Inflation and public debt management.
A time consistent approach.

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Abstract

The existence of long term or short term nominal debt implies different dicretionary monetary policies. In absence of any monetization, long term debt leads to a lower inflation than short term debt. If debt monetization is possible, the lower inflation of the long term case is traded off with the lower distortionary taxation in the short term case and no unambiguous conclusion can be reached.

1. Introduction.

This paper illustrates some dynamic linkages between monetary policy, public debt management and the institutional setup of an economic system, within a two period model.

A given government expenditure is financed by issuing government bonds. Debt management involves a twofold choice. On the one hand the decision is between nominal debt or debt indexed to the price level; on the other, the decision is between short-term or long-term debt. Further, the policymaker controls money growth rates and inflation rates. At the end of period 2, a distorsionary tax is levied and the revenues are used to repay the debt. The government's objective is to minimize the deadweight loss of the tax, and the inflation rate.

In equilibrium, the policymaker equalizes the marginal loss arising from the inflationary effects of using monetary instruments, and the loss due to taxation. With nominal debt and no precommitment on his future behaviour, the policymaker has an incentive to create an inflationary surprise that reduces the real value of the debt and, hence, the loss from distortionary taxation. The private sector anticipates this policy, and the nominal interest rate on the debt, incorporating expected inflation, rises. No surprise occurs and the outcome is an inflationary equilibrium.

Without monetization, the loss due to taxation is always the same for long or short term debt, while average inflation is lower if long-term bonds are issued. Hence, a debt with longer maturity is to be preferred. With indexed bonds or precommitment, the inflationary bias disappears and debt maturity is immaterial.

With debt monetization, the monetary instrument is effective ex-ante as well as ex-post, and current higher inflation can be exchanged for future lower taxation. The choice between long and short-term debt depends on the relative weights of the different objectives in the loss function of the policymaker. With long term debt there is a greater centralization of the policymaker problem (and a greater smoothing between distortionary instruments is possible), while with short term debt an additional instrument is available - monetization in period 1. This is so, because long term bonds come to maturity only in period 2 and anticipated repayment is not allowed. Contrary to some recent studies (e.g. Blanchard and Missale (1991)), our simulations suggest that in some cases short term bond issue is preferable with a moderate stock of debt, while with higher level of debt the result may be reversed.

Section 2 presents the debt mangement problem. In section 3 the solutions in the no monetization case are worked out. Section 4 briefly extends the analysis to the case of debt monetization. Section 6 offers conclusions.

2. The policy problem.

The private sector is composed of identical households who regard the policymaker's behaviour as given. Households want to maintain the real value of their assets (government bonds) constant throughout periods 1 and 2; they accordingly try to minimize their forecasting errors about inflation rates. Within this deterministic context, households form inflationary expectations at the beginning of each period, and perfectly anticipate the policymaker's future actions.

There is a unique policymaker, that manages both fiscal and monetary instruments. At the beginning of period 1, a certain amount of public debt is issued, and the policymaker must choose how to manage it. He minimizes a loss function, the arguments of which are the inflation rate and the tax rate of period 2. His choice is between indexed or nominal, and short or long-term bonds. Further, he sets the rate of money growth in each period (and, via a strict quantity money equation, the inflation rate). Since all debt must eventually be repaid, a distortionary tax is levied on households in period 2.

This problem is analyzed only in the polar cases of debt composition - viz. short or long-term debt and indexed or nominal debt (1).

We consider two different institutional setups. In the former situation, precommitments are possible. With perfect foresight this eliminates all the incentives to create inflationary surprises (that partly depend on policy decisions). Debt management is immaterial, since the amount of public debt in real terms is independent of the policymaker's choices.

In the alternative setup, no precommitments are possible, which raises the problem of the dynamic consistency of the monetary policy. Indeed, debt management modifies the policy incentives that lay behind the optimal money growth rate. In particular, the mechanism by which inflationary expectations are formed, is different for long, as against short, term bonds. This, in turn, alters the optimal time-consistent policy, and creates a role for debt management.

2.1 The structure of the economy.

We consider a small open economy, with perfect capital mobility and substitutability. The exchange rate is flexible, PPP holds, and the foreign price level is constant. An international capital market offers a real interest rate r per period. Goverment bonds yield a real interest rate equal to r (since in a certainty environment there is no risk premium). The yield of a one period government bond is therefore $(1+r)(1+\pi^e)$.

As we want to focus on the relationship between debt management and money growth, we ignore the goods market: production is held constant at its natural level. Only the monetary and financial features of the economy are analyzed.

Aggregate demand is described by the quantity theory. For the sake of simplicity, money demand is independent of the inflation rate (2). From

[1] $$M_t=kyP_t$$

(M_t being the money stock, P_t the price level and y the natural level of production), follows immediately

[2] $$\mu_t = \pi_t$$

(where μ_t is the money growth rate in period t and π_t the corresponding inflation rate, controlled by the policymaker). Since k and y are constant, the money stock in real terms is also constant.

The intertemporal budget contraint of the government puts forth the different debt-management options. Initially there is an exogenous deficit financed with the issue of an amount b_0 of public debt (in real terms). In the subsequent periods, deficits are zero.

Consider the case of nominal debt. At the beginning of period 1 the choice of the policymaker is between issuing short and long-term debt. In the former case, after (partial) monetization, debt is renewed at the beginning of period 2. In the latter case, all the choices are made in period 1 and no further policy is undertaken. At the end of a two-period programming interval, government debt comes to maturity and the distorsionary tax is levied.

Let ${}_i\pi_j{}^e$ denote the expectation of period j inflation, formed at the beginning of period i; if short term bonds are issued, the debt in nominal terms, at the end of period 1, is

[3] $$B_1 = B_0(1+r)(1+{}_1\pi_1{}^e) - (M_1-M_0).$$

The first term shows the amount of debt that should be paid back to the private sector, while the second term shows the effect of monetization.

At the end of period 2 the nominal amount of debt to be repaid is:

[4] $$B_2 = B_1(1+r)(1+{}_2\pi_2{}^e) - (M_2-M_1) =$$

$$= [B_0(1+r)(1+{}_1\pi_1{}^e) - (M_1-M_0)](1+r)(1+{}_2\pi_2{}^e) - (M_2-M_1).$$

From the money market equilibrium we get:

$$(M_2-M_1) = k\ y\ (P_2-P_1) = m(P_2-P_1).$$

The nominal interest rate equals the real interest rate corrected for expected inflation. The ex-post real yield is given by the ex-ante nominal interest rate less actual inflation. In equilibrium inflationary expectations are fullfilled.

We substitute the previous expression in [4] and obtain (S is for "short"):

[5] $${}_SB_2 = [B_0(1+r)(1+{}_1\pi_1{}^e) - ky(P_1-P_0)](1+r)(1+{}_2\pi_2{}^e) - ky(P_2-P_1).$$

Hence, the public debt in real terms is approximately:

[6] $${}_Sb_2 = [b_0(1+r+{}_1\pi_1{}^e-\pi_1)(1+r+{}_2\pi_2{}^e-\pi_2)-ky(1+r+{}_2\pi_2{}^e-\pi_2)\pi_1-ky\pi_2.$$

With long-term debt (and no monetization during period 1) the corresponding expression is (L is for "long"):

[7] $_Lb_2 = b_0(1+r+{}_1\pi_1^e-\pi_1)(1+r+{}_2\pi_2^e-\pi_2) - ky\pi_2.$

However, if the debt is indexed to the price level and offers a real yield equal to r, the constraint $_i\pi_j=\pi_j$ is imposed (ex-ante) on the previous expressions [6] and [7]. Since at most the whole debt can be monetized, there is an upper limit on monetization. As a consequence, our descriptive analysis will apply only for parameter values such that (3):

$$_Sb_1>0, \quad _Sb_2>0, \quad _Lb_2>0.$$

We exclude the cases in which the public sector is a net asset holder (4).

2.2. The objectives of economic policy.

The policymaker minimizes the loss function:

[8] $Z = \frac{1}{2}\pi_1^2 + \frac{1}{2}\delta\pi_2^2 + \delta\tau b_2,$ $\delta \in (0, 1]$, where δ is the intertemporal discount factor and τ the relative weight of the distorsionary tax used to repay the debt.

Because of the distortions due to the economization of real balances, π enters the loss function. At the beginning of each period, the policymaker tries to create an inflationary surprise, in order to lower the real value of the debt. This raises the loss from (costly) inflation. At the end of period 2 the debt must be redeemed and a distorsionary tax is levied on the private sector. We assume that the deadweight loss is proportional to the stock of government debt. We neglect tax smoothing problems: taxation comes into picture only in the last period and (apart from monetization) redistribution over time of taxation and anticipated repayment is not allowed.

3. Debt-management and inflation without monetization.

Assume that the repudiation of the public debt is not allowed. Further, no (direct) monetization of the debt is possible. However, there are indirect incentives to the creation of unexpected money growth, since unexpected inflation reduces the real value of the debt. Money is therefore injected into the system by means of "helicopters".

The government budget constraint simplifies to:

[9] $$b_2 = b_0(1+r+{}_1\pi_1^e-\pi_1)(1+r+{}_i\pi_2^e-\pi_2),$$

with i=1 for long-term debt and i=2 for short-term debt. Substituting [9] into the loss funtion, the problem is solved with respect to the policy instruments.

Now we analyze the problem under a different hypothesis concerning the institutional framework and the maturity of the debt.

3.1. Precommitment equilibrium.

Assume the policymaker can precommit himself to any future behaviour. He knows that his future strategy will be anticipated, and accordingly takes into account private agents' reactions:

[10] $$\begin{aligned} {}_1\pi_1^e &= \pi_1 \\ {}_1\pi_2^e = {}_2\pi_2^e &= \pi_2. \end{aligned}$$

[9] can be simplyfied to

[11] $$Z = \tfrac{1}{2}\pi_1^2+\tfrac{1}{2}\delta\pi_2^2+\tfrac{1}{2}\delta\tau b_0(1+r)^2.$$

The policymaker is the leader of a Stackelberg game with the private sector. The optimal inflation rates are

[12] $$\pi_1 = \pi_2 = 0,$$

since there are no incentives to money growth and the total loss is

$\frac{1}{2}\delta\tau b_0(1+r)^2$, independently of debt management.

Since there is a finite number of periods in which the game is repeated and a unique non-cooperative equilibrium in the one-shot game, the only outcome is the replication of the discretionary equilibrium.

3.2. Discretionary equilibrium with indexed bonds.

Without any precommitment, the policymaker considers the private sector's inflationary expectations as given.

With indexed debt, the loss function simplyfies again to [11], just as is the case of precommitment. Also in this case there is no distinction between short term and long term debt, and the optimal policy is again the same. The relevant equilibrium concept in this case, however, is the Nash equilibrium.

With the economy described by a Phillips curve and a further objective for the policymaker - the level of production higher than the inefficient natural rate - the outcome with indexed debt would be slightly different with respect to the precommitment case. In equilibrium, inflation would be positive since indexation removes the inflationary incentives only partially. The resulting outcome would be of the Barro-Gordon type. However, while, on the one hand, debt indexation removes some of the inflationary incentives with respect to the nominal debt, on the other hand it diminishes the cost of inflation in the loss function. This latter effect raises the equilibrium inflation.

Finally, we can easily trace the fiscal policy implicitly postulated in the background of the literature on the time consistency of monetary policy: there is an indexed debt growing at the same rate as production.

3.3. Discretionary equilibrium with nominal debt.

In this case both mechanisms are at work, which have been considered so far in the literature on time consistency. During each period, the policymaker has an incentive to create unexpected inflation in order to reduce the real value of the debt. Knowing that in equilibrium no surprise inflation is possible, he would like to bind himself to a predetermined low-inflation policy. Without exogenous precommitments such a strategy is not time-consistent, and the only outcome is the high-inflation Nash equilibrium: the policymaker selects his best policy taking ${}_i\pi_j{}^e$ as given.

Time-consistency as defined by Kydland-Prescott also involves an explicit time dimension. Loosely speaking, time consistency problems arise if the optimal policy strategy computed at time t is suboptimal if computed again in t', t'>t. Hence, the original strategy is not credible; the strategy actually selected is time-consistent, but sub-optimal. In our case, the period 2 policymaker takes the pre-existing situation as given (and hence his past policy as well) and selects his current optimal policy. In period 1, a forward-looking policymaker takes into account the future consequences of his current policy. He acts like a leader of a dynamic Stackelberg game, and his optimal policy is computed by working backwards from period 2.

3.4. Short term nominal debt.

The loss function to be minimized is:

[13] $$Z_2 = \tfrac{1}{2}\pi_2{}^2+\tau b_0(1+r+{}_1\pi_1{}^e-\pi_1)(1+r+{}_2\pi_2{}^e-\pi_2).$$ In period 2, taking ${}_2\pi_2{}^e$ as given, the first order condition is:

[14] $$\pi_2-\tau b_0(1+r+{}_1\pi_1{}^e-\pi_1) = 0,$$

from which:

[15] $$_S\pi_2 = \tau b_0(1+r+{}_1\pi_1{}^e-\pi_1).$$

From the point of view of the policymaker of period 1, $_2\pi_2{}^e$ and π_2 are still to be determined; indeed they depend on his actual choice. π_2 is inversely related to the debt at the end of period 1: since $_1\pi_1{}^e$ is given, an increase in current inflation implies an increase in surprise inflation and a decrease in b_1 that diminshes the inflationary incentives. The relevant equation for the period 1 policymaker is [15], from which we get (6):

[16] $$\frac{d_S\pi_2}{d_S\pi_1} = -\tau b_0.$$

This substitutability disappears, however, if the policymaker could endogenize period-1 expectations, as in the precommitment case. In equilibrium (ex-post) [15] reduces to:

[17] $$_S\pi_2 = \tau b_0(1+r).$$

In period 1 the optimal current policy tries to modify the inflationary incentives of the future policymaker. Due to the perfect foresight assumption, ex-ante we can impose $_2\pi_2{}^e = \pi_2$ since the market for future short-term bonds is not yet open and $_2\pi_2{}^e$ is still to be formed.

The loss function to be minimized is:

[18] $$Z = \tfrac{1}{2}\pi_1{}^2+\tfrac{1}{2}\delta\pi_2{}^2+\tfrac{1}{2}\delta\tau b_0(1+r+{}_1\pi_1{}^e-\pi_1)(1+r),$$

and the first order condition is therefore:

[19] $$\pi_1 + \delta\pi_2 \frac{d\pi_2}{d\pi_1} - \tau\delta b_0(1+r) = 0.$$

Substituting [16] and [17], the optimal value of π_1 with short term debt is:

[20] $$_S\pi_1 = \delta(1+r)[\tau b_0+(\tau b_0)^2].$$

A larger initial debt and a higher real interest rate imply a higher inflation rate in each period. The same effect stems from an increase in τ, that is, an increase in the distortionary effect of taxation (or in a lower cost of inflation). An increase in δ gives a greater weight to the period-2 loss, and this leads to an increase in π_1.

Period 1 inflation is higher than period-2 inflation, if $\delta(1+\tau b_0)>1$. If future is not discounted (and δ->1) this condition is satisfied: in period 1 a greater activism of the policymaker is justified, as he anticipates the ineffectiveness of the future policy. On the other hand, if the current policymaker disregards the future (δ->0), there are no incentives to create inflation in period 1. A short-sighted policymaker, paradoxically, leads to a better outcome. The same reasoning will apply to the case of long-term debt.

3.5. Long term nominal debt.

With long-term debt both policymakers regard $_1\pi_1^e$ and $_1\pi_2^e$ as given, since inflationary expectations are both set at the beginning of period 1, just before the implementation of period-1 policy. This implies a different optimal monetary growth rate with respect to the case of short term debt.

In period 2, the problem is analogous to the one previously described, since all existing debt is now a short-term debt. The loss function is:

[21] $$Z_2 = \tfrac{1}{2}\pi_2^2+\tau b_0(1+r+{}_1\pi_1^e-\pi_1)(1+r+{}_1\pi_2^e-\pi_2),$$

and the period 2 discretionary policy is:

$$_L\pi_2 = {_S\pi_2} = \tau b_0(1+r).$$

The marginal rate of substitution between π_2 and $_1\pi_2$ is given again by [16].

The optimal policy in period 1 can be computed by solving the problem backwards, as in the previous section. Since $_1\pi_1{}^e$ and $_1\pi_2{}^e$ are both given from the point of view of the policymaker of period 1, an increase in inflation in each period is an increase in surprise inflation. The loss function to be minimized is:

[22] $$Z = \tfrac{1}{2}\pi_1{}^2 + \tfrac{1}{2}\delta\pi_2{}^2 + \delta\tau b_0(1+r+{_1\pi_1}{}^e-\pi_1)(1+r+{_1\pi_2}{}^e-\pi_2).$$

The period 1 first order condition is therefore:

[23] $$\pi_1 - \delta\tau b_0(1+r+{_1\pi_2}-\pi_2) + [\delta\pi_2-\delta\tau b_0(1+r_1\pi_1{}^e-\pi_1)]\frac{d\pi_2}{d\pi_1} = 0.$$

Substituting [16] and [17] into [23] we can determine the (ex-post) equilibrium inflation rate for period 1:

[24] $$_L\pi_1 = \delta(1+r)\tau b_0.$$

If $0<\delta<1$, $_L\pi_1$ is lower than $_L\pi_2$, while if future is not discounted, inflation is constant. Indeed, from the point of view of the period-1 policymaker, future inflation is an instrument as effective and important as current inflation. The concentration of (ex-ante) surprise inflation in one particular period depends only on the relative cost of inflation in periods 1 and 2; with $\delta=1$ the cost is the same and the optimal policy requires a smoothing of inflation.

The effects on $_L\pi_2$ of an increase in b_0, r and τ are analogous to those already analyzed in the short-term case.

3.6. Debt-management without monetization: a comparison.

If the policymaker can precommit his future behaviour, debt management is irrelevant. The policymaker is the leader of the game and (ex ante) incorporates the rationality of inflationary expectations; hence, there are no incentives to create surprise inflation, and $\pi_1=\pi_2=0$.

Indexation provides a partial or total form of precommitment; again, debt maturity has no influence and, without Barro and Gordon-type inflationary incentives, there is zero inflation in equilibrium.

Without precommitment, the policymaker cannot escape from a high inflation equilibrium, even if he anticipates the ineffectiveness of his current and future actions.

In period 2 the existing debt is short-term debt. The optimal choice of the policymaker is therefore the same with long-term as with short-term bonds. In period 1, however, the policymaker faces different constraints. With short-term debt, policy decisions are completely decentralized. In the implementation procedures, the period-1 policymaker takes into account the ineffectiveness of period 2 monetary policy, and finds it optimal to substitute actual inflation for future inflation. With long-term bonds, period 2 monetary policy is not *a priori* ineffective, and the policymaker finds it optimal to smooth out inflation over time. Hence period 1 inflation is lower with long-term rather than with short-term debt, and so is the value of the loss function.

4. Public debt management and monetization.

4.1 Indexed debt.

Without precommitment, in period 2 the policymaker minimizes:

[25] $$Z_2=\frac{1}{2}\pi_2{}^2+\tau b_1(1+r)-\tau ky\pi_2.$$

The optimal rate of inflation for period 2, either with long-term or short-term bonds is (The index "i" stands for "indexed"):

[26] $$S.i\pi_2=L.i\pi_2=\tau ky.$$

In period 1 the policymaker takes into account the money growth rate of period 2. In the case of indexed debt, however, there are no intertemporal spillovers and the policies in the two periods are independent of each other.

With short-term debt the loss function is:

[27] $$Z=\frac{1}{2}\pi_1{}^2+\frac{1}{2}\delta\pi_2{}^2+\delta\tau b_0(1+r)^2-\delta\tau ky\pi_2-\delta\tau(1+r)ky\pi_1,$$

and the corresponding optimal inflation rate is:

[28] $$S.i\pi_1=\delta\tau ky(1+r).$$

With long-term bonds the loss function is:

[29] $$Z=\frac{1}{2}\pi_1{}^2+\frac{1}{2}\delta\pi_2{}^2+\delta\tau b_0(1+r)^2-\tau ky\pi_2,$$

and the corresponding optimal inflation rate is, obviously, zero:

[30] $$L.i\pi_1=0.$$

With long-term bonds, inflation rates are increasing because monetization is possible in period 2 only.

With short-term debt, inflation rates are both positive and increasing if $\delta>1/(1+r)$. If the policymaker is benevolent and adopts the same discount as the market, inflation rates are constant.

4.2. Institutional contraints on economic policy.

A precommitment is equivalent to the issue of indexed bonds. A committed central planner, that incorporates the rationality of expectations ex-ante, describes his policy by means of [26] and [28] or [30]. There are no time consistency problems and the command optimum is again described by two decentralized decisions.

4.3 Discretionary equilibrium with nominal debt.

The indirect inflationary incentives of the policymaker when nominal debt is issued have been already discussed. Now, in addition to this mechanism, we have to take into account the effects of debt monetization. With short-term bonds, monetization takes place in both periods, while with long-term bonds monetization takes place in period 2 only. Hence, the latter effect works in the opposite direction to the former. Therefore, if a substantial amount of monetization is possible, the unambiguous results of section 3 no longer hold.

4.4. Short-term nominal debt.

In period 2 the policymaker minimizes the loss:

[31] $$ {}_SZ_2 = \tfrac{1}{2}\pi_2^2 + \tau {}_Sb_2, $$

with ${}_Sb_2$ defined by [6]. From the first order condition we get:

$$ {}_S\pi_2 = \tau b_0(1+r+{}_1\pi_1^{e}-\pi_1)+\tau ky-\tau ky{}_S\pi_1, $$

that in equilibrium reduces to:

[32] $$ {}_S\pi_2 = \tau b_0(1+r)+\tau ky-\tau ky{}_S\pi_1. $$

This is a semi-reduced form, since ${}_S\pi_2$ depends (negatively) on past inflation.

The rate of substitution between the inflation rates is:

[33] $$ \frac{d\pi_2}{d\pi_1} = -\tau(b_0+ky). $$

Following the same line of reasoning we put forward in section 3.4, in period 1 the policymaker minimizes:

[34] $$_SZ_1 = \tfrac{1}{2}\pi_1{}^2 + \tfrac{1}{2}\delta\pi_2{}^2 + \delta\tau b_0(1+r)(1+r+{}_1\pi_1{}^e-\pi_1)-\tau ky(1+r)\pi_1-\tau ky\pi_2.$$

From the first order condition

[35] $$\pi_1+\delta\pi_2\frac{d\pi_2}{d\pi_1} - \delta\tau b_0(1+r)-\delta\tau ky(1+r)-\delta\tau ky\frac{d\pi_2}{d\pi_1} = 0,$$

after some algebra we get the optimal inflation rate for period 1:

[36] $$_S\pi_1 = \frac{\delta\tau(1+r)(1+\tau b_0)(b_0+ky)}{1+\delta\tau^2(b_0+ky)ky}.$$

Substituting back in [32], $_S\pi_1$ is therefore:

[37] $$_S\pi_2 = \tau ky-(1+r) + \frac{(1+r)(1+\tau b_0)}{1+\delta\tau^2(b_0+ky)ky}.$$

4.5. Long-term nominal debt.

At the beginning of period 2, the policymaker minimizes:

[38] $$_LZ_2 = \tfrac{1}{2}\pi_2{}^2+\tau_L b_2,$$

with $_Lb_2$ defined by [7]. With predetermined inflationary expectations, we get the optimal value of $_L\pi_2$,

[39] $$_L\pi_2=\tau b_0(1+r+{}_1\pi_1{}^e-\pi_1)+\tau ky,$$

that in equilibrium is:

[40] $$_L\pi_2=\tau b_0(1+r)+\tau ky.$$

In period 1 the policymaker computes the rate of substitution between π_1 and π_2 and, therefore, the future (ex-ante) effects of his current action on the period-2 policymaker:

[41] $$\frac{d\pi_2}{d\pi_1} = - \tau b_0.$$

In period 1 the policymaker minimizes:

[42] $$_LZ_1 = \tfrac{1}{2}\pi_1^2 + \tfrac{1}{2}\delta\pi_2^2 + \delta\tau b_2.$$

From the first order condition

[43] $$\pi_1+\delta\pi_2\frac{d\pi_2}{d\pi_1} -\delta\tau b_0(1+r+{}_1\pi_2{}^e-\pi_2) -\delta\tau b_0(1+r+{}_1\pi_1{}^e-\pi_1)\frac{d\pi_2}{d\pi_1} -\delta\tau ky\frac{d\pi_2}{d\pi_1}=0,$$

after some simplification, we get:

[44] $$_L\pi_1 = \delta\tau b_0(1+r).$$

In equilibrium, $_L\pi_1$ is the same as in section 3.5. With long-term debt no monetization is possible in period 1 and, furthermore, in equilibrium, the anticipation of the future monetization (and of the correspondingly higher inflation) is just balanced by the effect of the future lower public debt in real terms.

Without monetization ($ky=0$) the previous expressions reduce to the corresponding expressions of the no monetization case, given by [17], [20] and [24].

4.6. Contraints on the parameters.

The aim of this paper is to develop a positive analysis of some of the consequences of the debt management in developed countries. Since in these countries we observe positive amounts of public debt, the rates of money creation should be consistent with such figures.

With indexed debt the condition:

[45] $$b_0 > \frac{\tau k^2y^2}{(1+r)^2}[1+\delta(1+r)^2]$$

assures a positive amount of debt in period 2 (long term bonds) and in periods 1 and 2 (short term bonds).

For $_Lb_2>0$ we assume:

[46] $$b_0 > \frac{\tau k^2 y^2}{(1+r)(1+r-\tau ky)}.$$

Clearly, for b_0, $1+r-\tau ky>0$. For ${}_Sb_1>0$ we assume:

[47] $$b_0 > \frac{\delta\tau k^2 y^2}{1-\delta\tau ky},$$

for $1-\delta\tau ky>0$.

The last condition is for ${}_Sb_2>0$:

[48] $$b_0 > \frac{\tau k^2 y^2[1+\delta\tau^2 k^2 y^2+\delta(1+r)(1+r-\tau ky)]}{(1+r+\delta\tau^2 k^2 y^2)(1+r-\tau ky)-\delta\tau ky(1+r)^2}.$$

The denominator of [48] is positive if

[49] $$1-\delta\tau ky > \frac{\delta(\tau ky)^3}{(1+r)(1+r-\tau ky)},$$

and we assume this condition to be met. If [48] is satisfied, [45] - [47] are satisfied too. Hence, if debt is large enough it is never completely monetized.

4.7. A comparison between inflation rates.

We can unambiguously rank the inflation rates of period 2.

$${}_{S.i}\pi_2 = {}_{L.i}\pi_2 < {}_S\pi_2 < {}_L\pi_2.$$

The first result is obvious. The second makes use of condition [47]. From expressions [37] and [40] we get the third result.

These results have an intuitive explanation. With indexed debt, the inflationary incentive which comes from the reduction of the real value of the debt disappears. With long-term nominal debt, no monetization is possible in period 1. At the beginning of period 2 the debt is then greater than in the case of short-term debt, and the corresponding inflation rate higher.

We can unambiguosly rank the inflation rates also for period 1. If the initial stock of public debt is greater than the money stock ($b_0>m$) we get:

$$_{L.i}\pi_1<_{S.i}\pi_1<_L\pi_1<_S\pi_1$$

Unsurprisingly, $_{L.i}\pi_1<_{S.i}\pi_1$ and $_L\pi_1<_S\pi_1$ (since we assumed $1-\delta\tau ky>0$). These results come both from the impossibility of monetization in period 1 in the case of long-term debt, and from the (ex-ante) anticipation of the unavailability of inflationary surprises in period 2 if short-term bonds are issued.

4.8. A comparison between the losses.

If indexed debt is issued, the best result is reached with short-term bonds, since an extra instrument is available.

$$Z_{L.i}=\delta\tau b_0(1+r)^2-\tfrac{1}{2}\delta(\tau ky)^2,$$

$$Z_{S.i}=Z_{L.i}-\tfrac{1}{2}[\delta\tau ky(1+r)]^2.$$

In this deterministic framework, indexed debt is superior to nominal debt, because it eliminates the incentives to create surprise inflation - a costly and ineffective instrument (7).

With nominal debt the "inflationary incentives" effect favours long-term debt, while the "monetization" effect works in the opposite direction. Hence, the sign of the expression

[50] $$_SZ_1-_LZ_1=\tfrac{1}{2}(_S\pi_1{}^2-_L\pi_1{}^2)+[\tfrac{1}{2}\delta(_S\pi_2+_L\pi_2)+\delta\tau ky](_S\pi_2-_L\pi_2)-\delta\tau _S\pi_1(1+r)ky$$

depends on the specific values of the parameters. The following table presents a few simulations for $r=0.03$, $y=1.0$ and different values of τ, δ, k and b_0 (9).

(table 1)

With long-term debt, inflation is increasing, while with short-term debt it might not. Because of the different monetization procedures, ${}_Lb_2 > {}_Sb_2$.

Contrary to other results (e.g. Blanchard-Missale (1991)), in some cases a lower stock of debt seems to support the case for short-term bonds issue.

5. Conclusions.

The issue of a particular type of bond influences the choice of the time consistent monetary policy. The issue of nominal debt creates a role for surprise inflation, which lowers the real value of the debt and the corresponding distorsionary taxation, to be levied on the private agents.

Private agents have perfect foresight and in equilibrium no surprise inflation occurs ex-post. Without precommitment, however, the inflationary incentives are different for short-term or long-term nominal debt. In the first case the degree of the decentralization of policy is greater: the period-1 policymaker anticipates the ineffectiveness of the inflationary surprises in period 2. This leads to excess inflation in period 1. With long-term debt, in period 1 inflationary expectations are incorporated into the already issued public debt. The period-1 policymaker cannot anticipate ex-ante ineffectiveness of the period-2 policy. Hence the optimal amount of inflation is simply smoothed over two periods. In absence of any direct monetization of the debt, the deadweight loss due to distorsionary taxation is the same for long as the short-term debt, while the lower inflation in period 1 makes it more convenient to issue long term debt.

When debt monetization is possible the inflationary incentives (supporting long-term debt) and the monetization incentives (supporting short-term debt) work in opposite directions, and issuing short-term nominal debt may turn out to be preferable than issuing long-term debt.

However, in a deterministic framework the best result is reached with the issue of indexed debt, since all incentives for surprise inflation are then removed.

This conclusion is drawn from a two period model with a complete separation between real and financial variables. The extension to the multiperiod case, while analitically difficult, does not change the results.

Along these lines we could extend the analysis, introducing a more articulated set of policy instruments. The analogies with other problems of time consistency should now be apparent: some distortion in the system induces the policymaker to cheat the private sector. The lower the distortion and the higher the number of instruments used, the better the outcome of the discretionary equilibrium.

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Notes

(1) Gale (1990) shows that the debt-management is important if it modifies the opportunity set of the private sector. In this model, it happens to be the case. This represent a fruitful, albeit very simple, approach to the debt- management.

(2) This assumption rules out multiple equilibria.

(3) Our aim is to provide some insights on debt-management policies; hence we consider only the polar cases. This approach clarifies the effects of the different inflationary incentives at work. Our perspective hence requires the same number of instruments to be available to the policymaker in the different situations at issue, since an additional instrument makes it possible a greater smoothing (and this lowers the overall loss).

(4) It is immediately apparent that the analysis can be easily extended to a growing economy, dividing all variables by the GDP.

(5) This distinction is put forward, e.g., by Persson and Tabellini (1991), chapter 1.

(6) Expression [16] requires, indeed, a weaker condition. Because expectations are not predetermined, it is sufficient that $d_2\pi_2^e/d\pi_1=d\pi_2/d\pi_1$. This latter condition, however, is not robust to different specifications of the loss function.

(7) In a stochastic context with incomplete markets, the issue of nominal bonds can offer an insurance service. This justifies the existence of such debt. Cfr. Bohn (1988). In a deterministic context the latter argument cannot be adequately developed.

(8) Constraint [48] is binding, for example, if $\delta=0.8$, $\tau=0.8$, $k=0.4$, $r=0.03$ and $y=1.0$. Hence with short term debt, b_2 is positive only if $b_0>0.4$.

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Table 1.

Simulation results for inflation rates, debt and policymaker's loss for the nominal debt (no precommitment) case.

τ	0.2	0.2	0.2	0.2	0.2	0.2	0.5	0.5
δ	0.8	0.8	0.8	0.8	0.5	0.5	0.5	0.5
k	0.4	0.4	0.1	0.1	0.1	0.1	0.1	0.1
b_0	1.0	0.3	1.0	0.3	1.0	0.3	1.0	0.3
$L^{\pi}1$	16.48	4.94	16.48	4.94	10.30	3.09	25.75	7.73
$L^{\pi}2$	28.60	14.18	22.60	8.18	22.60	8.18	56.50	20.45
$S^{\pi}1$	27.20	12.12	21.68	6.98	13.57	4.36	41.91	11.79
$S^{\pi}2$	26.42	13.21	22.17	8.04	22.33	8.09	54.40	19.86
$L^{b}2$	0.947	0.262	1.038	0.310	1.038	0.310	1.004	0.298
$S^{b}2$	0.843	0.216	1.016	0.303	1.025	0.306	0.963	0.286
$L^{Z}1$	0.198	0.051	0.200	0.054	0.126	0.033	0.364	0.088
$S^{Z}1$	0.200	0.049	0.102	0.030	0.103	0.031	0.403	0.089
$S^{Z}1-L^{Z}1$	0.002	-0.002	-0.098	-0.024	-0.023	-0.002	0.039	0.001