# Vertical Differentiation and Import Reducing Tariff

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*Abstract*

*Two monopolists operate in two countries which differ only for their per capita income. Each firm sells a single product which is vertically differentiated. If trade opens, the firm operating in the poorer country starts to export to the richer. This might induce the government of the richer country to set an import reducing tariff that could, under certain conditions, benefit also the firm of the poorer country.*

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## 1. Introduction

Vertical differentiation has been thought of as one of the most prominent determinants of trade among similar countries in manufactured goods, since the pioneering book of Linder (1961). More recently, Krishna (1987, 1990) has tried to endogenize the choice of product quality by a monopolist on the trace of Spence (1975) model. Within a framework of oligopolistic strategic interaction Shaked and Sutton (1982, 1983), provided new insights on the choice of product quality under Bertrand competition in a closed economy. Shaked and Sutton (1984), and Motta (1992) extended this analysis to international trade, in order to investigate the effect of quality choice on firm profitability and the spectrum of goods being offered.

Following these contributions we attempt to analyse the twin problem of modelling price and quality choices when trade is open between two countries which differ only for the level of per capita income.

Our purpose is to investigate the "comparative advantage" that is brought about by price-quality choices by firms in their respective countries. We start by considering two single product monopolists operating in their domestic markets and choosing quality once and for all. In this sense we think of quality as of an irreversible commitment. Subsequently, the separation between the two markets vanishes, giving rise to a duopolistic market in which firms compete in prices. The kind of trade associated with this duopoly is a one way trade with no overlapping between imports and exports. This might induce the government of the importing country to set an import reducing tariff in order to protect the domestic firm.

## 2. The autarky model

We consider two monopolies each producing a manufactured good that can be vertically differentiated. Each monopolist operates in its domestic market and faces consumers who are heterogeneous in terms of their incomes. In order to describe the income distribution , we think of consumers as being uniformly distributed according to $f(\theta)$, defined over $[\underline{\theta},\overline{\theta}]$, $\underline{\theta} \geq 0, \overline{\theta} - \underline{\theta} \geq \frac{\overline{\theta}}{3}$. Parameter $\theta$ indicates the marginal willingness to pay for quality, which grows as income increases, and it can be interpreted as the reciprocal of the marginal utility of income. The population of consumers is normalised to $\overline{\theta} - \underline{\theta}$. Each consumer has unit demand, i.e., either he buys one unit of the differentiated good or nothing.[1]

We assume that each consumer has a surplus function which determines his choice between buying and not buying. He buys if the following condition is met:

$$U = \theta q - p \geq 0, \tag{1}$$

where $p$ is the price of the good, while $q$ is the quality.

For each price-quality pair set by the producer, we are able to derive the demand function:[2]

$$x = \overline{\theta} - \frac{p}{q} \tag{2}$$

On the production side we assume that costs are convex in quality[3] and linear in quantity:

$$C = q^2 x. \tag{3}$$

The monopolist maximizes his profit function with respect to quality and price:

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1. The reason for the restriction $\overline{\theta} - \underline{\theta} \geq \frac{\overline{\theta}}{3}$ comes from the need to avoid situations of excess supply by a monopolist in autarky. Since every consumer has unit demand, the normalised population has to be greater or equal to $\frac{\overline{\theta}}{3}$.

2. Since consumers are uniformly distributed over $[\underline{\theta}, \overline{\theta}]$, the value of $\theta$ associated with the last consumer who buys is $p/q$ (from condition (1) met as an equality). Therefore, demand is given by the difference between the $\theta$ of the richest consumer $(\overline{\theta})$ and the price-quality ratio offered by the monopolist $(p/q)$, multiplied by the cumulative uniform distribution $\int_{\underline{\theta}}^{\overline{\theta}} f(\theta) d\theta = 1$. This implies that the maximum value $\underline{\theta}$ can reach is 3, since the difference $\overline{\theta} - \underline{\theta}$ cannot be lower than $\frac{\overline{\theta}}{3}$ and population has been normalised to 1.

3. The assumption of convex costs is the only one giving rise to meaningful solutions when dealing with an oligopolistic market, that we shall consider in the next section. For the analysis of the relationship between product quality and the curvature of the cost function, see Lambertini (1993).

$$\pi_M = \left(\bar{\theta} - \frac{p}{q}\right)(p - q^2) \tag{4}$$

We can therefore derive two first order conditions:

$$\frac{\partial \pi_M}{\partial p} = \bar{\theta} - \frac{2p}{q} + q = 0 \tag{5}$$

$$\frac{\partial \pi_M}{\partial q} = p + \frac{p^2}{q^2} - 2\bar{\theta} q = 0, \tag{6}$$

from which we get the following solutions:

$$p_M^* = \frac{2}{9}\bar{\theta}^2 \tag{7}$$

$$q_M^* = \frac{\bar{\theta}}{3}. \tag{8}$$

Then maximum profit and optimal quantity will be:

$$\pi_M^* = \frac{\bar{\theta}^3}{27} \tag{9}$$

$$x_M^* = \frac{\bar{\theta}}{3} \tag{10}$$

We now calculate, using (1) and (3), the social welfare of the monopolistic market arrangement, as the sum of the consumer and producer surpluses:[4]

$$SW_M = \int_{\frac{2}{3}\bar{\theta}}^{\bar{\theta}} (\theta q - q^2) d\theta \tag{11}$$

that yields, in equilibrium:

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**4.** The lower limit of the integral is given by $\bar{\theta} - x_M^* = \frac{2}{3}\bar{\theta}$. The value of $SW$ over $[\underline{\theta}, \ \frac{2}{3}\bar{\theta}[$ is nil, since no consumer buys in this interval.

$$SW_M = \frac{3}{54}\overline{\theta}^3 \qquad (12)$$

while consumer surplus amounts to

$$CS_M = \frac{1}{54}\overline{\theta}^3 \qquad (13)$$

**3. The duopoly model**

The monopolist choice we have considered in the previous section may be thought of as the result of a decision taken in conditions of autarky. Now we wish to see what happens when trade opens. We consider two countries, whose only difference is the level of per capita income. Neither natural nor administrative barriers separate the two countries. A crucial assumption is made as to quality, which is chosen by the monopolists in autarky in an irreversible way.

The new market arrangement is a duopoly made up by the two former monopolists, whose only decision variable is now price.

We assume that country $D$ (domestic) is richer than country $F$ (foreign). Consumers have heterogeneous marginal willingness to pay for quality and the richest consumer of the poorer country has the same marginal willingness to pay of the poorest consumer of the richer country.[5] Hence, we can define the locus of indifference between the two goods, and the threshold beyond which the consumers buy the low quality good, and below which the consumers do not buy the good, since net surplus is negative.

In figure 1 we describe the distribution of consumers after the opening of trade.

INSERT FIGURE 1

5. This assumption is needed to avoid discontinuous demand functions, and implies that the support of $f_D(\theta)$ and $f_F(\theta)$ are contiguous, i.e., $\underline{\theta}_D = \overline{\theta}_F$.

In figure 1, $h$ and $k$ define the location in the price-quality space ($\theta$) of two sorts of indifferent consumers: in $k$, we have a consumer which is indifferent between either buying the low quality good or not to buy anything, while $h$ locates the consumer indifferent between the two goods. The value of $k$ is derived according to the following indifference condition:

$$\theta_F q_F - p_F = 0 \tag{14}$$

Solving (14), we get:

$$\theta_F = \frac{p_F}{q_F} = k \tag{15}$$

The value of $h$ is given by the solution of the following indifference condition:[6]

$$\theta q_F - p_F = \theta q_D - p_D \tag{16}$$

leading to:

$$\theta = \frac{3(p_D - p_F)}{\bar{\theta}_D - \bar{\theta}_F} = h \tag{17}$$

We are now able to derive the demand function for each duopolist:

$$x_D = \bar{\theta}_D - \frac{3(p_D - p_F)}{\bar{\theta}_D - \bar{\theta}_F} \tag{18}$$

$$x_F = \frac{3(p_D - p_F)}{\bar{\theta}_D - \bar{\theta}_F} - 3\frac{p_F}{\bar{\theta}_F} \tag{19}$$

Each duopolist maximizes profit using his price as a control variable. The solution concept is a Bertrand-Nash equilibrium. The profit functions are, respectively:

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6. Since we haven't any prior knowledge of the location of $h$, we do not use subscript for $\theta$ in (16).

$$\pi_D = \left(\overline{\theta}_D - \frac{3(p_D - p_F)}{\overline{\theta}_D - \overline{\theta}_F}\right)(p_D - q_D^2) \tag{20}$$

$$\pi_F = \left(\frac{3(p_D - p_F)}{\overline{\theta}_D - \overline{\theta}_F} - 3\frac{p_F}{\overline{\theta}_F}\right)(p_F - q_F^2) \tag{21}$$

The first order conditions are:

$$\frac{\partial \pi_D}{\partial p_D} = \frac{18\overline{\theta}_D p_F - 9\overline{\theta}_F p_D - \overline{\theta}_D \overline{\theta}_F^2}{3\overline{\theta}_F(\overline{\theta}_F - \overline{\theta}_D)} = 0 \tag{22}$$

$$\frac{\partial \pi_F}{\partial p_F} = \frac{4\overline{\theta}_D^2 + 9p_F - 18p_D - 3\overline{\theta}_D\overline{\theta}_F}{3(\overline{\theta}_D - \overline{\theta}_F)} = 0 \tag{23}$$

Solving the system (22-23) with respect to $p_D$ and $p_F$, we obtain the following equilibrium prices:

$$p_D^* = \frac{\overline{\theta}_D(2\overline{\theta}_D - \overline{\theta}_F)}{9} \tag{24}$$

$$p_F^* = \frac{\overline{\theta}_D\overline{\theta}_F}{9} \tag{25}$$

Hence, substituting the equilibrium prices (24-25) into the original profit functions (20-21), we get the following equilibrium profits:

$$\pi_D^* = \frac{\overline{\theta}_D^2(\overline{\theta}_D - \overline{\theta}_F)}{27} \tag{26}$$

$$\pi_F^* = \frac{\overline{\theta}_D\overline{\theta}_F(\overline{\theta}_D - \overline{\theta}_F)}{27} \tag{27}$$

It follows that:

$$\frac{\pi_D^*}{\pi_F^*} = \frac{\overline{\theta}_D}{\overline{\theta}_F} > 1 \tag{28}$$

while in the former autarky equilibrium, the ratio between the profits of the two monopolists is:

$$\frac{\pi^*_{DM}}{\pi^*_{FM}} = \left(\frac{\overline{\theta}_D}{\overline{\theta}_F}\right)^3 \tag{29}$$

In the duopoly arrangement, we have a new ratio between the profits of the two firms operating, respectively, in the richer and the poorer market. The firm operating in the richer market faces a relative decrease of its profit *vis à vis* the firm operating in the poorer market. This simple result points to some possible reaction to protect the looser, i.e., the firm in the richer market. In order to have a complete framework for the intervention of a public authority, we have to evaluate the effects of opening of trade also on consumers. To this purpose, we have to compute the consumer surplus.

In the poorer country, the consumer surplus is the following:

$$CS_F = \int_{\underline{\theta}_F}^{k} (\theta q_F - p_F) d\theta + \int_{k}^{\overline{\theta}_F} (\theta q_F - p_F) d\theta. \tag{30}$$

The first part of the right-hand side of (30) is equal to zero, because it covers the non-buying area of the poorer country. Taking into account (15), equation (30) becomes:

$$CS_F = \int_{\frac{\overline{\theta}_F}{3}}^{\overline{\theta}_F} (\theta q_F - p_F) d\theta, \tag{31}$$

which yields

$$CS_F = \frac{4}{9}\overline{\theta}_F^2 - \frac{2}{3}\overline{\theta}_F p_F \tag{32}$$

Substituting into (32) the equilibrium values of $q_F$ and $p_F$, we get

$$CS_F = \frac{2}{27}\overline{\theta}_F^2 (2\overline{\theta}_F - \overline{\theta}_D), \tag{33}$$

which is the consumer surplus of the poorer country. Adding (33) to (27), we get the social welfare of the poorer country:

$$SW_F = \frac{2}{27}\overline{\theta}_F^2(2\overline{\theta}_F - \overline{\theta}_D) + \frac{\overline{\theta}_D\overline{\theta}_F}{27}(\overline{\theta}_D - \overline{\theta}_F)$$

$$= \frac{\overline{\theta}_F}{27}\left(4\overline{\theta}_F^2 + \overline{\theta}_D^2 - 3\overline{\theta}_F\overline{\theta}_D\right) \tag{34}$$

We now proceed to compute the welfare of the domestic (richer) country. To this purpose, we first evaluate the consumer surplus:

$$CS_D = \int_{\underline{\theta}_D}^{h}(\theta q_F - p_F)d\theta + \int_{h}^{\overline{\theta}_D}(\theta q_D - p_D)d\theta, \tag{35}$$

which yields

$$CS_D = \frac{\overline{\theta}_D^3 - 9\overline{\theta}_F^3 + 2\overline{\theta}_D^2\overline{\theta}_F + 6\overline{\theta}_F^2\overline{\theta}_D}{54}. \tag{36}$$

Adding together (26) and (36), we get the social welfare of the domestic country:

$$SW_D = \frac{3\overline{\theta}_D^3 + 6\overline{\theta}_D\overline{\theta}_F^2 - 9\overline{\theta}_F^3}{54} \tag{37}$$

We now evaluate the difference between the levels of social welfare in the two countries. Before doing that, we have to establish a sort of viability condition, defining the lower bound of the ratio between $\overline{\theta}_D$ and $\overline{\theta}_F$, below which a disequilibrium in the richer country (excess supply) appears. This condition is the extension of the parallel condition stated in section 2 and explained in footnote 1:

$$\frac{\overline{\theta}_D}{\overline{\theta}_F} \geq \frac{3}{2}. \tag{38}$$

We are then in a position to state the following:

**PROPOSITION 1:** if we compare the social welfare levels of country $D$ and country $F$, there is no value of $\frac{\overline{\theta}_D}{\overline{\theta}_F}$ belonging to the viable region, such that the welfare of the richer country is lower than the welfare of the poorer country.

**Proof:** see appendix 1.

## 4. The structure and direction of trade flows

The opening of trade has to be accompanied by the analysis of trade flows between the two countries. To this aim we have to calculate the equilibrium values of $k$ and $h$, i.e. the two discriminant points in the price-quality spectrum of the duopoly market. As a matter of fact, $k$ defines the price-quality ratio corresponding to the last consumer who buys the low quality good. We see in which country this consumer lives, hence:

$$k = \frac{\overline{\theta}_D}{3} \tag{39}$$

Moreover, for the same reason, we have to locate the consumer who is indifferent between the two goods:

$$h = \frac{2}{3}\overline{\theta}_D \tag{40}$$

These two values are needed to derive the distribution of demand between the two countries and the two goods. Together with condition (38), condition (40) states that the market demand for the high quality good is expressed only by the richer country. This implies that the latter is the only consumer and producer of the high quality good which not traded. Moreover, the richer country might consume also the low quality good. This happens if

$$h > \overline{\theta}_F. \tag{41}$$

The intuition behind (41) is that, if the consumer indexed by $h$ lies within $]\overline{\theta}_F,\ \overline{\theta}_D]$,[7] the consumers who cannot afford the high quality good buy the imported low quality good. Notice that the demand for the high quality good is the same observed under autarky; however, under condition (41), the opening of trade allows the poorer consumers in the richer country to buy the imported good, giving rise to a trade flow. Hence, the poorer country exports to the richer country, while the latter does not export anything to the poorer country. The kind of trade we

7. We consider an interval open on the left, since we cannot have simultaneously trade and the coincidence of $h$ with $\overline{\theta}_F$.

obtain is a one way trade which is not consistent with the Intraindustry Trade of Linder type. This result might appear quite odd. It comes from the assumption that the spectrum of income distribution of one country does not overlap in any point the spectrum of income distribution of the other, i.e., the poorest consumer of the richer country is richer than the richest consumer of the poorer country. Under condition (41), the total demand of the richer country is:

$$x_D = \frac{\overline{\theta}_D}{3} + h - \overline{\theta}_F \tag{42}$$

Defining the production of the high quality good as $y_D$, (42) can be rewritten as:

$$x_D = y_D + h - \overline{\theta}_F. \tag{43}$$

Hence, $h - \overline{\theta}_F$ defines the demand for the imported good.

Let us turn to the poorer country and define its total demand. If condition (41) is met, the latter is:

$$x_F = \overline{\theta}_F - k = \overline{\theta}_F - \frac{\overline{\theta}_D}{3}. \tag{44}$$

In this case, exports amount to:

$$y_F - x_F = \frac{2}{3}\overline{\theta}_D - \overline{\theta}_F, \tag{45}$$

which, according to condition (38), must be non negative. If (45) is nil, trade disappears and the demand of the poorer country becomes:

$$x_F = h - k = \frac{\overline{\theta}_D}{3}. \tag{46}$$

After having analysed trade flows, we can derive some additional insights as to the benefits of the opening of trade. Quite a bizarre phenomenon arises since the pure opening of trade has some effect also if it does not give rise to trade flows. If we compare conditions (10) and (46), we realize that the total number of consumers being served in the poorer country, after the opening of trade, increases, because a lower price makes poorer consumers afford the low quality good, which was not affordable in autarky.

Under conditions (41) and (46), the richer market is being completely served under both regimes only with the high quality good. The only difference between the two regimes lies in the distribution of total surplus because of different prices. When the two countries have the same dimension in terms of population, i.e., $\overline{\theta}_F = \frac{\overline{\theta}_D}{2}$, the opening of trade increases the number of consumers being served in the poorer country, as compared to autarky. In this case, the total demand of the richer country is given by (43). Under conditions (44) and (45), the number of consumers being served in the poorer country increases if $\overline{\theta}_F > \frac{\overline{\theta}_D}{2}$.[8]

To sum up, trade, by and large, increases the number of consumers who buy in the poorer country, and ensures that all consumers in the richer country can buy either the high quality or the low quality good, while in autarky this latter result was not possible.

## 5. The analysis of welfare distribution in countries *D* and *F*

After having established proposition 1, we wish to compare the distribution of welfare in the two countries before and after the opening of trade.

First of all, we wish to compare the level of social welfare between autarky and trade. To do that, we have to see for which values of the relevant parameters trade is welfare-improving. It appears that

$$SW_D^T > SW_D^M \quad \textit{iff} \quad \overline{\theta}_D > \frac{3}{2}\overline{\theta}_F. \tag{47}$$

This condition holds since it corresponds to the viability condition already established in (38). Now we have to make the same comparison for country *F*:

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8. Using (10) and (44), we obtain the following inequality:

$$\overline{\theta}_F - \frac{\overline{\theta}_D}{3} - \frac{\overline{\theta}_F}{3} > 0,$$

which is satisfied for $\overline{\theta}_F > \frac{\overline{\theta}_D}{2}$.

$$SW_F^T > SW_F^M \quad iff \quad \frac{\overline{\theta}_F}{27}\left(4\overline{\theta}_F^2 + \overline{\theta}_D^2 - 3\overline{\theta}_D\overline{\theta}_F\right) - \frac{3}{54}\overline{\theta}_F^3 > 0, \tag{48}$$

which simplifies to:

$$\overline{\theta}_F\left(5\overline{\theta}_F^2 + 2\overline{\theta}_D^2 - 6\overline{\theta}_F\overline{\theta}_D\right) > 0 \tag{49}$$

This condition is met for all non negative values of $\overline{\theta}_D, \overline{\theta}_F$. We can therefore state the following:

**PROPOSITION 2:** the opening of trade increases the social welfare in both countries in the admissible range of the relevant parameters.

We can now turn to the analysis of the welfare distribution between consumers and producers, as a consequence of trade.

As far as the consumer surplus in the richer country is concerned, we have that:

$$CS_D^T > CS_D^M \quad iff \quad 2\overline{\theta}_D^2\overline{\theta}_F + 6\overline{\theta}_D\overline{\theta}_F^2 - 9\overline{\theta}_F^3 > 0, \tag{50}$$

which can be rewritten as:

$$\overline{\theta}_F\left(2\overline{\theta}_D^2 + 6\overline{\theta}_D\overline{\theta}_F - 9\overline{\theta}_F^2\right) > 0. \tag{51}$$

This holds by virtue of the viability condition (38).

In the poorer country,

$$CS_F^T > CS_F^M \quad iff \quad \frac{2}{27}\overline{\theta}_F^2(2\overline{\theta}_F - \overline{\theta}_D) - \frac{\overline{\theta}_F^3}{54} > 0, \tag{52}$$

which is met for

$$\frac{2}{3}\overline{\theta}_D \geq \overline{\theta}_F > \frac{4}{7}\overline{\theta}_D. \tag{53}$$

This implies that, if $\overline{\theta}_F \in ]0, \ \frac{4}{7}\overline{\theta}_D[$, which still belongs to the viable region, the consumer surplus of country *F* is lower after the opening of trade, because of a decrease in the number of consumers who buy the good. These results can be summarized in:

**PROPOSITION 3:** the opening of trade increases the consumer surplus in both countries if $\overline{\theta}_F \in ]\frac{4}{7}\overline{\theta}_D, \ \frac{2}{3}\overline{\theta}_D]$; if, instead, $\overline{\theta}_F \in ]0, \ \frac{4}{7}\overline{\theta}_D[$, the opening of trade increases the consumer surplus of the richer country while decreasing the consumer surplus of the poorer country.

The above Proposition can be interpreted in terms of the relative size of the two countries: if the poorer country is bigger than the richer, consumers of the poorer country will always benefit from trade.

After having examined consumers, we wish to see what happens to producer surpluses. If we consider the richer country, it is trivial to verify that:

$$\pi_D^M > \pi_D^T \quad iff \quad \frac{\overline{\theta}_D^3}{27} - \frac{\overline{\theta}_D^2}{27}(\overline{\theta}_D - \overline{\theta}_F) > 0, \tag{54}$$

which is always true. In other words, the monopolist of the richer country looses when turned into an international duopolist.

In the poorer country, we must verify that:

$$\pi_F^M > \pi_F^T \quad iff \quad \frac{\overline{\theta}_F^3}{27} - \frac{\overline{\theta}_D\overline{\theta}_F}{27}(\overline{\theta}_D - \overline{\theta}_F) > 0, \tag{55}$$

or

$$\overline{\theta}_F > \frac{(\sqrt{5}-1)}{2}\overline{\theta}_D = 0.618\overline{\theta}_D. \tag{56}$$

If $\overline{\theta}_F < 0.618\overline{\theta}_D$, the firm of country $F$ is better off after the opening of trade, since it esports a lot to the richer country. In the interval $]\frac{4}{7}\overline{\theta}_D, \ 0.618\overline{\theta}_D[$, the firm and the consumers of country $F$ are both better off. For $\overline{\theta}_F < \frac{4}{7}\overline{\theta}_D$, the effect of the opening of trade is positive for the firm but not for the consumers of the poorer country, because exports are high, and, as a consequence, many consumers of country $F$ are not served. In the interval $\left[0.618\overline{\theta}_D, \ \frac{2}{3}\overline{\theta}_D\right]$, consumers are better off because they face a higher quality-price ratio, while the firm of the poorer country is worse off because it exports less and at a lower price.[9]

We can finally state

9. We have to remember that in this model the price of a good is equalized across countries.

**PROPOSITION 4:** the opening of trade decreases the profit of both firms if $\overline{\theta}_F > 0.618\overline{\theta}_D$. Otherwise, it decreases the profit accruing to the firm of the richer country, while increasing the profit of the firm in the poorer country.

and

**PROPOSITION 5:** there is a region of the parameters, identified by $]\frac{4}{7}\overline{\theta}_D, \quad 0.618\overline{\theta}_D[$, in which both consumers and the producer of the poorer country gain from trade.

The intuition behind the above Propositions is that the firm of the poorer country gains from trade if either the poorer country is sufficiently poor or small. This is due the coincidence between the measure of per capita income and the size of total demand.

## 6. An import-reducing tariff

When the richer country faces a trade deficit $\left(\overline{\theta}_F < \frac{2}{3}\overline{\theta}_D\right)$, it might seem advisable for it to devise an import reducing tariff. We assume that a quantity tariff is levied on imports of the richer country. This gives rise to the following demand functions:

$$x_D = \overline{\theta}_D - h = \overline{\theta}_D - \frac{(p_D - p_F - t)}{q_D - q_F} \tag{57}$$

$$x_F = h - k = \frac{(p_D - p_F - t)}{q_D - q_F} - \frac{p_F}{q_F} \tag{58}$$

where $t$ is the unit tariff. Notice that the introduction of a tariff modifies only the location of $h$.

The two new profit functions are:

$$\pi_D = (p_D - q_D^2)\left(\overline{\theta}_D - \frac{(p_D - p_F - t)}{q_D - q_F}\right) \tag{59}$$

$$\pi_F = (p_F - q_F^2)\left(\overline{\theta}_F - \frac{p_F}{q_F}\right) + (p_F + t - q_F^2)\left(\frac{(p_D - p_F - t)}{q_D - q_F} - \overline{\theta}_F\right) \tag{60}$$

If we differentiate (59) and (60) with respect to prices we obtain the first order conditions yielding the following equilibrium prices:

$$p_D^* = \frac{6\overline{\theta}_F\overline{\theta}_D^2 - \overline{\theta}_D\overline{\theta}_F^2 - 8\overline{\theta}_D^3 + 18\overline{\theta}_F t - 18\overline{\theta}_D t}{9(\overline{\theta}_F - 4\overline{\theta}_D)} \tag{61}$$

$$p_F^* = \frac{\overline{\theta}_F(\overline{\theta}_D\overline{\theta}_F - 4\overline{\theta}_D^2 + 27t)}{9(\overline{\theta}_F - 4\overline{\theta}_D)} \tag{62}$$

We are now ready to produce some comparative statics on the tariff imposed by country $D$. We first work out prices:

$$\frac{\partial p_D}{\partial t} = \frac{2(\overline{\theta}_F - \overline{\theta}_D)}{\overline{\theta}_F - 4\overline{\theta}_D} > 0 \quad \forall \quad \overline{\theta}_F, \overline{\theta}_D \tag{63}$$

$$\frac{\partial p_F}{\partial t} = \frac{3\overline{\theta}_F}{\overline{\theta}_F - 4\overline{\theta}_D} < 0 \quad \forall \quad \overline{\theta}_F, \overline{\theta}_D \tag{64}$$

Then we look at the profits after substituting the equilibrium prices (61) and (62) into (59) and (60):

$$\pi_D^* = \frac{(\overline{\theta}_D - \overline{\theta}_F)(18t + 4\overline{\theta}_D^2 - \overline{\theta}_D\overline{\theta}_F)^2}{27(\overline{\theta}_F - 4\overline{\theta}_D)^2} \tag{65}$$

$$\begin{aligned}\pi_F^* = (&9\overline{\theta}_F^3\overline{\theta}_D^2 - \overline{\theta}_F^4\overline{\theta}_D - 24\overline{\theta}_F^2\overline{\theta}_D^3 + 16\overline{\theta}_F\overline{\theta}_D^4 - 18\overline{\theta}_F^3 t + 216\overline{\theta}_F^2\overline{\theta}_D t - \\ &-648\overline{\theta}_F\overline{\theta}_D^2 t + 288\overline{\theta}_D^3 t - 81\overline{\theta}_F t^2 - 648\overline{\theta}_D t^2)/\left(27(\overline{\theta}_F - 4\overline{\theta}_D)^2\right)\end{aligned} \tag{66}$$

and proceeding with comparative statics we can obtain:

$$\frac{\partial \pi_D^*}{\partial t} > 0 \quad \forall \quad \overline{\theta}_D, \overline{\theta}_F \tag{67}$$

As for the firm operating in the poorer country, we have:

$$\frac{\partial \pi_F^*}{\partial t} < 0 \quad \mathit{iff} \quad t > \frac{16\overline{\theta}_D^3 - \overline{\theta}_F^3 + 12\overline{\theta}_F^2\overline{\theta}_D - 36\overline{\theta}_F\overline{\theta}_D^2}{9(\overline{\theta}_F + 8\overline{\theta}_D)} \tag{68}$$

Setting $\overline{\theta}_D = 1$ we find a region in which a positive tariff can benefit also the exporting firm that is being hit by the tariff; in other words there is a positive $t$ for which $\frac{\partial \pi_F^*}{\partial t} > 0$. This region is indicated in figure 2 by the area $A$.

INSERT FIGURE 2

**PROPOSITION 6**: for $\overline{\theta}_D = 1$ and $\overline{\theta}_F < 0.5359$, any positive tariff benefits both firms.

Eventually, we calculate the level of $t$ that eliminates imports. This can be easily obtained by imposing:

$$\frac{p_D^* - p_F^* - t}{q_D - q_F} - \overline{\theta}_F = 0, \tag{69}$$

yielding :

$$t = \frac{(\overline{\theta}_F - 4\overline{\theta}_D)(3\overline{\theta}_F - 2\overline{\theta}_D)}{18}, \tag{70}$$

which is always positive.

## 7. Conclusions

We analysed trade in vertically differentiated goods between two countries whose only difference is the level of per capita income. Each country has one firm which competes with a foreign producer in a Bertrand fashion, after having chosen quality in an autarkic environment. The result is a one way trade with no overlapping, from the poorer to the richer country. Trade benefits consumers of the richer country in all circumstances, while it increases the surplus of consumers of the poorer country only if the poorer country is bigger than the richer country. The firm of the richer country always looses from trade, while the producer of the poorer country might gain from trade if the exogenous per capita income of the poorer country is sufficiently low.

An import reducing tariff can be set by the government of the richer country, who faces a trade deficit. Some odd effect appears, since the introduction of a quantity tariff may benefit also the exporting firm, provided the exporting country is sufficiently small or poor.