, it turns out that, while this choice is profitable under Cournot competition, it is not under Bertrand competition, as pointed out by Gal-Or (1985) and Dowrick (1986): under price competition, since the reaction functions in the price space are upwards sloping,[7] both firms would prefer to be the follower, i.e., in the jargon of this paper, both would prefer to be strictly profit-maximizers.

---

6. Cfr. Vickers (1985, p.142).

7. In other words, products are strategic complements in prices (Bulow et al., 1985).

5. Conclusions

The inclusion of sales in the objective function of a competitive firm operating in a market for differentiated goods has somewhat different implications, according to the kind of competition, i.e., in prices or in quantities.

Under quantity competition, by allowing managers to attribute a positive weight to sales, the shareholders can obtain the same profits they would gain should their firm maximize profits acting as a Stackelberg leader.

The same holds under price competition, but for the fact that, given the strategic complementarity in prices between goods, every firm would prefer to be a profit-maximizer, or, in the sequential game jargon, the follower.